Computer algebra independent integration tests

1-Algebraic-functions/1.1-Binomial-products/1.1.4-Improper/1.1.4.3-e-x-^m-a-x^j+b-x^k-^p-c+d-x^n-^q

Nasser M. Abbasi

May 13, 2020 Compiled on May 13, 2020 at 3:10pm

Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [298]. This is test number [31].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (298)	% 0. (0)
Mathematica	% 99.33 (296)	% 0.67 (2)
Maple	% 92.28 (275)	% 7.72 (23)
Maxima	% 35.91 (107)	% 64.09 (191)
Fricas	% 76.51 (228)	% 23.49 (70)
Sympy	% 40.27 (120)	% 59.73 (178)
Giac	% 69.8 (208)	% 30.2 (90)

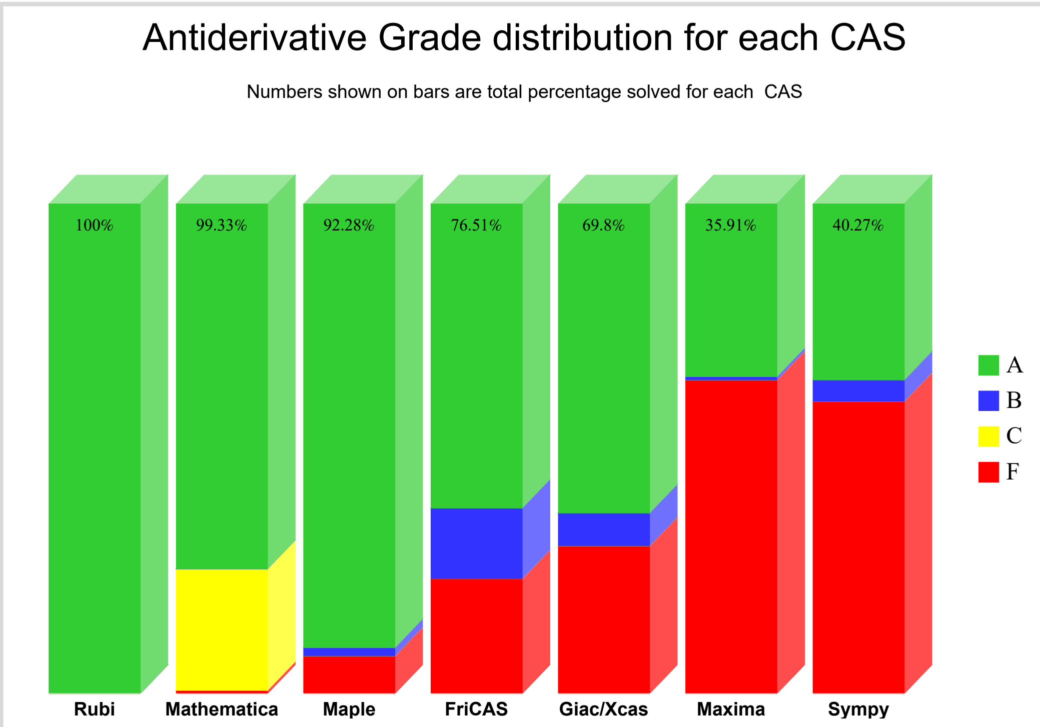
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

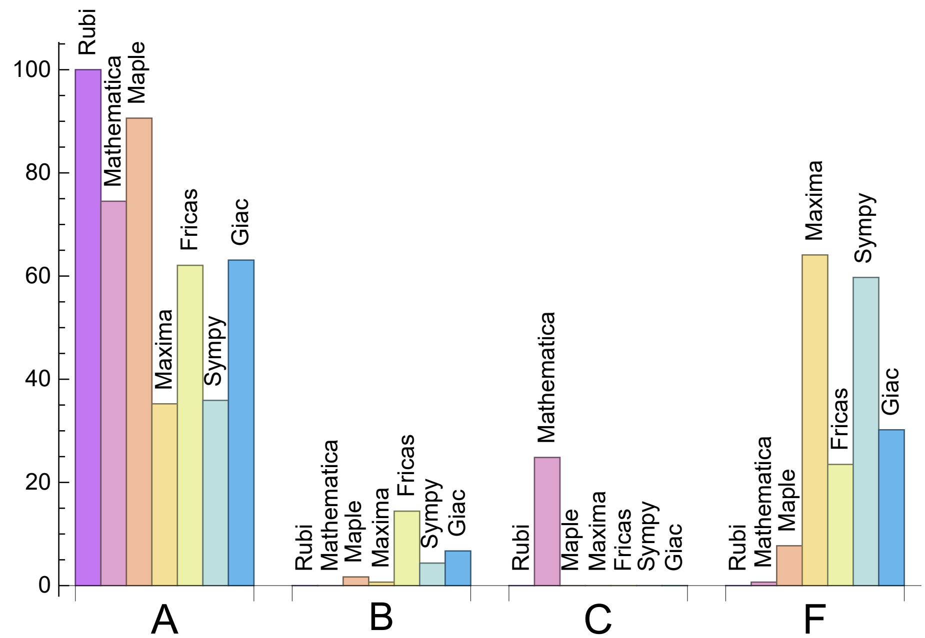
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	74.5	0.	24.83	0.67
Maple	90.6	1.68	0.	7.72
Maxima	35.23	0.67	0.	64.09
Fricas	62.08	14.43	0.	23.49
Sympy	35.91	4.36	0.	59.73
Giac	63.09	6.71	0.	30.2

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.18	145.03	1.	103.5	1.
Mathematica	0.09	99.23	0.83	85.	0.89
Maple	0.01	174.17	1.15	118.	1.15
Maxima	1.22	83.4	1.33	73.	1.32
Fricas	1.33	560.64	3.85	283.	3.25
Sympy	10.56	117.71	1.59	78.	1.12
Giac	1.35	195.12	1.69	123.5	1.39

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {277}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

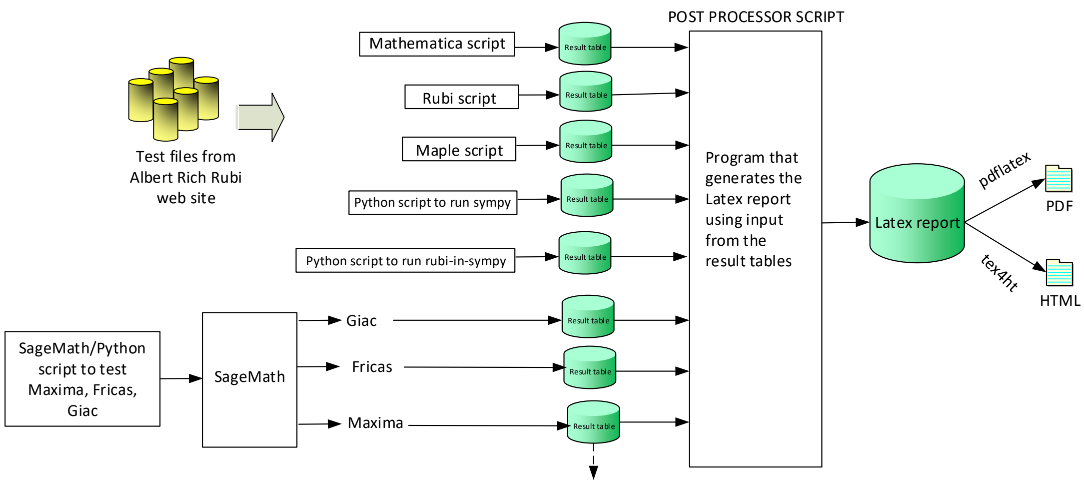
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

Nasser M. Abbasi
June 22, 2018

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 126, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 195, 197, 199, 201, 203, 205, 207, 209, 211, 213, 215, 217, 219, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 285, 286, 287, 288, 289, 290, 291, 293, 294, 295, 296, 297, 298 }

B grade: { }

C grade: { 112, 113, 125, 127, 128, 129, 157, 158, 191, 192, 193, 194, 196, 198, 200, 202, 204, 206, 208, 210, 212, 214, 216, 218, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 283 }

F grade: { 284, 292}

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 278, 280, 281 }

B grade: { 126, 269, 270, 271, 279 }

C grade: { }

F grade: { 272, 273, 274, 275, 276, 277, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 44, 46, 48, 50, 53, 55, 57, 59, 61, 63, 65, 67, 69, 71, 74, 76, 78, 80, 82, 84, 86, 88, 100, 101, 102, 119, 120, 121, 122, 139, 140, 141, 149, 153, 154, 155, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 275, 278, 280, 281

B grade: { 279, 283 }

C grade: { }

F grade: { 41, 43, 45, 47, 49, 51, 52, 54, 56, 58, 60, 62, 64, 66, 68, 70, 72, 73, 75, 77, 79, 81, 83, 85, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 142, 143, 144, 145, 146, 147, 148, 150, 151, 152, 156, 157, 158, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 276, 277, 282, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 275, 278, 279, 280, 281 }

B grade: { 84, 86, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 269, 270, 271, 283 }

C grade: { }

F grade: { 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 272, 273, 274, 276, 277, 282, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 50, 53, 55, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 68, 69, 71, 73, 74, 75, 76, 78, 79, 80, 81, 82, 84, 86, 87, 88, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 189, 190, 269, 270, 271, 278, 279, 280, 281 }

B grade: { 28, 45, 49, 51, 52, 54, 56, 66, 70, 72, 77, 83, 85 }

C grade: { }

F grade: { 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 175, 183, 184, 185, 186, 187, 188, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 272, 273, 274, 275, 276, 277, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 123, 124, 125, 126, 127, 128, 129, 132, 133, 134, 135, 136, 137, 138, 142, 149, 155, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 278, 279, 280, 281 }

B grade: { 28, 95, 96, 97, 98, 99, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 269, 270, 271, 283 }

C grade: { }

F grade: { 130, 131, 139, 140, 141, 143, 144, 145, 146, 147, 148, 150, 151, 152, 153, 154, 156, 157, 158, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 272, 273, 274, 275, 276, 277, 282, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	36	74	29	39
normalized size	1	1.	1.	0.85	1.09	2.24	0.88	1.18
time (sec)	N/A	0.027	0.006	0.043	1.125	0.397	0.061	1.277

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	36	74	29	39
normalized size	1	1.	1.	0.85	1.09	2.24	0.88	1.18
time (sec)	N/A	0.041	0.007	0.	1.039	0.415	0.059	1.226

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	36	74	29	39
normalized size	1	1.	1.	0.85	1.09	2.24	0.88	1.18
time (sec)	N/A	0.024	0.005	0.	1.136	0.395	0.057	1.157

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	36	66	29	39
normalized size	1	1.	1.	0.85	1.09	2.	0.88	1.18
time (sec)	N/A	0.038	0.007	0.	1.133	0.475	0.058	1.162

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	32	58	26	35
normalized size	1	1.	1.	0.89	1.14	2.07	0.93	1.25
time (sec)	N/A	0.019	0.005	0.001	1.112	0.476	0.057	1.123

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	28	38	65	27	41
normalized size	1	1.	1.	0.97	1.31	2.24	0.93	1.41
time (sec)	N/A	0.027	0.009	0.001	1.324	0.495	0.255	1.262

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	24	32	61	20	31
normalized size	1	1.	1.	0.92	1.23	2.35	0.77	1.19
time (sec)	N/A	0.023	0.009	0.003	1.412	0.478	0.252	1.202

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	26	38	70	26	57
normalized size	1	1.	1.	0.9	1.31	2.41	0.9	1.97
time (sec)	N/A	0.028	0.011	0.005	1.161	0.528	0.326	1.173

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	27	25	35	63	26	38
normalized size	1	1.	1.04	0.96	1.35	2.42	1.	1.46
time (sec)	N/A	0.022	0.011	0.005	1.123	0.449	0.342	1.201

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	31	28	41	73	27	53
normalized size	1	1.	1.07	0.97	1.41	2.52	0.93	1.83
time (sec)	N/A	0.027	0.017	0.004	1.092	0.495	0.497	1.214

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	33	28	39	70	32	42
normalized size	1	1.	1.06	0.9	1.26	2.26	1.03	1.35
time (sec)	N/A	0.022	0.011	0.004	1.129	0.491	0.504	1.146

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	52	69	131	56	72
normalized size	1	1.	1.	0.95	1.25	2.38	1.02	1.31
time (sec)	N/A	0.048	0.01	0.	1.143	0.419	0.071	1.206

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	52	69	120	53	72
normalized size	1	1.	1.	0.95	1.25	2.18	0.96	1.31
time (sec)	N/A	0.068	0.008	0.001	1.116	0.461	0.071	1.189

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	52	69	117	56	72
normalized size	1	1.	1.	0.95	1.25	2.13	1.02	1.31
time (sec)	N/A	0.039	0.009	0.001	1.064	0.51	0.069	1.263

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	51	52	69	117	53	72
normalized size	1	1.	1.21	1.24	1.64	2.79	1.26	1.71
time (sec)	N/A	0.07	0.013	0.001	1.108	0.458	0.069	1.265

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	65	109	53	68
normalized size	1	1.	1.	0.98	1.3	2.18	1.06	1.36
time (sec)	N/A	0.032	0.008	0.	1.125	0.48	0.068	1.335

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	51	51	70	116	49	72
normalized size	1	1.	1.19	1.19	1.63	2.7	1.14	1.67
time (sec)	N/A	0.04	0.016	0.002	1.108	0.451	0.289	1.272

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	49	65	116	48	65
normalized size	1	1.	1.	1.02	1.35	2.42	1.	1.35
time (sec)	N/A	0.037	0.017	0.003	1.14	0.442	0.286	1.231

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	49	50	70	122	48	95
normalized size	1	1.	0.96	0.98	1.37	2.39	0.94	1.86
time (sec)	N/A	0.053	0.028	0.005	1.089	0.506	0.372	1.213

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	50	46	68	109	49	68
normalized size	1	1.	1.04	0.96	1.42	2.27	1.02	1.42
time (sec)	N/A	0.037	0.019	0.006	1.137	0.497	0.387	1.306

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	50	51	73	122	49	97
normalized size	1	1.	0.98	1.	1.43	2.39	0.96	1.9
time (sec)	N/A	0.047	0.031	0.005	1.132	0.472	0.659	1.196

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	45	69	119	51	72
normalized size	1	1.	1.	0.94	1.44	2.48	1.06	1.5
time (sec)	N/A	0.037	0.02	0.004	1.161	0.445	0.734	1.243

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	53	52	74	127	53	89
normalized size	1	1.	1.04	1.02	1.45	2.49	1.04	1.75
time (sec)	N/A	0.045	0.026	0.005	1.121	0.47	1.165	1.253

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	59	48	72	126	56	74
normalized size	1	1.	1.11	0.91	1.36	2.38	1.06	1.4
time (sec)	N/A	0.036	0.017	0.006	1.168	0.48	1.169	1.249

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	75	76	99	169	80	104
normalized size	1	1.	1.	1.01	1.32	2.25	1.07	1.39
time (sec)	N/A	0.064	0.014	0.001	1.087	0.497	0.075	1.174

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	69	76	99	169	82	104
normalized size	1	1.	1.01	1.12	1.46	2.49	1.21	1.53
time (sec)	N/A	0.134	0.019	0.001	1.178	0.468	0.075	1.225

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	75	76	99	166	82	104
normalized size	1	1.	1.	1.01	1.32	2.21	1.09	1.39
time (sec)	N/A	0.055	0.012	0.001	1.129	0.442	0.075	1.213

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	69	76	99	166	80	104
normalized size	1	1.	1.64	1.81	2.36	3.95	1.9	2.48
time (sec)	N/A	0.071	0.019	0.001	1.132	0.482	0.075	1.208

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	70	73	95	155	76	99
normalized size	1	1.	1.	1.04	1.36	2.21	1.09	1.41
time (sec)	N/A	0.042	0.011	0.001	1.085	0.511	0.074	1.2

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	71	76	100	162	80	105
normalized size	1	1.	1.18	1.27	1.67	2.7	1.33	1.75
time (sec)	N/A	0.053	0.02	0.001	1.143	0.552	0.324	1.239

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	65	71	93	161	68	95
normalized size	1	1.	1.	1.09	1.43	2.48	1.05	1.46
time (sec)	N/A	0.045	0.022	0.004	1.168	0.481	0.314	1.163

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	73	75	100	171	78	131
normalized size	1	1.	1.03	1.06	1.41	2.41	1.1	1.85
time (sec)	N/A	0.079	0.027	0.006	1.183	0.498	0.41	1.236

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	71	70	99	162	73	100
normalized size	1	1.	1.03	1.01	1.43	2.35	1.06	1.45
time (sec)	N/A	0.05	0.023	0.006	1.131	0.46	0.427	1.185

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	73	76	103	163	73	132
normalized size	1	1.	1.01	1.06	1.43	2.26	1.01	1.83
time (sec)	N/A	0.072	0.03	0.007	1.144	0.43	0.727	1.291

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	68	64	99	162	75	101
normalized size	1	1.	1.	0.94	1.46	2.38	1.1	1.49
time (sec)	N/A	0.05	0.024	0.005	1.107	0.473	0.789	1.24

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	71	75	104	171	75	134
normalized size	1	1.	1.	1.06	1.46	2.41	1.06	1.89
time (sec)	N/A	0.064	0.037	0.006	1.059	0.469	1.416	1.186

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	66	63	99	163	75	104
normalized size	1	1.	1.	0.95	1.5	2.47	1.14	1.58
time (sec)	N/A	0.05	0.027	0.006	1.192	0.484	1.602	1.272

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	77	76	104	173	76	122
normalized size	1	1.	1.22	1.21	1.65	2.75	1.21	1.94
time (sec)	N/A	0.047	0.033	0.006	1.173	0.503	2.661	1.197

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	73	66	101	173	80	107
normalized size	1	1.	1.	0.9	1.38	2.37	1.1	1.47
time (sec)	N/A	0.049	0.028	0.006	1.194	0.497	2.69	1.207

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	78	66	101	166	80	107
normalized size	1	1.	1.59	1.35	2.06	3.39	1.63	2.18
time (sec)	N/A	0.04	0.019	0.004	1.086	0.476	4.433	1.252

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	119	140	0	581	194	180
normalized size	1	1.	1.	1.18	0.	4.88	1.63	1.51
time (sec)	N/A	0.087	0.078	0.006	0.	0.524	0.538	1.223

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	92	110	131	201	85	136
normalized size	1	1.	0.96	1.15	1.36	2.09	0.89	1.42
time (sec)	N/A	0.126	0.037	0.004	1.018	0.482	0.47	1.233

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	98	116	0	487	173	146
normalized size	1	1.	1.	1.18	0.	4.97	1.77	1.49
time (sec)	N/A	0.076	0.062	0.004	0.	0.556	0.508	1.227

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	71	86	100	155	65	104
normalized size	1	1.	0.95	1.15	1.33	2.07	0.87	1.39
time (sec)	N/A	0.094	0.029	0.004	1.507	0.49	0.446	1.245

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	77	92	0	381	150	115
normalized size	1	1.	1.	1.19	0.	4.95	1.95	1.49
time (sec)	N/A	0.066	0.05	0.003	0.	0.541	0.485	1.183

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	47	62	68	108	44	70
normalized size	1	1.	0.87	1.15	1.26	2.	0.81	1.3
time (sec)	N/A	0.069	0.019	0.002	2.023	0.497	0.423	1.251

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	57	68	0	277	90	77
normalized size	1	1.	0.98	1.17	0.	4.78	1.55	1.33
time (sec)	N/A	0.05	0.038	0.004	0.	0.589	0.462	1.349

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	31	40	42	65	27	43
normalized size	1	1.	0.89	1.14	1.2	1.86	0.77	1.23
time (sec)	N/A	0.043	0.011	0.003	1.157	0.607	0.389	1.287

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	45	0	223	82	46
normalized size	1	1.	1.	1.12	0.	5.58	2.05	1.15
time (sec)	N/A	0.028	0.022	0.002	0.	0.775	0.413	1.182

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	37	47	74	26	46
normalized size	1	1.	1.	1.09	1.38	2.18	0.76	1.35
time (sec)	N/A	0.041	0.012	0.004	1.21	0.772	0.654	1.258

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	48	0	228	82	49
normalized size	1	1.	1.	1.14	0.	5.43	1.95	1.17
time (sec)	N/A	0.032	0.025	0.005	0.	0.765	0.452	1.242

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	39	0	230	75	51
normalized size	1	1.	1.	0.95	0.	5.61	1.83	1.24
time (sec)	N/A	0.034	0.024	0.005	0.	0.748	0.468	1.259

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	56	65	111	41	96
normalized size	1	1.	1.	1.14	1.33	2.27	0.84	1.96
time (sec)	N/A	0.056	0.021	0.005	1.167	0.619	0.816	1.206

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	60	72	0	296	129	77
normalized size	1	1.	0.98	1.18	0.	4.85	2.11	1.26
time (sec)	N/A	0.056	0.053	0.006	0.	0.868	0.564	1.272

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	70	81	95	158	61	135
normalized size	1	1.	1.	1.16	1.36	2.26	0.87	1.93
time (sec)	N/A	0.072	0.029	0.007	2.59	0.793	0.997	1.144

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	78	96	0	398	163	109
normalized size	1	1.	1.	1.23	0.	5.1	2.09	1.4
time (sec)	N/A	0.068	0.053	0.006	0.	0.835	0.678	1.251

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	96	107	130	211	88	170
normalized size	1	1.	1.04	1.16	1.41	2.29	0.96	1.85
time (sec)	N/A	0.09	0.04	0.006	1.052	0.722	1.164	1.287

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	134	155	0	755	233	188
normalized size	1	1.	1.01	1.17	0.	5.68	1.75	1.41
time (sec)	N/A	0.166	0.104	0.01	0.	0.889	0.927	1.336

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	93	122	144	309	102	182
normalized size	1	1.	0.89	1.16	1.37	2.94	0.97	1.73
time (sec)	N/A	0.135	0.066	0.01	1.12	0.644	0.872	1.218

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	111	132	0	637	206	155
normalized size	1	1.	1.01	1.2	0.	5.79	1.87	1.41
time (sec)	N/A	0.118	0.083	0.01	0.	0.832	0.862	1.205

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	72	98	111	251	78	143
normalized size	1	1.	0.87	1.18	1.34	3.02	0.94	1.72
time (sec)	N/A	0.098	0.052	0.01	1.107	0.754	0.812	1.237

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	89	105	0	513	128	119
normalized size	1	1.	1.	1.18	0.	5.76	1.44	1.34
time (sec)	N/A	0.089	0.071	0.009	0.	0.778	0.785	1.271

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	50	74	81	165	56	95
normalized size	1	1.	0.82	1.21	1.33	2.7	0.92	1.56
time (sec)	N/A	0.07	0.036	0.008	1.265	0.69	0.715	1.298

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	68	82	0	433	114	80
normalized size	1	1.	1.	1.21	0.	6.37	1.68	1.18
time (sec)	N/A	0.061	0.051	0.008	0.	0.839	0.667	1.28

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	47	54	92	36	50
normalized size	1	1.	1.	1.15	1.32	2.24	0.88	1.22
time (sec)	N/A	0.047	0.012	0.007	2.768	0.789	0.506	1.261

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	68	0	381	112	77
normalized size	1	1.	1.	1.08	0.	6.05	1.78	1.22
time (sec)	N/A	0.033	0.046	0.008	0.	0.71	0.542	1.169

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	46	53	69	151	46	70
normalized size	1	1.	0.9	1.04	1.35	2.96	0.9	1.37
time (sec)	N/A	0.055	0.029	0.012	1.121	0.792	0.574	1.597

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	70	85	0	447	114	84
normalized size	1	1.	1.	1.21	0.	6.39	1.63	1.2
time (sec)	N/A	0.074	0.033	0.01	0.	0.927	0.653	1.457

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	64	86	103	248	70	108
normalized size	1	1.	0.88	1.18	1.41	3.4	0.96	1.48
time (sec)	N/A	0.08	0.052	0.014	1.212	0.867	1.021	1.328

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	90	110	0	532	184	115
normalized size	1	1.	1.	1.22	0.	5.91	2.04	1.28
time (sec)	N/A	0.117	0.072	0.013	0.	0.847	0.827	1.151

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	85	114	143	327	100	203
normalized size	1	1.	0.88	1.18	1.47	3.37	1.03	2.09
time (sec)	N/A	0.108	0.097	0.013	1.175	0.615	1.3	1.15

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	112	136	0	653	218	151
normalized size	1	1.	1.01	1.23	0.	5.88	1.96	1.36
time (sec)	N/A	0.198	0.09	0.013	0.	0.847	1.022	1.142

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	133	174	0	892	250	186
normalized size	1	1.	0.95	1.24	0.	6.37	1.79	1.33
time (sec)	N/A	0.235	0.104	0.012	0.	0.875	1.634	1.153

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	94	134	157	360	116	178
normalized size	1	1.	0.85	1.21	1.41	3.24	1.05	1.6
time (sec)	N/A	0.136	0.063	0.011	1.153	1.067	1.648	1.137

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	113	147	0	763	212	150
normalized size	1	1.	0.96	1.25	0.	6.47	1.8	1.27
time (sec)	N/A	0.162	0.085	0.01	0.	1.176	1.505	1.183

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	92	109	127	289	94	126
normalized size	1	1.	1.03	1.22	1.43	3.25	1.06	1.42
time (sec)	N/A	0.102	0.037	0.009	1.145	1.048	1.462	1.211

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	92	122	0	672	194	108
normalized size	1	1.	0.97	1.28	0.	7.07	2.04	1.14
time (sec)	N/A	0.099	0.071	0.009	0.	1.095	1.236	1.115

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	64	80	97	184	70	74
normalized size	1	1.	0.96	1.19	1.45	2.75	1.04	1.1
time (sec)	N/A	0.076	0.024	0.007	1.194	1.317	1.075	1.159

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	83	89	0	624	153	105
normalized size	1	1.	0.92	0.99	0.	6.93	1.7	1.17
time (sec)	N/A	0.076	0.083	0.008	0.	1.098	0.895	1.162

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	30	39	57	86	42	38
normalized size	1	1.	0.94	1.22	1.78	2.69	1.31	1.19
time (sec)	N/A	0.031	0.013	0.007	1.14	1.013	0.672	1.143

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	84	90	0	621	150	105
normalized size	1	1.	0.91	0.98	0.	6.75	1.63	1.14
time (sec)	N/A	0.045	0.059	0.007	0.	1.111	0.729	1.127

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	59	68	104	250	75	103
normalized size	1	1.	0.87	1.	1.53	3.68	1.1	1.51
time (sec)	N/A	0.073	0.046	0.011	1.188	1.009	0.747	1.153

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	96	125	0	686	194	111
normalized size	1	1.	1.	1.3	0.	7.15	2.02	1.16
time (sec)	N/A	0.118	0.062	0.013	0.	1.021	0.901	1.155

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	86	118	147	412	107	142
normalized size	1	1.	0.89	1.22	1.52	4.25	1.1	1.46
time (sec)	N/A	0.117	0.06	0.015	1.099	1.059	1.36	1.192

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	119	152	0	782	226	146
normalized size	1	1.	1.02	1.3	0.	6.68	1.93	1.25
time (sec)	N/A	0.177	0.07	0.013	0.	1.04	1.212	1.177

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	108	150	185	474	136	178
normalized size	1	1.	0.89	1.24	1.53	3.92	1.12	1.47
time (sec)	N/A	0.131	0.077	0.015	1.178	1.101	1.858	1.163

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	140	177	0	909	260	182
normalized size	1	1.	1.	1.26	0.	6.49	1.86	1.3
time (sec)	N/A	0.332	0.075	0.014	0.	1.079	1.692	1.158

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	135	180	230	560	165	271
normalized size	1	1.	0.91	1.22	1.55	3.78	1.11	1.83
time (sec)	N/A	0.173	0.12	0.017	1.372	1.012	2.57	1.126

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	193	290	0	844	0	331
normalized size	1	1.	0.89	1.33	0.	3.87	0.	1.52
time (sec)	N/A	0.381	0.294	0.028	0.	1.691	0.	1.177

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	173	248	0	737	0	285
normalized size	1	1.	0.96	1.37	0.	4.07	0.	1.57
time (sec)	N/A	0.334	0.273	0.008	0.	1.321	0.	1.154

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	151	206	0	609	0	239
normalized size	1	1.	1.21	1.65	0.	4.87	0.	1.91
time (sec)	N/A	0.199	0.201	0.009	0.	1.229	0.	1.229

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	129	164	0	497	0	189
normalized size	1	1.	1.21	1.53	0.	4.64	0.	1.77
time (sec)	N/A	0.154	0.179	0.009	0.	1.209	0.	1.327

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	91	124	0	392	0	139
normalized size	1	1.	0.91	1.24	0.	3.92	0.	1.39
time (sec)	N/A	0.197	0.144	0.006	0.	1.172	0.	1.348

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	78	130	0	363	0	124
normalized size	1	1.	0.8	1.34	0.	3.74	0.	1.28
time (sec)	N/A	0.213	0.143	0.008	0.	1.189	0.	1.232

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	86	109	0	363	0	220
normalized size	1	1.	1.08	1.36	0.	4.54	0.	2.75
time (sec)	N/A	0.196	0.122	0.01	0.	1.094	0.	1.549

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	44	48	0	131	0	338
normalized size	1	1.	0.72	0.79	0.	2.15	0.	5.54
time (sec)	N/A	0.162	0.018	0.006	0.	0.919	0.	1.807

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	66	70	0	184	0	419
normalized size	1	1.	0.69	0.73	0.	1.92	0.	4.36
time (sec)	N/A	0.211	0.025	0.005	0.	1.114	0.	2.313

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	88	94	0	238	0	500
normalized size	1	1.	0.66	0.71	0.	1.79	0.	3.76
time (sec)	N/A	0.245	0.029	0.006	0.	1.358	0.	3.139

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	94	118	0	298	0	581
normalized size	1	1.	0.55	0.69	0.	1.75	0.	3.42
time (sec)	N/A	0.3	0.068	0.006	0.	1.787	0.	4.422

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	82	91	143	230	0	180
normalized size	1	1.	0.63	0.69	1.09	1.76	0.	1.37
time (sec)	N/A	0.22	0.06	0.006	1.207	1.119	0.	1.169

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	64	67	112	177	0	142
normalized size	1	1.	0.68	0.71	1.19	1.88	0.	1.51
time (sec)	N/A	0.166	0.042	0.006	1.101	1.162	0.	1.161

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	41	45	69	124	0	99
normalized size	1	1.	0.67	0.74	1.13	2.03	0.	1.62
time (sec)	N/A	0.019	0.025	0.005	1.204	0.961	0.	1.215

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	84	85	0	359	0	157
normalized size	1	1.	1.08	1.09	0.	4.6	0.	2.01
time (sec)	N/A	0.146	0.077	0.007	0.	1.253	0.	1.158

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	94	135	0	375	0	103
normalized size	1	1.	0.94	1.35	0.	3.75	0.	1.03
time (sec)	N/A	0.161	0.046	0.01	0.	1.043	0.	1.241

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	95	174	0	437	0	178
normalized size	1	1.	0.92	1.69	0.	4.24	0.	1.73
time (sec)	N/A	0.158	0.107	0.01	0.	1.077	0.	1.25

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	215	328	0	984	0	378
normalized size	1	1.	0.96	1.47	0.	4.41	0.	1.7
time (sec)	N/A	0.404	0.325	0.032	0.	2.333	0.	1.166

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	193	286	0	853	0	332
normalized size	1	1.	1.16	1.71	0.	5.11	0.	1.99
time (sec)	N/A	0.248	0.283	0.011	0.	1.699	0.	1.186

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	171	244	0	717	0	279
normalized size	1	1.	1.16	1.65	0.	4.84	0.	1.89
time (sec)	N/A	0.191	0.232	0.01	0.	1.608	0.	1.17

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	151	202	0	616	0	240
normalized size	1	1.	1.05	1.4	0.	4.28	0.	1.67
time (sec)	N/A	0.266	0.219	0.009	0.	1.283	0.	1.192

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	130	162	0	504	0	192
normalized size	1	1.	0.95	1.18	0.	3.68	0.	1.4
time (sec)	N/A	0.296	0.118	0.004	0.	1.303	0.	1.184

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	96	174	0	467	0	170
normalized size	1	1.	0.75	1.36	0.	3.65	0.	1.33
time (sec)	N/A	0.277	0.174	0.009	0.	1.114	0.	1.238

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	98	219	0	435	0	304
normalized size	1	1.	0.72	1.61	0.	3.2	0.	2.24
time (sec)	N/A	0.274	0.051	0.01	0.	1.273	0.	1.456

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	94	153	0	471	0	343
normalized size	1	1.	0.9	1.47	0.	4.53	0.	3.3
time (sec)	N/A	0.251	0.047	0.011	0.	1.275	0.	1.894

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	44	48	0	178	0	500
normalized size	1	1.	0.72	0.79	0.	2.92	0.	8.2
time (sec)	N/A	0.173	0.024	0.004	0.	1.307	0.	3.539

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	66	70	0	236	0	581
normalized size	1	1.	0.69	0.73	0.	2.46	0.	6.05
time (sec)	N/A	0.233	0.031	0.005	0.	1.332	0.	4.782

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	89	94	0	304	0	662
normalized size	1	1.	0.67	0.71	0.	2.29	0.	4.98
time (sec)	N/A	0.279	0.034	0.006	0.	1.483	0.	4.92

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	95	118	0	356	0	743
normalized size	1	1.	0.56	0.69	0.	2.09	0.	4.37
time (sec)	N/A	0.321	0.07	0.006	0.	1.816	0.	5.158

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	89	142	0	409	0	786
normalized size	1	1.	0.43	0.69	0.	1.98	0.	3.8
time (sec)	N/A	0.352	0.067	0.007	0.	2.557	0.	6.311

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	113	115	203	350	0	440
normalized size	1	1.	0.67	0.68	1.21	2.08	0.	2.62
time (sec)	N/A	0.297	0.083	0.005	1.241	1.088	0.	1.25

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	92	91	173	294	0	363
normalized size	1	1.	0.7	0.69	1.32	2.24	0.	2.77
time (sec)	N/A	0.241	0.067	0.006	1.289	1.203	0.	1.224

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	71	67	142	228	0	288
normalized size	1	1.	0.74	0.7	1.48	2.38	0.	3.
time (sec)	N/A	0.07	0.047	0.004	1.277	1.088	0.	1.138

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	48	45	108	171	0	203
normalized size	1	1.	0.79	0.74	1.77	2.8	0.	3.33
time (sec)	N/A	0.159	0.031	0.003	1.245	1.108	0.	1.129

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	109	99	0	467	0	189
normalized size	1	1.	1.07	0.97	0.	4.58	0.	1.85
time (sec)	N/A	0.205	0.088	0.009	0.	1.168	0.	1.182

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	109	172	0	447	0	155
normalized size	1	1.	0.82	1.29	0.	3.36	0.	1.17
time (sec)	N/A	0.22	0.063	0.009	0.	1.111	0.	1.352

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	63	213	0	479	0	196
normalized size	1	1.	0.47	1.58	0.	3.55	0.	1.45
time (sec)	N/A	0.216	0.035	0.01	0.	1.149	0.	1.308

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	121	259	0	549	0	236
normalized size	1	1.	0.86	1.85	0.	3.92	0.	1.69
time (sec)	N/A	0.223	0.127	0.013	0.	1.165	0.	1.323

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	66	302	0	664	0	289
normalized size	1	1.	0.37	1.71	0.	3.75	0.	1.63
time (sec)	N/A	0.281	0.035	0.017	0.	1.233	0.	1.291

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	65	344	0	765	0	316
normalized size	1	1.	0.3	1.61	0.	3.57	0.	1.48
time (sec)	N/A	0.339	0.033	0.033	0.	1.433	0.	1.295

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	66	386	0	905	0	397
normalized size	1	1.	0.26	1.54	0.	3.61	0.	1.58
time (sec)	N/A	0.39	0.037	0.062	0.	1.714	0.	1.295

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	145	211	0	626	0	0
normalized size	1	1.	0.82	1.2	0.	3.56	0.	0.
time (sec)	N/A	0.328	0.168	0.013	0.	1.213	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	123	169	0	512	0	0
normalized size	1	1.	0.88	1.22	0.	3.68	0.	0.
time (sec)	N/A	0.275	0.114	0.008	0.	1.112	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	97	127	0	404	0	123
normalized size	1	1.	1.17	1.53	0.	4.87	0.	1.48
time (sec)	N/A	0.171	0.09	0.007	0.	1.067	0.	1.235

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	81	88	0	302	0	90
normalized size	1	1.	1.23	1.33	0.	4.58	0.	1.36
time (sec)	N/A	0.123	0.043	0.006	0.	1.154	0.	1.235

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	74	67	0	298	0	54
normalized size	1	1.	1.3	1.18	0.	5.23	0.	0.95
time (sec)	N/A	0.151	0.029	0.009	0.	1.038	0.	1.182

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	43	47	0	86	0	58
normalized size	1	1.	0.7	0.77	0.	1.41	0.	0.95
time (sec)	N/A	0.168	0.024	0.005	0.	1.075	0.	1.167

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	64	70	0	135	0	99
normalized size	1	1.	0.67	0.73	0.	1.41	0.	1.03
time (sec)	N/A	0.209	0.026	0.005	0.	1.184	0.	1.156

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	89	94	0	190	0	140
normalized size	1	1.	0.67	0.71	0.	1.43	0.	1.05
time (sec)	N/A	0.253	0.031	0.006	0.	1.154	0.	1.17

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	94	118	0	240	0	182
normalized size	1	1.	0.55	0.69	0.	1.41	0.	1.07
time (sec)	N/A	0.303	0.058	0.006	0.	1.266	0.	1.188

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	85	89	143	184	0	0
normalized size	1	1.	0.65	0.68	1.09	1.4	0.	0.
time (sec)	N/A	0.24	0.06	0.005	1.186	1.108	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	63	65	112	128	0	0
normalized size	1	1.	0.67	0.69	1.19	1.36	0.	0.
time (sec)	N/A	0.193	0.042	0.005	1.316	1.018	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	40	42	68	80	0	0
normalized size	1	1.	0.68	0.71	1.15	1.36	0.	0.
time (sec)	N/A	0.135	0.028	0.004	1.159	1.001	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	73	72	0	300	0	81
normalized size	1	1.	1.33	1.31	0.	5.45	0.	1.47
time (sec)	N/A	0.02	0.032	0.008	0.	1.068	0.	1.193

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	87	105	0	344	0	0
normalized size	1	1.	1.28	1.54	0.	5.06	0.	0.
time (sec)	N/A	0.117	0.052	0.009	0.	1.063	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	104	146	0	450	0	0
normalized size	1	1.	1.01	1.42	0.	4.37	0.	0.
time (sec)	N/A	0.165	0.15	0.01	0.	1.43	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	136	166	0	738	0	0
normalized size	1	1.	0.74	0.9	0.	4.01	0.	0.
time (sec)	N/A	0.333	0.179	0.015	0.	1.536	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	113	140	0	625	0	0
normalized size	1	1.	0.77	0.95	0.	4.25	0.	0.
time (sec)	N/A	0.28	0.137	0.009	0.	1.485	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	91	115	0	504	0	0
normalized size	1	1.	0.81	1.03	0.	4.5	0.	0.
time (sec)	N/A	0.242	0.113	0.008	0.	1.43	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	75	75	0	396	0	0
normalized size	1	1.	1.12	1.12	0.	5.91	0.	0.
time (sec)	N/A	0.177	0.07	0.005	0.	1.36	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	47	88	93	0	49
normalized size	1	1.	1.	1.27	2.38	2.51	0.	1.32
time (sec)	N/A	0.121	0.019	0.004	1.174	1.328	0.	1.187

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	64	66	0	149	0	0
normalized size	1	1.	0.97	1.	0.	2.26	0.	0.
time (sec)	N/A	0.164	0.022	0.005	0.	1.251	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	85	94	0	200	0	0
normalized size	1	1.	0.84	0.93	0.	1.98	0.	0.
time (sec)	N/A	0.22	0.027	0.006	0.	1.318	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	75	118	0	252	0	0
normalized size	1	1.	0.54	0.86	0.	1.83	0.	0.
time (sec)	N/A	0.267	0.034	0.006	0.	1.501	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	82	91	111	194	0	0
normalized size	1	1.	0.59	0.65	0.8	1.4	0.	0.
time (sec)	N/A	0.248	0.051	0.005	1.193	1.364	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	60	66	80	139	0	0
normalized size	1	1.	0.58	0.63	0.77	1.34	0.	0.
time (sec)	N/A	0.2	0.038	0.005	1.194	1.489	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	35	44	53	88	0	81
normalized size	1	1.	0.51	0.64	0.77	1.28	0.	1.17
time (sec)	N/A	0.15	0.023	0.003	1.213	1.279	0.	1.332

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	73	79	0	419	0	0
normalized size	1	1.	1.14	1.23	0.	6.55	0.	0.
time (sec)	N/A	0.139	0.031	0.009	0.	1.443	0.	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	61	129	0	549	0	0
normalized size	1	1.	0.43	0.91	0.	3.87	0.	0.
time (sec)	N/A	0.089	0.024	0.01	0.	1.482	0.	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	64	157	0	671	0	0
normalized size	1	1.	0.47	1.15	0.	4.9	0.	0.
time (sec)	N/A	0.192	0.027	0.01	0.	1.412	0.	0.

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	33	32	36	90	46	39
normalized size	1	1.	0.85	0.82	0.92	2.31	1.18	1.
time (sec)	N/A	0.023	0.016	0.004	1.155	1.299	27.759	1.157

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	33	32	36	89	46	39
normalized size	1	1.	0.85	0.82	0.92	2.28	1.18	1.
time (sec)	N/A	0.023	0.015	0.003	1.132	1.319	19.579	1.16

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	33	32	36	89	46	39
normalized size	1	1.	0.85	0.82	0.92	2.28	1.18	1.
time (sec)	N/A	0.023	0.016	0.003	1.065	1.556	10.291	1.137

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	33	32	36	88	37	39
normalized size	1	1.	0.85	0.82	0.92	2.26	0.95	1.
time (sec)	N/A	0.021	0.014	0.003	1.187	1.64	3.659	1.14

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	33	32	36	85	46	39
normalized size	1	1.	0.85	0.82	0.92	2.18	1.18	1.
time (sec)	N/A	0.021	0.014	0.003	1.024	1.529	3.317	1.148

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	33	32	36	81	46	39
normalized size	1	1.	0.85	0.82	0.92	2.08	1.18	1.
time (sec)	N/A	0.021	0.014	0.004	1.196	1.659	2.662	1.148

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	33	32	36	74	44	39
normalized size	1	1.	0.89	0.86	0.97	2.	1.19	1.05
time (sec)	N/A	0.021	0.014	0.003	1.127	1.537	4.549	1.131

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	35	32	36	74	44	39
normalized size	1	1.	0.95	0.86	0.97	2.	1.19	1.05
time (sec)	N/A	0.021	0.01	0.004	1.15	1.615	7.518	1.171

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	56	69	157	80	72
normalized size	1	1.	1.	0.89	1.1	2.49	1.27	1.14
time (sec)	N/A	0.041	0.032	0.005	1.1	1.575	96.261	1.178

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	53	56	69	150	80	72
normalized size	1	1.	0.84	0.89	1.1	2.38	1.27	1.14
time (sec)	N/A	0.039	0.03	0.005	1.152	1.7	53.763	1.166

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	56	69	151	80	72
normalized size	1	1.	1.	0.89	1.1	2.4	1.27	1.14
time (sec)	N/A	0.038	0.029	0.005	1.131	1.535	27.974	1.213

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	53	56	69	149	66	72
normalized size	1	1.	0.84	0.89	1.1	2.37	1.05	1.14
time (sec)	N/A	0.038	0.029	0.005	1.127	1.561	6.599	1.171

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	53	56	69	146	80	72
normalized size	1	1.	0.84	0.89	1.1	2.32	1.27	1.14
time (sec)	N/A	0.039	0.029	0.006	1.171	1.548	11.918	1.162

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	56	69	147	80	72
normalized size	1	1.	1.	0.89	1.1	2.33	1.27	1.14
time (sec)	N/A	0.039	0.028	0.005	1.171	1.563	13.452	1.122

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	53	56	69	143	80	72
normalized size	1	1.	0.84	0.89	1.1	2.27	1.27	1.14
time (sec)	N/A	0.038	0.03	0.005	1.139	1.597	16.498	1.132

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	53	56	69	136	80	72
normalized size	1	1.	0.84	0.89	1.1	2.16	1.27	1.14
time (sec)	N/A	0.038	0.03	0.006	1.169	1.466	21.516	1.191

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	85	80	99	217	0	104
normalized size	1	1.	1.	0.94	1.16	2.55	0.	1.22
time (sec)	N/A	0.052	0.047	0.006	1.081	1.495	0.	1.112

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	85	80	99	216	114	104
normalized size	1	1.	1.	0.94	1.16	2.54	1.34	1.22
time (sec)	N/A	0.05	0.043	0.007	1.116	1.683	165.782	1.12

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	85	80	99	213	114	104
normalized size	1	1.	1.	0.94	1.16	2.51	1.34	1.22
time (sec)	N/A	0.05	0.041	0.005	1.196	1.84	113.16	1.147

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	85	80	99	211	95	104
normalized size	1	1.	1.	0.94	1.16	2.48	1.12	1.22
time (sec)	N/A	0.05	0.04	0.005	1.127	1.821	18.293	1.12

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	85	80	99	209	114	104
normalized size	1	1.	1.	0.94	1.16	2.46	1.34	1.22
time (sec)	N/A	0.052	0.038	0.005	1.161	1.803	56.039	1.135

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	85	80	99	207	114	104
normalized size	1	1.	1.	0.94	1.16	2.44	1.34	1.22
time (sec)	N/A	0.05	0.037	0.005	1.107	1.858	59.921	1.139

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	85	80	99	205	114	104
normalized size	1	1.	1.	0.94	1.16	2.41	1.34	1.22
time (sec)	N/A	0.052	0.038	0.004	1.167	1.88	81.981	1.122

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	85	80	99	201	114	104
normalized size	1	1.	1.	0.94	1.16	2.36	1.34	1.22
time (sec)	N/A	0.051	0.037	0.007	1.098	1.862	112.692	1.137

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	133	336	0	1914	0	402
normalized size	1	1.	0.48	1.21	0.	6.88	0.	1.45
time (sec)	N/A	0.274	0.23	0.033	0.	2.269	0.	1.164

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	227	330	0	1477	0	402
normalized size	1	1.	0.82	1.2	0.	5.35	0.	1.46
time (sec)	N/A	0.247	0.276	0.008	0.	2.274	0.	1.163

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	110	308	0	1823	0	356
normalized size	1	1.	0.43	1.2	0.	7.09	0.	1.39
time (sec)	N/A	0.218	0.127	0.007	0.	2.408	0.	1.152

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	208	299	0	1365	0	355
normalized size	1	1.	0.82	1.17	0.	5.35	0.	1.39
time (sec)	N/A	0.209	0.202	0.008	0.	2.076	0.	1.13

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	95	280	0	1698	0	339
normalized size	1	1.	0.4	1.18	0.	7.16	0.	1.43
time (sec)	N/A	0.188	0.075	0.008	0.	2.335	0.	1.192

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	166	277	0	1347	0	339
normalized size	1	1.	0.71	1.18	0.	5.73	0.	1.44
time (sec)	N/A	0.184	0.133	0.007	0.	2.18	0.	1.199

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	74	277	0	1735	374	339
normalized size	1	1.	0.31	1.18	0.	7.38	1.59	1.44
time (sec)	N/A	0.189	0.088	0.01	0.	2.33	85.082	1.158

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	168	280	0	1347	374	339
normalized size	1	1.	0.71	1.18	0.	5.68	1.58	1.43
time (sec)	N/A	0.188	0.136	0.008	0.	2.295	160.822	1.185

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	46	299	0	1793	0	362
normalized size	1	1.	0.18	1.17	0.	7.03	0.	1.42
time (sec)	N/A	0.215	0.02	0.011	0.	2.264	0.	1.179

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	47	308	0	1453	0	347
normalized size	1	1.	0.18	1.2	0.	5.65	0.	1.35
time (sec)	N/A	0.214	0.018	0.011	0.	2.311	0.	1.186

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	47	330	0	1898	0	393
normalized size	1	1.	0.17	1.2	0.	6.88	0.	1.42
time (sec)	N/A	0.245	0.017	0.012	0.	2.639	0.	1.191

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	47	336	0	1540	0	393
normalized size	1	1.	0.17	1.21	0.	5.54	0.	1.41
time (sec)	N/A	0.241	0.016	0.011	0.	2.523	0.	1.219

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	332	332	417	372	0	1928	0	452
normalized size	1	1.	1.26	1.12	0.	5.81	0.	1.36
time (sec)	N/A	0.277	0.455	0.016	0.	2.427	0.	1.277

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	310	154	348	0	2390	0	404
normalized size	1	1.	0.5	1.12	0.	7.71	0.	1.3
time (sec)	N/A	0.245	0.221	0.015	0.	2.645	0.	1.241

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	310	385	339	0	1775	0	402
normalized size	1	1.	1.24	1.09	0.	5.73	0.	1.3
time (sec)	N/A	0.253	0.398	0.013	0.	2.468	0.	1.329

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	289	136	317	0	2141	0	382
normalized size	1	1.	0.47	1.1	0.	7.41	0.	1.32
time (sec)	N/A	0.229	0.198	0.013	0.	2.664	0.	1.349

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	289	354	323	0	1594	0	382
normalized size	1	1.	1.22	1.12	0.	5.52	0.	1.32
time (sec)	N/A	0.234	0.391	0.014	0.	2.523	0.	1.221

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	95	305	0	1968	0	369
normalized size	1	1.	0.36	1.17	0.	7.54	0.	1.41
time (sec)	N/A	0.2	0.128	0.013	0.	2.562	0.	1.208

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	203	305	0	1559	0	369
normalized size	1	1.	0.78	1.17	0.	5.97	0.	1.41
time (sec)	N/A	0.2	0.243	0.013	0.	2.475	0.	1.191

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	284	117	323	0	2026	0	375
normalized size	1	1.	0.41	1.14	0.	7.13	0.	1.32
time (sec)	N/A	0.23	0.214	0.016	0.	2.566	0.	1.278

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	289	355	317	0	1700	0	382
normalized size	1	1.	1.23	1.1	0.	5.88	0.	1.32
time (sec)	N/A	0.225	0.423	0.015	0.	2.493	0.	1.27

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	310	151	339	0	2310	0	409
normalized size	1	1.	0.49	1.09	0.	7.45	0.	1.32
time (sec)	N/A	0.271	0.446	0.017	0.	2.655	0.	1.356

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	310	385	348	0	1898	0	394
normalized size	1	1.	1.24	1.12	0.	6.12	0.	1.27
time (sec)	N/A	0.255	0.493	0.019	0.	2.459	0.	1.341

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	332	332	176	372	0	2475	0	443
normalized size	1	1.	0.53	1.12	0.	7.45	0.	1.33
time (sec)	N/A	0.286	0.414	0.018	0.	2.633	0.	1.376

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	343	435	381	0	1955	0	433
normalized size	1	1.	1.27	1.11	0.	5.7	0.	1.26
time (sec)	N/A	0.278	0.475	0.019	0.	2.574	0.	1.312

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	176	357	0	2367	0	410
normalized size	1	1.	0.55	1.11	0.	7.35	0.	1.27
time (sec)	N/A	0.247	0.316	0.018	0.	2.819	0.	1.181

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	403	363	0	1787	0	410
normalized size	1	1.	1.25	1.13	0.	5.55	0.	1.27
time (sec)	N/A	0.254	0.458	0.016	0.	2.6	0.	1.258

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	137	325	0	2226	0	396
normalized size	1	1.	0.47	1.11	0.	7.6	0.	1.35
time (sec)	N/A	0.223	0.224	0.015	0.	2.664	0.	1.328

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	389	334	0	1817	0	402
normalized size	1	1.	1.31	1.12	0.	6.1	0.	1.35
time (sec)	N/A	0.236	0.446	0.015	0.	2.604	0.	1.294

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	62	335	0	2276	0	402
normalized size	1	1.	0.21	1.12	0.	7.64	0.	1.35
time (sec)	N/A	0.229	0.055	0.016	0.	2.77	0.	1.393

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	230	325	0	1782	0	396
normalized size	1	1.	0.78	1.11	0.	6.08	0.	1.35
time (sec)	N/A	0.231	0.298	0.015	0.	2.542	0.	1.317

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	316	147	363	0	2244	0	405
normalized size	1	1.	0.47	1.15	0.	7.1	0.	1.28
time (sec)	N/A	0.256	0.205	0.018	0.	2.739	0.	1.391

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	400	357	0	1901	0	410
normalized size	1	1.	1.24	1.11	0.	5.9	0.	1.27
time (sec)	N/A	0.251	0.447	0.017	0.	2.753	0.	1.31

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	343	189	381	0	2507	0	440
normalized size	1	1.	0.55	1.11	0.	7.31	0.	1.28
time (sec)	N/A	0.275	0.477	0.021	0.	2.697	0.	1.351

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	343	433	390	0	2063	0	425
normalized size	1	1.	1.26	1.14	0.	6.01	0.	1.24
time (sec)	N/A	0.278	0.511	0.02	0.	2.73	0.	1.309

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	365	365	216	414	0	2684	0	474
normalized size	1	1.	0.59	1.13	0.	7.35	0.	1.3
time (sec)	N/A	0.32	0.502	0.021	0.	2.73	0.	1.381

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	365	365	467	420	0	2233	0	474
normalized size	1	1.	1.28	1.15	0.	6.12	0.	1.3
time (sec)	N/A	0.321	0.581	0.022	0.	2.595	0.	1.393

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	136	307	0	0	0	0
normalized size	1	1.	0.56	1.26	0.	0.	0.	0.
time (sec)	N/A	0.39	0.162	0.053	0.	0.	0.	0.

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	369	111	446	0	0	0	0
normalized size	1	1.	0.3	1.21	0.	0.	0.	0.
time (sec)	N/A	0.436	0.129	0.03	0.	0.	0.	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	111	283	0	0	0	0
normalized size	1	1.	0.54	1.39	0.	0.	0.	0.
time (sec)	N/A	0.3	0.13	0.023	0.	0.	0.	0.

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	326	326	94	422	0	0	0	0
normalized size	1	1.	0.29	1.29	0.	0.	0.	0.
time (sec)	N/A	0.375	0.082	0.021	0.	0.	0.	0.

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	94	257	0	0	0	0
normalized size	1	1.	0.57	1.56	0.	0.	0.	0.
time (sec)	N/A	0.254	0.05	0.02	0.	0.	0.	0.

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	323	97	399	0	0	0	0
normalized size	1	1.	0.3	1.24	0.	0.	0.	0.
time (sec)	N/A	0.362	0.043	0.03	0.	0.	0.	0.

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	97	239	0	0	0	0
normalized size	1	1.	0.6	1.47	0.	0.	0.	0.
time (sec)	N/A	0.243	0.044	0.025	0.	0.	0.	0.

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	328	328	96	422	0	0	0	0
normalized size	1	1.	0.29	1.29	0.	0.	0.	0.
time (sec)	N/A	0.374	0.042	0.032	0.	0.	0.	0.

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	98	255	0	0	0	0
normalized size	1	1.	0.59	1.53	0.	0.	0.	0.
time (sec)	N/A	0.25	0.042	0.029	0.	0.	0.	0.

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	369	99	452	0	0	0	0
normalized size	1	1.	0.27	1.22	0.	0.	0.	0.
time (sec)	N/A	0.436	0.044	0.032	0.	0.	0.	0.

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	98	283	0	0	0	0
normalized size	1	1.	0.48	1.39	0.	0.	0.	0.
time (sec)	N/A	0.308	0.045	0.03	0.	0.	0.	0.

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	486	486	160	518	0	0	0	0
normalized size	1	1.	0.33	1.07	0.	0.	0.	0.
time (sec)	N/A	0.671	0.215	0.04	0.	0.	0.	0.

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	321	321	160	355	0	0	0	0
normalized size	1	1.	0.5	1.11	0.	0.	0.	0.
time (sec)	N/A	0.49	0.207	0.033	0.	0.	0.	0.

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	447	447	138	494	0	0	0	0
normalized size	1	1.	0.31	1.11	0.	0.	0.	0.
time (sec)	N/A	0.59	0.175	0.014	0.	0.	0.	0.

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	282	138	331	0	0	0	0
normalized size	1	1.	0.49	1.17	0.	0.	0.	0.
time (sec)	N/A	0.425	0.169	0.015	0.	0.	0.	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	408	408	115	470	0	0	0	0
normalized size	1	1.	0.28	1.15	0.	0.	0.	0.
time (sec)	N/A	0.515	0.144	0.015	0.	0.	0.	0.

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	115	307	0	0	0	0
normalized size	1	1.	0.48	1.28	0.	0.	0.	0.
time (sec)	N/A	0.369	0.142	0.014	0.	0.	0.	0.

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	369	98	446	0	0	0	0
normalized size	1	1.	0.27	1.21	0.	0.	0.	0.
time (sec)	N/A	0.45	0.103	0.017	0.	0.	0.	0.

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	97	283	0	0	0	0
normalized size	1	1.	0.48	1.41	0.	0.	0.	0.
time (sec)	N/A	0.312	0.068	0.015	0.	0.	0.	0.

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	356	85	429	0	0	0	0
normalized size	1	1.	0.24	1.21	0.	0.	0.	0.
time (sec)	N/A	0.446	0.074	0.02	0.	0.	0.	0.

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	101	260	0	0	0	0
normalized size	1	1.	0.5	1.3	0.	0.	0.	0.
time (sec)	N/A	0.322	0.055	0.015	0.	0.	0.	0.

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	354	354	99	427	0	0	0	0
normalized size	1	1.	0.28	1.21	0.	0.	0.	0.
time (sec)	N/A	0.435	0.045	0.019	0.	0.	0.	0.

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	101	254	0	0	0	0
normalized size	1	1.	0.5	1.25	0.	0.	0.	0.
time (sec)	N/A	0.319	0.052	0.016	0.	0.	0.	0.

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	364	364	100	452	0	0	0	0
normalized size	1	1.	0.27	1.24	0.	0.	0.	0.
time (sec)	N/A	0.446	0.055	0.02	0.	0.	0.	0.

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	143	298	0	0	0	0
normalized size	1	1.	0.59	1.23	0.	0.	0.	0.
time (sec)	N/A	0.368	0.159	0.025	0.	0.	0.	0.

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	369	122	437	0	0	0	0
normalized size	1	1.	0.33	1.18	0.	0.	0.	0.
time (sec)	N/A	0.445	0.138	0.025	0.	0.	0.	0.

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	122	274	0	0	0	0
normalized size	1	1.	0.6	1.34	0.	0.	0.	0.
time (sec)	N/A	0.314	0.132	0.016	0.	0.	0.	0.

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	330	330	97	413	0	0	0	0
normalized size	1	1.	0.29	1.25	0.	0.	0.	0.
time (sec)	N/A	0.377	0.117	0.017	0.	0.	0.	0.

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	97	248	0	0	0	0
normalized size	1	1.	0.58	1.49	0.	0.	0.	0.
time (sec)	N/A	0.255	0.111	0.015	0.	0.	0.	0.

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	81	378	0	0	0	0
normalized size	1	1.	0.28	1.29	0.	0.	0.	0.
time (sec)	N/A	0.311	0.088	0.014	0.	0.	0.	0.

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	80	216	0	0	0	0
normalized size	1	1.	0.62	1.66	0.	0.	0.	0.
time (sec)	N/A	0.198	0.062	0.015	0.	0.	0.	0.

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	82	377	0	0	0	0
normalized size	1	1.	0.29	1.34	0.	0.	0.	0.
time (sec)	N/A	0.309	0.049	0.018	0.	0.	0.	0.

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	82	219	0	0	0	0
normalized size	1	1.	0.63	1.67	0.	0.	0.	0.
time (sec)	N/A	0.204	0.047	0.015	0.	0.	0.	0.

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	332	332	83	413	0	0	0	0
normalized size	1	1.	0.25	1.24	0.	0.	0.	0.
time (sec)	N/A	0.378	0.045	0.018	0.	0.	0.	0.

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	85	247	0	0	0	0
normalized size	1	1.	0.51	1.48	0.	0.	0.	0.
time (sec)	N/A	0.253	0.045	0.016	0.	0.	0.	0.

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	369	84	443	0	0	0	0
normalized size	1	1.	0.23	1.2	0.	0.	0.	0.
time (sec)	N/A	0.441	0.044	0.02	0.	0.	0.	0.

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	84	274	0	0	0	0
normalized size	1	1.	0.41	1.34	0.	0.	0.	0.
time (sec)	N/A	0.311	0.044	0.019	0.	0.	0.	0.

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	134	281	0	0	0	0
normalized size	1	1.	0.53	1.12	0.	0.	0.	0.
time (sec)	N/A	0.39	0.166	0.038	0.	0.	0.	0.

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	377	377	110	420	0	0	0	0
normalized size	1	1.	0.29	1.11	0.	0.	0.	0.
time (sec)	N/A	0.461	0.125	0.034	0.	0.	0.	0.

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	110	255	0	0	0	0
normalized size	1	1.	0.51	1.19	0.	0.	0.	0.
time (sec)	N/A	0.327	0.133	0.019	0.	0.	0.	0.

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	340	340	85	394	0	0	0	0
normalized size	1	1.	0.25	1.16	0.	0.	0.	0.
time (sec)	N/A	0.397	0.103	0.018	0.	0.	0.	0.

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	86	230	0	0	0	0
normalized size	1	1.	0.48	1.29	0.	0.	0.	0.
time (sec)	N/A	0.272	0.108	0.017	0.	0.	0.	0.

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	78	388	0	0	0	0
normalized size	1	1.	0.26	1.3	0.	0.	0.	0.
time (sec)	N/A	0.328	0.087	0.019	0.	0.	0.	0.

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	76	222	0	0	0	0
normalized size	1	1.	0.55	1.62	0.	0.	0.	0.
time (sec)	N/A	0.221	0.06	0.019	0.	0.	0.	0.

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	318	77	392	0	0	0	0
normalized size	1	1.	0.24	1.23	0.	0.	0.	0.
time (sec)	N/A	0.39	0.047	0.02	0.	0.	0.	0.

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	92	235	0	0	0	0
normalized size	1	1.	0.55	1.41	0.	0.	0.	0.
time (sec)	N/A	0.272	0.049	0.02	0.	0.	0.	0.

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	368	368	79	420	0	0	0	0
normalized size	1	1.	0.21	1.14	0.	0.	0.	0.
time (sec)	N/A	0.451	0.043	0.021	0.	0.	0.	0.

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	79	254	0	0	0	0
normalized size	1	1.	0.39	1.25	0.	0.	0.	0.
time (sec)	N/A	0.328	0.046	0.02	0.	0.	0.	0.

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	405	405	79	450	0	0	0	0
normalized size	1	1.	0.2	1.11	0.	0.	0.	0.
time (sec)	N/A	0.522	0.045	0.022	0.	0.	0.	0.

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	89	474	0	910	2077	814
normalized size	1	1.	0.93	4.94	0.	9.48	21.64	8.48
time (sec)	N/A	0.07	0.091	0.051	0.	2.293	11.235	1.293

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	66	262	0	512	1051	459
normalized size	1	1.	0.93	3.69	0.	7.21	14.8	6.46
time (sec)	N/A	0.052	0.057	0.006	0.	2.304	5.042	1.347

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	42	110	0	223	415	201
normalized size	1	1.	0.93	2.44	0.	4.96	9.22	4.47
time (sec)	N/A	0.03	0.04	0.003	0.	2.314	1.811	1.407

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	55	0	0	0	0	0
normalized size	1	1.	0.77	0.	0.	0.	0.	0.
time (sec)	N/A	0.047	0.062	0.234	0.	0.	0.	0.

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	92	80	0	0	0	0	0
normalized size	1	0.94	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	0.075	0.235	0.	0.	0.	0.

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	126	135	0	0	0	0	0
normalized size	1	0.9	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.137	0.098	0.313	0.	0.	0.	0.

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	63	0	151	300	0	0
normalized size	1	1.	0.66	0.	1.59	3.16	0.	0.
time (sec)	N/A	0.156	0.069	0.547	1.473	2.428	0.	0.

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	111	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.224	0.164	2.108	0.	0.	0.	0.

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	129	129	210	0	0	0	0	0
normalized size	1	1.	1.63	0.	0.	0.	0.	0.
time (sec)	N/A	0.357	0.212	0.823	0.	0.	0.	0.

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	20	46	17	23
normalized size	1	1.	1.	0.84	1.05	2.42	0.89	1.21
time (sec)	N/A	0.026	0.005	0.049	1.698	2.015	0.1	1.209

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	36	47	38	10	22
normalized size	1	1.	1.	2.4	3.13	2.53	0.67	1.47
time (sec)	N/A	0.022	0.005	0.055	1.764	2.119	0.134	1.247

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	43	14	22
normalized size	1	1.	1.	0.82	1.06	2.53	0.82	1.29
time (sec)	N/A	0.025	0.005	0.052	1.755	2.062	0.161	1.229

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	19	46	10	22
normalized size	1	1.	1.	0.94	1.19	2.88	0.62	1.38
time (sec)	N/A	0.014	0.003	0.051	1.122	2.062	0.147	1.243

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	52	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.05	0.019	0.767	0.	0.	0.	0.

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	116	0	50	88	0	65
normalized size	1	1.	6.44	0.	2.78	4.89	0.	3.61
time (sec)	N/A	0.038	0.17	0.745	1.438	2.545	0.	1.387

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	0.07	0.525	0.	0.	0.	0.

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	157	0	0	0	0	0
normalized size	1	1.	0.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.216	0.126	0.497	0.	0.	0.	0.

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	119	0	0	0	0	0
normalized size	1	1.	0.91	0.	0.	0.	0.	0.
time (sec)	N/A	0.097	0.063	0.495	0.	0.	0.	0.

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	97	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	0.03	0.498	0.	0.	0.	0.

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.015	0.531	0.	0.	0.	0.

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	77	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	0.023	0.511	0.	0.	0.	0.

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	112	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.096	0.094	0.52	0.	0.	0.	0.

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	184	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.158	0.163	0.511	0.	0.	0.	0.

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.071	0.071	0.561	0.	0.	0.	0.

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	179	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.239	0.167	0.53	0.	0.	0.	0.

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	120	0	0	0	0	0
normalized size	1	1.	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.115	0.11	0.507	0.	0.	0.	0.

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	57	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.033	0.022	0.497	0.	0.	0.	0.

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	88	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.068	0.052	0.52	0.	0.	0.	0.

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	125	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.131	0.072	0.535	0.	0.	0.	0.

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	182	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.264	0.199	0.528	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [207] had the largest ratio of [0.4231]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	2	1.	22	0.091
2	A	4	3	1.	20	0.15
3	A	3	2	1.	19	0.105
4	A	4	3	1.	22	0.136
5	A	3	2	1.	22	0.091
6	A	4	3	1.	22	0.136
7	A	3	2	1.	22	0.091
8	A	4	3	1.	22	0.136
9	A	3	2	1.	22	0.091
10	A	4	3	1.	22	0.136
11	A	3	2	1.	22	0.091
12	A	3	2	1.	21	0.095
13	A	4	3	1.	24	0.125
14	A	3	2	1.	24	0.083
15	A	4	3	1.	24	0.125
16	A	3	2	1.	24	0.083
17	A	5	4	1.	24	0.167
18	A	3	2	1.	24	0.083
19	A	4	3	1.	24	0.125
20	A	3	2	1.	24	0.083
21	A	4	3	1.	24	0.125
22	A	3	2	1.	24	0.083
23	A	4	3	1.	24	0.125
24	A	3	2	1.	24	0.083
25	A	3	2	1.	24	0.083
26	A	4	3	1.	24	0.125
27	A	3	2	1.	24	0.083
28	A	4	3	1.	24	0.125
29	A	3	2	1.	24	0.083
30	A	5	4	1.	24	0.167
31	A	3	2	1.	24	0.083

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
32	A	4	3	1.	24	0.125
33	A	3	2	1.	24	0.083
34	A	4	3	1.	24	0.125
35	A	3	2	1.	24	0.083
36	A	4	3	1.	24	0.125
37	A	3	2	1.	24	0.083
38	A	5	4	1.	24	0.167
39	A	3	2	1.	24	0.083
40	A	4	4	1.	24	0.167
41	A	5	4	1.	24	0.167
42	A	4	3	1.	24	0.125
43	A	5	4	1.	24	0.167
44	A	4	3	1.	24	0.125
45	A	5	4	1.	24	0.167
46	A	4	3	1.	24	0.125
47	A	4	4	1.	24	0.167
48	A	4	3	1.	24	0.125
49	A	3	3	1.	24	0.125
50	A	4	3	1.	22	0.136
51	A	3	3	1.	21	0.143
52	A	3	3	1.	22	0.136
53	A	4	3	1.	24	0.125
54	A	4	4	1.	24	0.167
55	A	4	3	1.	24	0.125
56	A	5	4	1.	24	0.167
57	A	4	3	1.	24	0.125
58	A	5	4	1.	24	0.167
59	A	4	3	1.	24	0.125
60	A	5	4	1.	24	0.167
61	A	4	3	1.	24	0.125
62	A	5	4	1.	24	0.167
63	A	4	3	1.	24	0.125

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
64	A	4	4	1.	24	0.167
65	A	4	3	1.	24	0.125
66	A	3	3	1.	24	0.125
67	A	4	3	1.	24	0.125
68	A	4	4	1.	24	0.167
69	A	4	3	1.	22	0.136
70	A	5	4	1.	21	0.19
71	A	4	3	1.	24	0.125
72	A	5	4	1.	24	0.167
73	A	6	5	1.	24	0.208
74	A	4	3	1.	24	0.125
75	A	6	5	1.	24	0.208
76	A	4	3	1.	24	0.125
77	A	5	5	1.	24	0.208
78	A	4	3	1.	24	0.125
79	A	4	4	1.	24	0.167
80	A	3	3	1.	24	0.125
81	A	4	4	1.	24	0.167
82	A	4	3	1.	24	0.125
83	A	5	4	1.	24	0.167
84	A	4	3	1.	24	0.125
85	A	6	5	1.	24	0.208
86	A	4	3	1.	22	0.136
87	A	6	5	1.	21	0.238
88	A	4	3	1.	24	0.125
89	A	8	7	1.	26	0.269
90	A	7	7	1.	26	0.269
91	A	5	5	1.	26	0.192
92	A	5	5	1.	24	0.208
93	A	5	5	1.	26	0.192
94	A	5	5	1.	26	0.192
95	A	5	5	1.	26	0.192

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	3	3	1.	26	0.115
97	A	4	4	1.	26	0.154
98	A	5	4	1.	26	0.154
99	A	6	4	1.	26	0.154
100	A	4	3	1.	26	0.115
101	A	3	3	1.	26	0.115
102	A	2	2	1.	23	0.087
103	A	4	4	1.	26	0.154
104	A	4	4	1.	26	0.154
105	A	4	4	1.	26	0.154
106	A	8	7	1.	26	0.269
107	A	6	5	1.	26	0.192
108	A	6	5	1.	24	0.208
109	A	6	6	1.	26	0.231
110	A	6	6	1.	26	0.231
111	A	6	5	1.	26	0.192
112	A	6	6	1.	26	0.231
113	A	6	5	1.	26	0.192
114	A	3	3	1.	26	0.115
115	A	4	4	1.	26	0.154
116	A	5	4	1.	26	0.154
117	A	6	4	1.	26	0.154
118	A	7	4	1.	26	0.154
119	A	5	4	1.	26	0.154
120	A	4	4	1.	26	0.154
121	A	3	3	1.	23	0.13
122	A	2	2	1.	26	0.077
123	A	5	4	1.	26	0.154
124	A	5	4	1.	26	0.154
125	A	5	5	1.	26	0.192
126	A	5	4	1.	26	0.154
127	A	6	5	1.	26	0.192

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
128	A	7	5	1.	26	0.192
129	A	8	5	1.	26	0.192
130	A	7	6	1.	26	0.231
131	A	6	6	1.	26	0.231
132	A	4	4	1.	26	0.154
133	A	4	4	1.	24	0.167
134	A	4	4	1.	26	0.154
135	A	3	3	1.	26	0.115
136	A	4	4	1.	26	0.154
137	A	5	4	1.	26	0.154
138	A	6	4	1.	26	0.154
139	A	4	3	1.	26	0.115
140	A	3	3	1.	26	0.115
141	A	2	2	1.	26	0.077
142	A	3	3	1.	23	0.13
143	A	3	3	1.	26	0.115
144	A	4	4	1.	26	0.154
145	A	7	6	1.	26	0.231
146	A	6	6	1.	26	0.231
147	A	5	5	1.	26	0.192
148	A	4	4	1.	26	0.154
149	A	2	2	1.	24	0.083
150	A	3	3	1.	26	0.115
151	A	4	4	1.	26	0.154
152	A	5	4	1.	26	0.154
153	A	4	3	1.	26	0.115
154	A	3	3	1.	26	0.115
155	A	2	2	1.	26	0.077
156	A	3	3	1.	26	0.115
157	A	5	5	1.	23	0.217
158	A	5	5	1.	26	0.192
159	A	3	2	1.	24	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
160	A	3	2	1.	24	0.083
161	A	3	2	1.	24	0.083
162	A	3	2	1.	24	0.083
163	A	3	2	1.	24	0.083
164	A	3	2	1.	24	0.083
165	A	3	2	1.	24	0.083
166	A	3	2	1.	24	0.083
167	A	3	2	1.	26	0.077
168	A	3	2	1.	26	0.077
169	A	3	2	1.	26	0.077
170	A	3	2	1.	26	0.077
171	A	3	2	1.	26	0.077
172	A	3	2	1.	26	0.077
173	A	3	2	1.	26	0.077
174	A	3	2	1.	26	0.077
175	A	3	2	1.	26	0.077
176	A	3	2	1.	26	0.077
177	A	3	2	1.	26	0.077
178	A	3	2	1.	26	0.077
179	A	3	2	1.	26	0.077
180	A	3	2	1.	26	0.077
181	A	3	2	1.	26	0.077
182	A	3	2	1.	26	0.077
183	A	14	10	1.	26	0.385
184	A	14	10	1.	26	0.385
185	A	13	10	1.	26	0.385
186	A	13	10	1.	26	0.385
187	A	12	9	1.	26	0.346
188	A	12	9	1.	26	0.346
189	A	12	9	1.	26	0.346
190	A	12	9	1.	26	0.346
191	A	13	10	1.	26	0.385

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
192	A	13	10	1.	26	0.385
193	A	14	10	1.	26	0.385
194	A	14	10	1.	26	0.385
195	A	15	10	1.	26	0.385
196	A	14	10	1.	26	0.385
197	A	14	10	1.	26	0.385
198	A	13	10	1.	26	0.385
199	A	13	10	1.	26	0.385
200	A	12	9	1.	26	0.346
201	A	12	9	1.	26	0.346
202	A	13	10	1.	26	0.385
203	A	13	10	1.	26	0.385
204	A	14	10	1.	26	0.385
205	A	14	10	1.	26	0.385
206	A	15	10	1.	26	0.385
207	A	15	11	1.	26	0.423
208	A	14	11	1.	26	0.423
209	A	14	11	1.	26	0.423
210	A	13	10	1.	26	0.385
211	A	13	10	1.	26	0.385
212	A	13	10	1.	26	0.385
213	A	13	10	1.	26	0.385
214	A	14	11	1.	26	0.423
215	A	14	11	1.	26	0.423
216	A	15	11	1.	26	0.423
217	A	15	11	1.	26	0.423
218	A	16	11	1.	26	0.423
219	A	16	11	1.	26	0.423
220	A	7	6	1.	28	0.214
221	A	8	8	1.	28	0.286
222	A	6	6	1.	28	0.214
223	A	7	7	1.	28	0.25

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
224	A	5	5	1.	28	0.179
225	A	7	7	1.	28	0.25
226	A	5	5	1.	28	0.179
227	A	7	7	1.	28	0.25
228	A	5	5	1.	28	0.179
229	A	8	8	1.	28	0.286
230	A	6	6	1.	28	0.214
231	A	11	8	1.	28	0.286
232	A	9	6	1.	28	0.214
233	A	10	8	1.	28	0.286
234	A	8	6	1.	28	0.214
235	A	9	8	1.	28	0.286
236	A	7	6	1.	28	0.214
237	A	8	7	1.	28	0.25
238	A	6	5	1.	28	0.179
239	A	8	7	1.	28	0.25
240	A	6	5	1.	28	0.179
241	A	8	8	1.	28	0.286
242	A	6	6	1.	28	0.214
243	A	8	7	1.	28	0.25
244	A	7	5	1.	28	0.179
245	A	8	7	1.	28	0.25
246	A	6	5	1.	28	0.179
247	A	7	7	1.	28	0.25
248	A	5	5	1.	28	0.179
249	A	6	6	1.	28	0.214
250	A	4	4	1.	28	0.143
251	A	6	6	1.	28	0.214
252	A	4	4	1.	28	0.143
253	A	7	7	1.	28	0.25
254	A	5	5	1.	28	0.179
255	A	8	7	1.	28	0.25

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
256	A	6	5	1.	28	0.179
257	A	7	5	1.	28	0.179
258	A	8	7	1.	28	0.25
259	A	6	5	1.	28	0.179
260	A	7	7	1.	28	0.25
261	A	5	5	1.	28	0.179
262	A	6	6	1.	28	0.214
263	A	4	4	1.	28	0.143
264	A	7	7	1.	28	0.25
265	A	5	5	1.	28	0.179
266	A	8	8	1.	28	0.286
267	A	6	6	1.	28	0.214
268	A	9	8	1.	28	0.286
269	A	3	2	1.	24	0.083
270	A	3	2	1.	24	0.083
271	A	3	2	1.	22	0.091
272	A	3	3	1.	24	0.125
273	A	3	3	0.94	24	0.125
274	A	4	4	0.9	24	0.167
275	A	2	2	1.	32	0.062
276	A	4	3	1.	30	0.1
277	A	4	3	1.	34	0.088
278	A	4	3	1.	19	0.158
279	A	4	3	1.	15	0.2
280	A	4	3	1.	19	0.158
281	A	3	2	1.	17	0.118
282	A	4	4	1.	25	0.16
283	A	2	2	1.	39	0.051
284	A	4	4	1.	20	0.2
285	A	7	7	1.	20	0.35
286	A	6	6	1.	18	0.333
287	A	5	5	1.	17	0.294

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
288	A	3	3	1.	20	0.15
289	A	4	4	1.	20	0.2
290	A	5	4	1.	20	0.2
291	A	5	4	1.	20	0.2
292	A	4	4	1.	20	0.2
293	A	7	7	1.	20	0.35
294	A	6	6	1.	18	0.333
295	A	3	3	1.	17	0.176
296	A	4	4	1.	20	0.2
297	A	5	5	1.	20	0.25
298	A	5	5	1.	20	0.25

Chapter 3

Listing of integrals

3.1 $\int x^2 \left(A+Bx^2\right) \left(bx^2+cx^4\right) dx$

Optimal. Leaf size=33

$$\frac{1}{7}x^7(Ac+bB)+\frac{1}{5}Abx^5+\frac{1}{9}Bcx^9$$

```
[Out] (A*b*x^5)/5 + ((b*B + A*c)*x^7)/7 + (B*c*x^9)/9
```

Rubi [A] time = 0.027026, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1584, 448}

$$\frac{1}{7}x^7(Ac+bB)+\frac{1}{5}Abx^5+\frac{1}{9}Bcx^9$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(A + B*x^2)*(b*x^2 + c*x^4),x]
```

```
[Out] (A*b*x^5)/5 + ((b*B + A*c)*x^7)/7 + (B*c*x^9)/9
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\left(A+Bx^2\right)\left(bx^2+cx^4\right)dx&=\int x^4\left(A+Bx^2\right)\left(b+cx^2\right)dx\\&=\int\left(Abx^4+(bB+Ac)x^6+Bcx^8\right)dx\\&=\frac{1}{5}Abx^5+\frac{1}{7}(bB+Ac)x^7+\frac{1}{9}Bcx^9\end{aligned}$$

Mathematica [A] time = 0.0061206, size = 33, normalized size = 1.

$$\frac{1}{7}x^7(Ac+bB)+\frac{1}{5}Abx^5+\frac{1}{9}Bcx^9$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(A + B*x^2)*(b*x^2 + c*x^4),x]
```

```
[Out] (A*b*x^5)/5 + ((b*B + A*c)*x^7)/7 + (B*c*x^9)/9
```

Maple [A] time = 0.043, size = 28, normalized size = 0.9

$$\frac{Abx^5}{5}+\frac{(Ac+Bb)\,x^7}{7}+\frac{Bcx^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)*(c*x^4+b*x^2),x)
```

```
[Out] 1/5*A*b*x^5+1/7*(A*c+B*b)*x^7+1/9*B*c*x^9
```

Maxima [A] time = 1.12523, size = 36, normalized size = 1.09

$$\frac{1}{9}\,Bcx^9 + \frac{1}{7}\,(Bb + Ac)x^7 + \frac{1}{5}\,Abx^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/9*B*c*x^9 + 1/7*(B*b + A*c)*x^7 + 1/5*A*b*x^5
```

Fricas [A] time = 0.396868, size = 74, normalized size = 2.24

$$\frac{1}{9}x^9cB + \frac{1}{7}x^7bB + \frac{1}{7}x^7cA + \frac{1}{5}x^5bA$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/9*x^9*c*B + 1/7*x^7*b*B + 1/7*x^7*c*A + 1/5*x^5*b*A
```

Sympy [A] time = 0.060546, size = 29, normalized size = 0.88

$$\frac{Abx^5}{5} + \frac{Bcx^9}{9} + x^7\left(\frac{Ac}{7} + \frac{Bb}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)*(c*x**4+b*x**2),x)
```

```
[Out] A*b*x**5/5 + B*c*x**9/9 + x**7*(A*c/7 + B*b/7)
```

Giac [A] time = 1.27726, size = 39, normalized size = 1.18

$$\frac{1}{9}\,Bcx^9 + \frac{1}{7}\,Bbx^7 + \frac{1}{7}\,Acx^7 + \frac{1}{5}\,Abx^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/9*B*c*x^9 + 1/7*B*b*x^7 + 1/7*A*c*x^7 + 1/5*A*b*x^5
```

3.2 $\int x\left(A+Bx^2\right)\left(bx^2+cx^4\right)\,dx$

Optimal. Leaf size=33

$$\frac{1}{6}x^6(Ac+bB)+\frac{1}{4}Abx^4+\frac{1}{8}Bcx^8$$

```
[Out] (A*b*x^4)/4 + ((b*B + A*c)*x^6)/6 + (B*c*x^8)/8
```

Rubi [A] time = 0.0405135, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {1584, 446, 76}

$$\frac{1}{6}x^6(Ac+bB)+\frac{1}{4}Abx^4+\frac{1}{8}Bcx^8$$

Antiderivative was successfully verified.

```
[In] Int[x*(A + B*x^2)*(b*x^2 + c*x^4),x]
```

```
[Out] (A*b*x^4)/4 + ((b*B + A*c)*x^6)/6 + (B*c*x^8)/8
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
```

```
qQ[p, 1])
```

Rubi steps

$$
\begin{aligned}
\int x\left(A+Bx^2\right)\left(bx^2+cx^4\right)\,dx &= \int x^3\left(A+Bx^2\right)\left(b+cx^2\right)\,dx\\
&= \frac{1}{2}\operatorname{Subst}\left(\int x(A+Bx)(b+cx)\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(Abx+(bB+Ac)x^2+Bcx^3\right)\,dx,x,x^2\right)\\
&= \frac{1}{4}Abx^4+\frac{1}{6}(bB+Ac)x^6+\frac{1}{8}Bcx^8
\end{aligned}
$$

Mathematica [A] time = 0.0073959, size = 33, normalized size = 1.

$$\frac{1}{6}x^6(Ac+bB)+\frac{1}{4}Abx^4+\frac{1}{8}Bcx^8$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(A + B*x^2)*(b*x^2 + c*x^4),x]
```

```
[Out] (A*b*x^4)/4 + ((b*B + A*c)*x^6)/6 + (B*c*x^8)/8
```

Maple [A] time = 0., size = 28, normalized size = 0.9

$$\frac{Abx^4}{4}+\frac{(Ac+Bb)\,x^6}{6}+\frac{Bcx^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)*(c*x^4+b*x^2),x)
```

```
[Out] 1/4*A*b*x^4+1/6*(A*c+B*b)*x^6+1/8*B*c*x^8
```

Maxima [A] time = 1.03862, size = 36, normalized size = 1.09

$$\frac{1}{8}\,Bcx^8+\frac{1}{6}\,(Bb+Ac)x^6+\frac{1}{4}\,Abx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/8*B*c*x^8 + 1/6*(B*b + A*c)*x^6 + 1/4*A*b*x^4
```

Fricas [A] time = 0.415189, size = 74, normalized size = 2.24

$$\frac{1}{8}x^8cB + \frac{1}{6}x^6bB + \frac{1}{6}x^6cA + \frac{1}{4}x^4bA$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/8*x^8*c*B + 1/6*x^6*b*B + 1/6*x^6*c*A + 1/4*x^4*b*A
```

Sympy [A] time = 0.058968, size = 29, normalized size = 0.88

$$\frac{Abx^4}{4} + \frac{Bcx^8}{8} + x^6\left(\frac{Ac}{6} + \frac{Bb}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)*(c*x**4+b*x**2),x)
```

```
[Out] A*b*x**4/4 + B*c*x**8/8 + x**6*(A*c/6 + B*b/6)
```

Giac [A] time = 1.22563, size = 39, normalized size = 1.18

$$\frac{1}{8}Bcx^8 + \frac{1}{6}Bbx^6 + \frac{1}{6}Acx^6 + \frac{1}{4}Abx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/8*B*c*x^8 + 1/6*B*b*x^6 + 1/6*A*c*x^6 + 1/4*A*b*x^4
```

3.3 $\int \left(A+Bx^2\right)\left(bx^2+cx^4\right)\,dx$

Optimal. Leaf size=33

$$\frac{1}{5}x^5(Ac+bB)+\frac{1}{3}Abx^3+\frac{1}{7}Bcx^7$$

```
[Out] (A*b*x^3)/3 + ((b*B + A*c)*x^5)/5 + (B*c*x^7)/7
```

Rubi [A] time = 0.0242327, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1593, 448}

$$\frac{1}{5}x^5(Ac+bB)+\frac{1}{3}Abx^3+\frac{1}{7}Bcx^7$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)*(b*x^2 + c*x^4),x]
```

```
[Out] (A*b*x^3)/3 + ((b*B + A*c)*x^5)/5 + (B*c*x^7)/7
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(A+Bx^2\right)\left(bx^2+cx^4\right)\,dx &= \int x^2\left(A+Bx^2\right)\left(b+cx^2\right)\,dx\\ &= \int \left(Abx^2+(bB+Ac)x^4+Bcx^6\right)\,dx\\ &= \frac{1}{3}Abx^3+\frac{1}{5}(bB+Ac)x^5+\frac{1}{7}Bcx^7\end{aligned}$$

Mathematica [A] time = 0.0053023, size = 33, normalized size = 1.

$$\frac{1}{5}x^5(Ac+bB)+\frac{1}{3}Abx^3+\frac{1}{7}Bcx^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)*(b*x^2 + c*x^4),x]
```

```
[Out] (A*b*x^3)/3 + ((b*B + A*c)*x^5)/5 + (B*c*x^7)/7
```

Maple [A] time = 0., size = 28, normalized size = 0.9

$$\frac{Abx^3}{3}+\frac{(Ac+Bb)\,x^5}{5}+\frac{Bcx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2),x)
```

```
[Out] 1/3*A*b*x^3+1/5*(A*c+B*b)*x^5+1/7*B*c*x^7
```

Maxima [A] time = 1.13554, size = 36, normalized size = 1.09

$$\frac{1}{7}\,Bcx^7+\frac{1}{5}\,(Bb+Ac)x^5+\frac{1}{3}\,Abx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/7*B*c*x^7 + 1/5*(B*b + A*c)*x^5 + 1/3*A*b*x^3
```

Fricas [A] time = 0.395012, size = 74, normalized size = 2.24

$$\frac{1}{7}x^7cB + \frac{1}{5}x^5bB + \frac{1}{5}x^5cA + \frac{1}{3}x^3bA$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*c*B + 1/5*x^5*b*B + 1/5*x^5*c*A + 1/3*x^3*b*A
```

Sympy [A] time = 0.057483, size = 29, normalized size = 0.88

$$\frac{Abx^3}{3} + \frac{Bcx^7}{7} + x^5\left(\frac{Ac}{5} + \frac{Bb}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2),x)
```

```
[Out] A*b*x**3/3 + B*c*x**7/7 + x**5*(A*c/5 + B*b/5)
```

Giac [A] time = 1.15677, size = 39, normalized size = 1.18

$$\frac{1}{7}Bcx^7 + \frac{1}{5}Bbx^5 + \frac{1}{5}Acx^5 + \frac{1}{3}Abx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/7*B*c*x^7 + 1/5*B*b*x^5 + 1/5*A*c*x^5 + 1/3*A*b*x^3
```

3.4 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{x} \, dx$

Optimal. Leaf size=33

$$\frac{1}{4}x^4(Ac+bB)+\frac{1}{2}Abx^2+\frac{1}{6}Bcx^6$$

```
[Out] (A*b*x^2)/2 + ((b*B + A*c)*x^4)/4 + (B*c*x^6)/6
```

Rubi [A] time = 0.0375626, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1584, 444, 43}

$$\frac{1}{4}x^4(Ac+bB)+\frac{1}{2}Abx^2+\frac{1}{6}Bcx^6$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4))/x,x]
```

```
[Out] (A*b*x^2)/2 + ((b*B + A*c)*x^4)/4 + (B*c*x^6)/6
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)}{x}\,dx &= \int x\left(A+Bx^2\right)\left(b+cx^2\right)\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int (A+Bx)(b+cx)\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(Ab+(bB+Ac)x+Bcx^2\right)\,dx,x,x^2\right)\\ &= \frac{1}{2}Abx^2+\frac{1}{4}(bB+Ac)x^4+\frac{1}{6}Bcx^6\end{aligned}$$

Mathematica [A] time = 0.0074786, size = 33, normalized size = 1.

$$\frac{1}{4}x^4(Ac+bB)+\frac{1}{2}Abx^2+\frac{1}{6}Bcx^6$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/x,x]
```

```
[Out] (A*b*x^2)/2 + ((b*B + A*c)*x^4)/4 + (B*c*x^6)/6
```

Maple [A] time = 0., size = 28, normalized size = 0.9

$$\frac{Abx^2}{2}+\frac{(Ac+Bb)\,x^4}{4}+\frac{Bcx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)/x,x)
```

```
[Out] 1/2*A*b*x^2+1/4*(A*c+B*b)*x^4+1/6*B*c*x^6
```

Maxima [A] time = 1.13298, size = 36, normalized size = 1.09

$$\frac{1}{6}\,Bcx^6+\frac{1}{4}\,(Bb+Ac)x^4+\frac{1}{2}\,Abx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x,x, algorithm="maxima")
```

```
[Out] 1/6*B*c*x^6 + 1/4*(B*b + A*c)*x^4 + 1/2*A*b*x^2
```

Fricas [A] time = 0.475083, size = 66, normalized size = 2.

$$\frac{1}{6}\,Bcx^6 + \frac{1}{4}\,(Bb + Ac)x^4 + \frac{1}{2}\,Abx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x,x, algorithm="fricas")
```

```
[Out] 1/6*B*c*x^6 + 1/4*(B*b + A*c)*x^4 + 1/2*A*b*x^2
```

Sympy [A] time = 0.058017, size = 29, normalized size = 0.88

$$\frac{Abx^2}{2} + \frac{Bcx^6}{6} + x^4\left(\frac{Ac}{4} + \frac{Bb}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)/x,x)
```

```
[Out] A*b*x**2/2 + B*c*x**6/6 + x**4*(A*c/4 + B*b/4)
```

Giac [A] time = 1.16247, size = 39, normalized size = 1.18

$$\frac{1}{6}\,Bcx^6 + \frac{1}{4}\,Bbx^4 + \frac{1}{4}\,Acx^4 + \frac{1}{2}\,Abx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x,x, algorithm="giac")
```

```
[Out] 1/6*B*c*x^6 + 1/4*B*b*x^4 + 1/4*A*c*x^4 + 1/2*A*b*x^2
```

3.5 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^2}\,dx$

Optimal. Leaf size=28

$$\frac{1}{3}x^3(Ac+bB)+Abx+\frac{1}{5}Bcx^5$$

```
[Out] A*b*x + ((b*B + A*c)*x^3)/3 + (B*c*x^5)/5
```

Rubi [A] time = 0.018882, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1584, 373}

$$\frac{1}{3}x^3(Ac+bB)+Abx+\frac{1}{5}Bcx^5$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4))/x^2,x]
```

```
[Out] A*b*x + ((b*B + A*c)*x^3)/3 + (B*c*x^5)/5
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^2}\,dx = \int (A+Bx^2)(b+cx^2)\,dx$$
$$= \int (Ab+(bB+Ac)x^2+Bcx^4)\,dx$$
$$= Abx+\frac{1}{3}(bB+Ac)x^3+\frac{1}{5}Bcx^5$$

Mathematica [A] time = 0.0051632, size = 28, normalized size = 1.

$$\frac{1}{3}x^3(Ac+bB)+Abx+\frac{1}{5}Bcx^5$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/x^2,x]
```

```
[Out] A*b*x + ((b*B + A*c)*x^3)/3 + (B*c*x^5)/5
```

Maple [A] time = 0.001, size = 25, normalized size = 0.9

$$Abx+\frac{(Ac+Bb)\,x^3}{3}+\frac{Bcx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)/x^2,x)
```

```
[Out] A*b*x+1/3*(A*c+B*b)*x^3+1/5*B*c*x^5
```

Maxima [A] time = 1.11203, size = 32, normalized size = 1.14

$$\frac{1}{5}\,Bcx^5+\frac{1}{3}\,(Bb+Ac)x^3+Abx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/5*B*c*x^5 + 1/3*(B*b + A*c)*x^3 + A*b*x
```

Fricas [A] time = 0.476049, size = 58, normalized size = 2.07

$$\frac{1}{5}\,Bcx^5 + \frac{1}{3}\,(Bb + Ac)x^3 + Abx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/5*B*c*x^5 + 1/3*(B*b + A*c)*x^3 + A*b*x
```

Sympy [A] time = 0.05726, size = 26, normalized size = 0.93

$$Abx + \frac{Bcx^5}{5} + x^3\left(\frac{Ac}{3} + \frac{Bb}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)/x**2,x)
```

```
[Out] A*b*x + B*c*x**5/5 + x**3*(A*c/3 + B*b/3)
```

Giac [A] time = 1.12314, size = 35, normalized size = 1.25

$$\frac{1}{5}\,Bcx^5 + \frac{1}{3}\,Bbx^3 + \frac{1}{3}\,Acx^3 + Abx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^2,x, algorithm="giac")
```

```
[Out] 1/5*B*c*x^5 + 1/3*B*b*x^3 + 1/3*A*c*x^3 + A*b*x
```

3.6 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^3}\,dx$

Optimal. Leaf size=29

$$\frac{1}{2}x^2(Ac+bB)+Ab\log(x)+\frac{1}{4}Bcx^4$$

```
[Out] ((b*B + A*c)*x^2)/2 + (B*c*x^4)/4 + A*b*Log[x]
```

Rubi [A] time = 0.027211, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1584, 446, 76}

$$\frac{1}{2}x^2(Ac+bB)+Ab\log(x)+\frac{1}{4}Bcx^4$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4))/x^3,x]
```

```
[Out] ((b*B + A*c)*x^2)/2 + (B*c*x^4)/4 + A*b*Log[x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
```

```
qQ[p, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)}{x^3}\,dx &= \int \frac{\left(A+Bx^2\right)\left(b+cx^2\right)}{x}\,dx\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{(A+Bx)(b+cx)}{x}\,dx, x, x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(bB+Ac+\frac{Ab}{x}+Bcx\right)dx, x, x^2\right)\\
&= \frac{1}{2}(bB+Ac)x^2+\frac{1}{4}Bcx^4+Ab\log(x)
\end{aligned}
$$

Mathematica [A] time = 0.0086492, size = 29, normalized size = 1.

$$\frac{1}{2}x^2(Ac+bB)+Ab\log(x)+\frac{1}{4}Bcx^4$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/x^3,x]
```

```
[Out] ((b*B + A*c)*x^2)/2 + (B*c*x^4)/4 + A*b*Log[x]
```

Maple [A] time = 0.001, size = 28, normalized size = 1.

$$\frac{Bcx^4}{4}+\frac{Ax^2c}{2}+\frac{Bx^2b}{2}+Ab\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)/x^3,x)
```

```
[Out] 1/4*B*c*x^4+1/2*A*x^2*c+1/2*B*x^2*b+A*b*ln(x)
```

Maxima [A] time = 1.32364, size = 38, normalized size = 1.31

$$\frac{1}{4}Bcx^4+\frac{1}{2}(Bb+Ac)x^2+\frac{1}{2}Ab\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*B*c*x^4 + 1/2*(B*b + A*c)*x^2 + 1/2*A*b*log(x^2)
```

Fricas [A] time = 0.495015, size = 65, normalized size = 2.24

$$\frac{1}{4}Bcx^4 + \frac{1}{2}(Bb + Ac)x^2 + Ab\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*B*c*x^4 + 1/2*(B*b + A*c)*x^2 + A*b*log(x)
```

Sympy [A] time = 0.255003, size = 27, normalized size = 0.93

$$Ab\log(x) + \frac{Bcx^4}{4} + x^2\left(\frac{Ac}{2} + \frac{Bb}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)/x**3,x)
```

```
[Out] A*b*log(x) + B*c*x**4/4 + x**2*(A*c/2 + B*b/2)
```

Giac [A] time = 1.26228, size = 41, normalized size = 1.41

$$\frac{1}{4}Bcx^4 + \frac{1}{2}Bbx^2 + \frac{1}{2}Acx^2 + \frac{1}{2}Ab\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*B*c*x^4 + 1/2*B*b*x^2 + 1/2*A*c*x^2 + 1/2*A*b*log(x^2)
```

3.7 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^4} \, dx$

Optimal. Leaf size=26

$$x(Ac+bB)-\frac{Ab}{x}+\frac{1}{3}Bcx^3$$

```
[Out] -((A*b)/x) + (b*B + A*c)*x + (B*c*x^3)/3
```

Rubi [A] time = 0.0234336, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1584, 448}

$$x(Ac+bB)-\frac{Ab}{x}+\frac{1}{3}Bcx^3$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4))/x^4,x]
```

```
[Out] -((A*b)/x) + (b*B + A*c)*x + (B*c*x^3)/3
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^4}\,dx = \int \frac{(A+Bx^2)(b+cx^2)}{x^2}\,dx$$
$$= \int \left(bB\left(1+\frac{Ac}{bB}\right)+\frac{Ab}{x^2}+Bcx^2\right)dx$$
$$= -\frac{Ab}{x}+(bB+Ac)x+\frac{1}{3}Bcx^3$$

Mathematica [A] time = 0.0090501, size = 26, normalized size = 1.

$$x(Ac+bB)-\frac{Ab}{x}+\frac{1}{3}Bcx^3$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/x^4,x]
```

```
[Out] -((A*b)/x) + (b*B + A*c)*x + (B*c*x^3)/3
```

Maple [A] time = 0.003, size = 24, normalized size = 0.9

$$\frac{Bcx^3}{3}+Acx+Bbx-\frac{Ab}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)/x^4,x)
```

```
[Out] 1/3*B*c*x^3+A*c*x+B*b*x-A*b/x
```

Maxima [A] time = 1.41248, size = 32, normalized size = 1.23

$$\frac{1}{3}Bcx^3+(Bb+Ac)x-\frac{Ab}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*B*c*x^3 + (B*b + A*c)*x - A*b/x
```

Fricas [A] time = 0.477661, size = 61, normalized size = 2.35

$$\frac{Bcx^4 + 3\,(Bb + Ac)x^2 - 3\,Ab}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(B*c*x^4 + 3*(B*b + A*c)*x^2 - 3*A*b)/x
```

Sympy [A] time = 0.252091, size = 20, normalized size = 0.77

$$-\frac{Ab}{x} + \frac{Bcx^3}{3} + x\,(Ac + Bb)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)/x**4,x)
```

```
[Out] -A*b/x + B*c*x**3/3 + x*(A*c + B*b)
```

Giac [A] time = 1.20154, size = 31, normalized size = 1.19

$$\frac{1}{3}\,Bcx^3 + Bbx + Acx - \frac{Ab}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^4,x, algorithm="giac")
```

```
[Out] 1/3*B*c*x^3 + B*b*x + A*c*x - A*b/x
```

3.8 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^5} dx$

Optimal. Leaf size=29

$$\log(x)(Ac+bB)-\frac{Ab}{2x^2}+\frac{1}{2}Bcx^2$$

```
[Out] -(A*b)/(2*x^2) + (B*c*x^2)/2 + (b*B + A*c)*Log[x]
```

Rubi [A] time = 0.0275993, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1584, 446, 76}

$$\log(x)(Ac+bB)-\frac{Ab}{2x^2}+\frac{1}{2}Bcx^2$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4))/x^5,x]
```

```
[Out] -(A*b)/(2*x^2) + (B*c*x^2)/2 + (b*B + A*c)*Log[x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
```

```
qQ[p, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)}{x^5}\,dx &= \int \frac{\left(A+Bx^2\right)\left(b+cx^2\right)}{x^3}\,dx\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{(A+Bx)(b+cx)}{x^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(Bc+\frac{Ab}{x^2}+\frac{bB+Ac}{x}\right)dx,x,x^2\right)\\
&= -\frac{Ab}{2x^2}+\frac{1}{2}Bcx^2+(bB+Ac)\log(x)
\end{aligned}
$$

Mathematica [A] time = 0.0108132, size = 29, normalized size = 1.

$$\log(x)(Ac+bB)-\frac{Ab}{2x^2}+\frac{1}{2}Bcx^2$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/x^5,x]
```

```
[Out] -(A*b)/(2*x^2) + (B*c*x^2)/2 + (b*B + A*c)*Log[x]
```

Maple [A] time = 0.005, size = 26, normalized size = 0.9

$$\frac{Bcx^2}{2}+A\ln(x)\,c+B\ln(x)\,b-\frac{Ab}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)/x^5,x)
```

```
[Out] 1/2*B*c*x^2+A*ln(x)*c+B*ln(x)*b-1/2*A*b/x^2
```

Maxima [A] time = 1.16114, size = 38, normalized size = 1.31

$$\frac{1}{2}Bcx^2 + \frac{1}{2}(Bb + Ac)\log\left(x^2\right) - \frac{Ab}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*B*c*x^2 + 1/2*(B*b + A*c)*log(x^2) - 1/2*A*b/x^2
```

Fricas [A] time = 0.52777, size = 70, normalized size = 2.41

$$\frac{Bcx^4 + 2(Bb + Ac)x^2\log(x) - Ab}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/2*(B*c*x^4 + 2*(B*b + A*c)*x^2*log(x) - A*b)/x^2
```

Sympy [A] time = 0.326274, size = 26, normalized size = 0.9

$$-\frac{Ab}{2x^2} + \frac{Bcx^2}{2} + (Ac + Bb)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)/x**5,x)
```

```
[Out] -A*b/(2*x**2) + B*c*x**2/2 + (A*c + B*b)*log(x)
```

Giac [A] time = 1.17334, size = 57, normalized size = 1.97

$$\frac{1}{2}Bcx^2 + \frac{1}{2}(Bb + Ac)\log\left(x^2\right) - \frac{Bbx^2 + Acx^2 + Ab}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^5,x, algorithm="giac")
```

```
[Out] 1/2*B*c*x^2 + 1/2*(B*b + A*c)*log(x^2) - 1/2*(B*b*x^2 + A*c*x^2 + A*b)/x^2
```

3.9 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^6} dx$

Optimal. Leaf size=26

$$-\frac{Ac+bB}{x}-\frac{Ab}{3x^3}+Bcx$$

```
[Out] -(A*b)/(3*x^3) - (b*B + A*c)/x + B*c*x
```

Rubi [A] time = 0.0220519, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1584, 448}

$$-\frac{Ac+bB}{x}-\frac{Ab}{3x^3}+Bcx$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4))/x^6,x]
```

```
[Out] -(A*b)/(3*x^3) - (b*B + A*c)/x + B*c*x
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^6}\,dx = \int \frac{(A+Bx^2)(b+cx^2)}{x^4}\,dx$$
$$= \int \left(Bc + \frac{Ab}{x^4} + \frac{bB+Ac}{x^2}\right) dx$$
$$= -\frac{Ab}{3x^3} - \frac{bB+Ac}{x} + Bcx$$

Mathematica [A] time = 0.0111177, size = 27, normalized size = 1.04

$$\frac{-Ac-bB}{x} - \frac{Ab}{3x^3} + Bcx$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/x^6,x]
```

```
[Out] -(A*b)/(3*x^3) + (-(b*B) - A*c)/x + B*c*x
```

Maple [A] time = 0.005, size = 25, normalized size = 1.

$$Bcx - \frac{Ab}{3\,x^3} - \frac{Ac+Bb}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)/x^6,x)
```

```
[Out] B*c*x-1/3*A*b/x^3-(A*c+B*b)/x
```

Maxima [A] time = 1.12318, size = 35, normalized size = 1.35

$$Bcx - \frac{3\,(Bb+Ac)x^2 + Ab}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^6,x, algorithm="maxima")
```

```
[Out] B*c*x - 1/3*(3*(B*b + A*c)*x^2 + A*b)/x^3
```

Fricas [A] time = 0.448699, size = 63, normalized size = 2.42

$$\frac{3\,Bcx^4 - 3\,(Bb + Ac)x^2 - Ab}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^6,x, algorithm="fricas")
```

```
[Out] 1/3*(3*B*c*x^4 - 3*(B*b + A*c)*x^2 - A*b)/x^3
```

Sympy [A] time = 0.341664, size = 26, normalized size = 1.

$$Bcx - \frac{Ab + x^2\,(3Ac + 3Bb)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)/x**6,x)
```

```
[Out] B*c*x - (A*b + x**2*(3*A*c + 3*B*b))/(3*x**3)
```

Giac [A] time = 1.20146, size = 38, normalized size = 1.46

$$Bcx - \frac{3\,Bbx^2 + 3\,Acx^2 + Ab}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^6,x, algorithm="giac")
```

```
[Out] B*c*x - 1/3*(3*B*b*x^2 + 3*A*c*x^2 + A*b)/x^3
```

3.10 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^7}\,dx$

Optimal. Leaf size=29

$$-\frac{Ac+bB}{2x^2}-\frac{Ab}{4x^4}+Bc\log(x)$$

```
[Out] -(A*b)/(4*x^4) - (b*B + A*c)/(2*x^2) + B*c*Log[x]
```

Rubi [A] time = 0.0269632, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1584, 446, 76}

$$-\frac{Ac+bB}{2x^2}-\frac{Ab}{4x^4}+Bc\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4))/x^7,x]
```

```
[Out] -(A*b)/(4*x^4) - (b*B + A*c)/(2*x^2) + B*c*Log[x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
```

```
qQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)}{x^7}\,dx &= \int \frac{\left(A+Bx^2\right)\left(b+cx^2\right)}{x^5}\,dx\\ &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(A+Bx)(b+cx)}{x^3}\,dx,x,x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{Ab}{x^3}+\frac{bB+Ac}{x^2}+\frac{Bc}{x}\right)dx,x,x^2\right)\\ &= -\frac{Ab}{4x^4}-\frac{bB+Ac}{2x^2}+Bc\log(x)\end{aligned}$$

Mathematica [A] time = 0.0168233, size = 31, normalized size = 1.07

$$\frac{-Ac-bB}{2x^2}-\frac{Ab}{4x^4}+Bc\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/x^7,x]
```

```
[Out] -(A*b)/(4*x^4) + (-(b*B) - A*c)/(2*x^2) + B*c*Log[x]
```

Maple [A] time = 0.004, size = 28, normalized size = 1.

$$Bc\ln(x)-\frac{Ab}{4\,x^4}-\frac{Ac}{2\,x^2}-\frac{Bb}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)/x^7,x)
```

```
[Out] B*c*ln(x)-1/4*A*b/x^4-1/2/x^2*A*c-1/2/x^2*B*b
```

Maxima [A] time = 1.09194, size = 41, normalized size = 1.41

$$\frac{1}{2}\,Bc\log\left(x^2\right) - \frac{2\,(Bb + Ac)x^2 + Ab}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^7,x, algorithm="maxima")
```

```
[Out] 1/2*B*c*log(x^2) - 1/4*(2*(B*b + A*c)*x^2 + A*b)/x^4
```

Fricas [A] time = 0.495456, size = 73, normalized size = 2.52

$$\frac{4\,Bcx^4\log\left(x\right) - 2\,(Bb + Ac)x^2 - Ab}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/4*(4*B*c*x^4*log(x) - 2*(B*b + A*c)*x^2 - A*b)/x^4
```

Sympy [A] time = 0.496881, size = 27, normalized size = 0.93

$$Bc\log\left(x\right) - \frac{Ab + x^2\,(2Ac + 2Bb)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)/x**7,x)
```

```
[Out] B*c*log(x) - (A*b + x**2*(2*A*c + 2*B*b))/(4*x**4)
```

Giac [A] time = 1.21355, size = 53, normalized size = 1.83

$$\frac{1}{2}\,Bc\log\left(x^2\right) - \frac{3\,Bcx^4 + 2\,Bbx^2 + 2\,Acx^2 + Ab}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^7,x, algorithm="giac")
```

```
[Out] 1/2*B*c*log(x^2) - 1/4*(3*B*c*x^4 + 2*B*b*x^2 + 2*A*c*x^2 + A*b)/x^4
```

3.11 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^8}\,dx$

Optimal. Leaf size=31

$$-\frac{Ac+bB}{3x^3}-\frac{Ab}{5x^5}-\frac{Bc}{x}$$

```
[Out] -(A*b)/(5*x^5) - (b*B + A*c)/(3*x^3) - (B*c)/x
```

Rubi [A] time = 0.0218235, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1584, 448}

$$-\frac{Ac+bB}{3x^3}-\frac{Ab}{5x^5}-\frac{Bc}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4))/x^8,x]
```

```
[Out] -(A*b)/(5*x^5) - (b*B + A*c)/(3*x^3) - (B*c)/x
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^8}\,dx = \int \frac{(A+Bx^2)(b+cx^2)}{x^6}\,dx$$
$$= \int \left(\frac{Ab}{x^6} + \frac{bB+Ac}{x^4} + \frac{Bc}{x^2}\right) dx$$
$$= -\frac{Ab}{5x^5} - \frac{bB+Ac}{3x^3} - \frac{Bc}{x}$$

Mathematica [A] time = 0.0108171, size = 33, normalized size = 1.06

$$\frac{-Ac-bB}{3x^3} - \frac{Ab}{5x^5} - \frac{Bc}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/x^8,x]
```

```
[Out] -(A*b)/(5*x^5) + (-(b*B) - A*c)/(3*x^3) - (B*c)/x
```

Maple [A] time = 0.004, size = 28, normalized size = 0.9

$$-\frac{Ac+Bb}{3\,x^3} - \frac{Ab}{5\,x^5} - \frac{Bc}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)/x^8,x)
```

```
[Out] -1/3*(A*c+B*b)/x^3-1/5*A*b/x^5-B*c/x
```

Maxima [A] time = 1.12884, size = 39, normalized size = 1.26

$$-\frac{15\,Bcx^4 + 5\,(Bb+Ac)x^2 + 3\,Ab}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^8,x, algorithm="maxima")
```

```
[Out] -1/15*(15*B*c*x^4 + 5*(B*b + A*c)*x^2 + 3*A*b)/x^5
```

Fricas [A] time = 0.491082, size = 70, normalized size = 2.26

$$-\frac{15\,Bcx^4+5\,(Bb+Ac)x^2+3\,Ab}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^8,x, algorithm="fricas")
```

```
[Out] -1/15*(15*B*c*x^4 + 5*(B*b + A*c)*x^2 + 3*A*b)/x^5
```

Sympy [A] time = 0.504007, size = 32, normalized size = 1.03

$$-\frac{3Ab+15Bcx^4+x^2\,(5Ac+5Bb)}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)/x**8,x)
```

```
[Out] -(3*A*b + 15*B*c*x**4 + x**2*(5*A*c + 5*B*b))/(15*x**5)
```

Giac [A] time = 1.14603, size = 42, normalized size = 1.35

$$-\frac{15\,Bcx^4+5\,Bbx^2+5\,Acx^2+3\,Ab}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^8,x, algorithm="giac")
```

```
[Out] -1/15*(15*B*c*x^4 + 5*B*b*x^2 + 5*A*c*x^2 + 3*A*b)/x^5
```

3.12 $\int \left(A+Bx^2\right)\left(bx^2+cx^4\right)^2\,dx$

Optimal. Leaf size=55

$$\frac{1}{5}Ab^2x^5+\frac{1}{9}cx^9(Ac+2bB)+\frac{1}{7}bx^7(2Ac+bB)+\frac{1}{11}Bc^2x^{11}$$

```
[Out] (A*b^2*x^5)/5 + (b*(b*B + 2*A*c)*x^7)/7 + (c*(2*b*B + A*c)*x^9)/9 + (B*c^2*
x^11)/11
```

Rubi [A] time = 0.0475207, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1593, 448}

$$\frac{1}{5}Ab^2x^5+\frac{1}{9}cx^9(Ac+2bB)+\frac{1}{7}bx^7(2Ac+bB)+\frac{1}{11}Bc^2x^{11}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (A*b^2*x^5)/5 + (b*(b*B + 2*A*c)*x^7)/7 + (c*(2*b*B + A*c)*x^9)/9 + (B*c^2*
x^11)/11
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \left(A+Bx^2\right)\left(bx^2+cx^4\right)^2\,dx=\int x^4\left(A+Bx^2\right)\left(b+cx^2\right)^2\,dx$$
$$=\int \left(Ab^2x^4+b(bB+2Ac)x^6+c(2bB+Ac)x^8+Bc^2x^{10}\right)dx$$
$$=\frac{1}{5}Ab^2x^5+\frac{1}{7}b(bB+2Ac)x^7+\frac{1}{9}c(2bB+Ac)x^9+\frac{1}{11}Bc^2x^{11}$$

Mathematica [A] time = 0.0096894, size = 55, normalized size = 1.

$$\frac{1}{5}Ab^2x^5+\frac{1}{9}cx^9(Ac+2bB)+\frac{1}{7}bx^7(2Ac+bB)+\frac{1}{11}Bc^2x^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (A*b^2*x^5)/5 + (b*(b*B + 2*A*c)*x^7)/7 + (c*(2*b*B + A*c)*x^9)/9 + (B*c^2*
x^11)/11
```

Maple [A] time = 0., size = 52, normalized size = 1.

$$\frac{Bc^2x^{11}}{11}+\frac{\left(Ac^2+2\,Bbc\right)x^9}{9}+\frac{\left(2\,Abc+Bb^2\right)x^7}{7}+\frac{Ab^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/11*B*c^2*x^11+1/9*(A*c^2+2*B*b*c)*x^9+1/7*(2*A*b*c+B*b^2)*x^7+1/5*A*b^2*x
^5
```

Maxima [A] time = 1.1433, size = 69, normalized size = 1.25

$$\frac{1}{11}\,Bc^2x^{11}+\frac{1}{9}\left(2\,Bbc+Ac^2\right)x^9+\frac{1}{5}\,Ab^2x^5+\frac{1}{7}\left(Bb^2+2\,Abc\right)x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/11*B*c^2*x^11 + 1/9*(2*B*b*c + A*c^2)*x^9 + 1/5*A*b^2*x^5 + 1/7*(B*b^2 +
2*A*b*c)*x^7
```

Fricas [A] time = 0.41931, size = 131, normalized size = 2.38

$$\frac{1}{11}x^{11}c^2B + \frac{2}{9}x^9cbB + \frac{1}{9}x^9c^2A + \frac{1}{7}x^7b^2B + \frac{2}{7}x^7cbA + \frac{1}{5}x^5b^2A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*c^2*B + 2/9*x^9*c*b*B + 1/9*x^9*c^2*A + 1/7*x^7*b^2*B + 2/7*x^7*c
*b*A + 1/5*x^5*b^2*A
```

Sympy [A] time = 0.070626, size = 56, normalized size = 1.02

$$\frac{Ab^2x^5}{5} + \frac{Bc^2x^{11}}{11} + x^9\left(\frac{Ac^2}{9} + \frac{2Bbc}{9}\right) + x^7\left(\frac{2Abc}{7} + \frac{Bb^2}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2,x)
```

```
[Out] A*b**2*x**5/5 + B*c**2*x**11/11 + x**9*(A*c**2/9 + 2*B*b*c/9) + x**7*(2*A*b
*c/7 + B*b**2/7)
```

Giac [A] time = 1.20551, size = 72, normalized size = 1.31

$$\frac{1}{11}\,Bc^2x^{11} + \frac{2}{9}\,Bbcx^9 + \frac{1}{9}\,Ac^2x^9 + \frac{1}{7}\,Bb^2x^7 + \frac{2}{7}\,Abcx^7 + \frac{1}{5}\,Ab^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/11*B*c^2*x^11 + 2/9*B*b*c*x^9 + 1/9*A*c^2*x^9 + 1/7*B*b^2*x^7 + 2/7*A*b*c
*x^7 + 1/5*A*b^2*x^5
```

3.13 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x} dx$

Optimal. Leaf size=55

$$\frac{1}{4}Ab^2x^4 + \frac{1}{8}cx^8(Ac + 2bB) + \frac{1}{6}bx^6(2Ac + bB) + \frac{1}{10}Bc^2x^{10}$$

```
[Out] (A*b^2*x^4)/4 + (b*(b*B + 2*A*c)*x^6)/6 + (c*(2*b*B + A*c)*x^8)/8 + (B*c^2*
x^10)/10
```

Rubi [A] time = 0.0676805, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 446, 76}

$$\frac{1}{4}Ab^2x^4 + \frac{1}{8}cx^8(Ac + 2bB) + \frac{1}{6}bx^6(2Ac + bB) + \frac{1}{10}Bc^2x^{10}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x,x]
```

```
[Out] (A*b^2*x^4)/4 + (b*(b*B + 2*A*c)*x^6)/6 + (c*(2*b*B + A*c)*x^8)/8 + (B*c^2*
x^10)/10
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
```

```
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^2}{x}\,dx &= \int x^3\left(A+Bx^2\right)\left(b+cx^2\right)^2\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int x(A+Bx)(b+cx)^2\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int\left(Ab^2x+b(bB+2Ac)x^2+c(2bB+Ac)x^3+Bc^2x^4\right)dx,x,x^2\right)\\ &= \frac{1}{4}Ab^2x^4+\frac{1}{6}b(bB+2Ac)x^6+\frac{1}{8}c(2bB+Ac)x^8+\frac{1}{10}Bc^2x^{10}\end{aligned}$$

Mathematica [A] time = 0.0079181, size = 55, normalized size = 1.

$$\frac{1}{4}Ab^2x^4+\frac{1}{8}cx^8(Ac+2bB)+\frac{1}{6}bx^6(2Ac+bB)+\frac{1}{10}Bc^2x^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x,x]
```

```
[Out] (A*b^2*x^4)/4 + (b*(b*B + 2*A*c)*x^6)/6 + (c*(2*b*B + A*c)*x^8)/8 + (B*c^2*
x^10)/10
```

Maple [A] time = 0.001, size = 52, normalized size = 1.

$$\frac{Bc^2x^{10}}{10}+\frac{\left(Ac^2+2\,Bbc\right)x^8}{8}+\frac{\left(2\,Abc+Bb^2\right)x^6}{6}+\frac{Ab^2x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x,x)
```

```
[Out] 1/10*B*c^2*x^10+1/8*(A*c^2+2*B*b*c)*x^8+1/6*(2*A*b*c+B*b^2)*x^6+1/4*A*b^2*x
^4
```

Maxima [A] time = 1.11603, size = 69, normalized size = 1.25

$$\frac{1}{10}\,Bc^2x^{10} + \frac{1}{8}\left(2\,Bbc + Ac^2\right)x^8 + \frac{1}{4}\,Ab^2x^4 + \frac{1}{6}\left(Bb^2 + 2\,Abc\right)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x,x, algorithm="maxima")
```

```
[Out] 1/10*B*c^2*x^10 + 1/8*(2*B*b*c + A*c^2)*x^8 + 1/4*A*b^2*x^4 + 1/6*(B*b^2 +
2*A*b*c)*x^6
```

Fricas [A] time = 0.461019, size = 120, normalized size = 2.18

$$\frac{1}{10}\,Bc^2x^{10} + \frac{1}{8}\left(2\,Bbc + Ac^2\right)x^8 + \frac{1}{4}\,Ab^2x^4 + \frac{1}{6}\left(Bb^2 + 2\,Abc\right)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x,x, algorithm="fricas")
```

```
[Out] 1/10*B*c^2*x^10 + 1/8*(2*B*b*c + A*c^2)*x^8 + 1/4*A*b^2*x^4 + 1/6*(B*b^2 +
2*A*b*c)*x^6
```

Sympy [A] time = 0.070706, size = 53, normalized size = 0.96

$$\frac{Ab^2x^4}{4} + \frac{Bc^2x^{10}}{10} + x^8\left(\frac{Ac^2}{8} + \frac{Bbc}{4}\right) + x^6\left(\frac{Abc}{3} + \frac{Bb^2}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x,x)
```

```
[Out] A*b**2*x**4/4 + B*c**2*x**10/10 + x**8*(A*c**2/8 + B*b*c/4) + x**6*(A*b*c/3
 + B*b**2/6)
```

Giac [A] time = 1.18926, size = 72, normalized size = 1.31

$$\frac{1}{10}\,Bc^2x^{10} + \frac{1}{4}\,Bbcx^8 + \frac{1}{8}\,Ac^2x^8 + \frac{1}{6}\,Bb^2x^6 + \frac{1}{3}\,Abcx^6 + \frac{1}{4}\,Ab^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x,x, algorithm="giac")
```

```
[Out] 1/10*B*c^2*x^10 + 1/4*B*b*c*x^8 + 1/8*A*c^2*x^8 + 1/6*B*b^2*x^6 + 1/3*A*b*c
*x^6 + 1/4*A*b^2*x^4
```

3.14 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^2} \, dx$

Optimal. Leaf size=55

$$\frac{1}{3}Ab^2x^3 + \frac{1}{7}cx^7(Ac + 2bB) + \frac{1}{5}bx^5(2Ac + bB) + \frac{1}{9}Bc^2x^9$$

```
[Out] (A*b^2*x^3)/3 + (b*(b*B + 2*A*c)*x^5)/5 + (c*(2*b*B + A*c)*x^7)/7 + (B*c^2*
x^9)/9
```

Rubi [A] time = 0.038727, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1584, 448}

$$\frac{1}{3}Ab^2x^3 + \frac{1}{7}cx^7(Ac + 2bB) + \frac{1}{5}bx^5(2Ac + bB) + \frac{1}{9}Bc^2x^9$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^2,x]
```

```
[Out] (A*b^2*x^3)/3 + (b*(b*B + 2*A*c)*x^5)/5 + (c*(2*b*B + A*c)*x^7)/7 + (B*c^2*
x^9)/9
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^2}{x^2}\,dx = \int x^2\left(A+Bx^2\right)\left(b+cx^2\right)^2\,dx$$
$$= \int \left(Ab^2x^2+b(bB+2Ac)x^4+c(2bB+Ac)x^6+Bc^2x^8\right)dx$$
$$= \frac{1}{3}Ab^2x^3+\frac{1}{5}b(bB+2Ac)x^5+\frac{1}{7}c(2bB+Ac)x^7+\frac{1}{9}Bc^2x^9$$

Mathematica [A] time = 0.0089094, size = 55, normalized size = 1.

$$\frac{1}{3}Ab^2x^3+\frac{1}{7}cx^7(Ac+2bB)+\frac{1}{5}bx^5(2Ac+bB)+\frac{1}{9}Bc^2x^9$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^2,x]
```

```
[Out] (A*b^2*x^3)/3 + (b*(b*B + 2*A*c)*x^5)/5 + (c*(2*b*B + A*c)*x^7)/7 + (B*c^2*
x^9)/9
```

Maple [A] time = 0.001, size = 52, normalized size = 1.

$$\frac{Bc^2x^9}{9}+\frac{\left(Ac^2+2\,Bbc\right)x^7}{7}+\frac{\left(2\,Abc+Bb^2\right)x^5}{5}+\frac{Ab^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^2,x)
```

```
[Out] 1/9*B*c^2*x^9+1/7*(A*c^2+2*B*b*c)*x^7+1/5*(2*A*b*c+B*b^2)*x^5+1/3*A*b^2*x^3
```

Maxima [A] time = 1.0644, size = 69, normalized size = 1.25

$$\frac{1}{9}\,Bc^2x^9+\frac{1}{7}\left(2\,Bbc+Ac^2\right)x^7+\frac{1}{3}\,Ab^2x^3+\frac{1}{5}\left(Bb^2+2\,Abc\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/9*B*c^2*x^9 + 1/7*(2*B*b*c + A*c^2)*x^7 + 1/3*A*b^2*x^3 + 1/5*(B*b^2 + 2*
A*b*c)*x^5
```

Fricas [A] time = 0.509525, size = 117, normalized size = 2.13

$$\frac{1}{9}\,Bc^2x^9 + \frac{1}{7}\left(2\,Bbc + Ac^2\right)x^7 + \frac{1}{3}\,Ab^2x^3 + \frac{1}{5}\left(Bb^2 + 2\,Abc\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/9*B*c^2*x^9 + 1/7*(2*B*b*c + A*c^2)*x^7 + 1/3*A*b^2*x^3 + 1/5*(B*b^2 + 2*
A*b*c)*x^5
```

Sympy [A] time = 0.068828, size = 56, normalized size = 1.02

$$\frac{Ab^2x^3}{3} + \frac{Bc^2x^9}{9} + x^7\left(\frac{Ac^2}{7} + \frac{2Bbc}{7}\right) + x^5\left(\frac{2Abc}{5} + \frac{Bb^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**2,x)
```

```
[Out] A*b**2*x**3/3 + B*c**2*x**9/9 + x**7*(A*c**2/7 + 2*B*b*c/7) + x**5*(2*A*b*c
/5 + B*b**2/5)
```

Giac [A] time = 1.26311, size = 72, normalized size = 1.31

$$\frac{1}{9}\,Bc^2x^9 + \frac{2}{7}\,Bbcx^7 + \frac{1}{7}\,Ac^2x^7 + \frac{1}{5}\,Bb^2x^5 + \frac{2}{5}\,Abcx^5 + \frac{1}{3}\,Ab^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/9*B*c^2*x^9 + 2/7*B*b*c*x^7 + 1/7*A*c^2*x^7 + 1/5*B*b^2*x^5 + 2/5*A*b*c*x
^5 + 1/3*A*b^2*x^3
```

3.15 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^3} dx$

Optimal. Leaf size=42

$$\frac{B(b+cx^2)^4}{8c^2} - \frac{(b+cx^2)^3(bB-Ac)}{6c^2}$$

```
[Out] -((b*B - A*c)*(b + c*x^2)^3)/(6*c^2) + (B*(b + c*x^2)^4)/(8*c^2)
```

Rubi [A] time = 0.0696529, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 444, 43}

$$\frac{B(b+cx^2)^4}{8c^2} - \frac{(b+cx^2)^3(bB-Ac)}{6c^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^3,x]
```

```
[Out] -((b*B - A*c)*(b + c*x^2)^3)/(6*c^2) + (B*(b + c*x^2)^4)/(8*c^2)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^2}{x^3}\,dx &= \int x\left(A+Bx^2\right)\left(b+cx^2\right)^2\,dx\\
&= \frac{1}{2}\,\text{Subst}\left(\int (A+Bx)(b+cx)^2\,dx, x, x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{(-bB+Ac)(b+cx)^2}{c}+\frac{B(b+cx)^3}{c}\right)dx, x, x^2\right)\\
&= -\frac{(bB-Ac)\left(b+cx^2\right)^3}{6c^2}+\frac{B\left(b+cx^2\right)^4}{8c^2}
\end{aligned}
$$

Mathematica [A] time = 0.0128354, size = 51, normalized size = 1.21

$$\frac{1}{24}x^2\left(12Ab^2+4cx^4(Ac+2bB)+6bx^2(2Ac+bB)+3Bc^2x^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^3,x]
```

```
[Out] (x^2*(12*A*b^2 + 6*b*(b*B + 2*A*c)*x^2 + 4*c*(2*b*B + A*c)*x^4 + 3*B*c^2*x^
6))/24
```

Maple [A] time = 0.001, size = 52, normalized size = 1.2

$$\frac{Bc^2x^8}{8}+\frac{\left(Ac^2+2\,Bbc\right)x^6}{6}+\frac{\left(2\,Abc+Bb^2\right)x^4}{4}+\frac{Ab^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^3,x)
```

```
[Out] 1/8*B*c^2*x^8+1/6*(A*c^2+2*B*b*c)*x^6+1/4*(2*A*b*c+B*b^2)*x^4+1/2*A*b^2*x^2
```

Maxima [A] time = 1.10798, size = 69, normalized size = 1.64

$$\frac{1}{8}Bc^2x^8 + \frac{1}{6}\left(2Bbc + Ac^2\right)x^6 + \frac{1}{2}Ab^2x^2 + \frac{1}{4}\left(Bb^2 + 2Abc\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*B*c^2*x^8 + 1/6*(2*B*b*c + A*c^2)*x^6 + 1/2*A*b^2*x^2 + 1/4*(B*b^2 + 2*
A*b*c)*x^4
```

Fricas [A] time = 0.457673, size = 117, normalized size = 2.79

$$\frac{1}{8}Bc^2x^8 + \frac{1}{6}\left(2Bbc + Ac^2\right)x^6 + \frac{1}{2}Ab^2x^2 + \frac{1}{4}\left(Bb^2 + 2Abc\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/8*B*c^2*x^8 + 1/6*(2*B*b*c + A*c^2)*x^6 + 1/2*A*b^2*x^2 + 1/4*(B*b^2 + 2*
A*b*c)*x^4
```

Sympy [A] time = 0.069064, size = 53, normalized size = 1.26

$$\frac{Ab^2x^2}{2} + \frac{Bc^2x^8}{8} + x^6\left(\frac{Ac^2}{6} + \frac{Bbc}{3}\right) + x^4\left(\frac{Abc}{2} + \frac{Bb^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**3,x)
```

```
[Out] A*b**2*x**2/2 + B*c**2*x**8/8 + x**6*(A*c**2/6 + B*b*c/3) + x**4*(A*b*c/2 +
 B*b**2/4)
```

Giac [A] time = 1.26475, size = 72, normalized size = 1.71

$$\frac{1}{8}Bc^2x^8 + \frac{1}{3}Bbcx^6 + \frac{1}{6}Ac^2x^6 + \frac{1}{4}Bb^2x^4 + \frac{1}{2}Abcx^4 + \frac{1}{2}Ab^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^3,x, algorithm="giac")
```

```
[Out] 1/8*B*c^2*x^8 + 1/3*B*b*c*x^6 + 1/6*A*c^2*x^6 + 1/4*B*b^2*x^4 + 1/2*A*b*c*x
^4 + 1/2*A*b^2*x^2
```

3.16 $\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^2}{x^4}\,dx$

Optimal. Leaf size=50

$$Ab^2x+\frac{1}{5}cx^5(Ac+2bB)+\frac{1}{3}bx^3(2Ac+bB)+\frac{1}{7}Bc^2x^7$$

```
[Out] A*b^2*x + (b*(b*B + 2*A*c)*x^3)/3 + (c*(2*b*B + A*c)*x^5)/5 + (B*c^2*x^7)/7
```

Rubi [A] time = 0.0322901, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1584, 373}

$$Ab^2x+\frac{1}{5}cx^5(Ac+2bB)+\frac{1}{3}bx^3(2Ac+bB)+\frac{1}{7}Bc^2x^7$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^4,x]
```

```
[Out] A*b^2*x + (b*(b*B + 2*A*c)*x^3)/3 + (c*(2*b*B + A*c)*x^5)/5 + (B*c^2*x^7)/7
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^4}\,dx = \int (A+Bx^2)(b+cx^2)^2\,dx$$
$$= \int \left(Ab^2+b(bB+2Ac)x^2+c(2bB+Ac)x^4+Bc^2x^6\right)dx$$
$$= Ab^2x+\frac{1}{3}b(bB+2Ac)x^3+\frac{1}{5}c(2bB+Ac)x^5+\frac{1}{7}Bc^2x^7$$

Mathematica [A] time = 0.0084935, size = 50, normalized size = 1.

$$Ab^2x+\frac{1}{5}cx^5(Ac+2bB)+\frac{1}{3}bx^3(2Ac+bB)+\frac{1}{7}Bc^2x^7$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^4,x]
```

```
[Out] A*b^2*x + (b*(b*B + 2*A*c)*x^3)/3 + (c*(2*b*B + A*c)*x^5)/5 + (B*c^2*x^7)/7
```

Maple [A] time = 0., size = 49, normalized size = 1.

$$\frac{Bc^2x^7}{7}+\frac{(Ac^2+2\,Bbc)\,x^5}{5}+\frac{(2\,Abc+Bb^2)\,x^3}{3}+Ab^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^4,x)
```

```
[Out] 1/7*B*c^2*x^7+1/5*(A*c^2+2*B*b*c)*x^5+1/3*(2*A*b*c+B*b^2)*x^3+A*b^2*x
```

Maxima [A] time = 1.1253, size = 65, normalized size = 1.3

$$\frac{1}{7}\,Bc^2x^7+\frac{1}{5}\left(2\,Bbc+Ac^2\right)x^5+Ab^2x+\frac{1}{3}\left(Bb^2+2\,Abc\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/7*B*c^2*x^7 + 1/5*(2*B*b*c + A*c^2)*x^5 + A*b^2*x + 1/3*(B*b^2 + 2*A*b*c)
*x^3
```

Fricas [A] time = 0.480369, size = 109, normalized size = 2.18

$$\frac{1}{7}Bc^2x^7 + \frac{1}{5}\left(2\,Bbc + Ac^2\right)x^5 + Ab^2x + \frac{1}{3}\left(Bb^2 + 2\,Abc\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^4,x, algorithm="fricas")
```

```
[Out] 1/7*B*c^2*x^7 + 1/5*(2*B*b*c + A*c^2)*x^5 + A*b^2*x + 1/3*(B*b^2 + 2*A*b*c)
*x^3
```

Sympy [A] time = 0.068293, size = 53, normalized size = 1.06

$$Ab^2x + \frac{Bc^2x^7}{7} + x^5\left(\frac{Ac^2}{5} + \frac{2Bbc}{5}\right) + x^3\left(\frac{2Abc}{3} + \frac{Bb^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**4,x)
```

```
[Out] A*b**2*x + B*c**2*x**7/7 + x**5*(A*c**2/5 + 2*B*b*c/5) + x**3*(2*A*b*c/3 +
B*b**2/3)
```

Giac [A] time = 1.3346, size = 68, normalized size = 1.36

$$\frac{1}{7}Bc^2x^7 + \frac{2}{5}Bbcx^5 + \frac{1}{5}Ac^2x^5 + \frac{1}{3}Bb^2x^3 + \frac{2}{3}Abcx^3 + Ab^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^4,x, algorithm="giac")
```

```
[Out] 1/7*B*c^2*x^7 + 2/5*B*b*c*x^5 + 1/5*A*c^2*x^5 + 1/3*B*b^2*x^3 + 2/3*A*b*c*x
^3 + A*b^2*x
```

3.17 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^5} dx$

Optimal. Leaf size=43

$$Ab^2 \log(x) + Abcx^2 + \frac{1}{4}Ac^2x^4 + \frac{B(b+cx^2)^3}{6c}$$

```
[Out] A*b*c*x^2 + (A*c^2*x^4)/4 + (B*(b + c*x^2)^3)/(6*c) + A*b^2*Log[x]
```

Rubi [A] time = 0.0402506, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1584, 446, 80, 43}

$$Ab^2 \log(x) + Abcx^2 + \frac{1}{4}Ac^2x^4 + \frac{B(b+cx^2)^3}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^5,x]
```

```
[Out] A*b*c*x^2 + (A*c^2*x^4)/4 + (B*(b + c*x^2)^3)/(6*c) + A*b^2*Log[x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
```

```
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^2}{x^5}\,dx &= \int \frac{\left(A+Bx^2\right)\left(b+cx^2\right)^2}{x}\,dx \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(A+Bx)(b+cx)^2}{x}\,dx, x, x^2\right) \\
&= \frac{B\left(b+cx^2\right)^3}{6c} + \frac{1}{2}A\operatorname{Subst}\left(\int \frac{(b+cx)^2}{x}\,dx, x, x^2\right) \\
&= \frac{B\left(b+cx^2\right)^3}{6c} + \frac{1}{2}A\operatorname{Subst}\left(\int \left(2bc+\frac{b^2}{x}+c^2x\right)dx, x, x^2\right) \\
&= Abcx^2 + \frac{1}{4}Ac^2x^4 + \frac{B\left(b+cx^2\right)^3}{6c} + Ab^2\log(x)
\end{aligned}
$$

Mathematica [A] time = 0.0163226, size = 51, normalized size = 1.19

$$
Ab^2\log(x) + \frac{1}{4}cx^4(Ac+2bB) + \frac{1}{2}bx^2(2Ac+bB) + \frac{1}{6}Bc^2x^6
$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^5,x]
```

```
[Out] (b*(b*B + 2*A*c)*x^2)/2 + (c*(2*b*B + A*c)*x^4)/4 + (B*c^2*x^6)/6 + A*b^2*L
og[x]
```

Maple [A] time = 0.002, size = 51, normalized size = 1.2

$$
\frac{Bc^2x^6}{6} + \frac{Ac^2x^4}{4} + \frac{Bx^4bc}{2} + Abcx^2 + \frac{Bx^2b^2}{2} + Ab^2\ln(x)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^5,x)
```

```
[Out] 1/6*B*c^2*x^6+1/4*A*c^2*x^4+1/2*B*x^4*b*c+A*b*c*x^2+1/2*B*x^2*b^2+A*b^2*ln(
x)
```

Maxima [A] time = 1.1075, size = 70, normalized size = 1.63

$$\frac{1}{6}Bc^2x^6 + \frac{1}{4}\left(2Bbc + Ac^2\right)x^4 + \frac{1}{2}Ab^2\log\left(x^2\right) + \frac{1}{2}\left(Bb^2 + 2Abc\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/6*B*c^2*x^6 + 1/4*(2*B*b*c + A*c^2)*x^4 + 1/2*A*b^2*log(x^2) + 1/2*(B*b^2
 + 2*A*b*c)*x^2
```

Fricas [A] time = 0.450696, size = 116, normalized size = 2.7

$$\frac{1}{6}Bc^2x^6 + \frac{1}{4}\left(2Bbc + Ac^2\right)x^4 + Ab^2\log(x) + \frac{1}{2}\left(Bb^2 + 2Abc\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/6*B*c^2*x^6 + 1/4*(2*B*b*c + A*c^2)*x^4 + A*b^2*log(x) + 1/2*(B*b^2 + 2*A
*b*c)*x^2
```

Sympy [A] time = 0.289352, size = 49, normalized size = 1.14

$$Ab^2\log(x) + \frac{Bc^2x^6}{6} + x^4\left(\frac{Ac^2}{4} + \frac{Bbc}{2}\right) + x^2\left(Abc + \frac{Bb^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**5,x)
```

```
[Out] A*b**2*log(x) + B*c**2*x**6/6 + x**4*(A*c**2/4 + B*b*c/2) + x**2*(A*b*c + B
*b**2/2)
```

Giac [A] time = 1.27175, size = 72, normalized size = 1.67

$$\frac{1}{6}Bc^2x^6 + \frac{1}{2}Bbcx^4 + \frac{1}{4}Ac^2x^4 + \frac{1}{2}Bb^2x^2 + Abcx^2 + \frac{1}{2}Ab^2\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^5,x, algorithm="giac")
```

```
[Out] 1/6*B*c^2*x^6 + 1/2*B*b*c*x^4 + 1/4*A*c^2*x^4 + 1/2*B*b^2*x^2 + A*b*c*x^2 +
 1/2*A*b^2*log(x^2)
```

3.18 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^6}\,dx$

Optimal. Leaf size=48

$$-\frac{Ab^2}{x}+\frac{1}{3}cx^3(Ac+2bB)+bx(2Ac+bB)+\frac{1}{5}Bc^2x^5$$

```
[Out] -((A*b^2)/x) + b*(b*B + 2*A*c)*x + (c*(2*b*B + A*c)*x^3)/3 + (B*c^2*x^5)/5
```

Rubi [A] time = 0.037229, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1584, 448}

$$-\frac{Ab^2}{x}+\frac{1}{3}cx^3(Ac+2bB)+bx(2Ac+bB)+\frac{1}{5}Bc^2x^5$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^6,x]
```

```
[Out] -((A*b^2)/x) + b*(b*B + 2*A*c)*x + (c*(2*b*B + A*c)*x^3)/3 + (B*c^2*x^5)/5
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^6}\,dx = \int \frac{(A+Bx^2)(b+cx^2)^2}{x^2}\,dx$$
$$= \int \left(b(bB+2Ac)+\frac{Ab^2}{x^2}+c(2bB+Ac)x^2+Bc^2x^4\right)dx$$
$$= -\frac{Ab^2}{x}+b(bB+2Ac)x+\frac{1}{3}c(2bB+Ac)x^3+\frac{1}{5}Bc^2x^5$$

Mathematica [A] time = 0.0174977, size = 48, normalized size = 1.

$$-\frac{Ab^2}{x}+\frac{1}{3}cx^3(Ac+2bB)+bx(2Ac+bB)+\frac{1}{5}Bc^2x^5$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^6,x]

[Out] -((A*b^2)/x) + b*(b*B + 2*A*c)*x + (c*(2*b*B + A*c)*x^3)/3 + (B*c^2*x^5)/5
```

Maple [A] time = 0.003, size = 49, normalized size = 1.

$$\frac{Bc^2x^5}{5}+\frac{Ax^3c^2}{3}+\frac{2\,Bx^3bc}{3}+2\,Abcx+Bb^2x-\frac{Ab^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^6,x)

[Out] 1/5*B*c^2*x^5+1/3*A*x^3*c^2+2/3*B*x^3*b*c+2*A*b*c*x+B*b^2*x-A*b^2/x
```

Maxima [A] time = 1.14033, size = 65, normalized size = 1.35

$$\frac{1}{5}\,Bc^2x^5+\frac{1}{3}\left(2\,Bbc+Ac^2\right)x^3-\frac{Ab^2}{x}+\left(Bb^2+2\,Abc\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^6,x, algorithm="maxima")
```

```
[Out] 1/5*B*c^2*x^5 + 1/3*(2*B*b*c + A*c^2)*x^3 - A*b^2/x + (B*b^2 + 2*A*b*c)*x
```

Fricas [A] time = 0.442007, size = 116, normalized size = 2.42

$$\frac{3 Bc^2x^6 + 5\left(2 Bbc + Ac^2\right)x^4 - 15 Ab^2 + 15\left(Bb^2 + 2 Abc\right)x^2}{15 x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^6,x, algorithm="fricas")
```

```
[Out] 1/15*(3*B*c^2*x^6 + 5*(2*B*b*c + A*c^2)*x^4 - 15*A*b^2 + 15*(B*b^2 + 2*A*b*
c)*x^2)/x
```

Sympy [A] time = 0.286375, size = 48, normalized size = 1.

$$-\frac{Ab^2}{x} + \frac{Bc^2x^5}{5} + x^3\left(\frac{Ac^2}{3} + \frac{2Bbc}{3}\right) + x\left(2Abc + Bb^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**6,x)
```

```
[Out] -A*b**2/x + B*c**2*x**5/5 + x**3*(A*c**2/3 + 2*B*b*c/3) + x*(2*A*b*c + B*b*
*2)
```

Giac [A] time = 1.23109, size = 65, normalized size = 1.35

$$\frac{1}{5} Bc^2x^5 + \frac{2}{3} Bbcx^3 + \frac{1}{3} Ac^2x^3 + Bb^2x + 2 Abcx - \frac{Ab^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^6,x, algorithm="giac")
```

```
[Out] 1/5*B*c^2*x^5 + 2/3*B*b*c*x^3 + 1/3*A*c^2*x^3 + B*b^2*x + 2*A*b*c*x - A*b^2
/x
```

3.19 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^7} \, dx$

Optimal. Leaf size=51

$$-\frac{Ab^2}{2x^2}+\frac{1}{2}cx^2(Ac+2bB)+b\log(x)(2Ac+bB)+\frac{1}{4}Bc^2x^4$$

```
[Out] -(A*b^2)/(2*x^2) + (c*(2*b*B + A*c)*x^2)/2 + (B*c^2*x^4)/4 + b*(b*B + 2*A*c
)*Log[x]
```

Rubi [A] time = 0.053257, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 446, 76}

$$-\frac{Ab^2}{2x^2}+\frac{1}{2}cx^2(Ac+2bB)+b\log(x)(2Ac+bB)+\frac{1}{4}Bc^2x^4$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^7,x]
```

```
[Out] -(A*b^2)/(2*x^2) + (c*(2*b*B + A*c)*x^2)/2 + (B*c^2*x^4)/4 + b*(b*B + 2*A*c
)*Log[x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
```

```
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^7}\,dx &= \int \frac{(A+Bx^2)(b+cx^2)^2}{x^3}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(A+Bx)(b+cx)^2}{x^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(c(2bB+Ac)+\frac{Ab^2}{x^2}+\frac{b(bB+2Ac)}{x}+Bc^2x\right)dx,x,x^2\right)\\ &= -\frac{Ab^2}{2x^2}+\frac{1}{2}c(2bB+Ac)x^2+\frac{1}{4}Bc^2x^4+b(bB+2Ac)\log(x)\end{aligned}$$

Mathematica [A] time = 0.0279669, size = 49, normalized size = 0.96

$$\frac{1}{4}\left(-\frac{2Ab^2}{x^2}+2cx^2(Ac+2bB)+4b\log(x)(2Ac+bB)+Bc^2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^7,x]
```

```
[Out] ((-2*A*b^2)/x^2 + 2*c*(2*b*B + A*c)*x^2 + B*c^2*x^4 + 4*b*(b*B + 2*A*c)*Log
[x])/4
```

Maple [A] time = 0.005, size = 50, normalized size = 1.

$$\frac{Bc^2x^4}{4}+\frac{Ax^2c^2}{2}+Bx^2bc+2\,A\ln(x)\,bc+B\ln(x)\,b^2-\frac{Ab^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^7,x)
```

```
[Out] 1/4*B*c^2*x^4+1/2*A*x^2*c^2+B*x^2*b*c+2*A*ln(x)*b*c+B*ln(x)*b^2-1/2*A*b^2/x
^2
```

Maxima [A] time = 1.08898, size = 70, normalized size = 1.37

$$\frac{1}{4}\,Bc^2x^4 + \frac{1}{2}\left(2\,Bbc + Ac^2\right)x^2 + \frac{1}{2}\left(Bb^2 + 2\,Abc\right)\log\left(x^2\right) - \frac{Ab^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^7,x, algorithm="maxima")
```

```
[Out] 1/4*B*c^2*x^4 + 1/2*(2*B*b*c + A*c^2)*x^2 + 1/2*(B*b^2 + 2*A*b*c)*log(x^2)
- 1/2*A*b^2/x^2
```

Fricas [A] time = 0.506127, size = 122, normalized size = 2.39

$$\frac{Bc^2x^6 + 2\left(2\,Bbc + Ac^2\right)x^4 + 4\left(Bb^2 + 2\,Abc\right)x^2\log\left(x\right) - 2\,Ab^2}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^7,x, algorithm="fricas")
```

```
[Out] 1/4*(B*c^2*x^6 + 2*(2*B*b*c + A*c^2)*x^4 + 4*(B*b^2 + 2*A*b*c)*x^2*log(x) -
 2*A*b^2)/x^2
```

Sympy [A] time = 0.371721, size = 48, normalized size = 0.94

$$-\frac{Ab^2}{2x^2} + \frac{Bc^2x^4}{4} + b\left(2Ac + Bb\right)\log\left(x\right) + x^2\left(\frac{Ac^2}{2} + Bbc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**7,x)
```

```
[Out] -A*b**2/(2*x**2) + B*c**2*x**4/4 + b*(2*A*c + B*b)*log(x) + x**2*(A*c**2/2
+ B*b*c)
```

Giac [A] time = 1.2133, size = 95, normalized size = 1.86

$$\frac{1}{4}\,Bc^2x^4 + Bbcx^2 + \frac{1}{2}\,Ac^2x^2 + \frac{1}{2}\left(Bb^2 + 2\,Abc\right)\log\left(x^2\right) - \frac{Bb^2x^2 + 2\,Abcx^2 + Ab^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^7,x, algorithm="giac")
```

```
[Out] 1/4*B*c^2*x^4 + B*b*c*x^2 + 1/2*A*c^2*x^2 + 1/2*(B*b^2 + 2*A*b*c)*log(x^2)
- 1/2*(B*b^2*x^2 + 2*A*b*c*x^2 + A*b^2)/x^2
```

3.20 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^8} dx$

Optimal. Leaf size=48

$$-\frac{Ab^2}{3x^3}+cx(Ac+2bB)-\frac{b(2Ac+bB)}{x}+\frac{1}{3}Bc^2x^3$$

```
[Out] -(A*b^2)/(3*x^3) - (b*(b*B + 2*A*c))/x + c*(2*b*B + A*c)*x + (B*c^2*x^3)/3
```

Rubi [A] time = 0.0368726, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1584, 448}

$$-\frac{Ab^2}{3x^3}+cx(Ac+2bB)-\frac{b(2Ac+bB)}{x}+\frac{1}{3}Bc^2x^3$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^8,x]
```

```
[Out] -(A*b^2)/(3*x^3) - (b*(b*B + 2*A*c))/x + c*(2*b*B + A*c)*x + (B*c^2*x^3)/3
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^8}\,dx = \int \frac{(A+Bx^2)(b+cx^2)^2}{x^4}\,dx$$
$$= \int \left(c(2bB+Ac)+\frac{Ab^2}{x^4}+\frac{b(bB+2Ac)}{x^2}+Bc^2x^2\right)dx$$
$$= -\frac{Ab^2}{3x^3}-\frac{b(bB+2Ac)}{x}+c(2bB+Ac)x+\frac{1}{3}Bc^2x^3$$

Mathematica [A] time = 0.0185551, size = 50, normalized size = 1.04

$$\frac{b^2(-B)-2Abc}{x}-\frac{Ab^2}{3x^3}+cx(Ac+2bB)+\frac{1}{3}Bc^2x^3$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^8,x]
```

```
[Out] -(A*b^2)/(3*x^3) + (-(b^2*B) - 2*A*b*c)/x + c*(2*b*B + A*c)*x + (B*c^2*x^3)
/3
```

Maple [A] time = 0.006, size = 46, normalized size = 1.

$$\frac{Bc^2x^3}{3}+Ac^2x+2\,Bbcx-\frac{Ab^2}{3\,x^3}-\frac{b\,(2\,Ac+Bb)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^8,x)
```

```
[Out] 1/3*B*c^2*x^3+A*c^2*x+2*B*b*c*x-1/3*A*b^2/x^3-b*(2*A*c+B*b)/x
```

Maxima [A] time = 1.13744, size = 68, normalized size = 1.42

$$\frac{1}{3}\,Bc^2x^3+\left(2\,Bbc+Ac^2\right)x-\frac{Ab^2+3\left(Bb^2+2\,Abc\right)x^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^8,x, algorithm="maxima")
```

```
[Out] 1/3*B*c^2*x^3 + (2*B*b*c + A*c^2)*x - 1/3*(A*b^2 + 3*(B*b^2 + 2*A*b*c)*x^2)
/x^3
```

Fricas [A] time = 0.496618, size = 109, normalized size = 2.27

$$\frac{Bc^2x^6 + 3\left(2\,Bbc + Ac^2\right)x^4 - Ab^2 - 3\left(Bb^2 + 2\,Abc\right)x^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^8,x, algorithm="fricas")
```

```
[Out] 1/3*(B*c^2*x^6 + 3*(2*B*b*c + A*c^2)*x^4 - A*b^2 - 3*(B*b^2 + 2*A*b*c)*x^2)
/x^3
```

Sympy [A] time = 0.386567, size = 49, normalized size = 1.02

$$\frac{Bc^2x^3}{3} + x\left(Ac^2 + 2Bbc\right) - \frac{Ab^2 + x^2\left(6Abc + 3Bb^2\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**8,x)
```

```
[Out] B*c**2*x**3/3 + x*(A*c**2 + 2*B*b*c) - (A*b**2 + x**2*(6*A*b*c + 3*B*b**2))
/(3*x**3)
```

Giac [A] time = 1.30564, size = 68, normalized size = 1.42

$$\frac{1}{3}\,Bc^2x^3 + 2\,Bbcx + Ac^2x - \frac{3\,Bb^2x^2 + 6\,Abcx^2 + Ab^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^8,x, algorithm="giac")
```

```
[Out] 1/3*B*c^2*x^3 + 2*B*b*c*x + A*c^2*x - 1/3*(3*B*b^2*x^2 + 6*A*b*c*x^2 + A*b^
2)/x^3
```

3.21 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^9} dx$

Optimal. Leaf size=51

$$-\frac{Ab^2}{4x^4} - \frac{b(2Ac+bB)}{2x^2} + c\log(x)(Ac+2bB) + \frac{1}{2}Bc^2x^2$$

```
[Out] -(A*b^2)/(4*x^4) - (b*(b*B + 2*A*c))/(2*x^2) + (B*c^2*x^2)/2 + c*(2*b*B + A
*c)*Log[x]
```

Rubi [A] time = 0.0473707, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 446, 76}

$$-\frac{Ab^2}{4x^4} - \frac{b(2Ac+bB)}{2x^2} + c\log(x)(Ac+2bB) + \frac{1}{2}Bc^2x^2$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^9,x]
```

```
[Out] -(A*b^2)/(4*x^4) - (b*(b*B + 2*A*c))/(2*x^2) + (B*c^2*x^2)/2 + c*(2*b*B + A
*c)*Log[x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
```

```
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^2}{x^9}\,dx &= \int \frac{\left(A+Bx^2\right)\left(b+cx^2\right)^2}{x^5}\,dx\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{(A+Bx)(b+cx)^2}{x^3}\,dx,x,x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(Bc^2+\frac{Ab^2}{x^3}+\frac{b(bB+2Ac)}{x^2}+\frac{c(2bB+Ac)}{x}\right)dx,x,x^2\right)\\
&= -\frac{Ab^2}{4x^4}-\frac{b(bB+2Ac)}{2x^2}+\frac{1}{2}Bc^2x^2+c(2bB+Ac)\log(x)
\end{aligned}
$$

Mathematica [A] time = 0.0311582, size = 50, normalized size = 0.98

$$
c\log(x)(Ac+2bB)-\frac{Ab\left(b+4cx^2\right)+2Bx^2\left(b^2-c^2x^4\right)}{4x^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^9,x]
```

```
[Out] -(A*b*(b + 4*c*x^2) + 2*B*x^2*(b^2 - c^2*x^4))/(4*x^4) + c*(2*b*B + A*c)*Lo
g[x]
```

Maple [A] time = 0.005, size = 51, normalized size = 1.

$$
\frac{Bc^2x^2}{2}+A\ln(x)\,c^2+2\,B\ln(x)\,bc-\frac{Ab^2}{4\,x^4}-\frac{Abc}{x^2}-\frac{Bb^2}{2\,x^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^9,x)
```

```
[Out] 1/2*B*c^2*x^2+A*ln(x)*c^2+2*B*ln(x)*b*c-1/4*A*b^2/x^4-1/x^2*b*A*c-1/2/x^2*b
^2*B
```

Maxima [A] time = 1.13199, size = 73, normalized size = 1.43

$$\frac{1}{2}Bc^2x^2 + \frac{1}{2}\left(2Bbc + Ac^2\right)\log\left(x^2\right) - \frac{Ab^2 + 2\left(Bb^2 + 2Abc\right)x^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^9,x, algorithm="maxima")
```

```
[Out] 1/2*B*c^2*x^2 + 1/2*(2*B*b*c + A*c^2)*log(x^2) - 1/4*(A*b^2 + 2*(B*b^2 + 2*
A*b*c)*x^2)/x^4
```

Fricas [A] time = 0.47153, size = 122, normalized size = 2.39

$$\frac{2Bc^2x^6 + 4\left(2Bbc + Ac^2\right)x^4\log(x) - Ab^2 - 2\left(Bb^2 + 2Abc\right)x^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^9,x, algorithm="fricas")
```

```
[Out] 1/4*(2*B*c^2*x^6 + 4*(2*B*b*c + A*c^2)*x^4*log(x) - A*b^2 - 2*(B*b^2 + 2*A*
b*c)*x^2)/x^4
```

Sympy [A] time = 0.658919, size = 49, normalized size = 0.96

$$\frac{Bc^2x^2}{2} + c(Ac + 2Bb)\log(x) - \frac{Ab^2 + x^2\left(4Abc + 2Bb^2\right)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**9,x)
```

```
[Out] B*c**2*x**2/2 + c*(A*c + 2*B*b)*log(x) - (A*b**2 + x**2*(4*A*b*c + 2*B*b**2
))/(4*x**4)
```

Giac [A] time = 1.19601, size = 97, normalized size = 1.9

$$\frac{1}{2}Bc^2x^2 + \frac{1}{2}\left(2Bbc + Ac^2\right)\log\left(x^2\right) - \frac{6Bbcx^4 + 3Ac^2x^4 + 2Bb^2x^2 + 4Abcx^2 + Ab^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^9,x, algorithm="giac")
```

```
[Out] 1/2*B*c^2*x^2 + 1/2*(2*B*b*c + A*c^2)*log(x^2) - 1/4*(6*B*b*c*x^4 + 3*A*c^2
*x^4 + 2*B*b^2*x^2 + 4*A*b*c*x^2 + A*b^2)/x^4
```

3.22 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{10}} dx$

Optimal. Leaf size=48

$$-\frac{Ab^2}{5x^5}-\frac{b(2Ac+bB)}{3x^3}-\frac{c(Ac+2bB)}{x}+Bc^2x$$

```
[Out] -(A*b^2)/(5*x^5) - (b*(b*B + 2*A*c))/(3*x^3) - (c*(2*b*B + A*c))/x + B*c^2*
x
```

Rubi [A] time = 0.036689, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1584, 448}

$$-\frac{Ab^2}{5x^5}-\frac{b(2Ac+bB)}{3x^3}-\frac{c(Ac+2bB)}{x}+Bc^2x$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^10,x]
```

```
[Out] -(A*b^2)/(5*x^5) - (b*(b*B + 2*A*c))/(3*x^3) - (c*(2*b*B + A*c))/x + B*c^2*
x
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{10}}\,dx = \int \frac{(A+Bx^2)(b+cx^2)^2}{x^6}\,dx$$
$$= \int \left(Bc^2 + \frac{Ab^2}{x^6} + \frac{b(bB+2Ac)}{x^4} + \frac{c(2bB+Ac)}{x^2}\right) dx$$
$$= -\frac{Ab^2}{5x^5} - \frac{b(bB+2Ac)}{3x^3} - \frac{c(2bB+Ac)}{x} + Bc^2x$$

Mathematica [A] time = 0.0199065, size = 48, normalized size = 1.

$$-\frac{Ab^2}{5x^5} - \frac{b(2Ac+bB)}{3x^3} - \frac{c(Ac+2bB)}{x} + Bc^2x$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^10,x]
```

```
[Out] -(A*b^2)/(5*x^5) - (b*(b*B + 2*A*c))/(3*x^3) - (c*(2*b*B + A*c))/x + B*c^2*
x
```

Maple [A] time = 0.004, size = 45, normalized size = 0.9

$$-\frac{Ab^2}{5\,x^5} - \frac{b\,(2\,Ac+Bb)}{3\,x^3} - \frac{c\,(Ac+2\,Bb)}{x} + Bc^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^10,x)
```

```
[Out] -1/5*A*b^2/x^5-1/3*b*(2*A*c+B*b)/x^3-c*(A*c+2*B*b)/x+B*c^2*x
```

Maxima [A] time = 1.16067, size = 69, normalized size = 1.44

$$Bc^2x - \frac{15\left(2\,Bbc + Ac^2\right)x^4 + 3\,Ab^2 + 5\left(Bb^2 + 2\,Abc\right)x^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^10,x, algorithm="maxima")
```

```
[Out] B*c^2*x - 1/15*(15*(2*B*b*c + A*c^2)*x^4 + 3*A*b^2 + 5*(B*b^2 + 2*A*b*c)*x^
2)/x^5
```

Fricas [A] time = 0.445129, size = 119, normalized size = 2.48

$$\frac{15\,Bc^2x^6 - 15\left(2\,Bbc + Ac^2\right)x^4 - 3\,Ab^2 - 5\left(Bb^2 + 2\,Abc\right)x^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^10,x, algorithm="fricas")
```

```
[Out] 1/15*(15*B*c^2*x^6 - 15*(2*B*b*c + A*c^2)*x^4 - 3*A*b^2 - 5*(B*b^2 + 2*A*b*
c)*x^2)/x^5
```

Sympy [A] time = 0.733644, size = 51, normalized size = 1.06

$$Bc^2x - \frac{3Ab^2 + x^4\left(15Ac^2 + 30Bbc\right) + x^2\left(10Abc + 5Bb^2\right)}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**10,x)
```

```
[Out] B*c**2*x - (3*A*b**2 + x**4*(15*A*c**2 + 30*B*b*c) + x**2*(10*A*b*c + 5*B*b
**2))/(15*x**5)
```

Giac [A] time = 1.24291, size = 72, normalized size = 1.5

$$Bc^2x - \frac{30\,Bbcx^4 + 15\,Ac^2x^4 + 5\,Bb^2x^2 + 10\,Abcx^2 + 3\,Ab^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^10,x, algorithm="giac")
```

```
[Out] B*c^2*x - 1/15*(30*B*b*c*x^4 + 15*A*c^2*x^4 + 5*B*b^2*x^2 + 10*A*b*c*x^2 +
3*A*b^2)/x^5
```

3.23 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{11}} \, dx$

Optimal. Leaf size=51

$$-\frac{Ab^2}{6x^6}-\frac{b(2Ac+bB)}{4x^4}-\frac{c(Ac+2bB)}{2x^2}+Bc^2\log(x)$$

```
[Out] -(A*b^2)/(6*x^6) - (b*(b*B + 2*A*c))/(4*x^4) - (c*(2*b*B + A*c))/(2*x^2) +
B*c^2*Log[x]
```

Rubi [A] time = 0.044925, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 446, 76}

$$-\frac{Ab^2}{6x^6}-\frac{b(2Ac+bB)}{4x^4}-\frac{c(Ac+2bB)}{2x^2}+Bc^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^11,x]
```

```
[Out] -(A*b^2)/(6*x^6) - (b*(b*B + 2*A*c))/(4*x^4) - (c*(2*b*B + A*c))/(2*x^2) +
B*c^2*Log[x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
```

```
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^2}{x^{11}}\,dx &= \int \frac{\left(A+Bx^2\right)\left(b+cx^2\right)^2}{x^7}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(A+Bx)(b+cx)^2}{x^4}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{Ab^2}{x^4}+\frac{b(bB+2Ac)}{x^3}+\frac{c(2bB+Ac)}{x^2}+\frac{Bc^2}{x}\right)dx,x,x^2\right)\\ &= -\frac{Ab^2}{6x^6}-\frac{b(bB+2Ac)}{4x^4}-\frac{c(2bB+Ac)}{2x^2}+Bc^2\log(x)\end{aligned}$$

Mathematica [A] time = 0.0261184, size = 53, normalized size = 1.04

$$Bc^2\log(x)-\frac{2A\left(b^2+3bcx^2+3c^2x^4\right)+3bBx^2\left(b+4cx^2\right)}{12x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^11,x]
```

```
[Out] -(3*b*B*x^2*(b + 4*c*x^2) + 2*A*(b^2 + 3*b*c*x^2 + 3*c^2*x^4))/(12*x^6) + B
*c^2*Log[x]
```

Maple [A] time = 0.005, size = 52, normalized size = 1.

$$Bc^2\ln(x)-\frac{Abc}{2\,x^4}-\frac{Bb^2}{4\,x^4}-\frac{Ac^2}{2\,x^2}-\frac{Bbc}{x^2}-\frac{Ab^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^11,x)
```

```
[Out] B*c^2*ln(x)-1/2*b/x^4*A*c-1/4*b^2/x^4*B-1/2*c^2/x^2*A-c/x^2*B*b-1/6*A*b^2/x
^6
```

Maxima [A] time = 1.12125, size = 74, normalized size = 1.45

$$\frac{1}{2}\,Bc^2\log\left(x^2\right)-\frac{6\left(2\,Bbc+Ac^2\right)x^4+2\,Ab^2+3\left(Bb^2+2\,Abc\right)x^2}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^11,x, algorithm="maxima")
```

```
[Out] 1/2*B*c^2*log(x^2) - 1/12*(6*(2*B*b*c + A*c^2)*x^4 + 2*A*b^2 + 3*(B*b^2 + 2
*A*b*c)*x^2)/x^6
```

Fricas [A] time = 0.470435, size = 127, normalized size = 2.49

$$\frac{12\,Bc^2x^6\log\left(x\right)-6\left(2\,Bbc+Ac^2\right)x^4-2\,Ab^2-3\left(Bb^2+2\,Abc\right)x^2}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^11,x, algorithm="fricas")
```

```
[Out] 1/12*(12*B*c^2*x^6*log(x) - 6*(2*B*b*c + A*c^2)*x^4 - 2*A*b^2 - 3*(B*b^2 +
2*A*b*c)*x^2)/x^6
```

Sympy [A] time = 1.16495, size = 53, normalized size = 1.04

$$Bc^2\log\left(x\right)-\frac{2Ab^2+x^4\left(6Ac^2+12Bbc\right)+x^2\left(6Abc+3Bb^2\right)}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**11,x)
```

```
[Out] B*c**2*log(x) - (2*A*b**2 + x**4*(6*A*c**2 + 12*B*b*c) + x**2*(6*A*b*c + 3*
B*b**2))/(12*x**6)
```

Giac [A] time = 1.25292, size = 89, normalized size = 1.75

$$\frac{1}{2}\,Bc^2\log\left(x^2\right)-\frac{11\,Bc^2x^6+12\,Bbcx^4+6\,Ac^2x^4+3\,Bb^2x^2+6\,Abcx^2+2\,Ab^2}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^11,x, algorithm="giac")
```

```
[Out] 1/2*B*c^2*log(x^2) - 1/12*(11*B*c^2*x^6 + 12*B*b*c*x^4 + 6*A*c^2*x^4 + 3*B*
b^2*x^2 + 6*A*b*c*x^2 + 2*A*b^2)/x^6
```

3.24 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{12}}\,dx$

Optimal. Leaf size=53

$$-\frac{Ab^2}{7x^7}-\frac{b(2Ac+bB)}{5x^5}-\frac{c(Ac+2bB)}{3x^3}-\frac{Bc^2}{x}$$

```
[Out] -(A*b^2)/(7*x^7) - (b*(b*B + 2*A*c))/(5*x^5) - (c*(2*b*B + A*c))/(3*x^3) -
(B*c^2)/x
```

Rubi [A] time = 0.0358008, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1584, 448}

$$-\frac{Ab^2}{7x^7}-\frac{b(2Ac+bB)}{5x^5}-\frac{c(Ac+2bB)}{3x^3}-\frac{Bc^2}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^12,x]
```

```
[Out] -(A*b^2)/(7*x^7) - (b*(b*B + 2*A*c))/(5*x^5) - (c*(2*b*B + A*c))/(3*x^3) -
(B*c^2)/x
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^2}{x^{12}}\,dx = \int \frac{\left(A+Bx^2\right)\left(b+cx^2\right)^2}{x^8}\,dx$$
$$= \int \left(\frac{Ab^2}{x^8}+\frac{b(bB+2Ac)}{x^6}+\frac{c(2bB+Ac)}{x^4}+\frac{Bc^2}{x^2}\right)dx$$
$$= -\frac{Ab^2}{7x^7}-\frac{b(bB+2Ac)}{5x^5}-\frac{c(2bB+Ac)}{3x^3}-\frac{Bc^2}{x}$$

Mathematica [A] time = 0.0168103, size = 59, normalized size = 1.11

$$\frac{b^2(-B)-2Abc}{5x^5}-\frac{Ab^2}{7x^7}+\frac{-Ac^2-2bBc}{3x^3}-\frac{Bc^2}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^12,x]
```

```
[Out] -(A*b^2)/(7*x^7) + (-(b^2*B) - 2*A*b*c)/(5*x^5) + (-2*b*B*c - A*c^2)/(3*x^3
) - (B*c^2)/x
```

Maple [A] time = 0.006, size = 48, normalized size = 0.9

$$-\frac{Ab^2}{7\,x^7}-\frac{b\,(2\,Ac+Bb)}{5\,x^5}-\frac{c\,(Ac+2\,Bb)}{3\,x^3}-\frac{Bc^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^12,x)
```

```
[Out] -1/7*A*b^2/x^7-1/5*b*(2*A*c+B*b)/x^5-1/3*c*(A*c+2*B*b)/x^3-B*c^2/x
```

Maxima [A] time = 1.16759, size = 72, normalized size = 1.36

$$-\frac{105\,Bc^2x^6+35\left(2\,Bbc+Ac^2\right)x^4+15\,Ab^2+21\left(Bb^2+2\,Abc\right)x^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^12,x, algorithm="maxima")
```

```
[Out] -1/105*(105*B*c^2*x^6 + 35*(2*B*b*c + A*c^2)*x^4 + 15*A*b^2 + 21*(B*b^2 + 2
*A*b*c)*x^2)/x^7
```

Fricas [A] time = 0.479817, size = 126, normalized size = 2.38

$$-\frac{105\,Bc^2x^6 + 35\left(2\,Bbc + Ac^2\right)x^4 + 15\,Ab^2 + 21\left(Bb^2 + 2\,Abc\right)x^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^12,x, algorithm="fricas")
```

```
[Out] -1/105*(105*B*c^2*x^6 + 35*(2*B*b*c + A*c^2)*x^4 + 15*A*b^2 + 21*(B*b^2 + 2
*A*b*c)*x^2)/x^7
```

Sympy [A] time = 1.16941, size = 56, normalized size = 1.06

$$-\frac{15Ab^2 + 105Bc^2x^6 + x^4\left(35Ac^2 + 70Bbc\right) + x^2\left(42Abc + 21Bb^2\right)}{105x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**12,x)
```

```
[Out] -(15*A*b**2 + 105*B*c**2*x**6 + x**4*(35*A*c**2 + 70*B*b*c) + x**2*(42*A*b*
c + 21*B*b**2))/(105*x**7)
```

Giac [A] time = 1.24865, size = 74, normalized size = 1.4

$$-\frac{105\,Bc^2x^6 + 70\,Bbcx^4 + 35\,Ac^2x^4 + 21\,Bb^2x^2 + 42\,Abcx^2 + 15\,Ab^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^12,x, algorithm="giac")
```

```
[Out] -1/105*(105*B*c^2*x^6 + 70*B*b*c*x^4 + 35*A*c^2*x^4 + 21*B*b^2*x^2 + 42*A*b
*c*x^2 + 15*A*b^2)/x^7
```

3.25 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^2}\,dx$

Optimal. Leaf size=75

$$\frac{1}{7}b^2x^7(3Ac+bB)+\frac{1}{5}Ab^3x^5+\frac{1}{11}c^2x^{11}(Ac+3bB)+\frac{1}{3}bcx^9(Ac+bB)+\frac{1}{13}Bc^3x^{13}$$

```
[Out] (A*b^3*x^5)/5 + (b^2*(b*B + 3*A*c)*x^7)/7 + (b*c*(b*B + A*c)*x^9)/3 + (c^2*
(3*b*B + A*c)*x^11)/11 + (B*c^3*x^13)/13
```

Rubi [A] time = 0.0643977, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1584, 448}

$$\frac{1}{7}b^2x^7(3Ac+bB)+\frac{1}{5}Ab^3x^5+\frac{1}{11}c^2x^{11}(Ac+3bB)+\frac{1}{3}bcx^9(Ac+bB)+\frac{1}{13}Bc^3x^{13}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^2,x]
```

```
[Out] (A*b^3*x^5)/5 + (b^2*(b*B + 3*A*c)*x^7)/7 + (b*c*(b*B + A*c)*x^9)/3 + (c^2*
(3*b*B + A*c)*x^11)/11 + (B*c^3*x^13)/13
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^3}{x^2}\,dx = \int x^4\left(A+Bx^2\right)\left(b+cx^2\right)^3\,dx$$
$$= \int \left(Ab^3x^4 + b^2(bB+3Ac)x^6 + 3bc(bB+Ac)x^8 + c^2(3bB+Ac)x^{10} + Bc^3x^{12}\right)dx$$
$$= \frac{1}{5}Ab^3x^5 + \frac{1}{7}b^2(bB+3Ac)x^7 + \frac{1}{3}bc(bB+Ac)x^9 + \frac{1}{11}c^2(3bB+Ac)x^{11} + \frac{1}{13}Bc^3x^{13}$$

Mathematica [A] time = 0.0137938, size = 75, normalized size = 1.

$$\frac{1}{7}b^2x^7(3Ac+bB) + \frac{1}{5}Ab^3x^5 + \frac{1}{11}c^2x^{11}(Ac+3bB) + \frac{1}{3}bcx^9(Ac+bB) + \frac{1}{13}Bc^3x^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^2,x]
```

```
[Out] (A*b^3*x^5)/5 + (b^2*(b*B + 3*A*c)*x^7)/7 + (b*c*(b*B + A*c)*x^9)/3 + (c^2*
(3*b*B + A*c)*x^11)/11 + (B*c^3*x^13)/13
```

Maple [A] time = 0.001, size = 76, normalized size = 1.

$$\frac{Bc^3x^{13}}{13} + \frac{\left(Ac^3+3\,Bbc^2\right)x^{11}}{11} + \frac{\left(3\,Abc^2+3\,Bb^2c\right)x^9}{9} + \frac{\left(3\,Ab^2c+Bb^3\right)x^7}{7} + \frac{Ab^3x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^2,x)
```

```
[Out] 1/13*B*c^3*x^13+1/11*(A*c^3+3*B*b*c^2)*x^11+1/9*(3*A*b*c^2+3*B*b^2*c)*x^9+1
/7*(3*A*b^2*c+B*b^3)*x^7+1/5*A*b^3*x^5
```

Maxima [A] time = 1.08715, size = 99, normalized size = 1.32

$$\frac{1}{13}\,Bc^3x^{13} + \frac{1}{11}\left(3\,Bbc^2+Ac^3\right)x^{11} + \frac{1}{3}\left(Bb^2c+Abc^2\right)x^9 + \frac{1}{5}\,Ab^3x^5 + \frac{1}{7}\left(Bb^3+3\,Ab^2c\right)x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^2,x, algorithm="maxima")
```

```
[Out] 1/13*B*c^3*x^13 + 1/11*(3*B*b*c^2 + A*c^3)*x^11 + 1/3*(B*b^2*c + A*b*c^2)*x
^9 + 1/5*A*b^3*x^5 + 1/7*(B*b^3 + 3*A*b^2*c)*x^7
```

Fricas [A] time = 0.496914, size = 169, normalized size = 2.25

$$\frac{1}{13}Bc^3x^{13} + \frac{1}{11}\left(3Bbc^2 + Ac^3\right)x^{11} + \frac{1}{3}\left(Bb^2c + Abc^2\right)x^9 + \frac{1}{5}Ab^3x^5 + \frac{1}{7}\left(Bb^3 + 3Ab^2c\right)x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^2,x, algorithm="fricas")
```

```
[Out] 1/13*B*c^3*x^13 + 1/11*(3*B*b*c^2 + A*c^3)*x^11 + 1/3*(B*b^2*c + A*b*c^2)*x
^9 + 1/5*A*b^3*x^5 + 1/7*(B*b^3 + 3*A*b^2*c)*x^7
```

Sympy [A] time = 0.075443, size = 80, normalized size = 1.07

$$\frac{Ab^3x^5}{5} + \frac{Bc^3x^{13}}{13} + x^{11}\left(\frac{Ac^3}{11} + \frac{3Bbc^2}{11}\right) + x^9\left(\frac{Abc^2}{3} + \frac{Bb^2c}{3}\right) + x^7\left(\frac{3Ab^2c}{7} + \frac{Bb^3}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**2,x)
```

```
[Out] A*b**3*x**5/5 + B*c**3*x**13/13 + x**11*(A*c**3/11 + 3*B*b*c**2/11) + x**9*
(A*b*c**2/3 + B*b**2*c/3) + x**7*(3*A*b**2*c/7 + B*b**3/7)
```

Giac [A] time = 1.17397, size = 104, normalized size = 1.39

$$\frac{1}{13}Bc^3x^{13} + \frac{3}{11}Bbc^2x^{11} + \frac{1}{11}Ac^3x^{11} + \frac{1}{3}Bb^2cx^9 + \frac{1}{3}Abc^2x^9 + \frac{1}{7}Bb^3x^7 + \frac{3}{7}Ab^2cx^7 + \frac{1}{5}Ab^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^2,x, algorithm="giac")
```

```
[Out] 1/13*B*c^3*x^13 + 3/11*B*b*c^2*x^11 + 1/11*A*c^3*x^11 + 1/3*B*b^2*c*x^9 + 1
/3*A*b*c^2*x^9 + 1/7*B*b^3*x^7 + 3/7*A*b^2*c*x^7 + 1/5*A*b^3*x^5
```

3.26 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^3}\,dx$

Optimal. Leaf size=68

$$-\frac{\left(b+cx^2\right)^5(2bB-Ac)}{10c^3}+\frac{b\left(b+cx^2\right)^4(bB-Ac)}{8c^3}+\frac{B\left(b+cx^2\right)^6}{12c^3}$$

```
[Out] (b*(b*B - A*c)*(b + c*x^2)^4)/(8*c^3) - ((2*b*B - A*c)*(b + c*x^2)^5)/(10*c
^3) + (B*(b + c*x^2)^6)/(12*c^3)
```

Rubi [A] time = 0.134347, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 446, 76}

$$-\frac{\left(b+cx^2\right)^5(2bB-Ac)}{10c^3}+\frac{b\left(b+cx^2\right)^4(bB-Ac)}{8c^3}+\frac{B\left(b+cx^2\right)^6}{12c^3}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^3,x]
```

```
[Out] (b*(b*B - A*c)*(b + c*x^2)^4)/(8*c^3) - ((2*b*B - A*c)*(b + c*x^2)^5)/(10*c
^3) + (B*(b + c*x^2)^6)/(12*c^3)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
```

```
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^3}{x^3}\,dx &= \int x^3\left(A+Bx^2\right)\left(b+cx^2\right)^3\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int x(A+Bx)(b+cx)^3\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{b(bB-Ac)(b+cx)^3}{c^2}+\frac{(-2bB+Ac)(b+cx)^4}{c^2}+\frac{B(b+cx)^5}{c^2}\right)dx,x,x^2\right)\\ &= \frac{b(bB-Ac)\left(b+cx^2\right)^4}{8c^3}-\frac{(2bB-Ac)\left(b+cx^2\right)^5}{10c^3}+\frac{B\left(b+cx^2\right)^6}{12c^3}\end{aligned}$$

Mathematica [A] time = 0.0186531, size = 69, normalized size = 1.01

$$\frac{1}{120}x^4\left(20b^2x^2(3Ac+bB)+30Ab^3+12c^2x^6(Ac+3bB)+45bcx^4(Ac+bB)+10Bc^3x^8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^3,x]
```

```
[Out] (x^4*(30*A*b^3 + 20*b^2*(b*B + 3*A*c)*x^2 + 45*b*c*(b*B + A*c)*x^4 + 12*c^2
*(3*b*B + A*c)*x^6 + 10*B*c^3*x^8))/120
```

Maple [A] time = 0.001, size = 76, normalized size = 1.1

$$\frac{Bc^3x^{12}}{12}+\frac{\left(Ac^3+3\,Bbc^2\right)x^{10}}{10}+\frac{\left(3\,Abc^2+3\,Bb^2c\right)x^8}{8}+\frac{\left(3\,Ab^2c+Bb^3\right)x^6}{6}+\frac{Ab^3x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^3,x)
```

```
[Out] 1/12*B*c^3*x^12+1/10*(A*c^3+3*B*b*c^2)*x^10+1/8*(3*A*b*c^2+3*B*b^2*c)*x^8+1
/6*(3*A*b^2*c+B*b^3)*x^6+1/4*A*b^3*x^4
```

Maxima [A] time = 1.17759, size = 99, normalized size = 1.46

$$\frac{1}{12}\,Bc^3x^{12} + \frac{1}{10}\left(3\,Bbc^2 + Ac^3\right)x^{10} + \frac{3}{8}\left(Bb^2c + Abc^2\right)x^8 + \frac{1}{4}\,Ab^3x^4 + \frac{1}{6}\left(Bb^3 + 3\,Ab^2c\right)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^3,x, algorithm="maxima")
```

```
[Out] 1/12*B*c^3*x^12 + 1/10*(3*B*b*c^2 + A*c^3)*x^10 + 3/8*(B*b^2*c + A*b*c^2)*x
^8 + 1/4*A*b^3*x^4 + 1/6*(B*b^3 + 3*A*b^2*c)*x^6
```

Fricas [A] time = 0.468432, size = 169, normalized size = 2.49

$$\frac{1}{12}\,Bc^3x^{12} + \frac{1}{10}\left(3\,Bbc^2 + Ac^3\right)x^{10} + \frac{3}{8}\left(Bb^2c + Abc^2\right)x^8 + \frac{1}{4}\,Ab^3x^4 + \frac{1}{6}\left(Bb^3 + 3\,Ab^2c\right)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/12*B*c^3*x^12 + 1/10*(3*B*b*c^2 + A*c^3)*x^10 + 3/8*(B*b^2*c + A*b*c^2)*x
^8 + 1/4*A*b^3*x^4 + 1/6*(B*b^3 + 3*A*b^2*c)*x^6
```

Sympy [A] time = 0.075238, size = 82, normalized size = 1.21

$$\frac{Ab^3x^4}{4} + \frac{Bc^3x^{12}}{12} + x^{10}\left(\frac{Ac^3}{10} + \frac{3Bbc^2}{10}\right) + x^8\left(\frac{3Abc^2}{8} + \frac{3Bb^2c}{8}\right) + x^6\left(\frac{Ab^2c}{2} + \frac{Bb^3}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**3,x)
```

```
[Out] A*b**3*x**4/4 + B*c**3*x**12/12 + x**10*(A*c**3/10 + 3*B*b*c**2/10) + x**8*
(3*A*b*c**2/8 + 3*B*b**2*c/8) + x**6*(A*b**2*c/2 + B*b**3/6)
```

Giac [A] time = 1.22454, size = 104, normalized size = 1.53

$$\frac{1}{12}Bc^3x^{12} + \frac{3}{10}Bbc^2x^{10} + \frac{1}{10}Ac^3x^{10} + \frac{3}{8}Bb^2cx^8 + \frac{3}{8}Abc^2x^8 + \frac{1}{6}Bb^3x^6 + \frac{1}{2}Ab^2cx^6 + \frac{1}{4}Ab^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^3,x, algorithm="giac")
```

```
[Out] 1/12*B*c^3*x^12 + 3/10*B*b*c^2*x^10 + 1/10*A*c^3*x^10 + 3/8*B*b^2*c*x^8 + 3
/8*A*b*c^2*x^8 + 1/6*B*b^3*x^6 + 1/2*A*b^2*c*x^6 + 1/4*A*b^3*x^4
```

3.27 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^4}\,dx$

Optimal. Leaf size=75

$$\frac{1}{5}b^2x^5(3Ac+bB)+\frac{1}{3}Ab^3x^3+\frac{1}{9}c^2x^9(Ac+3bB)+\frac{3}{7}bcx^7(Ac+bB)+\frac{1}{11}Bc^3x^{11}$$

```
[Out] (A*b^3*x^3)/3 + (b^2*(b*B + 3*A*c)*x^5)/5 + (3*b*c*(b*B + A*c)*x^7)/7 + (c^
2*(3*b*B + A*c)*x^9)/9 + (B*c^3*x^11)/11
```

Rubi [A] time = 0.0552321, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1584, 448}

$$\frac{1}{5}b^2x^5(3Ac+bB)+\frac{1}{3}Ab^3x^3+\frac{1}{9}c^2x^9(Ac+3bB)+\frac{3}{7}bcx^7(Ac+bB)+\frac{1}{11}Bc^3x^{11}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^4,x]
```

```
[Out] (A*b^3*x^3)/3 + (b^2*(b*B + 3*A*c)*x^5)/5 + (3*b*c*(b*B + A*c)*x^7)/7 + (c^
2*(3*b*B + A*c)*x^9)/9 + (B*c^3*x^11)/11
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^4}\,dx = \int x^2(A+Bx^2)(b+cx^2)^3\,dx$$
$$= \int \left(Ab^3x^2+b^2(bB+3Ac)x^4+3bc(bB+Ac)x^6+c^2(3bB+Ac)x^8+Bc^3x^{10}\right)dx$$
$$= \frac{1}{3}Ab^3x^3+\frac{1}{5}b^2(bB+3Ac)x^5+\frac{3}{7}bc(bB+Ac)x^7+\frac{1}{9}c^2(3bB+Ac)x^9+\frac{1}{11}Bc^3x^{11}$$

Mathematica [A] time = 0.0118488, size = 75, normalized size = 1.

$$\frac{1}{5}b^2x^5(3Ac+bB)+\frac{1}{3}Ab^3x^3+\frac{1}{9}c^2x^9(Ac+3bB)+\frac{3}{7}bcx^7(Ac+bB)+\frac{1}{11}Bc^3x^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^4,x]
```

```
[Out] (A*b^3*x^3)/3 + (b^2*(b*B + 3*A*c)*x^5)/5 + (3*b*c*(b*B + A*c)*x^7)/7 + (c^
2*(3*b*B + A*c)*x^9)/9 + (B*c^3*x^11)/11
```

Maple [A] time = 0.001, size = 76, normalized size = 1.

$$\frac{Bc^3x^{11}}{11}+\frac{(Ac^3+3\,Bbc^2)\,x^9}{9}+\frac{(3\,Abc^2+3\,Bb^2c)\,x^7}{7}+\frac{(3\,Ab^2c+Bb^3)\,x^5}{5}+\frac{Ab^3x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^4,x)
```

```
[Out] 1/11*B*c^3*x^11+1/9*(A*c^3+3*B*b*c^2)*x^9+1/7*(3*A*b*c^2+3*B*b^2*c)*x^7+1/5
*(3*A*b^2*c+B*b^3)*x^5+1/3*A*b^3*x^3
```

Maxima [A] time = 1.12863, size = 99, normalized size = 1.32

$$\frac{1}{11}\,Bc^3x^{11}+\frac{1}{9}\left(3\,Bbc^2+Ac^3\right)x^9+\frac{3}{7}\left(Bb^2c+Abc^2\right)x^7+\frac{1}{3}\,Ab^3x^3+\frac{1}{5}\left(Bb^3+3\,Ab^2c\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^4,x, algorithm="maxima")
```

```
[Out] 1/11*B*c^3*x^11 + 1/9*(3*B*b*c^2 + A*c^3)*x^9 + 3/7*(B*b^2*c + A*b*c^2)*x^7
 + 1/3*A*b^3*x^3 + 1/5*(B*b^3 + 3*A*b^2*c)*x^5
```

Fricas [A] time = 0.442372, size = 166, normalized size = 2.21

$$\frac{1}{11}Bc^3x^{11} + \frac{1}{9}\left(3Bbc^2 + Ac^3\right)x^9 + \frac{3}{7}\left(Bb^2c + Abc^2\right)x^7 + \frac{1}{3}Ab^3x^3 + \frac{1}{5}\left(Bb^3 + 3Ab^2c\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^4,x, algorithm="fricas")
```

```
[Out] 1/11*B*c^3*x^11 + 1/9*(3*B*b*c^2 + A*c^3)*x^9 + 3/7*(B*b^2*c + A*b*c^2)*x^7
 + 1/3*A*b^3*x^3 + 1/5*(B*b^3 + 3*A*b^2*c)*x^5
```

Sympy [A] time = 0.074925, size = 82, normalized size = 1.09

$$\frac{Ab^3x^3}{3} + \frac{Bc^3x^{11}}{11} + x^9\left(\frac{Ac^3}{9} + \frac{Bbc^2}{3}\right) + x^7\left(\frac{3Abc^2}{7} + \frac{3Bb^2c}{7}\right) + x^5\left(\frac{3Ab^2c}{5} + \frac{Bb^3}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**4,x)
```

```
[Out] A*b**3*x**3/3 + B*c**3*x**11/11 + x**9*(A*c**3/9 + B*b*c**2/3) + x**7*(3*A*
b*c**2/7 + 3*B*b**2*c/7) + x**5*(3*A*b**2*c/5 + B*b**3/5)
```

Giac [A] time = 1.21324, size = 104, normalized size = 1.39

$$\frac{1}{11}Bc^3x^{11} + \frac{1}{3}Bbc^2x^9 + \frac{1}{9}Ac^3x^9 + \frac{3}{7}Bb^2cx^7 + \frac{3}{7}Abc^2x^7 + \frac{1}{5}Bb^3x^5 + \frac{3}{5}Ab^2cx^5 + \frac{1}{3}Ab^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^4,x, algorithm="giac")
```

```
[Out] 1/11*B*c^3*x^11 + 1/3*B*b*c^2*x^9 + 1/9*A*c^3*x^9 + 3/7*B*b^2*c*x^7 + 3/7*A
*b*c^2*x^7 + 1/5*B*b^3*x^5 + 3/5*A*b^2*c*x^5 + 1/3*A*b^3*x^3
```

3.28 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^5} \, dx$

Optimal. Leaf size=42

$$\frac{B(b+cx^2)^5}{10c^2} - \frac{(b+cx^2)^4 (bB-Ac)}{8c^2}$$

```
[Out] -((b*B - A*c)*(b + c*x^2)^4)/(8*c^2) + (B*(b + c*x^2)^5)/(10*c^2)
```

Rubi [A] time = 0.0706387, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 444, 43}

$$\frac{B(b+cx^2)^5}{10c^2} - \frac{(b+cx^2)^4 (bB-Ac)}{8c^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^5,x]
```

```
[Out] -((b*B - A*c)*(b + c*x^2)^4)/(8*c^2) + (B*(b + c*x^2)^5)/(10*c^2)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^3}{x^5}\,dx &= \int x\left(A+Bx^2\right)\left(b+cx^2\right)^3\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int (A+Bx)(b+cx)^3\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{(-bB+Ac)(b+cx)^3}{c}+\frac{B(b+cx)^4}{c}\right)dx,x,x^2\right)\\ &= -\frac{(bB-Ac)\left(b+cx^2\right)^4}{8c^2}+\frac{B\left(b+cx^2\right)^5}{10c^2}\end{aligned}$$

Mathematica [A] time = 0.018608, size = 69, normalized size = 1.64

$$\frac{1}{40}x^2\left(10b^2x^2(3Ac+bB)+20Ab^3+5c^2x^6(Ac+3bB)+20bcx^4(Ac+bB)+4Bc^3x^8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^5,x]
```

```
[Out] (x^2*(20*A*b^3 + 10*b^2*(b*B + 3*A*c)*x^2 + 20*b*c*(b*B + A*c)*x^4 + 5*c^2*
(3*b*B + A*c)*x^6 + 4*B*c^3*x^8))/40
```

Maple [A] time = 0.001, size = 76, normalized size = 1.8

$$\frac{Bc^3x^{10}}{10}+\frac{\left(Ac^3+3\,Bbc^2\right)x^8}{8}+\frac{\left(3\,Abc^2+3\,Bb^2c\right)x^6}{6}+\frac{\left(3\,Ab^2c+Bb^3\right)x^4}{4}+\frac{Ab^3x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^5,x)
```

```
[Out] 1/10*B*c^3*x^10+1/8*(A*c^3+3*B*b*c^2)*x^8+1/6*(3*A*b*c^2+3*B*b^2*c)*x^6+1/4
*(3*A*b^2*c+B*b^3)*x^4+1/2*A*b^3*x^2
```

Maxima [A] time = 1.13199, size = 99, normalized size = 2.36

$$\frac{1}{10}\,Bc^3x^{10} + \frac{1}{8}\left(3\,Bbc^2 + Ac^3\right)x^8 + \frac{1}{2}\left(Bb^2c + Abc^2\right)x^6 + \frac{1}{2}\,Ab^3x^2 + \frac{1}{4}\left(Bb^3 + 3\,Ab^2c\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^5,x, algorithm="maxima")
```

```
[Out] 1/10*B*c^3*x^10 + 1/8*(3*B*b*c^2 + A*c^3)*x^8 + 1/2*(B*b^2*c + A*b*c^2)*x^6
 + 1/2*A*b^3*x^2 + 1/4*(B*b^3 + 3*A*b^2*c)*x^4
```

Fricas [A] time = 0.482166, size = 166, normalized size = 3.95

$$\frac{1}{10}\,Bc^3x^{10} + \frac{1}{8}\left(3\,Bbc^2 + Ac^3\right)x^8 + \frac{1}{2}\left(Bb^2c + Abc^2\right)x^6 + \frac{1}{2}\,Ab^3x^2 + \frac{1}{4}\left(Bb^3 + 3\,Ab^2c\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^5,x, algorithm="fricas")
```

```
[Out] 1/10*B*c^3*x^10 + 1/8*(3*B*b*c^2 + A*c^3)*x^8 + 1/2*(B*b^2*c + A*b*c^2)*x^6
 + 1/2*A*b^3*x^2 + 1/4*(B*b^3 + 3*A*b^2*c)*x^4
```

Sympy [B] time = 0.075218, size = 80, normalized size = 1.9

$$\frac{Ab^3x^2}{2} + \frac{Bc^3x^{10}}{10} + x^8\left(\frac{Ac^3}{8} + \frac{3Bbc^2}{8}\right) + x^6\left(\frac{Abc^2}{2} + \frac{Bb^2c}{2}\right) + x^4\left(\frac{3Ab^2c}{4} + \frac{Bb^3}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**5,x)
```

```
[Out] A*b**3*x**2/2 + B*c**3*x**10/10 + x**8*(A*c**3/8 + 3*B*b*c**2/8) + x**6*(A*
b*c**2/2 + B*b**2*c/2) + x**4*(3*A*b**2*c/4 + B*b**3/4)
```

Giac [B] time = 1.20848, size = 104, normalized size = 2.48

$$\frac{1}{10} Bc^3x^{10} + \frac{3}{8} Bbc^2x^8 + \frac{1}{8} Ac^3x^8 + \frac{1}{2} Bb^2cx^6 + \frac{1}{2} Abc^2x^6 + \frac{1}{4} Bb^3x^4 + \frac{3}{4} Ab^2cx^4 + \frac{1}{2} Ab^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^5,x, algorithm="giac")
```

```
[Out] 1/10*B*c^3*x^10 + 3/8*B*b*c^2*x^8 + 1/8*A*c^3*x^8 + 1/2*B*b^2*c*x^6 + 1/2*A
*b*c^2*x^6 + 1/4*B*b^3*x^4 + 3/4*A*b^2*c*x^4 + 1/2*A*b^3*x^2
```

3.29 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^6} dx$

Optimal. Leaf size=70

$$\frac{1}{3}b^2x^3(3Ac+bB)+Ab^3x+\frac{1}{7}c^2x^7(Ac+3bB)+\frac{3}{5}bcx^5(Ac+bB)+\frac{1}{9}Bc^3x^9$$

```
[Out] A*b^3*x + (b^2*(b*B + 3*A*c)*x^3)/3 + (3*b*c*(b*B + A*c)*x^5)/5 + (c^2*(3*b
*B + A*c)*x^7)/7 + (B*c^3*x^9)/9
```

Rubi [A] time = 0.0416425, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1584, 373}

$$\frac{1}{3}b^2x^3(3Ac+bB)+Ab^3x+\frac{1}{7}c^2x^7(Ac+3bB)+\frac{3}{5}bcx^5(Ac+bB)+\frac{1}{9}Bc^3x^9$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^6,x]
```

```
[Out] A*b^3*x + (b^2*(b*B + 3*A*c)*x^3)/3 + (3*b*c*(b*B + A*c)*x^5)/5 + (c^2*(3*b
*B + A*c)*x^7)/7 + (B*c^3*x^9)/9
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^6} dx = \int (A+Bx^2)(b+cx^2)^3 dx$$
$$= \int \left(Ab^3 + b^2(bB+3Ac)x^2 + 3bc(bB+Ac)x^4 + c^2(3bB+Ac)x^6 + Bc^3x^8\right) dx$$
$$= Ab^3x + \frac{1}{3}b^2(bB+3Ac)x^3 + \frac{3}{5}bc(bB+Ac)x^5 + \frac{1}{7}c^2(3bB+Ac)x^7 + \frac{1}{9}Bc^3x^9$$

Mathematica [A] time = 0.0107059, size = 70, normalized size = 1.

$$\frac{1}{3}b^2x^3(3Ac+bB) + Ab^3x + \frac{1}{7}c^2x^7(Ac+3bB) + \frac{3}{5}bcx^5(Ac+bB) + \frac{1}{9}Bc^3x^9$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^6,x]
```

```
[Out] A*b^3*x + (b^2*(b*B + 3*A*c)*x^3)/3 + (3*b*c*(b*B + A*c)*x^5)/5 + (c^2*(3*b
*B + A*c)*x^7)/7 + (B*c^3*x^9)/9
```

Maple [A] time = 0.001, size = 73, normalized size = 1.

$$\frac{Bc^3x^9}{9} + \frac{(Ac^3+3Bbc^2)x^7}{7} + \frac{(3Abc^2+3Bb^2c)x^5}{5} + \frac{(3Ab^2c+Bb^3)x^3}{3} + Ab^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^6,x)
```

```
[Out] 1/9*B*c^3*x^9+1/7*(A*c^3+3*B*b*c^2)*x^7+1/5*(3*A*b*c^2+3*B*b^2*c)*x^5+1/3*(
3*A*b^2*c+B*b^3)*x^3+A*b^3*x
```

Maxima [A] time = 1.08482, size = 95, normalized size = 1.36

$$\frac{1}{9}Bc^3x^9 + \frac{1}{7}(3Bbc^2+Ac^3)x^7 + \frac{3}{5}(Bb^2c+Abc^2)x^5 + Ab^3x + \frac{1}{3}(Bb^3+3Ab^2c)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^6,x, algorithm="maxima")
```

```
[Out] 1/9*B*c^3*x^9 + 1/7*(3*B*b*c^2 + A*c^3)*x^7 + 3/5*(B*b^2*c + A*b*c^2)*x^5 +
 A*b^3*x + 1/3*(B*b^3 + 3*A*b^2*c)*x^3
```

Fricas [A] time = 0.510629, size = 155, normalized size = 2.21

$$\frac{1}{9}Bc^3x^9 + \frac{1}{7}\left(3Bbc^2 + Ac^3\right)x^7 + \frac{3}{5}\left(Bb^2c + Abc^2\right)x^5 + Ab^3x + \frac{1}{3}\left(Bb^3 + 3Ab^2c\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^6,x, algorithm="fricas")
```

```
[Out] 1/9*B*c^3*x^9 + 1/7*(3*B*b*c^2 + A*c^3)*x^7 + 3/5*(B*b^2*c + A*b*c^2)*x^5 +
 A*b^3*x + 1/3*(B*b^3 + 3*A*b^2*c)*x^3
```

Sympy [A] time = 0.074467, size = 76, normalized size = 1.09

$$Ab^3x + \frac{Bc^3x^9}{9} + x^7\left(\frac{Ac^3}{7} + \frac{3Bbc^2}{7}\right) + x^5\left(\frac{3Abc^2}{5} + \frac{3Bb^2c}{5}\right) + x^3\left(Ab^2c + \frac{Bb^3}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**6,x)
```

```
[Out] A*b**3*x + B*c**3*x**9/9 + x**7*(A*c**3/7 + 3*B*b*c**2/7) + x**5*(3*A*b*c**
2/5 + 3*B*b**2*c/5) + x**3*(A*b**2*c + B*b**3/3)
```

Giac [A] time = 1.19985, size = 99, normalized size = 1.41

$$\frac{1}{9}Bc^3x^9 + \frac{3}{7}Bbc^2x^7 + \frac{1}{7}Ac^3x^7 + \frac{3}{5}Bb^2cx^5 + \frac{3}{5}Abc^2x^5 + \frac{1}{3}Bb^3x^3 + Ab^2cx^3 + Ab^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^6,x, algorithm="giac")
```

```
[Out] 1/9*B*c^3*x^9 + 3/7*B*b*c^2*x^7 + 1/7*A*c^3*x^7 + 3/5*B*b^2*c*x^5 + 3/5*A*b
*c^2*x^5 + 1/3*B*b^3*x^3 + A*b^2*c*x^3 + A*b^3*x
```

3.30 $\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^3}{x^7}\,dx$

Optimal. Leaf size=60

$$\frac{3}{2}Ab^2cx^2+Ab^3\log(x)+\frac{3}{4}Abc^2x^4+\frac{1}{6}Ac^3x^6+\frac{B\left(b+cx^2\right)^4}{8c}$$

```
[Out] (3*A*b^2*c*x^2)/2 + (3*A*b*c^2*x^4)/4 + (A*c^3*x^6)/6 + (B*(b + c*x^2)^4)/(
8*c) + A*b^3*Log[x]
```

Rubi [A] time = 0.0528008, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1584, 446, 80, 43}

$$\frac{3}{2}Ab^2cx^2+Ab^3\log(x)+\frac{3}{4}Abc^2x^4+\frac{1}{6}Ac^3x^6+\frac{B\left(b+cx^2\right)^4}{8c}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^7,x]
```

```
[Out] (3*A*b^2*c*x^2)/2 + (3*A*b*c^2*x^4)/4 + (A*c^3*x^6)/6 + (B*(b + c*x^2)^4)/(
8*c) + A*b^3*Log[x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
```

```
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^3}{x^7}\,dx &= \int \frac{\left(A+Bx^2\right)\left(b+cx^2\right)^3}{x}\,dx \\
&= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{(A+Bx)(b+cx)^3}{x}\,dx, x, x^2\right) \\
&= \frac{B\left(b+cx^2\right)^4}{8c}+\frac{1}{2}A\,\mathrm{Subst}\left(\int \frac{(b+cx)^3}{x}\,dx, x, x^2\right) \\
&= \frac{B\left(b+cx^2\right)^4}{8c}+\frac{1}{2}A\,\mathrm{Subst}\left(\int \left(3b^2c+\frac{b^3}{x}+3bc^2x+c^3x^2\right)dx, x, x^2\right) \\
&= \frac{3}{2}Ab^2cx^2+\frac{3}{4}Abc^2x^4+\frac{1}{6}Ac^3x^6+\frac{B\left(b+cx^2\right)^4}{8c}+Ab^3\log(x)
\end{aligned}
$$

Mathematica [A] time = 0.0204635, size = 71, normalized size = 1.18

$$
\frac{1}{2}b^2x^2(3Ac+bB)+Ab^3\log(x)+\frac{1}{6}c^2x^6(Ac+3bB)+\frac{3}{4}bcx^4(Ac+bB)+\frac{1}{8}Bc^3x^8
$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^7,x]
```

```
[Out] (b^2*(b*B + 3*A*c)*x^2)/2 + (3*b*c*(b*B + A*c)*x^4)/4 + (c^2*(3*b*B + A*c)*
x^6)/6 + (B*c^3*x^8)/8 + A*b^3*Log[x]
```

Maple [A] time = 0.001, size = 76, normalized size = 1.3

$$\frac{Bc^3x^8}{8} + \frac{Ac^3x^6}{6} + \frac{Bx^6bc^2}{2} + \frac{3\,Abc^2x^4}{4} + \frac{3\,Bx^4b^2c}{4} + \frac{3\,Ab^2cx^2}{2} + \frac{Bx^2b^3}{2} + Ab^3\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^7,x)
```

```
[Out] 1/8*B*c^3*x^8+1/6*A*c^3*x^6+1/2*B*x^6*b*c^2+3/4*A*b*c^2*x^4+3/4*B*x^4*b^2*c
+3/2*A*b^2*c*x^2+1/2*B*x^2*b^3+A*b^3*ln(x)
```

Maxima [A] time = 1.14292, size = 100, normalized size = 1.67

$$\frac{1}{8}Bc^3x^8 + \frac{1}{6}\left(3\,Bbc^2 + Ac^3\right)x^6 + \frac{3}{4}\left(Bb^2c + Abc^2\right)x^4 + \frac{1}{2}Ab^3\log\left(x^2\right) + \frac{1}{2}\left(Bb^3 + 3\,Ab^2c\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^7,x, algorithm="maxima")
```

```
[Out] 1/8*B*c^3*x^8 + 1/6*(3*B*b*c^2 + A*c^3)*x^6 + 3/4*(B*b^2*c + A*b*c^2)*x^4 +
 1/2*A*b^3*log(x^2) + 1/2*(B*b^3 + 3*A*b^2*c)*x^2
```

Fricas [A] time = 0.552365, size = 162, normalized size = 2.7

$$\frac{1}{8}Bc^3x^8 + \frac{1}{6}\left(3\,Bbc^2 + Ac^3\right)x^6 + \frac{3}{4}\left(Bb^2c + Abc^2\right)x^4 + Ab^3\log(x) + \frac{1}{2}\left(Bb^3 + 3\,Ab^2c\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^7,x, algorithm="fricas")
```

```
[Out] 1/8*B*c^3*x^8 + 1/6*(3*B*b*c^2 + A*c^3)*x^6 + 3/4*(B*b^2*c + A*b*c^2)*x^4 +
 A*b^3*log(x) + 1/2*(B*b^3 + 3*A*b^2*c)*x^2
```

Sympy [A] time = 0.324098, size = 80, normalized size = 1.33

$$Ab^3\log(x) + \frac{Bc^3x^8}{8} + x^6\left(\frac{Ac^3}{6} + \frac{Bbc^2}{2}\right) + x^4\left(\frac{3Abc^2}{4} + \frac{3Bb^2c}{4}\right) + x^2\left(\frac{3Ab^2c}{2} + \frac{Bb^3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**7,x)
```

```
[Out] A*b**3*log(x) + B*c**3*x**8/8 + x**6*(A*c**3/6 + B*b*c**2/2) + x**4*(3*A*b*
c**2/4 + 3*B*b**2*c/4) + x**2*(3*A*b**2*c/2 + B*b**3/2)
```

Giac [A] time = 1.23927, size = 105, normalized size = 1.75

$$\frac{1}{8}Bc^3x^8 + \frac{1}{2}Bbc^2x^6 + \frac{1}{6}Ac^3x^6 + \frac{3}{4}Bb^2cx^4 + \frac{3}{4}Abc^2x^4 + \frac{1}{2}Bb^3x^2 + \frac{3}{2}Ab^2cx^2 + \frac{1}{2}Ab^3\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^7,x, algorithm="giac")
```

```
[Out] 1/8*B*c^3*x^8 + 1/2*B*b*c^2*x^6 + 1/6*A*c^3*x^6 + 3/4*B*b^2*c*x^4 + 3/4*A*b
*c^2*x^4 + 1/2*B*b^3*x^2 + 3/2*A*b^2*c*x^2 + 1/2*A*b^3*log(x^2)
```

3.31 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^8} \, dx$

Optimal. Leaf size=65

$$b^2x(3Ac+bB)-\frac{Ab^3}{x}+\frac{1}{5}c^2x^5(Ac+3bB)+bcx^3(Ac+bB)+\frac{1}{7}Bc^3x^7$$

```
[Out] -((A*b^3)/x) + b^2*(b*B + 3*A*c)*x + b*c*(b*B + A*c)*x^3 + (c^2*(3*b*B + A*
c)*x^5)/5 + (B*c^3*x^7)/7
```

Rubi [A] time = 0.0454822, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1584, 448}

$$b^2x(3Ac+bB)-\frac{Ab^3}{x}+\frac{1}{5}c^2x^5(Ac+3bB)+bcx^3(Ac+bB)+\frac{1}{7}Bc^3x^7$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^8,x]
```

```
[Out] -((A*b^3)/x) + b^2*(b*B + 3*A*c)*x + b*c*(b*B + A*c)*x^3 + (c^2*(3*b*B + A*
c)*x^5)/5 + (B*c^3*x^7)/7
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^8}\,dx = \int \frac{(A+Bx^2)(b+cx^2)^3}{x^2}\,dx$$
$$= \int \left(b^2(bB+3Ac) + \frac{Ab^3}{x^2} + 3bc(bB+Ac)x^2 + c^2(3bB+Ac)x^4 + Bc^3x^6\right) dx$$
$$= -\frac{Ab^3}{x} + b^2(bB+3Ac)x + bc(bB+Ac)x^3 + \frac{1}{5}c^2(3bB+Ac)x^5 + \frac{1}{7}Bc^3x^7$$

Mathematica [A] time = 0.0222336, size = 65, normalized size = 1.

$$b^2x(3Ac+bB) - \frac{Ab^3}{x} + \frac{1}{5}c^2x^5(Ac+3bB) + bcx^3(Ac+bB) + \frac{1}{7}Bc^3x^7$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^8,x]
```

```
[Out] -((A*b^3)/x) + b^2*(b*B + 3*A*c)*x + b*c*(b*B + A*c)*x^3 + (c^2*(3*b*B + A*
c)*x^5)/5 + (B*c^3*x^7)/7
```

Maple [A] time = 0.004, size = 71, normalized size = 1.1

$$\frac{Bc^3x^7}{7} + \frac{Ax^5c^3}{5} + \frac{3\,Bx^5bc^2}{5} + Ax^3bc^2 + Bx^3b^2c + 3\,Ab^2cx + Bb^3x - \frac{Ab^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^8,x)
```

```
[Out] 1/7*B*c^3*x^7+1/5*A*x^5*c^3+3/5*B*x^5*b*c^2+A*x^3*b*c^2+B*x^3*b^2*c+3*A*b^2
*c*x+B*b^3*x-A*b^3/x
```

Maxima [A] time = 1.16804, size = 93, normalized size = 1.43

$$\frac{1}{7}\,Bc^3x^7 + \frac{1}{5}\left(3\,Bbc^2 + Ac^3\right)x^5 + \left(Bb^2c + Abc^2\right)x^3 - \frac{Ab^3}{x} + \left(Bb^3 + 3\,Ab^2c\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^8,x, algorithm="maxima")
```

```
[Out] 1/7*B*c^3*x^7 + 1/5*(3*B*b*c^2 + A*c^3)*x^5 + (B*b^2*c + A*b*c^2)*x^3 - A*b
^3/x + (B*b^3 + 3*A*b^2*c)*x
```

Fricas [A] time = 0.481409, size = 161, normalized size = 2.48

$$\frac{5\,Bc^3x^8 + 7\left(3\,Bbc^2 + Ac^3\right)x^6 + 35\left(Bb^2c + Abc^2\right)x^4 - 35\,Ab^3 + 35\left(Bb^3 + 3\,Ab^2c\right)x^2}{35\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^8,x, algorithm="fricas")
```

```
[Out] 1/35*(5*B*c^3*x^8 + 7*(3*B*b*c^2 + A*c^3)*x^6 + 35*(B*b^2*c + A*b*c^2)*x^4
- 35*A*b^3 + 35*(B*b^3 + 3*A*b^2*c)*x^2)/x
```

Sympy [A] time = 0.313686, size = 68, normalized size = 1.05

$$-\frac{Ab^3}{x} + \frac{Bc^3x^7}{7} + x^5\left(\frac{Ac^3}{5} + \frac{3Bbc^2}{5}\right) + x^3\left(Abc^2 + Bb^2c\right) + x\left(3Ab^2c + Bb^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**8,x)
```

```
[Out] -A*b**3/x + B*c**3*x**7/7 + x**5*(A*c**3/5 + 3*B*b*c**2/5) + x**3*(A*b*c**2
 + B*b**2*c) + x*(3*A*b**2*c + B*b**3)
```

Giac [A] time = 1.1633, size = 95, normalized size = 1.46

$$\frac{1}{7}\,Bc^3x^7 + \frac{3}{5}\,Bbc^2x^5 + \frac{1}{5}\,Ac^3x^5 + Bb^2cx^3 + Abc^2x^3 + Bb^3x + 3\,Ab^2cx - \frac{Ab^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^8,x, algorithm="giac")
```

```
[Out] 1/7*B*c^3*x^7 + 3/5*B*b*c^2*x^5 + 1/5*A*c^3*x^5 + B*b^2*c*x^3 + A*b*c^2*x^3
 + B*b^3*x + 3*A*b^2*c*x - A*b^3/x
```

3.32 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^9} dx$

Optimal. Leaf size=71

$$b^2\log(x)(3Ac+bB)-\frac{Ab^3}{2x^2}+\frac{1}{4}c^2x^4(Ac+3bB)+\frac{3}{2}bcx^2(Ac+bB)+\frac{1}{6}Bc^3x^6$$

```
[Out] -(A*b^3)/(2*x^2) + (3*b*c*(b*B + A*c)*x^2)/2 + (c^2*(3*b*B + A*c)*x^4)/4 +
(B*c^3*x^6)/6 + b^2*(b*B + 3*A*c)*Log[x]
```

Rubi [A] time = 0.0789122, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 446, 76}

$$b^2\log(x)(3Ac+bB)-\frac{Ab^3}{2x^2}+\frac{1}{4}c^2x^4(Ac+3bB)+\frac{3}{2}bcx^2(Ac+bB)+\frac{1}{6}Bc^3x^6$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^9,x]
```

```
[Out] -(A*b^3)/(2*x^2) + (3*b*c*(b*B + A*c)*x^2)/2 + (c^2*(3*b*B + A*c)*x^4)/4 +
(B*c^3*x^6)/6 + b^2*(b*B + 3*A*c)*Log[x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
```

```
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^3}{x^9}\,dx &= \int \frac{\left(A+Bx^2\right)\left(b+cx^2\right)^3}{x^3}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(A+Bx)(b+cx)^3}{x^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(3bc(bB+Ac)+\frac{Ab^3}{x^2}+\frac{b^2(bB+3Ac)}{x}+c^2(3bB+Ac)x+Bc^3x^2\right)dx,x,x\right.\\ &= -\frac{Ab^3}{2x^2}+\frac{3}{2}bc(bB+Ac)x^2+\frac{1}{4}c^2(3bB+Ac)x^4+\frac{1}{6}Bc^3x^6+b^2(bB+3Ac)\log(x)\end{aligned}$$

Mathematica [A] time = 0.0273874, size = 73, normalized size = 1.03

$$\log(x)\left(3Ab^2c+b^3B\right)-\frac{Ab^3}{2x^2}+\frac{1}{4}c^2x^4(Ac+3bB)+\frac{3}{2}bcx^2(Ac+bB)+\frac{1}{6}Bc^3x^6$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^9,x]
```

```
[Out] -(A*b^3)/(2*x^2) + (3*b*c*(b*B + A*c)*x^2)/2 + (c^2*(3*b*B + A*c)*x^4)/4 +
(B*c^3*x^6)/6 + (b^3*B + 3*A*b^2*c)*Log[x]
```

Maple [A] time = 0.006, size = 75, normalized size = 1.1

$$\frac{Bc^3x^6}{6}+\frac{Ax^4c^3}{4}+\frac{3\,Bx^4bc^2}{4}+\frac{3\,Ax^2bc^2}{2}+\frac{3\,Bx^2b^2c}{2}+3\,A\ln(x)\,b^2c+B\ln(x)\,b^3-\frac{Ab^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^9,x)
```

```
[Out] 1/6*B*c^3*x^6+1/4*A*x^4*c^3+3/4*B*x^4*b*c^2+3/2*A*x^2*b*c^2+3/2*B*x^2*b^2*c
+3*A*ln(x)*b^2*c+B*ln(x)*b^3-1/2*A*b^3/x^2
```

Maxima [A] time = 1.18279, size = 100, normalized size = 1.41

$$\frac{1}{6}Bc^3x^6 + \frac{1}{4}\left(3Bbc^2 + Ac^3\right)x^4 + \frac{3}{2}\left(Bb^2c + Abc^2\right)x^2 - \frac{Ab^3}{2x^2} + \frac{1}{2}\left(Bb^3 + 3Ab^2c\right)\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^9,x, algorithm="maxima")
```

```
[Out] 1/6*B*c^3*x^6 + 1/4*(3*B*b*c^2 + A*c^3)*x^4 + 3/2*(B*b^2*c + A*b*c^2)*x^2 -
 1/2*A*b^3/x^2 + 1/2*(B*b^3 + 3*A*b^2*c)*log(x^2)
```

Fricas [A] time = 0.497962, size = 171, normalized size = 2.41

$$\frac{2Bc^3x^8 + 3\left(3Bbc^2 + Ac^3\right)x^6 + 18\left(Bb^2c + Abc^2\right)x^4 - 6Ab^3 + 12\left(Bb^3 + 3Ab^2c\right)x^2\log(x)}{12x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^9,x, algorithm="fricas")
```

```
[Out] 1/12*(2*B*c^3*x^8 + 3*(3*B*b*c^2 + A*c^3)*x^6 + 18*(B*b^2*c + A*b*c^2)*x^4
- 6*A*b^3 + 12*(B*b^3 + 3*A*b^2*c)*x^2*log(x))/x^2
```

Sympy [A] time = 0.409838, size = 78, normalized size = 1.1

$$-\frac{Ab^3}{2x^2} + \frac{Bc^3x^6}{6} + b^2(3Ac + Bb)\log(x) + x^4\left(\frac{Ac^3}{4} + \frac{3Bbc^2}{4}\right) + x^2\left(\frac{3Abc^2}{2} + \frac{3Bb^2c}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**9,x)
```

```
[Out] -A*b**3/(2*x**2) + B*c**3*x**6/6 + b**2*(3*A*c + B*b)*log(x) + x**4*(A*c**3
/4 + 3*B*b*c**2/4) + x**2*(3*A*b*c**2/2 + 3*B*b**2*c/2)
```

Giac [A] time = 1.23608, size = 131, normalized size = 1.85

$$\frac{1}{6}Bc^3x^6 + \frac{3}{4}Bbc^2x^4 + \frac{1}{4}Ac^3x^4 + \frac{3}{2}Bb^2cx^2 + \frac{3}{2}Abc^2x^2 + \frac{1}{2}\left(Bb^3 + 3\,Ab^2c\right)\log\left(x^2\right) - \frac{Bb^3x^2 + 3\,Ab^2cx^2 + Ab^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^9,x, algorithm="giac")
```

```
[Out] 1/6*B*c^3*x^6 + 3/4*B*b*c^2*x^4 + 1/4*A*c^3*x^4 + 3/2*B*b^2*c*x^2 + 3/2*A*b
*c^2*x^2 + 1/2*(B*b^3 + 3*A*b^2*c)*log(x^2) - 1/2*(B*b^3*x^2 + 3*A*b^2*c*x^
2 + A*b^3)/x^2
```

3.33 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{10}} \, dx$

Optimal. Leaf size=69

$$-\frac{b^2(3Ac+bB)}{x}-\frac{Ab^3}{3x^3}+\frac{1}{3}c^2x^3(Ac+3bB)+3bcx(Ac+bB)+\frac{1}{5}Bc^3x^5$$

```
[Out] -(A*b^3)/(3*x^3) - (b^2*(b*B + 3*A*c))/x + 3*b*c*(b*B + A*c)*x + (c^2*(3*b*
B + A*c)*x^3)/3 + (B*c^3*x^5)/5
```

Rubi [A] time = 0.0495511, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1584, 448}

$$-\frac{b^2(3Ac+bB)}{x}-\frac{Ab^3}{3x^3}+\frac{1}{3}c^2x^3(Ac+3bB)+3bcx(Ac+bB)+\frac{1}{5}Bc^3x^5$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^10,x]
```

```
[Out] -(A*b^3)/(3*x^3) - (b^2*(b*B + 3*A*c))/x + 3*b*c*(b*B + A*c)*x + (c^2*(3*b*
B + A*c)*x^3)/3 + (B*c^3*x^5)/5
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{10}}\,dx = \int \frac{(A+Bx^2)(b+cx^2)^3}{x^4}\,dx$$
$$= \int \left(3bc(bB+Ac)+\frac{Ab^3}{x^4}+\frac{b^2(bB+3Ac)}{x^2}+c^2(3bB+Ac)x^2+Bc^3x^4\right)dx$$
$$= -\frac{Ab^3}{3x^3}-\frac{b^2(bB+3Ac)}{x}+3bc(bB+Ac)x+\frac{1}{3}c^2(3bB+Ac)x^3+\frac{1}{5}Bc^3x^5$$

Mathematica [A] time = 0.0228669, size = 71, normalized size = 1.03

$$\frac{b^3(-B)-3Ab^2c}{x}-\frac{Ab^3}{3x^3}+\frac{1}{3}c^2x^3(Ac+3bB)+3bcx(Ac+bB)+\frac{1}{5}Bc^3x^5$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^10,x]
```

```
[Out] -(A*b^3)/(3*x^3) + (-(b^3*B) - 3*A*b^2*c)/x + 3*b*c*(b*B + A*c)*x + (c^2*(3
*b*B + A*c)*x^3)/3 + (B*c^3*x^5)/5
```

Maple [A] time = 0.006, size = 70, normalized size = 1.

$$\frac{Bc^3x^5}{5}+\frac{Ax^3c^3}{3}+Bx^3bc^2+3\,Abc^2x+3\,Bb^2cx-\frac{Ab^3}{3\,x^3}-\frac{b^2\,(3\,Ac+Bb)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^10,x)
```

```
[Out] 1/5*B*c^3*x^5+1/3*A*x^3*c^3+B*x^3*b*c^2+3*A*b*c^2*x+3*B*b^2*c*x-1/3*A*b^3/x
^3-b^2*(3*A*c+B*b)/x
```

Maxima [A] time = 1.13104, size = 99, normalized size = 1.43

$$\frac{1}{5}\,Bc^3x^5+\frac{1}{3}\left(3\,Bbc^2+Ac^3\right)x^3+3\left(Bb^2c+Abc^2\right)x-\frac{Ab^3+3\left(Bb^3+3\,Ab^2c\right)x^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^10,x, algorithm="maxima")
```

```
[Out] 1/5*B*c^3*x^5 + 1/3*(3*B*b*c^2 + A*c^3)*x^3 + 3*(B*b^2*c + A*b*c^2)*x - 1/3
*(A*b^3 + 3*(B*b^3 + 3*A*b^2*c)*x^2)/x^3
```

Fricas [A] time = 0.459542, size = 162, normalized size = 2.35

$$\frac{3\,Bc^3x^8 + 5\left(3\,Bbc^2 + Ac^3\right)x^6 + 45\left(Bb^2c + Abc^2\right)x^4 - 5\,Ab^3 - 15\left(Bb^3 + 3\,Ab^2c\right)x^2}{15\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^10,x, algorithm="fricas")
```

```
[Out] 1/15*(3*B*c^3*x^8 + 5*(3*B*b*c^2 + A*c^3)*x^6 + 45*(B*b^2*c + A*b*c^2)*x^4
- 5*A*b^3 - 15*(B*b^3 + 3*A*b^2*c)*x^2)/x^3
```

Sympy [A] time = 0.427238, size = 73, normalized size = 1.06

$$\frac{Bc^3x^5}{5} + x^3\left(\frac{Ac^3}{3} + Bbc^2\right) + x\left(3Abc^2 + 3Bb^2c\right) - \frac{Ab^3 + x^2\left(9Ab^2c + 3Bb^3\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**10,x)
```

```
[Out] B*c**3*x**5/5 + x**3*(A*c**3/3 + B*b*c**2) + x*(3*A*b*c**2 + 3*B*b**2*c) -
(A*b**3 + x**2*(9*A*b**2*c + 3*B*b**3))/(3*x**3)
```

Giac [A] time = 1.18482, size = 100, normalized size = 1.45

$$\frac{1}{5}\,Bc^3x^5 + Bbc^2x^3 + \frac{1}{3}\,Ac^3x^3 + 3\,Bb^2cx + 3\,Abc^2x - \frac{3\,Bb^3x^2 + 9\,Ab^2cx^2 + Ab^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^10,x, algorithm="giac")
```

```
[Out] 1/5*B*c^3*x^5 + B*b*c^2*x^3 + 1/3*A*c^3*x^3 + 3*B*b^2*c*x + 3*A*b*c^2*x - 1
/3*(3*B*b^3*x^2 + 9*A*b^2*c*x^2 + A*b^3)/x^3
```

3.34 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{11}}\,dx$

Optimal. Leaf size=72

$$-\frac{b^2(3Ac+bB)}{2x^2}-\frac{Ab^3}{4x^4}+\frac{1}{2}c^2x^2(Ac+3bB)+3bc\log(x)(Ac+bB)+\frac{1}{4}Bc^3x^4$$

```
[Out] -(A*b^3)/(4*x^4) - (b^2*(b*B + 3*A*c))/(2*x^2) + (c^2*(3*b*B + A*c)*x^2)/2
+ (B*c^3*x^4)/4 + 3*b*c*(b*B + A*c)*Log[x]
```

Rubi [A] time = 0.0719043, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 446, 76}

$$-\frac{b^2(3Ac+bB)}{2x^2}-\frac{Ab^3}{4x^4}+\frac{1}{2}c^2x^2(Ac+3bB)+3bc\log(x)(Ac+bB)+\frac{1}{4}Bc^3x^4$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^11,x]
```

```
[Out] -(A*b^3)/(4*x^4) - (b^2*(b*B + 3*A*c))/(2*x^2) + (c^2*(3*b*B + A*c)*x^2)/2
+ (B*c^3*x^4)/4 + 3*b*c*(b*B + A*c)*Log[x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
```

```
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{11}}\,dx &= \int \frac{(A+Bx^2)(b+cx^2)^3}{x^5}\,dx\\ &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(A+Bx)(b+cx)^3}{x^3}\,dx,x,x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(c^2(3bB+Ac)+\frac{Ab^3}{x^3}+\frac{b^2(bB+3Ac)}{x^2}+\frac{3bc(bB+Ac)}{x}+Bc^3x\right)dx,x,x^2\right)\\ &= -\frac{Ab^3}{4x^4}-\frac{b^2(bB+3Ac)}{2x^2}+\frac{1}{2}c^2(3bB+Ac)x^2+\frac{1}{4}Bc^3x^4+3bc(bB+Ac)\log(x)\end{aligned}$$

Mathematica [A] time = 0.0304726, size = 73, normalized size = 1.01

$$\frac{Bx^2\left(-2b^3+6bc^2x^4+c^3x^6\right)-A\left(6b^2cx^2+b^3-2c^3x^6\right)}{4x^4}+3bc\log(x)(Ac+bB)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^11,x]
```

```
[Out] (-(A*(b^3 + 6*b^2*c*x^2 - 2*c^3*x^6)) + B*x^2*(-2*b^3 + 6*b*c^2*x^4 + c^3*x
^6))/(4*x^4) + 3*b*c*(b*B + A*c)*Log[x]
```

Maple [A] time = 0.007, size = 76, normalized size = 1.1

$$\frac{Bc^3x^4}{4}+\frac{Ax^2c^3}{2}+\frac{3\,Bx^2bc^2}{2}+3\,A\ln(x)\,bc^2+3\,B\ln(x)\,b^2c-\frac{Ab^3}{4\,x^4}-\frac{3\,Ab^2c}{2\,x^2}-\frac{Bb^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^11,x)
```

```
[Out] 1/4*B*c^3*x^4+1/2*A*x^2*c^3+3/2*B*x^2*b*c^2+3*A*ln(x)*b*c^2+3*B*ln(x)*b^2*c
-1/4*A*b^3/x^4-3/2*b^2/x^2*A*c-1/2*b^3/x^2*B
```

Maxima [A] time = 1.14411, size = 103, normalized size = 1.43

$$\frac{1}{4}Bc^3x^4 + \frac{1}{2}\left(3Bbc^2 + Ac^3\right)x^2 + \frac{3}{2}\left(Bb^2c + Abc^2\right)\log\left(x^2\right) - \frac{Ab^3 + 2\left(Bb^3 + 3Ab^2c\right)x^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^11,x, algorithm="maxima")
```

```
[Out] 1/4*B*c^3*x^4 + 1/2*(3*B*b*c^2 + A*c^3)*x^2 + 3/2*(B*b^2*c + A*b*c^2)*log(x
^2) - 1/4*(A*b^3 + 2*(B*b^3 + 3*A*b^2*c)*x^2)/x^4
```

Fricas [A] time = 0.4301, size = 163, normalized size = 2.26

$$\frac{Bc^3x^8 + 2\left(3Bbc^2 + Ac^3\right)x^6 + 12\left(Bb^2c + Abc^2\right)x^4\log(x) - Ab^3 - 2\left(Bb^3 + 3Ab^2c\right)x^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^11,x, algorithm="fricas")
```

```
[Out] 1/4*(B*c^3*x^8 + 2*(3*B*b*c^2 + A*c^3)*x^6 + 12*(B*b^2*c + A*b*c^2)*x^4*log
(x) - A*b^3 - 2*(B*b^3 + 3*A*b^2*c)*x^2)/x^4
```

Sympy [A] time = 0.72699, size = 73, normalized size = 1.01

$$\frac{Bc^3x^4}{4} + 3bc\left(Ac + Bb\right)\log(x) + x^2\left(\frac{Ac^3}{2} + \frac{3Bbc^2}{2}\right) - \frac{Ab^3 + x^2\left(6Ab^2c + 2Bb^3\right)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**11,x)
```

```
[Out] B*c**3*x**4/4 + 3*b*c*(A*c + B*b)*log(x) + x**2*(A*c**3/2 + 3*B*b*c**2/2) -
 (A*b**3 + x**2*(6*A*b**2*c + 2*B*b**3))/(4*x**4)
```

Giac [A] time = 1.2913, size = 132, normalized size = 1.83

$$\frac{1}{4}Bc^3x^4 + \frac{3}{2}Bbc^2x^2 + \frac{1}{2}Ac^3x^2 + \frac{3}{2}\left(Bb^2c + Abc^2\right)\log\left(x^2\right) - \frac{9\,Bb^2cx^4 + 9\,Abc^2x^4 + 2\,Bb^3x^2 + 6\,Ab^2cx^2 + Ab^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^11,x, algorithm="giac")
```

```
[Out] 1/4*B*c^3*x^4 + 3/2*B*b*c^2*x^2 + 1/2*A*c^3*x^2 + 3/2*(B*b^2*c + A*b*c^2)*l
og(x^2) - 1/4*(9*B*b^2*c*x^4 + 9*A*b*c^2*x^4 + 2*B*b^3*x^2 + 6*A*b^2*c*x^2
+ A*b^3)/x^4
```

3.35 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{12}} \, dx$

Optimal. Leaf size=68

$$-\frac{b^2(3Ac+bB)}{3x^3}-\frac{Ab^3}{5x^5}+c^2x(Ac+3bB)-\frac{3bc(Ac+bB)}{x}+\frac{1}{3}Bc^3x^3$$

```
[Out] -(A*b^3)/(5*x^5) - (b^2*(b*B + 3*A*c))/(3*x^3) - (3*b*c*(b*B + A*c))/x + c^
2*(3*b*B + A*c)*x + (B*c^3*x^3)/3
```

Rubi [A] time = 0.0495744, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1584, 448}

$$-\frac{b^2(3Ac+bB)}{3x^3}-\frac{Ab^3}{5x^5}+c^2x(Ac+3bB)-\frac{3bc(Ac+bB)}{x}+\frac{1}{3}Bc^3x^3$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^12,x]
```

```
[Out] -(A*b^3)/(5*x^5) - (b^2*(b*B + 3*A*c))/(3*x^3) - (3*b*c*(b*B + A*c))/x + c^
2*(3*b*B + A*c)*x + (B*c^3*x^3)/3
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{12}}\,dx &= \int \frac{(A+Bx^2)(b+cx^2)^3}{x^6}\,dx \\
&= \int \left(c^2(3bB+Ac)+\frac{Ab^3}{x^6}+\frac{b^2(bB+3Ac)}{x^4}+\frac{3bc(bB+Ac)}{x^2}+Bc^3x^2\right)dx \\
&= -\frac{Ab^3}{5x^5}-\frac{b^2(bB+3Ac)}{3x^3}-\frac{3bc(bB+Ac)}{x}+c^2(3bB+Ac)x+\frac{1}{3}Bc^3x^3
\end{aligned}
$$

Mathematica [A] time = 0.0244062, size = 68, normalized size = 1.

$$
-\frac{b^2(3Ac+bB)}{3x^3}-\frac{Ab^3}{5x^5}+c^2x(Ac+3bB)-\frac{3bc(Ac+bB)}{x}+\frac{1}{3}Bc^3x^3
$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^12,x]
```

```
[Out] -(A*b^3)/(5*x^5) - (b^2*(b*B + 3*A*c))/(3*x^3) - (3*b*c*(b*B + A*c))/x + c^
2*(3*b*B + A*c)*x + (B*c^3*x^3)/3
```

Maple [A] time = 0.005, size = 64, normalized size = 0.9

$$
\frac{Bc^3x^3}{3}+Ac^3x+3\,Bbc^2x-\frac{b^2\,(3\,Ac+Bb)}{3\,x^3}-\frac{Ab^3}{5\,x^5}-3\,\frac{bc\,(Ac+Bb)}{x}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^12,x)
```

```
[Out] 1/3*B*c^3*x^3+A*c^3*x+3*B*b*c^2*x-1/3*b^2*(3*A*c+B*b)/x^3-1/5*A*b^3/x^5-3*b
*c*(A*c+B*b)/x
```

Maxima [A] time = 1.10651, size = 99, normalized size = 1.46

$$
\frac{1}{3}\,Bc^3x^3+\left(3\,Bbc^2+Ac^3\right)x-\frac{45\left(Bb^2c+Abc^2\right)x^4+3\,Ab^3+5\left(Bb^3+3\,Ab^2c\right)x^2}{15\,x^5}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^12,x, algorithm="maxima")
```

```
[Out] 1/3*B*c^3*x^3 + (3*B*b*c^2 + A*c^3)*x - 1/15*(45*(B*b^2*c + A*b*c^2)*x^4 +
3*A*b^3 + 5*(B*b^3 + 3*A*b^2*c)*x^2)/x^5
```

Fricas [A] time = 0.472586, size = 162, normalized size = 2.38

$$\frac{5\,Bc^3x^8 + 15\left(3\,Bbc^2 + Ac^3\right)x^6 - 45\left(Bb^2c + Abc^2\right)x^4 - 3\,Ab^3 - 5\left(Bb^3 + 3\,Ab^2c\right)x^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^12,x, algorithm="fricas")
```

```
[Out] 1/15*(5*B*c^3*x^8 + 15*(3*B*b*c^2 + A*c^3)*x^6 - 45*(B*b^2*c + A*b*c^2)*x^4
 - 3*A*b^3 - 5*(B*b^3 + 3*A*b^2*c)*x^2)/x^5
```

Sympy [A] time = 0.789304, size = 75, normalized size = 1.1

$$\frac{Bc^3x^3}{3} + x\left(Ac^3 + 3Bbc^2\right) - \frac{3Ab^3 + x^4\left(45Abc^2 + 45Bb^2c\right) + x^2\left(15Ab^2c + 5Bb^3\right)}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**12,x)
```

```
[Out] B*c**3*x**3/3 + x*(A*c**3 + 3*B*b*c**2) - (3*A*b**3 + x**4*(45*A*b*c**2 + 4
5*B*b**2*c) + x**2*(15*A*b**2*c + 5*B*b**3))/(15*x**5)
```

Giac [A] time = 1.23963, size = 101, normalized size = 1.49

$$\frac{1}{3}\,Bc^3x^3 + 3\,Bbc^2x + Ac^3x - \frac{45\,Bb^2cx^4 + 45\,Abc^2x^4 + 5\,Bb^3x^2 + 15\,Ab^2cx^2 + 3\,Ab^3}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^12,x, algorithm="giac")
```

```
[Out] 1/3*B*c^3*x^3 + 3*B*b*c^2*x + A*c^3*x - 1/15*(45*B*b^2*c*x^4 + 45*A*b*c^2*x
^4 + 5*B*b^3*x^2 + 15*A*b^2*c*x^2 + 3*A*b^3)/x^5
```

3.36 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{13}} dx$

Optimal. Leaf size=71

$$-\frac{b^2(3Ac+bB)}{4x^4}-\frac{Ab^3}{6x^6}+c^2\log(x)(Ac+3bB)-\frac{3bc(Ac+bB)}{2x^2}+\frac{1}{2}Bc^3x^2$$

```
[Out] -(A*b^3)/(6*x^6) - (b^2*(b*B + 3*A*c))/(4*x^4) - (3*b*c*(b*B + A*c))/(2*x^2
) + (B*c^3*x^2)/2 + c^2*(3*b*B + A*c)*Log[x]
```

Rubi [A] time = 0.0640838, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 446, 76}

$$-\frac{b^2(3Ac+bB)}{4x^4}-\frac{Ab^3}{6x^6}+c^2\log(x)(Ac+3bB)-\frac{3bc(Ac+bB)}{2x^2}+\frac{1}{2}Bc^3x^2$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^13,x]
```

```
[Out] -(A*b^3)/(6*x^6) - (b^2*(b*B + 3*A*c))/(4*x^4) - (3*b*c*(b*B + A*c))/(2*x^2
) + (B*c^3*x^2)/2 + c^2*(3*b*B + A*c)*Log[x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
```

```
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{13}}\,dx &= \int \frac{(A+Bx^2)(b+cx^2)^3}{x^7}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(A+Bx)(b+cx)^3}{x^4}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(Bc^3+\frac{Ab^3}{x^4}+\frac{b^2(bB+3Ac)}{x^3}+\frac{3bc(bB+Ac)}{x^2}+\frac{c^2(3bB+Ac)}{x}\right)dx,x,x^2\right)\\ &= -\frac{Ab^3}{6x^6}-\frac{b^2(bB+3Ac)}{4x^4}-\frac{3bc(bB+Ac)}{2x^2}+\frac{1}{2}Bc^3x^2+c^2(3bB+Ac)\log(x)\end{aligned}$$

Mathematica [A] time = 0.037125, size = 71, normalized size = 1.

$$-\frac{b^2(3Ac+bB)}{4x^4}-\frac{Ab^3}{6x^6}+c^2\log(x)(Ac+3bB)-\frac{3bc(Ac+bB)}{2x^2}+\frac{1}{2}Bc^3x^2$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^13,x]
```

```
[Out] -(A*b^3)/(6*x^6) - (b^2*(b*B + 3*A*c))/(4*x^4) - (3*b*c*(b*B + A*c))/(2*x^2
) + (B*c^3*x^2)/2 + c^2*(3*b*B + A*c)*Log[x]
```

Maple [A] time = 0.006, size = 75, normalized size = 1.1

$$\frac{Bc^3x^2}{2}+A\ln(x)c^3+3B\ln(x)bc^2-\frac{3Ab^2c}{4x^4}-\frac{Bb^3}{4x^4}-\frac{3Abc^2}{2x^2}-\frac{3Bb^2c}{2x^2}-\frac{Ab^3}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^13,x)
```

```
[Out] 1/2*B*c^3*x^2+A*ln(x)*c^3+3*B*ln(x)*b*c^2-3/4*b^2/x^4*A*c-1/4*b^3/x^4*B-3/2
*b*c^2/x^2*A-3/2*b^2*c/x^2*B-1/6*A*b^3/x^6
```

Maxima [A] time = 1.05915, size = 104, normalized size = 1.46

$$\frac{1}{2}\,Bc^3x^2 + \frac{1}{2}\left(3\,Bbc^2 + Ac^3\right)\log\left(x^2\right) - \frac{18\left(Bb^2c + Abc^2\right)x^4 + 2\,Ab^3 + 3\left(Bb^3 + 3\,Ab^2c\right)x^2}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^13,x, algorithm="maxima")
```

```
[Out] 1/2*B*c^3*x^2 + 1/2*(3*B*b*c^2 + A*c^3)*log(x^2) - 1/12*(18*(B*b^2*c + A*b*
c^2)*x^4 + 2*A*b^3 + 3*(B*b^3 + 3*A*b^2*c)*x^2)/x^6
```

Fricas [A] time = 0.469042, size = 171, normalized size = 2.41

$$\frac{6\,Bc^3x^8 + 12\left(3\,Bbc^2 + Ac^3\right)x^6\log\left(x\right) - 18\left(Bb^2c + Abc^2\right)x^4 - 2\,Ab^3 - 3\left(Bb^3 + 3\,Ab^2c\right)x^2}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^13,x, algorithm="fricas")
```

```
[Out] 1/12*(6*B*c^3*x^8 + 12*(3*B*b*c^2 + A*c^3)*x^6*log(x) - 18*(B*b^2*c + A*b*c
^2)*x^4 - 2*A*b^3 - 3*(B*b^3 + 3*A*b^2*c)*x^2)/x^6
```

Sympy [A] time = 1.41572, size = 75, normalized size = 1.06

$$\frac{Bc^3x^2}{2} + c^2\left(Ac + 3Bb\right)\log\left(x\right) - \frac{2Ab^3 + x^4\left(18Abc^2 + 18Bb^2c\right) + x^2\left(9Ab^2c + 3Bb^3\right)}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**13,x)
```

```
[Out] B*c**3*x**2/2 + c**2*(A*c + 3*B*b)*log(x) - (2*A*b**3 + x**4*(18*A*b*c**2 +
 18*B*b**2*c) + x**2*(9*A*b**2*c + 3*B*b**3))/(12*x**6)
```

Giac [A] time = 1.18589, size = 134, normalized size = 1.89

$$\frac{1}{2}Bc^3x^2 + \frac{1}{2}\left(3Bbc^2 + Ac^3\right)\log\left(x^2\right) - \frac{33Bbc^2x^6 + 11Ac^3x^6 + 18Bb^2cx^4 + 18Abc^2x^4 + 3Bb^3x^2 + 9Ab^2cx^2 + 2Ab^3}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^13,x, algorithm="giac")
```

```
[Out] 1/2*B*c^3*x^2 + 1/2*(3*B*b*c^2 + A*c^3)*log(x^2) - 1/12*(33*B*b*c^2*x^6 + 1
1*A*c^3*x^6 + 18*B*b^2*c*x^4 + 18*A*b*c^2*x^4 + 3*B*b^3*x^2 + 9*A*b^2*c*x^2
 + 2*A*b^3)/x^6
```

3.37 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{14}} \, dx$

Optimal. Leaf size=66

$$-\frac{b^2(3Ac+bB)}{5x^5}-\frac{Ab^3}{7x^7}-\frac{c^2(Ac+3bB)}{x}-\frac{bc(Ac+bB)}{x^3}+Bc^3x$$

```
[Out] -(A*b^3)/(7*x^7) - (b^2*(b*B + 3*A*c))/(5*x^5) - (b*c*(b*B + A*c))/x^3 - (c
^2*(3*b*B + A*c))/x + B*c^3*x
```

Rubi [A] time = 0.0495761, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1584, 448}

$$-\frac{b^2(3Ac+bB)}{5x^5}-\frac{Ab^3}{7x^7}-\frac{c^2(Ac+3bB)}{x}-\frac{bc(Ac+bB)}{x^3}+Bc^3x$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^14,x]
```

```
[Out] -(A*b^3)/(7*x^7) - (b^2*(b*B + 3*A*c))/(5*x^5) - (b*c*(b*B + A*c))/x^3 - (c
^2*(3*b*B + A*c))/x + B*c^3*x
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^3}{x^{14}}\,dx = \int \frac{\left(A+Bx^2\right)\left(b+cx^2\right)^3}{x^8}\,dx$$
$$= \int \left(Bc^3 + \frac{Ab^3}{x^8} + \frac{b^2(bB+3Ac)}{x^6} + \frac{3bc(bB+Ac)}{x^4} + \frac{c^2(3bB+Ac)}{x^2}\right) dx$$
$$= -\frac{Ab^3}{7x^7} - \frac{b^2(bB+3Ac)}{5x^5} - \frac{bc(bB+Ac)}{x^3} - \frac{c^2(3bB+Ac)}{x} + Bc^3x$$

Mathematica [A] time = 0.0274184, size = 66, normalized size = 1.

$$-\frac{b^2(3Ac+bB)}{5x^5} - \frac{Ab^3}{7x^7} - \frac{c^2(Ac+3bB)}{x} - \frac{bc(Ac+bB)}{x^3} + Bc^3x$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^14,x]
```

```
[Out] -(A*b^3)/(7*x^7) - (b^2*(b*B + 3*A*c))/(5*x^5) - (b*c*(b*B + A*c))/x^3 - (c
^2*(3*b*B + A*c))/x + B*c^3*x
```

Maple [A] time = 0.006, size = 63, normalized size = 1.

$$-\frac{Ab^3}{7\,x^7} - \frac{b^2\,(3\,Ac+Bb)}{5\,x^5} - \frac{bc\,(Ac+Bb)}{x^3} - \frac{c^2\,(Ac+3\,Bb)}{x} + Bc^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^14,x)
```

```
[Out] -1/7*A*b^3/x^7-1/5*b^2*(3*A*c+B*b)/x^5-b*c*(A*c+B*b)/x^3-c^2*(A*c+3*B*b)/x+
B*c^3*x
```

Maxima [A] time = 1.19158, size = 99, normalized size = 1.5

$$Bc^3x - \frac{35\left(3\,Bbc^2+Ac^3\right)x^6 + 35\left(Bb^2c+Abc^2\right)x^4 + 5\,Ab^3 + 7\left(Bb^3+3\,Ab^2c\right)x^2}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^14,x, algorithm="maxima")
```

```
[Out] B*c^3*x - 1/35*(35*(3*B*b*c^2 + A*c^3)*x^6 + 35*(B*b^2*c + A*b*c^2)*x^4 + 5
*A*b^3 + 7*(B*b^3 + 3*A*b^2*c)*x^2)/x^7
```

Fricas [A] time = 0.483685, size = 163, normalized size = 2.47

$$\frac{35\,Bc^3x^8 - 35\left(3\,Bbc^2 + Ac^3\right)x^6 - 35\left(Bb^2c + Abc^2\right)x^4 - 5\,Ab^3 - 7\left(Bb^3 + 3\,Ab^2c\right)x^2}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^14,x, algorithm="fricas")
```

```
[Out] 1/35*(35*B*c^3*x^8 - 35*(3*B*b*c^2 + A*c^3)*x^6 - 35*(B*b^2*c + A*b*c^2)*x^
4 - 5*A*b^3 - 7*(B*b^3 + 3*A*b^2*c)*x^2)/x^7
```

Sympy [A] time = 1.60242, size = 75, normalized size = 1.14

$$Bc^3x - \frac{5Ab^3 + x^6\left(35Ac^3 + 105Bbc^2\right) + x^4\left(35Abc^2 + 35Bb^2c\right) + x^2\left(21Ab^2c + 7Bb^3\right)}{35x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**14,x)
```

```
[Out] B*c**3*x - (5*A*b**3 + x**6*(35*A*c**3 + 105*B*b*c**2) + x**4*(35*A*b*c**2
+ 35*B*b**2*c) + x**2*(21*A*b**2*c + 7*B*b**3))/(35*x**7)
```

Giac [A] time = 1.27154, size = 104, normalized size = 1.58

$$Bc^3x - \frac{105\,Bbc^2x^6 + 35\,Ac^3x^6 + 35\,Bb^2cx^4 + 35\,Abc^2x^4 + 7\,Bb^3x^2 + 21\,Ab^2cx^2 + 5\,Ab^3}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^14,x, algorithm="giac")
```

```
[Out] B*c^3*x - 1/35*(105*B*b*c^2*x^6 + 35*A*c^3*x^6 + 35*B*b^2*c*x^4 + 35*A*b*c^
2*x^4 + 7*B*b^3*x^2 + 21*A*b^2*c*x^2 + 5*A*b^3)/x^7
```

3.38 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{15}} \, dx$

Optimal. Leaf size=63

$$-\frac{A(b+cx^2)^4}{8bx^8}-\frac{3b^2Bc}{4x^4}-\frac{b^3B}{6x^6}-\frac{3bBc^2}{2x^2}+Bc^3\log(x)$$

```
[Out] -(b^3*B)/(6*x^6) - (3*b^2*B*c)/(4*x^4) - (3*b*B*c^2)/(2*x^2) - (A*(b + c*x^
2)^4)/(8*b*x^8) + B*c^3*Log[x]
```

Rubi [A] time = 0.0467981, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1584, 446, 78, 43}

$$-\frac{A(b+cx^2)^4}{8bx^8}-\frac{3b^2Bc}{4x^4}-\frac{b^3B}{6x^6}-\frac{3bBc^2}{2x^2}+Bc^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^15,x]
```



```
[Out] -(b^3*B)/(6*x^6) - (3*b^2*B*c)/(4*x^4) - (3*b*B*c^2)/(2*x^2) - (A*(b + c*x^
2)^4)/(8*b*x^8) + B*c^3*Log[x]
```


Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
```

```
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{15}}\,dx &= \int \frac{(A+Bx^2)(b+cx^2)^3}{x^9}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(A+Bx)(b+cx)^3}{x^5}\,dx, x, x^2\right)\\ &= -\frac{A(b+cx^2)^4}{8bx^8}+\frac{1}{2}B\operatorname{Subst}\left(\int \frac{(b+cx)^3}{x^4}\,dx, x, x^2\right)\\ &= -\frac{A(b+cx^2)^4}{8bx^8}+\frac{1}{2}B\operatorname{Subst}\left(\int \left(\frac{b^3}{x^4}+\frac{3b^2c}{x^3}+\frac{3bc^2}{x^2}+\frac{c^3}{x}\right)dx, x, x^2\right)\\ &= -\frac{b^3B}{6x^6}-\frac{3b^2Bc}{4x^4}-\frac{3bBc^2}{2x^2}-\frac{A(b+cx^2)^4}{8bx^8}+Bc^3\log(x)\end{aligned}$$

Mathematica [A] time = 0.0327315, size = 77, normalized size = 1.22

$$Bc^3\log(x)-\frac{3A\left(4b^2cx^2+b^3+6bc^2x^4+4c^3x^6\right)+2bBx^2\left(2b^2+9bcx^2+18c^2x^4\right)}{24x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^15,x]
```

```
[Out] -(2*b*B*x^2*(2*b^2 + 9*b*c*x^2 + 18*c^2*x^4) + 3*A*(b^3 + 4*b^2*c*x^2 + 6*b
*c^2*x^4 + 4*c^3*x^6))/(24*x^8) + B*c^3*Log[x]
```

Maple [A] time = 0.006, size = 76, normalized size = 1.2

$$Bc^3\ln(x) - \frac{3Abc^2}{4x^4} - \frac{3Bb^2c}{4x^4} - \frac{Ac^3}{2x^2} - \frac{3Bbc^2}{2x^2} - \frac{Ab^3}{8x^8} - \frac{Ab^2c}{2x^6} - \frac{Bb^3}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^15,x)
```

```
[Out] B*c^3*ln(x)-3/4*b*c^2/x^4*A-3/4*b^2*B*c/x^4-1/2*c^3/x^2*A-3/2*b*B*c^2/x^2-1
/8*A*b^3/x^8-1/2*b^2/x^6*A*c-1/6*b^3*B/x^6
```

Maxima [A] time = 1.17294, size = 104, normalized size = 1.65

$$\frac{1}{2}Bc^3\log\left(x^2\right) - \frac{12\left(3Bbc^2 + Ac^3\right)x^6 + 18\left(Bb^2c + Abc^2\right)x^4 + 3Ab^3 + 4\left(Bb^3 + 3Ab^2c\right)x^2}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^15,x, algorithm="maxima")
```

```
[Out] 1/2*B*c^3*log(x^2) - 1/24*(12*(3*B*b*c^2 + A*c^3)*x^6 + 18*(B*b^2*c + A*b*c
^2)*x^4 + 3*A*b^3 + 4*(B*b^3 + 3*A*b^2*c)*x^2)/x^8
```

Fricas [A] time = 0.502951, size = 173, normalized size = 2.75

$$\frac{24Bc^3x^8\log(x) - 12\left(3Bbc^2 + Ac^3\right)x^6 - 18\left(Bb^2c + Abc^2\right)x^4 - 3Ab^3 - 4\left(Bb^3 + 3Ab^2c\right)x^2}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^15,x, algorithm="fricas")
```

```
[Out] 1/24*(24*B*c^3*x^8*log(x) - 12*(3*B*b*c^2 + A*c^3)*x^6 - 18*(B*b^2*c + A*b*
c^2)*x^4 - 3*A*b^3 - 4*(B*b^3 + 3*A*b^2*c)*x^2)/x^8
```

Sympy [A] time = 2.66069, size = 76, normalized size = 1.21

$$Bc^3\log(x) - \frac{3Ab^3 + x^6\left(12Ac^3 + 36Bbc^2\right) + x^4\left(18Abc^2 + 18Bb^2c\right) + x^2\left(12Ab^2c + 4Bb^3\right)}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**15,x)
```

```
[Out] B*c**3*log(x) - (3*A*b**3 + x**6*(12*A*c**3 + 36*B*b*c**2) + x**4*(18*A*b*c
**2 + 18*B*b**2*c) + x**2*(12*A*b**2*c + 4*B*b**3))/(24*x**8)
```

Giac [A] time = 1.19666, size = 122, normalized size = 1.94

$$\frac{1}{2}Bc^3\log\left(x^2\right) - \frac{25\,Bc^3x^8 + 36\,Bbc^2x^6 + 12\,Ac^3x^6 + 18\,Bb^2cx^4 + 18\,Abc^2x^4 + 4\,Bb^3x^2 + 12\,Ab^2cx^2 + 3\,Ab^3}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^15,x, algorithm="giac")
```

```
[Out] 1/2*B*c^3*log(x^2) - 1/24*(25*B*c^3*x^8 + 36*B*b*c^2*x^6 + 12*A*c^3*x^6 + 1
8*B*b^2*c*x^4 + 18*A*b*c^2*x^4 + 4*B*b^3*x^2 + 12*A*b^2*c*x^2 + 3*A*b^3)/x^
8
```

3.39 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{16}}\,dx$

Optimal. Leaf size=73

$$-\frac{b^2(3Ac+bB)}{7x^7}-\frac{Ab^3}{9x^9}-\frac{c^2(Ac+3bB)}{3x^3}-\frac{3bc(Ac+bB)}{5x^5}-\frac{Bc^3}{x}$$

```
[Out] -(A*b^3)/(9*x^9) - (b^2*(b*B + 3*A*c))/(7*x^7) - (3*b*c*(b*B + A*c))/(5*x^5
) - (c^2*(3*b*B + A*c))/(3*x^3) - (B*c^3)/x
```

Rubi [A] time = 0.0490503, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1584, 448}

$$-\frac{b^2(3Ac+bB)}{7x^7}-\frac{Ab^3}{9x^9}-\frac{c^2(Ac+3bB)}{3x^3}-\frac{3bc(Ac+bB)}{5x^5}-\frac{Bc^3}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^16,x]
```

```
[Out] -(A*b^3)/(9*x^9) - (b^2*(b*B + 3*A*c))/(7*x^7) - (3*b*c*(b*B + A*c))/(5*x^5
) - (c^2*(3*b*B + A*c))/(3*x^3) - (B*c^3)/x
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^3}{x^{16}}\,dx = \int \frac{\left(A+Bx^2\right)\left(b+cx^2\right)^3}{x^{10}}\,dx$$
$$= \int \left(\frac{Ab^3}{x^{10}} + \frac{b^2(bB+3Ac)}{x^8} + \frac{3bc(bB+Ac)}{x^6} + \frac{c^2(3bB+Ac)}{x^4} + \frac{Bc^3}{x^2}\right) dx$$
$$= -\frac{Ab^3}{9x^9} - \frac{b^2(bB+3Ac)}{7x^7} - \frac{3bc(bB+Ac)}{5x^5} - \frac{c^2(3bB+Ac)}{3x^3} - \frac{Bc^3}{x}$$

Mathematica [A] time = 0.0277864, size = 73, normalized size = 1.

$$-\frac{b^2(3Ac+bB)}{7x^7} - \frac{Ab^3}{9x^9} - \frac{c^2(Ac+3bB)}{3x^3} - \frac{3bc(Ac+bB)}{5x^5} - \frac{Bc^3}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^16,x]
```

```
[Out] -(A*b^3)/(9*x^9) - (b^2*(b*B + 3*A*c))/(7*x^7) - (3*b*c*(b*B + A*c))/(5*x^5
) - (c^2*(3*b*B + A*c))/(3*x^3) - (B*c^3)/x
```

Maple [A] time = 0.006, size = 66, normalized size = 0.9

$$-\frac{Ab^3}{9\,x^9} - \frac{b^2\,(3\,Ac+Bb)}{7\,x^7} - \frac{3\,bc\,(Ac+Bb)}{5\,x^5} - \frac{c^2\,(Ac+3\,Bb)}{3\,x^3} - \frac{Bc^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^16,x)
```

```
[Out] -1/9*A*b^3/x^9-1/7*b^2*(3*A*c+B*b)/x^7-3/5*b*c*(A*c+B*b)/x^5-1/3*c^2*(A*c+3
*B*b)/x^3-B*c^3/x
```

Maxima [A] time = 1.19394, size = 101, normalized size = 1.38

$$-\frac{315\,Bc^3x^8 + 105\left(3\,Bbc^2 + Ac^3\right)x^6 + 189\left(Bb^2c + Abc^2\right)x^4 + 35\,Ab^3 + 45\left(Bb^3 + 3\,Ab^2c\right)x^2}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^16,x, algorithm="maxima")
```

```
[Out] -1/315*(315*B*c^3*x^8 + 105*(3*B*b*c^2 + A*c^3)*x^6 + 189*(B*b^2*c + A*b*c^
2)*x^4 + 35*A*b^3 + 45*(B*b^3 + 3*A*b^2*c)*x^2)/x^9
```

Fricas [A] time = 0.496981, size = 173, normalized size = 2.37

$$-\frac{315\,Bc^3x^8+105\left(3\,Bbc^2+Ac^3\right)x^6+189\left(Bb^2c+Abc^2\right)x^4+35\,Ab^3+45\left(Bb^3+3\,Ab^2c\right)x^2}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^16,x, algorithm="fricas")
```

```
[Out] -1/315*(315*B*c^3*x^8 + 105*(3*B*b*c^2 + A*c^3)*x^6 + 189*(B*b^2*c + A*b*c^
2)*x^4 + 35*A*b^3 + 45*(B*b^3 + 3*A*b^2*c)*x^2)/x^9
```

Sympy [A] time = 2.69023, size = 80, normalized size = 1.1

$$-\frac{35Ab^3+315Bc^3x^8+x^6\left(105Ac^3+315Bbc^2\right)+x^4\left(189Abc^2+189Bb^2c\right)+x^2\left(135Ab^2c+45Bb^3\right)}{315x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**16,x)
```

```
[Out] -(35*A*b**3 + 315*B*c**3*x**8 + x**6*(105*A*c**3 + 315*B*b*c**2) + x**4*(18
9*A*b*c**2 + 189*B*b**2*c) + x**2*(135*A*b**2*c + 45*B*b**3))/(315*x**9)
```

Giac [A] time = 1.20664, size = 107, normalized size = 1.47

$$-\frac{315\,Bc^3x^8+315\,Bbc^2x^6+105\,Ac^3x^6+189\,Bb^2cx^4+189\,Abc^2x^4+45\,Bb^3x^2+135\,Ab^2cx^2+35\,Ab^3}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^16,x, algorithm="giac")
```

```
[Out] -1/315*(315*B*c^3*x^8 + 315*B*b*c^2*x^6 + 105*A*c^3*x^6 + 189*B*b^2*c*x^4 +
 189*A*b*c^2*x^4 + 45*B*b^3*x^2 + 135*A*b^2*c*x^2 + 35*A*b^3)/x^9
```

3.40 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{17}} dx$

Optimal. Leaf size=49

$$-\frac{(b+cx^2)^4 (5bB-Ac)}{40b^2x^8}-\frac{A(b+cx^2)^4}{10bx^{10}}$$

```
[Out] -(A*(b + c*x^2)^4)/(10*b*x^10) - ((5*b*B - A*c)*(b + c*x^2)^4)/(40*b^2*x^8)
```

Rubi [A] time = 0.0397545, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1584, 446, 78, 37}

$$-\frac{(b+cx^2)^4 (5bB-Ac)}{40b^2x^8}-\frac{A(b+cx^2)^4}{10bx^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^17,x]
```

```
[Out] -(A*(b + c*x^2)^4)/(10*b*x^10) - ((5*b*B - A*c)*(b + c*x^2)^4)/(40*b^2*x^8)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
```

```
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{17}}\,dx &= \int \frac{(A+Bx^2)(b+cx^2)^3}{x^{11}}\,dx \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(A+Bx)(b+cx)^3}{x^6}\,dx, x, x^2\right) \\
&= -\frac{A(b+cx^2)^4}{10bx^{10}} + \frac{(5bB-Ac)\operatorname{Subst}\left(\int \frac{(b+cx)^3}{x^5}\,dx, x, x^2\right)}{10b} \\
&= -\frac{A(b+cx^2)^4}{10bx^{10}} - \frac{(5bB-Ac)(b+cx^2)^4}{40b^2x^8}
\end{aligned}
$$

Mathematica [A] time = 0.0194762, size = 78, normalized size = 1.59

$$
-\frac{A\left(15b^2cx^2+4b^3+20bc^2x^4+10c^3x^6\right)+5Bx^2\left(4b^2cx^2+b^3+6bc^2x^4+4c^3x^6\right)}{40x^{10}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^17,x]
```

```
[Out] -(5*B*x^2*(b^3 + 4*b^2*c*x^2 + 6*b*c^2*x^4 + 4*c^3*x^6) + A*(4*b^3 + 15*b^2
*c*x^2 + 20*b*c^2*x^4 + 10*c^3*x^6))/(40*x^10)
```

Maple [A] time = 0.004, size = 66, normalized size = 1.4

$$
-\frac{c^2(Ac+3Bb)}{4x^4} - \frac{Bc^3}{2x^2} - \frac{b^2(3Ac+Bb)}{8x^8} - \frac{Ab^3}{10x^{10}} - \frac{bc(Ac+Bb)}{2x^6}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^17,x)
```

```
[Out] -1/4*c^2*(A*c+3*B*b)/x^4-1/2*B*c^3/x^2-1/8*b^2*(3*A*c+B*b)/x^8-1/10*A*b^3/x
^10-1/2*b*c*(A*c+B*b)/x^6
```

Maxima [A] time = 1.08587, size = 101, normalized size = 2.06

$$-\frac{20\,Bc^3x^8+10\left(3\,Bbc^2+Ac^3\right)x^6+20\left(Bb^2c+Abc^2\right)x^4+4\,Ab^3+5\left(Bb^3+3\,Ab^2c\right)x^2}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^17,x, algorithm="maxima")
```

```
[Out] -1/40*(20*B*c^3*x^8 + 10*(3*B*b*c^2 + A*c^3)*x^6 + 20*(B*b^2*c + A*b*c^2)*x
^4 + 4*A*b^3 + 5*(B*b^3 + 3*A*b^2*c)*x^2)/x^10
```

Fricas [A] time = 0.475784, size = 166, normalized size = 3.39

$$-\frac{20\,Bc^3x^8+10\left(3\,Bbc^2+Ac^3\right)x^6+20\left(Bb^2c+Abc^2\right)x^4+4\,Ab^3+5\left(Bb^3+3\,Ab^2c\right)x^2}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^17,x, algorithm="fricas")
```

```
[Out] -1/40*(20*B*c^3*x^8 + 10*(3*B*b*c^2 + A*c^3)*x^6 + 20*(B*b^2*c + A*b*c^2)*x
^4 + 4*A*b^3 + 5*(B*b^3 + 3*A*b^2*c)*x^2)/x^10
```

Sympy [A] time = 4.43304, size = 80, normalized size = 1.63

$$-\frac{4Ab^3+20Bc^3x^8+x^6\left(10Ac^3+30Bbc^2\right)+x^4\left(20Abc^2+20Bb^2c\right)+x^2\left(15Ab^2c+5Bb^3\right)}{40x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**17,x)
```

```
[Out] -(4*A*b**3 + 20*B*c**3*x**8 + x**6*(10*A*c**3 + 30*B*b*c**2) + x**4*(20*A*b
*c**2 + 20*B*b**2*c) + x**2*(15*A*b**2*c + 5*B*b**3))/(40*x**10)
```

Giac [A] time = 1.25236, size = 107, normalized size = 2.18

$$-\frac{20\,Bc^{3}x^{8}+30\,Bbc^{2}x^{6}+10\,Ac^{3}x^{6}+20\,Bb^{2}cx^{4}+20\,Abc^{2}x^{4}+5\,Bb^{3}x^{2}+15\,Ab^{2}cx^{2}+4\,Ab^{3}}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^17,x, algorithm="giac")
```

```
[Out] -1/40*(20*B*c^3*x^8 + 30*B*b*c^2*x^6 + 10*A*c^3*x^6 + 20*B*b^2*c*x^4 + 20*A
*b*c^2*x^4 + 5*B*b^3*x^2 + 15*A*b^2*c*x^2 + 4*A*b^3)/x^10
```

3.41 $\int \frac{x^{10}(A+Bx^2)}{bx^2+cx^4}\,dx$

Optimal. Leaf size=119

$$-\frac{b^2x^3(bB-Ac)}{3c^4}+\frac{b^3x(bB-Ac)}{c^5}-\frac{b^{7/2}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{11/2}}-\frac{x^7(bB-Ac)}{7c^2}+\frac{bx^5(bB-Ac)}{5c^3}+\frac{Bx^9}{9c}$$

```
[Out] (b^3*(b*B - A*c)*x)/c^5 - (b^2*(b*B - A*c)*x^3)/(3*c^4) + (b*(b*B - A*c)*x^
5)/(5*c^3) - ((b*B - A*c)*x^7)/(7*c^2) + (B*x^9)/(9*c) - (b^(7/2)*(b*B - A*
c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(11/2)
```

Rubi [A] time = 0.0874827, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1584, 459, 302, 205}

$$-\frac{b^2x^3(bB-Ac)}{3c^4}+\frac{b^3x(bB-Ac)}{c^5}-\frac{b^{7/2}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{11/2}}-\frac{x^7(bB-Ac)}{7c^2}+\frac{bx^5(bB-Ac)}{5c^3}+\frac{Bx^9}{9c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^10*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (b^3*(b*B - A*c)*x)/c^5 - (b^2*(b*B - A*c)*x^3)/(3*c^4) + (b*(b*B - A*c)*x^
5)/(5*c^3) - ((b*B - A*c)*x^7)/(7*c^2) + (B*x^9)/(9*c) - (b^(7/2)*(b*B - A*
c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(11/2)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{10}\left(A+Bx^2\right)}{bx^2+cx^4}\,dx &= \int \frac{x^8\left(A+Bx^2\right)}{b+cx^2}\,dx \\
&= \frac{Bx^9}{9c}-\frac{(9bB-9Ac)\int \frac{x^8}{b+cx^2}\,dx}{9c} \\
&= \frac{Bx^9}{9c}-\frac{(9bB-9Ac)\int \left(-\frac{b^3}{c^4}+\frac{b^2x^2}{c^3}-\frac{bx^4}{c^2}+\frac{x^6}{c}+\frac{b^4}{c^4\left(b+cx^2\right)}\right)dx}{9c} \\
&= \frac{b^3(bB-Ac)x}{c^5}-\frac{b^2(bB-Ac)x^3}{3c^4}+\frac{b(bB-Ac)x^5}{5c^3}-\frac{(bB-Ac)x^7}{7c^2}+\frac{Bx^9}{9c}-\frac{\left(b^4(bB-Ac)\right)\int \frac{1}{b+cx^2}\,d
\end{aligned}$$

$$= \frac{b^3(bB-Ac)x}{c^5}-\frac{b^2(bB-Ac)x^3}{3c^4}+\frac{b(bB-Ac)x^5}{5c^3}-\frac{(bB-Ac)x^7}{7c^2}+\frac{Bx^9}{9c}-\frac{b^{7/2}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.}{c^{11/2}}$$

Mathematica [A] time = 0.0781213, size = 119, normalized size = 1.

$$-\frac{b^2x^3(bB-Ac)}{3c^4}+\frac{b^3x(bB-Ac)}{c^5}-\frac{b^{7/2}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{11/2}}+\frac{x^7(Ac-bB)}{7c^2}+\frac{bx^5(bB-Ac)}{5c^3}+\frac{Bx^9}{9c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^10*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (b^3*(b*B - A*c)*x)/c^5 - (b^2*(b*B - A*c)*x^3)/(3*c^4) + (b*(b*B - A*c)*x^
5)/(5*c^3) + ((-(b*B) + A*c)*x^7)/(7*c^2) + (B*x^9)/(9*c) - (b^(7/2)*(b*B -
 A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(11/2)
```

Maple [A] time = 0.006, size = 140, normalized size = 1.2

$$\frac{Bx^9}{9c}+\frac{Ax^7}{7c}-\frac{Bx^7b}{7c^2}-\frac{Abx^5}{5c^2}+\frac{Bx^5b^2}{5c^3}+\frac{Ab^2x^3}{3c^3}-\frac{Bx^3b^3}{3c^4}-\frac{Ab^3x}{c^4}+\frac{Bb^4x}{c^5}+\frac{b^4A}{c^4}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}-\frac{Bb^5}{c^5}\arctan\left(cx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(B*x^2+A)/(c*x^4+b*x^2),x)
```

```
[Out] 1/9*B*x^9/c+1/7/c*A*x^7-1/7/c^2*B*x^7*b-1/5/c^2*A*x^5*b+1/5/c^3*B*x^5*b^2+1
/3/c^3*A*x^3*b^2-1/3/c^4*B*x^3*b^3-1/c^4*A*b^3*x+1/c^5*B*b^4*x+b^4/c^4/(b*c
)^(1/2)*arctan(x*c/(b*c)^(1/2))*A-b^5/c^5/(b*c)^(1/2)*arctan(x*c/(b*c)^(1/2
))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.524356, size = 581, normalized size = 4.88

$$\left[\frac{70\,Bc^4x^9-90\left(Bbc^3-Ac^4\right)x^7+126\left(Bb^2c^2-Abc^3\right)x^5-210\left(Bb^3c-Ab^2c^2\right)x^3-315\left(Bb^4-Ab^3c\right)\sqrt{-\frac{b}{c}}\log\left(\frac{cx^2+2\,cx\sqrt{-\frac{b}{c}}}{cx^2+b}\right.}{630\,c^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [1/630*(70*B*c^4*x^9 - 90*(B*b*c^3 - A*c^4)*x^7 + 126*(B*b^2*c^2 - A*b*c^3)
*x^5 - 210*(B*b^3*c - A*b^2*c^2)*x^3 - 315*(B*b^4 - A*b^3*c)*sqrt(-b/c)*log
((c*x^2 + 2*c*x*sqrt(-b/c) - b)/(c*x^2 + b)) + 630*(B*b^4 - A*b^3*c)*x)/c^5
```

```
, 1/315*(35*B*c^4*x^9 - 45*(B*b*c^3 - A*c^4)*x^7 + 63*(B*b^2*c^2 - A*b*c^3)
*x^5 - 105*(B*b^3*c - A*b^2*c^2)*x^3 - 315*(B*b^4 - A*b^3*c)*sqrt(b/c)*arct
an(c*x*sqrt(b/c)/b) + 315*(B*b^4 - A*b^3*c)*x)/c^5]
```

Sympy [A] time = 0.537546, size = 194, normalized size = 1.63

$$\frac{Bx^9}{9c} + \frac{\sqrt{-\frac{b^7}{c^{11}}}\,(-Ac+Bb)\log\left(-\frac{c^5\sqrt{-\frac{b^7}{c^{11}}}(-Ac+Bb)}{-Ab^3c+Bb^4}+x\right)}{2} - \frac{\sqrt{-\frac{b^7}{c^{11}}}\,(-Ac+Bb)\log\left(\frac{c^5\sqrt{-\frac{b^7}{c^{11}}}(-Ac+Bb)}{-Ab^3c+Bb^4}+x\right)}{2} - \frac{x^7\,(-Ac+Bb)}{7c^2} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] B*x**9/(9*c) + sqrt(-b**7/c**11)*(-A*c + B*b)*log(-c**5*sqrt(-b**7/c**11)*(
-A*c + B*b)/(-A*b**3*c + B*b**4) + x)/2 - sqrt(-b**7/c**11)*(-A*c + B*b)*lo
g(c**5*sqrt(-b**7/c**11)*(-A*c + B*b)/(-A*b**3*c + B*b**4) + x)/2 - x**7*(-
A*c + B*b)/(7*c**2) + x**5*(-A*b*c + B*b**2)/(5*c**3) - x**3*(-A*b**2*c + B
*b**3)/(3*c**4) + x*(-A*b**3*c + B*b**4)/c**5
```

Giac [A] time = 1.22282, size = 180, normalized size = 1.51

$$-\frac{\left(Bb^5 - Ab^4c\right)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}c^5} + \frac{35\,Bc^8x^9 - 45\,Bbc^7x^7 + 45\,Ac^8x^7 + 63\,Bb^2c^6x^5 - 63\,Abc^7x^5 - 105\,Bb^3c^5x^3 + 105\,Ab^2c^6}{315\,c^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -(B*b^5 - A*b^4*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^5) + 1/315*(35*B*c^8*
x^9 - 45*B*b*c^7*x^7 + 45*A*c^8*x^7 + 63*B*b^2*c^6*x^5 - 63*A*b*c^7*x^5 - 1
05*B*b^3*c^5*x^3 + 105*A*b^2*c^6*x^3 + 315*B*b^4*c^4*x - 315*A*b^3*c^5*x)/c
^9
```

3.42 $\int \frac{x^9(A+Bx^2)}{bx^2+cx^4} dx$

Optimal. Leaf size=96

$$-\frac{b^2x^2(bB-Ac)}{2c^4}+\frac{b^3(bB-Ac)\log\left(b+cx^2\right)}{2c^5}-\frac{x^6(bB-Ac)}{6c^2}+\frac{bx^4(bB-Ac)}{4c^3}+\frac{Bx^8}{8c}$$

```
[Out] -(b^2*(b*B - A*c)*x^2)/(2*c^4) + (b*(b*B - A*c)*x^4)/(4*c^3) - ((b*B - A*c)
*x^6)/(6*c^2) + (B*x^8)/(8*c) + (b^3*(b*B - A*c)*Log[b + c*x^2])/(2*c^5)
```

Rubi [A] time = 0.126403, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 446, 77}

$$-\frac{b^2x^2(bB-Ac)}{2c^4}+\frac{b^3(bB-Ac)\log\left(b+cx^2\right)}{2c^5}-\frac{x^6(bB-Ac)}{6c^2}+\frac{bx^4(bB-Ac)}{4c^3}+\frac{Bx^8}{8c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^9*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] -(b^2*(b*B - A*c)*x^2)/(2*c^4) + (b*(b*B - A*c)*x^4)/(4*c^3) - ((b*B - A*c)
*x^6)/(6*c^2) + (B*x^8)/(8*c) + (b^3*(b*B - A*c)*Log[b + c*x^2])/(2*c^5)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
```

```
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^9\left(A+Bx^2\right)}{bx^2+cx^4}\,dx &= \int \frac{x^7\left(A+Bx^2\right)}{b+cx^2}\,dx\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^3(A+Bx)}{b+cx}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{b^2(bB-Ac)}{c^4}+\frac{b(bB-Ac)x}{c^3}+\frac{(-bB+Ac)x^2}{c^2}+\frac{Bx^3}{c}+\frac{b^3(bB-Ac)}{c^4(b+cx)}\right)dx, x, x^2\right)\\
&= -\frac{b^2(bB-Ac)x^2}{2c^4}+\frac{b(bB-Ac)x^4}{4c^3}-\frac{(bB-Ac)x^6}{6c^2}+\frac{Bx^8}{8c}+\frac{b^3(bB-Ac)\log\left(b+cx^2\right)}{2c^5}
\end{aligned}$$

Mathematica [A] time = 0.0366463, size = 92, normalized size = 0.96

$$\frac{cx^2\left(6b^2c\left(2A+Bx^2\right)-2bc^2x^2\left(3A+2Bx^2\right)+c^3x^4\left(4A+3Bx^2\right)-12b^3B\right)+12b^3(bB-Ac)\log\left(b+cx^2\right)}{24c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^9*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (c*x^2*(-12*b^3*B + 6*b^2*c*(2*A + B*x^2) - 2*b*c^2*x^2*(3*A + 2*B*x^2) + c
^3*x^4*(4*A + 3*B*x^2)) + 12*b^3*(b*B - A*c)*Log[b + c*x^2])/(24*c^5)
```

Maple [A] time = 0.004, size = 110, normalized size = 1.2

$$\frac{Bx^8}{8\,c}+\frac{Ax^6}{6\,c}-\frac{Bx^6b}{6\,c^2}-\frac{Abx^4}{4\,c^2}+\frac{Bx^4b^2}{4\,c^3}+\frac{Ab^2x^2}{2\,c^3}-\frac{Bx^2b^3}{2\,c^4}-\frac{b^3\ln\left(cx^2+b\right)A}{2\,c^4}+\frac{b^4\ln\left(cx^2+b\right)B}{2\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(B*x^2+A)/(c*x^4+b*x^2),x)
```

```
[Out] 1/8*B*x^8/c+1/6/c*A*x^6-1/6/c^2*B*x^6*b-1/4/c^2*A*x^4*b+1/4/c^3*B*x^4*b^2+1
/2/c^3*A*x^2*b^2-1/2/c^4*B*x^2*b^3-1/2*b^3/c^4*ln(c*x^2+b)*A+1/2*b^4/c^5*ln
(c*x^2+b)*B
```

Maxima [A] time = 1.01772, size = 131, normalized size = 1.36

$$\frac{3\,Bc^3x^8 - 4\left(Bbc^2 - Ac^3\right)x^6 + 6\left(Bb^2c - Abc^2\right)x^4 - 12\left(Bb^3 - Ab^2c\right)x^2}{24\,c^4} + \frac{\left(Bb^4 - Ab^3c\right)\log\left(cx^2 + b\right)}{2\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/24*(3*B*c^3*x^8 - 4*(B*b*c^2 - A*c^3)*x^6 + 6*(B*b^2*c - A*b*c^2)*x^4 - 1
2*(B*b^3 - A*b^2*c)*x^2)/c^4 + 1/2*(B*b^4 - A*b^3*c)*log(c*x^2 + b)/c^5
```

Fricas [A] time = 0.482255, size = 201, normalized size = 2.09

$$\frac{3\,Bc^4x^8 - 4\left(Bbc^3 - Ac^4\right)x^6 + 6\left(Bb^2c^2 - Abc^3\right)x^4 - 12\left(Bb^3c - Ab^2c^2\right)x^2 + 12\left(Bb^4 - Ab^3c\right)\log\left(cx^2 + b\right)}{24\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/24*(3*B*c^4*x^8 - 4*(B*b*c^3 - A*c^4)*x^6 + 6*(B*b^2*c^2 - A*b*c^3)*x^4 -
 12*(B*b^3*c - A*b^2*c^2)*x^2 + 12*(B*b^4 - A*b^3*c)*log(c*x^2 + b))/c^5
```

Sympy [A] time = 0.46994, size = 85, normalized size = 0.89

$$\frac{Bx^8}{8c} + \frac{b^3\left(-Ac + Bb\right)\log\left(b + cx^2\right)}{2c^5} - \frac{x^6\left(-Ac + Bb\right)}{6c^2} + \frac{x^4\left(-Abc + Bb^2\right)}{4c^3} - \frac{x^2\left(-Ab^2c + Bb^3\right)}{2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] B*x**8/(8*c) + b**3*(-A*c + B*b)*log(b + c*x**2)/(2*c**5) - x**6*(-A*c + B*
b)/(6*c**2) + x**4*(-A*b*c + B*b**2)/(4*c**3) - x**2*(-A*b**2*c + B*b**3)/(
2*c**4)
```

Giac [A] time = 1.23302, size = 136, normalized size = 1.42

$$\frac{3\,Bc^3x^8 - 4\,Bbc^2x^6 + 4\,Ac^3x^6 + 6\,Bb^2cx^4 - 6\,Abc^2x^4 - 12\,Bb^3x^2 + 12\,Ab^2cx^2}{24\,c^4} + \frac{\left(Bb^4 - Ab^3c\right)\log\left(\left|cx^2 + b\right|\right)}{2\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/24*(3*B*c^3*x^8 - 4*B*b*c^2*x^6 + 4*A*c^3*x^6 + 6*B*b^2*c*x^4 - 6*A*b*c^2
*x^4 - 12*B*b^3*x^2 + 12*A*b^2*c*x^2)/c^4 + 1/2*(B*b^4 - A*b^3*c)*log(abs(c
*x^2 + b))/c^5
```

3.43 $\int \frac{x^8(A+Bx^2)}{bx^2+cx^4} \, dx$

Optimal. Leaf size=98

$$-\frac{b^2x(bB-Ac)}{c^4}+\frac{b^{5/2}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{9/2}}-\frac{x^5(bB-Ac)}{5c^2}+\frac{bx^3(bB-Ac)}{3c^3}+\frac{Bx^7}{7c}$$

```
[Out] -((b^2*(b*B - A*c)*x)/c^4) + (b*(b*B - A*c)*x^3)/(3*c^3) - ((b*B - A*c)*x^5
)/(5*c^2) + (B*x^7)/(7*c) + (b^(5/2)*(b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]
])/c^(9/2)
```

Rubi [A] time = 0.0762564, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1584, 459, 302, 205}

$$-\frac{b^2x(bB-Ac)}{c^4}+\frac{b^{5/2}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{9/2}}-\frac{x^5(bB-Ac)}{5c^2}+\frac{bx^3(bB-Ac)}{3c^3}+\frac{Bx^7}{7c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] -((b^2*(b*B - A*c)*x)/c^4) + (b*(b*B - A*c)*x^3)/(3*c^3) - ((b*B - A*c)*x^5
)/(5*c^2) + (B*x^7)/(7*c) + (b^(5/2)*(b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]
])/c^(9/2)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8\left(A+Bx^2\right)}{bx^2+cx^4}\,dx &= \int \frac{x^6\left(A+Bx^2\right)}{b+cx^2}\,dx \\
&= \frac{Bx^7}{7c}-\frac{(7bB-7Ac)\int \frac{x^6}{b+cx^2}\,dx}{7c} \\
&= \frac{Bx^7}{7c}-\frac{(7bB-7Ac)\int \left(\frac{b^2}{c^3}-\frac{bx^2}{c^2}+\frac{x^4}{c}-\frac{b^3}{c^3\left(b+cx^2\right)}\right)dx}{7c} \\
&= -\frac{b^2(bB-Ac)x}{c^4}+\frac{b(bB-Ac)x^3}{3c^3}-\frac{(bB-Ac)x^5}{5c^2}+\frac{Bx^7}{7c}+\frac{\left(b^3(bB-Ac)\right)\int \frac{1}{b+cx^2}\,dx}{c^4} \\
&= -\frac{b^2(bB-Ac)x}{c^4}+\frac{b(bB-Ac)x^3}{3c^3}-\frac{(bB-Ac)x^5}{5c^2}+\frac{Bx^7}{7c}+\frac{b^{5/2}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{9/2}}
\end{aligned}$$

Mathematica [A] time = 0.0618947, size = 98, normalized size = 1.

$$-\frac{b^2x(bB-Ac)}{c^4}+\frac{b^{5/2}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{9/2}}+\frac{x^5(Ac-bB)}{5c^2}+\frac{bx^3(bB-Ac)}{3c^3}+\frac{Bx^7}{7c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] -((b^2*(b*B - A*c)*x)/c^4) + (b*(b*B - A*c)*x^3)/(3*c^3) + ((-(b*B) + A*c)*
x^5)/(5*c^2) + (B*x^7)/(7*c) + (b^(5/2)*(b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt
[b]])/c^(9/2)
```

Maple [A] time = 0.004, size = 116, normalized size = 1.2

$$\frac{Bx^7}{7c}+\frac{Ax^5}{5c}-\frac{Bx^5b}{5c^2}-\frac{Abx^3}{3c^2}+\frac{Bx^3b^2}{3c^3}+\frac{Ab^2x}{c^3}-\frac{Bb^3x}{c^4}-\frac{b^3A}{c^3}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}+\frac{Bb^4}{c^4}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(B*x^2+A)/(c*x^4+b*x^2),x)
```

```
[Out] 1/7*B*x^7/c+1/5/c*A*x^5-1/5/c^2*B*x^5*b-1/3/c^2*A*x^3*b+1/3/c^3*B*x^3*b^2+1
/c^3*A*b^2*x-1/c^4*B*b^3*x-b^3/c^3/(b*c)^(1/2)*arctan(x*c/(b*c)^(1/2))*A+b^
4/c^4/(b*c)^(1/2)*arctan(x*c/(b*c)^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.556467, size = 487, normalized size = 4.97

$$\left[\frac{30\,Bc^3x^7-42\left(Bbc^2-Ac^3\right)x^5+70\left(Bb^2c-Abc^2\right)x^3-105\left(Bb^3-Ab^2c\right)\sqrt{-\frac{b}{c}}\log\left(\frac{cx^2-2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)-210\left(Bb^3-Ab^2c\right)x}{210\,c^4}, \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [1/210*(30*B*c^3*x^7 - 42*(B*b*c^2 - A*c^3)*x^5 + 70*(B*b^2*c - A*b*c^2)*x^
3 - 105*(B*b^3 - A*b^2*c)*sqrt(-b/c)*log((c*x^2 - 2*c*x*sqrt(-b/c) - b)/(c*
x^2 + b)) - 210*(B*b^3 - A*b^2*c)*x)/c^4, 1/105*(15*B*c^3*x^7 - 21*(B*b*c^2
 - A*c^3)*x^5 + 35*(B*b^2*c - A*b*c^2)*x^3 + 105*(B*b^3 - A*b^2*c)*sqrt(b/c
```

```
)*arctan(c*x*sqrt(b/c)/b) - 105*(B*b^3 - A*b^2*c)*x)/c^4]
```

Sympy [A] time = 0.50813, size = 173, normalized size = 1.77

$$\frac{Bx^7}{7c} - \frac{\sqrt{-\frac{b^5}{c^9}}\,(-Ac+Bb)\log\left(-\frac{c^4\sqrt{-\frac{b^5}{c^9}}(-Ac+Bb)}{-Ab^2c+Bb^3}+x\right)}{2} + \frac{\sqrt{-\frac{b^5}{c^9}}\,(-Ac+Bb)\log\left(\frac{c^4\sqrt{-\frac{b^5}{c^9}}(-Ac+Bb)}{-Ab^2c+Bb^3}+x\right)}{2} - \frac{x^5\,(-Ac+Bb)}{5c^2} + \frac{x}{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] B*x**7/(7*c) - sqrt(-b**5/c**9)*(-A*c + B*b)*log(-c**4*sqrt(-b**5/c**9)*(-A
*c + B*b)/(-A*b**2*c + B*b**3) + x)/2 + sqrt(-b**5/c**9)*(-A*c + B*b)*log(c
**4*sqrt(-b**5/c**9)*(-A*c + B*b)/(-A*b**2*c + B*b**3) + x)/2 - x**5*(-A*c
+ B*b)/(5*c**2) + x**3*(-A*b*c + B*b**2)/(3*c**3) - x*(-A*b**2*c + B*b**3)/
c**4
```

Giac [A] time = 1.22651, size = 146, normalized size = 1.49

$$\frac{\left(Bb^4 - Ab^3c\right)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}c^4} + \frac{15\,Bc^6x^7 - 21\,Bbc^5x^5 + 21\,Ac^6x^5 + 35\,Bb^2c^4x^3 - 35\,Abc^5x^3 - 105\,Bb^3c^3x + 105\,Ab^2c^4x}{105\,c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] (B*b^4 - A*b^3*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^4) + 1/105*(15*B*c^6*x
^7 - 21*B*b*c^5*x^5 + 21*A*c^6*x^5 + 35*B*b^2*c^4*x^3 - 35*A*b*c^5*x^3 - 10
5*B*b^3*c^3*x + 105*A*b^2*c^4*x)/c^7
```

3.44 $\int \frac{x^7(A+Bx^2)}{bx^2+cx^4}\,dx$

Optimal. Leaf size=75

$$-\frac{b^2(bB-Ac)\log\left(b+cx^2\right)}{2c^4}-\frac{x^4(bB-Ac)}{4c^2}+\frac{bx^2(bB-Ac)}{2c^3}+\frac{Bx^6}{6c}$$

```
[Out] (b*(b*B - A*c)*x^2)/(2*c^3) - ((b*B - A*c)*x^4)/(4*c^2) + (B*x^6)/(6*c) - (
b^2*(b*B - A*c)*Log[b + c*x^2])/(2*c^4)
```

Rubi [A] time = 0.0938306, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 446, 77}

$$-\frac{b^2(bB-Ac)\log\left(b+cx^2\right)}{2c^4}-\frac{x^4(bB-Ac)}{4c^2}+\frac{bx^2(bB-Ac)}{2c^3}+\frac{Bx^6}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (b*(b*B - A*c)*x^2)/(2*c^3) - ((b*B - A*c)*x^4)/(4*c^2) + (B*x^6)/(6*c) - (
b^2*(b*B - A*c)*Log[b + c*x^2])/(2*c^4)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
```

```
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{x^7\left(A+Bx^2\right)}{bx^2+cx^4}\,dx &= \int \frac{x^5\left(A+Bx^2\right)}{b+cx^2}\,dx\\ &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{x^2(A+Bx)}{b+cx}\,dx,x,x^2\right)\\ &= \frac{1}{2}\,\mathrm{Subst}\left(\int \left(\frac{b(bB-Ac)}{c^3}+\frac{(-bB+Ac)x}{c^2}+\frac{Bx^2}{c}-\frac{b^2(bB-Ac)}{c^3(b+cx)}\right)dx,x,x^2\right)\\ &= \frac{b(bB-Ac)x^2}{2c^3}-\frac{(bB-Ac)x^4}{4c^2}+\frac{Bx^6}{6c}-\frac{b^2(bB-Ac)\log\left(b+cx^2\right)}{2c^4}\end{aligned}$$

Mathematica [A] time = 0.0289057, size = 71, normalized size = 0.95

$$\frac{cx^2\left(-3bc\left(2A+Bx^2\right)+c^2x^2\left(3A+2Bx^2\right)+6b^2B\right)+6b^2(Ac-bB)\log\left(b+cx^2\right)}{12c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (c*x^2*(6*b^2*B - 3*b*c*(2*A + B*x^2) + c^2*x^2*(3*A + 2*B*x^2)) + 6*b^2*(-
(b*B) + A*c)*Log[b + c*x^2])/(12*c^4)
```

Maple [A] time = 0.004, size = 86, normalized size = 1.2

$$\frac{Bx^6}{6\,c}+\frac{Ax^4}{4\,c}-\frac{Bx^4b}{4\,c^2}-\frac{Abx^2}{2\,c^2}+\frac{Bx^2b^2}{2\,c^3}+\frac{b^2\ln\left(cx^2+b\right)A}{2\,c^3}-\frac{b^3\ln\left(cx^2+b\right)B}{2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(B*x^2+A)/(c*x^4+b*x^2),x)
```

```
[Out] 1/6*B*x^6/c+1/4/c*A*x^4-1/4/c^2*B*x^4*b-1/2/c^2*A*x^2*b+1/2/c^3*B*x^2*b^2+1
/2*b^2/c^3*ln(c*x^2+b)*A-1/2*b^3/c^4*ln(c*x^2+b)*B
```

Maxima [A] time = 1.50688, size = 100, normalized size = 1.33

$$\frac{2 Bc^2x^6 - 3\left(Bbc - Ac^2\right)x^4 + 6\left(Bb^2 - Abc\right)x^2}{12 c^3} - \frac{\left(Bb^3 - Ab^2c\right)\log\left(cx^2 + b\right)}{2 c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/12*(2*B*c^2*x^6 - 3*(B*b*c - A*c^2)*x^4 + 6*(B*b^2 - A*b*c)*x^2)/c^3 - 1/
2*(B*b^3 - A*b^2*c)*log(c*x^2 + b)/c^4
```

Fricas [A] time = 0.489839, size = 155, normalized size = 2.07

$$\frac{2 Bc^3x^6 - 3\left(Bbc^2 - Ac^3\right)x^4 + 6\left(Bb^2c - Abc^2\right)x^2 - 6\left(Bb^3 - Ab^2c\right)\log\left(cx^2 + b\right)}{12 c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/12*(2*B*c^3*x^6 - 3*(B*b*c^2 - A*c^3)*x^4 + 6*(B*b^2*c - A*b*c^2)*x^2 - 6
*(B*b^3 - A*b^2*c)*log(c*x^2 + b))/c^4
```

Sympy [A] time = 0.445902, size = 65, normalized size = 0.87

$$\frac{Bx^6}{6c} - \frac{b^2\left(-Ac + Bb\right)\log\left(b + cx^2\right)}{2c^4} - \frac{x^4\left(-Ac + Bb\right)}{4c^2} + \frac{x^2\left(-Abc + Bb^2\right)}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] B*x**6/(6*c) - b**2*(-A*c + B*b)*log(b + c*x**2)/(2*c**4) - x**4*(-A*c + B*
b)/(4*c**2) + x**2*(-A*b*c + B*b**2)/(2*c**3)
```

Giac [A] time = 1.24536, size = 104, normalized size = 1.39

$$\frac{2\,Bc^2x^6 - 3\,Bbcx^4 + 3\,Ac^2x^4 + 6\,Bb^2x^2 - 6\,Abcx^2}{12\,c^3} - \frac{\left(Bb^3 - Ab^2c\right)\log\left(\left|cx^2 + b\right|\right)}{2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/12*(2*B*c^2*x^6 - 3*B*b*c*x^4 + 3*A*c^2*x^4 + 6*B*b^2*x^2 - 6*A*b*c*x^2)/
c^3 - 1/2*(B*b^3 - A*b^2*c)*log(abs(c*x^2 + b))/c^4
```

3.45 $\int \frac{x^6(A+Bx^2)}{bx^2+cx^4} dx$

Optimal. Leaf size=77

$$-\frac{b^{3/2}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{7/2}}-\frac{x^3(bB-Ac)}{3c^2}+\frac{bx(bB-Ac)}{c^3}+\frac{Bx^5}{5c}$$

```
[Out] (b*(b*B - A*c)*x)/c^3 - ((b*B - A*c)*x^3)/(3*c^2) + (B*x^5)/(5*c) - (b^(3/2
)*(b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(7/2)
```

Rubi [A] time = 0.0657095, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1584, 459, 302, 205}

$$-\frac{b^{3/2}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{7/2}}-\frac{x^3(bB-Ac)}{3c^2}+\frac{bx(bB-Ac)}{c^3}+\frac{Bx^5}{5c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (b*(b*B - A*c)*x)/c^3 - ((b*B - A*c)*x^3)/(3*c^2) + (B*x^5)/(5*c) - (b^(3/2
)*(b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(7/2)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6\left(A+Bx^2\right)}{bx^2+cx^4}\,dx &= \int \frac{x^4\left(A+Bx^2\right)}{b+cx^2}\,dx\\
&= \frac{Bx^5}{5c}-\frac{(5bB-5Ac)\int \frac{x^4}{b+cx^2}\,dx}{5c}\\
&= \frac{Bx^5}{5c}-\frac{(5bB-5Ac)\int \left(-\frac{b}{c^2}+\frac{x^2}{c}+\frac{b^2}{c^2\left(b+cx^2\right)}\right)dx}{5c}\\
&= \frac{b(bB-Ac)x}{c^3}-\frac{(bB-Ac)x^3}{3c^2}+\frac{Bx^5}{5c}-\frac{\left(b^2(bB-Ac)\right)\int \frac{1}{b+cx^2}\,dx}{c^3}\\
&= \frac{b(bB-Ac)x}{c^3}-\frac{(bB-Ac)x^3}{3c^2}+\frac{Bx^5}{5c}-\frac{b^{3/2}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{7/2}}
\end{aligned}$$

Mathematica [A] time = 0.0498843, size = 77, normalized size = 1.

$$-\frac{b^{3/2}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{7/2}}+\frac{x^3(Ac-bB)}{3c^2}+\frac{bx(bB-Ac)}{c^3}+\frac{Bx^5}{5c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (b*(b*B - A*c)*x)/c^3 + ((-(b*B) + A*c)*x^3)/(3*c^2) + (B*x^5)/(5*c) - (b^(
3/2)*(b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(7/2)
```

Maple [A] time = 0.003, size = 92, normalized size = 1.2

$$\frac{Bx^5}{5c}+\frac{Ax^3}{3c}-\frac{Bx^3b}{3c^2}-\frac{Abx}{c^2}+\frac{Bb^2x}{c^3}+\frac{b^2A}{c^2}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}-\frac{Bb^3}{c^3}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(B*x^2+A)/(c*x^4+b*x^2),x)
```

```
[Out] 1/5*B*x^5/c+1/3/c*A*x^3-1/3/c^2*B*x^3*b-1/c^2*A*b*x+1/c^3*B*b^2*x+b^2/c^2/(
b*c)^(1/2)*arctan(x*c/(b*c)^(1/2))*A-b^3/c^3/(b*c)^(1/2)*arctan(x*c/(b*c)^(
1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.541433, size = 381, normalized size = 4.95

$$\left[\frac{6\,Bc^2x^5-10\left(Bbc-Ac^2\right)x^3-15\left(Bb^2-Abc\right)\sqrt{-\frac{b}{c}}\log\left(\frac{cx^2+2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)+30\left(Bb^2-Abc\right)x}{30\,c^3},\ \frac{3\,Bc^2x^5-5\left(Bbc-Ac^2\right)x^3-1\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [1/30*(6*B*c^2*x^5 - 10*(B*b*c - A*c^2)*x^3 - 15*(B*b^2 - A*b*c)*sqrt(-b/c)
*log((c*x^2 + 2*c*x*sqrt(-b/c) - b)/(c*x^2 + b)) + 30*(B*b^2 - A*b*c)*x)/c^
3, 1/15*(3*B*c^2*x^5 - 5*(B*b*c - A*c^2)*x^3 - 15*(B*b^2 - A*b*c)*sqrt(b/c)
*arctan(c*x*sqrt(b/c)/b) + 15*(B*b^2 - A*b*c)*x)/c^3]
```

Sympy [B] time = 0.484553, size = 150, normalized size = 1.95

$$\frac{Bx^5}{5c} + \frac{\sqrt{-\frac{b^3}{c^7}}\,(-Ac+Bb)\log\left(-\frac{c^3\sqrt{-\frac{b^3}{c^7}}(-Ac+Bb)}{-Abc+Bb^2}+x\right)}{2} - \frac{\sqrt{-\frac{b^3}{c^7}}\,(-Ac+Bb)\log\left(\frac{c^3\sqrt{-\frac{b^3}{c^7}}(-Ac+Bb)}{-Abc+Bb^2}+x\right)}{2} - \frac{x^3(-Ac+Bb)}{3c^2} + \frac{x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] B*x**5/(5*c) + sqrt(-b**3/c**7)*(-A*c + B*b)*log(-c**3*sqrt(-b**3/c**7)*(-A
*c + B*b)/(-A*b*c + B*b**2) + x)/2 - sqrt(-b**3/c**7)*(-A*c + B*b)*log(c**3
*sqrt(-b**3/c**7)*(-A*c + B*b)/(-A*b*c + B*b**2) + x)/2 - x**3*(-A*c + B*b)
/(3*c**2) + x*(-A*b*c + B*b**2)/c**3
```

Giac [A] time = 1.18289, size = 115, normalized size = 1.49

$$-\frac{\left(Bb^3 - Ab^2c\right)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}c^3} + \frac{3\,Bc^4x^5 - 5\,Bbc^3x^3 + 5\,Ac^4x^3 + 15\,Bb^2c^2x - 15\,Abc^3x}{15\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -(B*b^3 - A*b^2*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^3) + 1/15*(3*B*c^4*x^
5 - 5*B*b*c^3*x^3 + 5*A*c^4*x^3 + 15*B*b^2*c^2*x - 15*A*b*c^3*x)/c^5
```

3.46 $\int \frac{x^5(A+Bx^2)}{bx^2+cx^4} \, dx$

Optimal. Leaf size=54

$$-\frac{x^2(bB-Ac)}{2c^2}+\frac{b(bB-Ac)\log\left(b+cx^2\right)}{2c^3}+\frac{Bx^4}{4c}$$

```
[Out] -((b*B - A*c)*x^2)/(2*c^2) + (B*x^4)/(4*c) + (b*(b*B - A*c)*Log[b + c*x^2])
/(2*c^3)
```

Rubi [A] time = 0.0687733, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 446, 77}

$$-\frac{x^2(bB-Ac)}{2c^2}+\frac{b(bB-Ac)\log\left(b+cx^2\right)}{2c^3}+\frac{Bx^4}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] -((b*B - A*c)*x^2)/(2*c^2) + (B*x^4)/(4*c) + (b*(b*B - A*c)*Log[b + c*x^2])
/(2*c^3)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
```

```
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{x^5\left(A+Bx^2\right)}{bx^2+cx^4}\,dx &= \int \frac{x^3\left(A+Bx^2\right)}{b+cx^2}\,dx\\ &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x(A+Bx)}{b+cx}\,dx, x, x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{-bB+Ac}{c^2}+\frac{Bx}{c}+\frac{b(bB-Ac)}{c^2(b+cx)}\right)dx, x, x^2\right)\\ &= -\frac{(bB-Ac)x^2}{2c^2}+\frac{Bx^4}{4c}+\frac{b(bB-Ac)\log\left(b+cx^2\right)}{2c^3}\end{aligned}$$

Mathematica [A] time = 0.0187509, size = 47, normalized size = 0.87

$$\frac{cx^2\left(2Ac-2bB+Bcx^2\right)+2b(bB-Ac)\log\left(b+cx^2\right)}{4c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (c*x^2*(-2*b*B + 2*A*c + B*c*x^2) + 2*b*(b*B - A*c)*Log[b + c*x^2])/(4*c^3)
```

Maple [A] time = 0.002, size = 62, normalized size = 1.2

$$\frac{Bx^4}{4\,c}+\frac{Ax^2}{2\,c}-\frac{Bx^2b}{2\,c^2}-\frac{b\ln\left(cx^2+b\right)A}{2\,c^2}+\frac{b^2\ln\left(cx^2+b\right)B}{2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)/(c*x^4+b*x^2),x)
```

```
[Out] 1/4*B*x^4/c+1/2/c*A*x^2-1/2/c^2*B*x^2*b-1/2*b/c^2*ln(c*x^2+b)*A+1/2*b^2/c^3
*ln(c*x^2+b)*B
```

Maxima [A] time = 2.02292, size = 68, normalized size = 1.26

$$\frac{Bcx^4 - 2(Bb - Ac)x^2}{4c^2} + \frac{(Bb^2 - Abc)\log(cx^2 + b)}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/4*(B*c*x^4 - 2*(B*b - A*c)*x^2)/c^2 + 1/2*(B*b^2 - A*b*c)*log(c*x^2 + b)/
c^3
```

Fricas [A] time = 0.496878, size = 108, normalized size = 2.

$$\frac{Bc^2x^4 - 2(Bbc - Ac^2)x^2 + 2(Bb^2 - Abc)\log(cx^2 + b)}{4c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/4*(B*c^2*x^4 - 2*(B*b*c - A*c^2)*x^2 + 2*(B*b^2 - A*b*c)*log(c*x^2 + b))/
c^3
```

Sympy [A] time = 0.423355, size = 44, normalized size = 0.81

$$\frac{Bx^4}{4c} + \frac{b(-Ac + Bb)\log(b + cx^2)}{2c^3} - \frac{x^2(-Ac + Bb)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] B*x**4/(4*c) + b*(-A*c + B*b)*log(b + c*x**2)/(2*c**3) - x**2*(-A*c + B*b)/
(2*c**2)
```

Giac [A] time = 1.25056, size = 70, normalized size = 1.3

$$\frac{Bcx^4 - 2Bbx^2 + 2Acx^2}{4c^2} + \frac{(Bb^2 - Abc)\log(|cx^2 + b|)}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/4*(B*c*x^4 - 2*B*b*x^2 + 2*A*c*x^2)/c^2 + 1/2*(B*b^2 - A*b*c)*log(abs(c*x
^2 + b))/c^3
```

3.47 $\int \frac{x^4(A+Bx^2)}{bx^2+cx^4} dx$

Optimal. Leaf size=58

$$-\frac{x(bB-Ac)}{c^2}+\frac{\sqrt{b}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{5/2}}+\frac{Bx^3}{3c}$$

```
[Out] -(((b*B - A*c)*x)/c^2) + (B*x^3)/(3*c) + (Sqrt[b]*(b*B - A*c)*ArcTan[(Sqrt[
c]*x)/Sqrt[b]])/c^(5/2)
```

Rubi [A] time = 0.0498338, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1584, 459, 321, 205}

$$-\frac{x(bB-Ac)}{c^2}+\frac{\sqrt{b}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{5/2}}+\frac{Bx^3}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] -(((b*B - A*c)*x)/c^2) + (B*x^3)/(3*c) + (Sqrt[b]*(b*B - A*c)*ArcTan[(Sqrt[
c]*x)/Sqrt[b]])/c^(5/2)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4\left(A+Bx^2\right)}{bx^2+cx^4}\,dx &= \int \frac{x^2\left(A+Bx^2\right)}{b+cx^2}\,dx\\ &= \frac{Bx^3}{3c}-\frac{(3bB-3Ac)\int \frac{x^2}{b+cx^2}\,dx}{3c}\\ &= -\frac{(bB-Ac)x}{c^2}+\frac{Bx^3}{3c}+\frac{(b(bB-Ac))\int \frac{1}{b+cx^2}\,dx}{c^2}\\ &= -\frac{(bB-Ac)x}{c^2}+\frac{Bx^3}{3c}+\frac{\sqrt{b}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.0384189, size = 57, normalized size = 0.98

$$\frac{x(Ac-bB)}{c^2}+\frac{\sqrt{b}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{5/2}}+\frac{Bx^3}{3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] ((-(b*B) + A*c)*x)/c^2 + (B*x^3)/(3*c) + (Sqrt[b]*(b*B - A*c)*ArcTan[(Sqrt[
c]*x)/Sqrt[b]])/c^(5/2)
```

Maple [A] time = 0.004, size = 68, normalized size = 1.2

$$\frac{Bx^3}{3\,c}+\frac{Ax}{c}-\frac{Bbx}{c^2}-\frac{Ab}{c}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}+\frac{Bb^2}{c^2}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^2+A)/(c*x^4+b*x^2),x)
```

```
[Out] 1/3*B*x^3/c+1/c*A*x-1/c^2*B*b*x-b/c/(b*c)^(1/2)*arctan(x*c/(b*c)^(1/2))*A+b
^2/c^2/(b*c)^(1/2)*arctan(x*c/(b*c)^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.589389, size = 277, normalized size = 4.78

$$\left[\frac{2\,Bcx^3 - 3\,(Bb - Ac)\sqrt{-\frac{b}{c}}\log\left(\frac{cx^2 - 2\,cx\sqrt{-\frac{b}{c}} - b}{cx^2 + b}\right) - 6\,(Bb - Ac)x}{6\,c^2}, \frac{Bcx^3 + 3\,(Bb - Ac)\sqrt{\frac{b}{c}}\arctan\left(\frac{cx\sqrt{\frac{b}{c}}}{b}\right) - 3\,(Bb - Ac)x}{3\,c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [1/6*(2*B*c*x^3 - 3*(B*b - A*c)*sqrt(-b/c)*log((c*x^2 - 2*c*x*sqrt(-b/c) -
b)/(c*x^2 + b)) - 6*(B*b - A*c)*x)/c^2, 1/3*(B*c*x^3 + 3*(B*b - A*c)*sqrt(b
/c)*arctan(c*x*sqrt(b/c)/b) - 3*(B*b - A*c)*x)/c^2]
```

Sympy [A] time = 0.461501, size = 90, normalized size = 1.55

$$\frac{Bx^3}{3c} - \frac{\sqrt{-\frac{b}{c^5}}\,(-Ac + Bb)\log\left(-c^2\sqrt{-\frac{b}{c^5}} + x\right)}{2} + \frac{\sqrt{-\frac{b}{c^5}}\,(-Ac + Bb)\log\left(c^2\sqrt{-\frac{b}{c^5}} + x\right)}{2} - \frac{x\,(-Ac + Bb)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] B*x**3/(3*c) - sqrt(-b/c**5)*(-A*c + B*b)*log(-c**2*sqrt(-b/c**5) + x)/2 +
sqrt(-b/c**5)*(-A*c + B*b)*log(c**2*sqrt(-b/c**5) + x)/2 - x*(-A*c + B*b)/c
**2
```

Giac [A] time = 1.34879, size = 77, normalized size = 1.33

$$\frac{\left(Bb^2 - Abc\right)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}c^2} + \frac{Bc^2x^3 - 3\,Bbcx + 3\,Ac^2x}{3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] (B*b^2 - A*b*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^2) + 1/3*(B*c^2*x^3 - 3*
B*b*c*x + 3*A*c^2*x)/c^3
```

3.48 $\int \frac{x^3(A+Bx^2)}{bx^2+cx^4}\,dx$

Optimal. Leaf size=35

$$\frac{Bx^2}{2c} - \frac{(bB - Ac)\log\left(b + cx^2\right)}{2c^2}$$

```
[Out] (B*x^2)/(2*c) - ((b*B - A*c)*Log[b + c*x^2])/(2*c^2)
```

Rubi [A] time = 0.0431874, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 444, 43}

$$\frac{Bx^2}{2c} - \frac{(bB - Ac)\log\left(b + cx^2\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (B*x^2)/(2*c) - ((b*B - A*c)*Log[b + c*x^2])/(2*c^2)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3 (A+Bx^2)}{bx^2+cx^4}\,dx &= \int \frac{x (A+Bx^2)}{b+cx^2}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{b+cx}\,dx, x, x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{B}{c}+\frac{-bB+Ac}{c(b+cx)}\right)dx, x, x^2\right)\\ &= \frac{Bx^2}{2c}-\frac{(bB-Ac)\log(b+cx^2)}{2c^2}\end{aligned}$$

Mathematica [A] time = 0.0110185, size = 31, normalized size = 0.89

$$\frac{(Ac-bB)\log(b+cx^2)+Bcx^2}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^2))/(b*x^2 + c*x^4),x]

[Out] (B*c*x^2 + (-(b*B) + A*c)*Log[b + c*x^2])/(2*c^2)
```

Maple [A] time = 0.003, size = 40, normalized size = 1.1

$$\frac{Bx^2}{2c}+\frac{\ln(cx^2+b)A}{2c}-\frac{\ln(cx^2+b)Bb}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)/(c*x^4+b*x^2),x)

[Out] 1/2*B*x^2/c+1/2/c*ln(c*x^2+b)*A-1/2/c^2*ln(c*x^2+b)*B*b
```

Maxima [A] time = 1.15695, size = 42, normalized size = 1.2

$$\frac{Bx^2}{2\,c} - \frac{(Bb - Ac)\log\left(cx^2 + b\right)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/2*B*x^2/c - 1/2*(B*b - A*c)*log(c*x^2 + b)/c^2
```

Fricas [A] time = 0.607129, size = 65, normalized size = 1.86

$$\frac{Bcx^2 - (Bb - Ac)\log\left(cx^2 + b\right)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/2*(B*c*x^2 - (B*b - A*c)*log(c*x^2 + b))/c^2
```

Sympy [A] time = 0.388506, size = 27, normalized size = 0.77

$$\frac{Bx^2}{2c} - \frac{(-Ac + Bb)\log\left(b + cx^2\right)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] B*x**2/(2*c) - (-A*c + B*b)*log(b + c*x**2)/(2*c**2)
```

Giac [A] time = 1.28704, size = 43, normalized size = 1.23

$$\frac{Bx^2}{2\,c} - \frac{(Bb - Ac)\log\left(\left|cx^2 + b\right|\right)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*B*x^2/c - 1/2*(B*b - A*c)*log(abs(c*x^2 + b))/c^2
```

3.49 $\int \frac{x^2(A+Bx^2)}{bx^2+cx^4} \, dx$

Optimal. Leaf size=40

$$\frac{Bx}{c} - \frac{(bB - Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{\sqrt{b}c^{3/2}}$$

```
[Out] (B*x)/c - ((b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(Sqrt[b]*c^(3/2))
```

Rubi [A] time = 0.0280687, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 388, 205}

$$\frac{Bx}{c} - \frac{(bB - Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{\sqrt{b}c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (B*x)/c - ((b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(Sqrt[b]*c^(3/2))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(A+Bx^2\right)}{bx^2+cx^4}\,dx &= \int \frac{A+Bx^2}{b+cx^2}\,dx\\ &= \frac{Bx}{c}-\frac{(bB-Ac)\int \frac{1}{b+cx^2}\,dx}{c}\\ &= \frac{Bx}{c}-\frac{(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{\sqrt{b}c^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0220838, size = 40, normalized size = 1.

$$\frac{Bx}{c}-\frac{(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{\sqrt{b}c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (B*x)/c - ((b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(Sqrt[b]*c^(3/2))
```

Maple [A] time = 0.002, size = 45, normalized size = 1.1

$$\frac{Bx}{c}+A\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}-\frac{Bb}{c}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)/(c*x^4+b*x^2),x)
```

```
[Out] B*x/c+1/(b*c)^(1/2)*arctan(x*c/(b*c)^(1/2))*A-1/c/(b*c)^(1/2)*arctan(x*c/(b
*c)^(1/2))*B*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.775171, size = 223, normalized size = 5.58

$$\left[\frac{2\,Bbcx + (Bb - Ac)\sqrt{-bc}\log\left(\frac{cx^2 - 2\,\sqrt{-bc}x - b}{cx^2 + b}\right)}{2\,bc^2}, \frac{Bbcx - (Bb - Ac)\sqrt{bc}\arctan\left(\frac{\sqrt{bc}x}{b}\right)}{bc^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*B*b*c*x + (B*b - A*c)*sqrt(-b*c)*log((c*x^2 - 2*sqrt(-b*c)*x - b)/(
c*x^2 + b)))/(b*c^2), (B*b*c*x - (B*b - A*c)*sqrt(b*c)*arctan(sqrt(b*c)*x/b
))/(b*c^2)]
```

Sympy [B] time = 0.412816, size = 82, normalized size = 2.05

$$\frac{Bx}{c} + \frac{\sqrt{-\frac{1}{bc^3}}\,(-Ac + Bb)\log\left(-bc\sqrt{-\frac{1}{bc^3}} + x\right)}{2} - \frac{\sqrt{-\frac{1}{bc^3}}\,(-Ac + Bb)\log\left(bc\sqrt{-\frac{1}{bc^3}} + x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] B*x/c + sqrt(-1/(b*c**3))*(-A*c + B*b)*log(-b*c*sqrt(-1/(b*c**3)) + x)/2 -
sqrt(-1/(b*c**3))*(-A*c + B*b)*log(b*c*sqrt(-1/(b*c**3)) + x)/2
```

Giac [A] time = 1.1817, size = 46, normalized size = 1.15

$$\frac{Bx}{c} - \frac{(Bb - Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] B*x/c - (B*b - A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c)
```

3.50 $\int \frac{x(A+Bx^2)}{bx^2+cx^4}\,dx$

Optimal. Leaf size=34

$$\frac{(bB-Ac)\log\left(b+cx^2\right)}{2bc}+\frac{A\log(x)}{b}$$

```
[Out] (A*Log[x])/b + ((b*B - A*c)*Log[b + c*x^2])/(2*b*c)
```

Rubi [A] time = 0.0407605, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1584, 446, 72}

$$\frac{(bB-Ac)\log\left(b+cx^2\right)}{2bc}+\frac{A\log(x)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (A*Log[x])/b + ((b*B - A*c)*Log[b + c*x^2])/(2*b*c)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(A+Bx^2\right)}{bx^2+cx^4}\,dx &= \int \frac{A+Bx^2}{x\left(b+cx^2\right)}\,dx\\ &= \frac{1}{2}\,\text{Subst}\left(\int \frac{A+Bx}{x(b+cx)}\,dx,x,x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{A}{bx}+\frac{bB-Ac}{b(b+cx)}\right)dx,x,x^2\right)\\ &= \frac{A\log(x)}{b}+\frac{(bB-Ac)\log\left(b+cx^2\right)}{2bc}\end{aligned}$$

Mathematica [A] time = 0.0118481, size = 34, normalized size = 1.

$$\frac{(bB-Ac)\log\left(b+cx^2\right)}{2bc}+\frac{A\log(x)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^2))/(b*x^2 + c*x^4),x]

[Out] (A*Log[x])/b + ((b*B - A*c)*Log[b + c*x^2])/(2*b*c)
```

Maple [A] time = 0.004, size = 37, normalized size = 1.1

$$\frac{A\ln(x)}{b}-\frac{\ln\left(cx^2+b\right)A}{2\,b}+\frac{\ln\left(cx^2+b\right)B}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)/(c*x^4+b*x^2),x)

[Out] A*ln(x)/b-1/2/b*ln(c*x^2+b)*A+1/2/c*ln(c*x^2+b)*B
```

Maxima [A] time = 1.2096, size = 47, normalized size = 1.38

$$\frac{A\log\left(x^2\right)}{2\,b}+\frac{(Bb-Ac)\log\left(cx^2+b\right)}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/2*A*log(x^2)/b + 1/2*(B*b - A*c)*log(c*x^2 + b)/(b*c)
```

Fricas [A] time = 0.772343, size = 74, normalized size = 2.18

$$\frac{2\,Ac\log\left(x\right) + \left(Bb - Ac\right)\log\left(cx^{2} + b\right)}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*A*c*log(x) + (B*b - A*c)*log(c*x^2 + b))/(b*c)
```

Sympy [A] time = 0.65411, size = 26, normalized size = 0.76

$$\frac{A\log\left(x\right)}{b} + \frac{\left(-Ac + Bb\right)\log\left(\frac{b}{c} + x^{2}\right)}{2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] A*log(x)/b + (-A*c + B*b)*log(b/c + x**2)/(2*b*c)
```

Giac [A] time = 1.25781, size = 46, normalized size = 1.35

$$\frac{A\log\left(|x|\right)}{b} + \frac{\left(Bb - Ac\right)\log\left(\left|cx^{2} + b\right|\right)}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] A*log(abs(x))/b + 1/2*(B*b - A*c)*log(abs(c*x^2 + b))/(b*c)
```

3.51 $\int \frac{A+Bx^2}{bx^2+cx^4} \, dx$

Optimal. Leaf size=42

$$\frac{(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{3/2}\sqrt{c}}-\frac{A}{bx}$$

```
[Out] -(A/(b*x)) + ((b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(b^(3/2)*Sqrt[c])
```

Rubi [A] time = 0.0318716, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1593, 453, 205}

$$\frac{(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{3/2}\sqrt{c}}-\frac{A}{bx}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(b*x^2 + c*x^4),x]
```

```
[Out] -(A/(b*x)) + ((b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(b^(3/2)*Sqrt[c])
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{bx^2+cx^4}\,dx &= \int \frac{A+Bx^2}{x^2\left(b+cx^2\right)}\,dx\\ &= -\frac{A}{bx}-\frac{(-bB+Ac)\int \frac{1}{b+cx^2}\,dx}{b}\\ &= -\frac{A}{bx}+\frac{(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{3/2}\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.0249807, size = 42, normalized size = 1.

$$\frac{(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{3/2}\sqrt{c}}-\frac{A}{bx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(b*x^2 + c*x^4),x]
```

```
[Out] -(A/(b*x)) + ((b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(b^(3/2)*Sqrt[c])
```

Maple [A] time = 0.005, size = 48, normalized size = 1.1

$$-\frac{A}{bx}-\frac{Ac}{b}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}+B\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(c*x^4+b*x^2),x)
```

```
[Out] -A/b/x-1/b/(b*c)^(1/2)*arctan(x*c/(b*c)^(1/2))*A*c+1/(b*c)^(1/2)*arctan(x*c
/(b*c)^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.764951, size = 228, normalized size = 5.43

$$\left[\frac{(Bb-Ac)\sqrt{-bc}x\log\left(\frac{cx^2+2\sqrt{-bc}x-b}{cx^2+b}\right)-2Abc}{2b^2cx}, \frac{(Bb-Ac)\sqrt{bc}x\arctan\left(\frac{\sqrt{bc}x}{b}\right)-Abc}{b^2cx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*((B*b - A*c)*sqrt(-b*c)*x*log((c*x^2 + 2*sqrt(-b*c)*x - b)/(c*x^2 + b)
) - 2*A*b*c)/(b^2*c*x), ((B*b - A*c)*sqrt(b*c)*x*arctan(sqrt(b*c)*x/b) - A*
b*c)/(b^2*c*x)]
```

Sympy [B] time = 0.452095, size = 82, normalized size = 1.95

$$-\frac{A}{bx}-\frac{\sqrt{-\frac{1}{b^3c}}(-Ac+Bb)\log\left(-b^2\sqrt{-\frac{1}{b^3c}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{b^3c}}(-Ac+Bb)\log\left(b^2\sqrt{-\frac{1}{b^3c}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] -A/(b*x) - sqrt(-1/(b**3*c))*(-A*c + B*b)*log(-b**2*sqrt(-1/(b**3*c)) + x)/
2 + sqrt(-1/(b**3*c))*(-A*c + B*b)*log(b**2*sqrt(-1/(b**3*c)) + x)/2
```

<u>Giac</u> [A] time = 1.24151, size = 49, normalized size = 1.17

$$\frac{(Bb - Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}b} - \frac{A}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] (B*b - A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b) - A/(b*x)
```

3.52 $\int \frac{A+Bx^2}{bx^2-cx^4} \, dx$

Optimal. Leaf size=41

$$\frac{(Ac+bB)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{3/2}\sqrt{c}}-\frac{A}{bx}$$

```
[Out] -(A/(b*x)) + ((b*B + A*c)*ArcTanh[(Sqrt[c]*x)/Sqrt[b]])/(b^(3/2)*Sqrt[c])
```

Rubi [A] time = 0.0344379, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1593, 453, 208}

$$\frac{(Ac+bB)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{3/2}\sqrt{c}}-\frac{A}{bx}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(b*x^2 - c*x^4),x]
```

```
[Out] -(A/(b*x)) + ((b*B + A*c)*ArcTanh[(Sqrt[c]*x)/Sqrt[b]])/(b^(3/2)*Sqrt[c])
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{bx^2-cx^4}\,dx &= \int \frac{A+Bx^2}{x^2\left(b-cx^2\right)}\,dx\\ &= -\frac{A}{bx}+\frac{(bB+Ac)\int \frac{1}{b-cx^2}\,dx}{b}\\ &= -\frac{A}{bx}+\frac{(bB+Ac)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{3/2}\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.023628, size = 41, normalized size = 1.

$$\frac{(Ac+bB)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{3/2}\sqrt{c}}-\frac{A}{bx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(b*x^2 - c*x^4),x]
```

```
[Out] -(A/(b*x)) + ((b*B + A*c)*ArcTanh[(Sqrt[c]*x)/Sqrt[b]])/(b^(3/2)*Sqrt[c])
```

Maple [A] time = 0.005, size = 39, normalized size = 1.

$$-\frac{-Ac-Bb}{b}\mathrm{Artanh}\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}-\frac{A}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(-c*x^4+b*x^2),x)
```

```
[Out] -(-A*c-B*b)/b/(b*c)^(1/2)*arctanh(x*c/(b*c)^(1/2))-A/b/x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(-c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.747638, size = 230, normalized size = 5.61

$$\left[\frac{(Bb+Ac)\sqrt{bc}x\log\left(\frac{cx^2+2\sqrt{bc}x+b}{cx^2-b}\right)-2\,Abc}{2\,b^2cx},-\frac{(Bb+Ac)\sqrt{-bc}x\arctan\left(\frac{\sqrt{-bc}x}{b}\right)+Abc}{b^2cx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(-c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*((B*b + A*c)*sqrt(b*c)*x*log((c*x^2 + 2*sqrt(b*c)*x + b)/(c*x^2 - b))
- 2*A*b*c)/(b^2*c*x), -((B*b + A*c)*sqrt(-b*c)*x*arctan(sqrt(-b*c)*x/b) + A
*b*c)/(b^2*c*x)]
```

Sympy [B] time = 0.467934, size = 75, normalized size = 1.83

$$-\frac{A}{bx}-\frac{\sqrt{\frac{1}{b^3c}}\,(Ac+Bb)\log\left(-b^2\sqrt{\frac{1}{b^3c}}+x\right)}{2}+\frac{\sqrt{\frac{1}{b^3c}}\,(Ac+Bb)\log\left(b^2\sqrt{\frac{1}{b^3c}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(-c*x**4+b*x**2),x)
```

```
[Out] -A/(b*x) - sqrt(1/(b**3*c))*(A*c + B*b)*log(-b**2*sqrt(1/(b**3*c)) + x)/2 +
 sqrt(1/(b**3*c))*(A*c + B*b)*log(b**2*sqrt(1/(b**3*c)) + x)/2
```

Giac [A] time = 1.25921, size = 51, normalized size = 1.24

$$-\frac{(Bb+Ac)\arctan\left(\frac{cx}{\sqrt{-bc}}\right)}{\sqrt{-bc}b}-\frac{A}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(-c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -(B*b + A*c)*arctan(c*x/sqrt(-b*c))/(sqrt(-b*c)*b) - A/(b*x)
```

3.53 $\int \frac{A+Bx^2}{x(bx^2+cx^4)}\,dx$

Optimal. Leaf size=49

$$-\frac{(bB-Ac)\log\left(b+cx^2\right)}{2b^2}+\frac{\log(x)(bB-Ac)}{b^2}-\frac{A}{2bx^2}$$

```
[Out] -A/(2*b*x^2) + ((b*B - A*c)*Log[x])/b^2 - ((b*B - A*c)*Log[b + c*x^2])/(2*b
^2)
```

Rubi [A] time = 0.0557434, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 446, 77}

$$-\frac{(bB-Ac)\log\left(b+cx^2\right)}{2b^2}+\frac{\log(x)(bB-Ac)}{b^2}-\frac{A}{2bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x*(b*x^2 + c*x^4)),x]
```

```
[Out] -A/(2*b*x^2) + ((b*B - A*c)*Log[x])/b^2 - ((b*B - A*c)*Log[b + c*x^2])/(2*b
^2)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
```

```
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)}\,dx &= \int \frac{A+Bx^2}{x^3\left(b+cx^2\right)}\,dx\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{x^2(b+cx)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{A}{bx^2}+\frac{bB-Ac}{b^2x}-\frac{c(bB-Ac)}{b^2(b+cx)}\right)dx, x, x^2\right)\\
&= -\frac{A}{2bx^2}+\frac{(bB-Ac)\log(x)}{b^2}-\frac{(bB-Ac)\log\left(b+cx^2\right)}{2b^2}
\end{aligned}$$

Mathematica [A] time = 0.020931, size = 49, normalized size = 1.

$$\frac{(Ac-bB)\log\left(b+cx^2\right)}{2b^2}+\frac{\log(x)(bB-Ac)}{b^2}-\frac{A}{2bx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x*(b*x^2 + c*x^4)),x]
```

```
[Out] -A/(2*b*x^2) + ((b*B - A*c)*Log[x])/b^2 + ((-(b*B) + A*c)*Log[b + c*x^2])/(
2*b^2)
```

Maple [A] time = 0.005, size = 56, normalized size = 1.1

$$-\frac{A}{2\,bx^2}-\frac{A\ln(x)\,c}{b^2}+\frac{\ln(x)\,B}{b}+\frac{\ln\left(cx^2+b\right)Ac}{2\,b^2}-\frac{\ln\left(cx^2+b\right)B}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x/(c*x^4+b*x^2),x)
```

```
[Out] -1/2*A/b/x^2-1/b^2*ln(x)*A*c+1/b*ln(x)*B+1/2/b^2*ln(c*x^2+b)*A*c-1/2/b*ln(c
*x^2+b)*B
```

Maxima [A] time = 1.16714, size = 65, normalized size = 1.33

$$-\frac{(Bb-Ac)\log\left(cx^2+b\right)}{2\,b^2}+\frac{(Bb-Ac)\log\left(x^2\right)}{2\,b^2}-\frac{A}{2\,bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -1/2*(B*b - A*c)*log(c*x^2 + b)/b^2 + 1/2*(B*b - A*c)*log(x^2)/b^2 - 1/2*A/
(b*x^2)
```

Fricas [A] time = 0.619287, size = 111, normalized size = 2.27

$$-\frac{(Bb-Ac)x^2\log\left(cx^2+b\right)-2\,(Bb-Ac)x^2\log\left(x\right)+Ab}{2\,b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] -1/2*((B*b - A*c)*x^2*log(c*x^2 + b) - 2*(B*b - A*c)*x^2*log(x) + A*b)/(b^2
*x^2)
```

Sympy [A] time = 0.815655, size = 41, normalized size = 0.84

$$-\frac{A}{2bx^2}+\frac{(-Ac+Bb)\log\left(x\right)}{b^2}-\frac{(-Ac+Bb)\log\left(\frac{b}{c}+x^2\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x/(c*x**4+b*x**2),x)
```

```
[Out] -A/(2*b*x**2) + (-A*c + B*b)*log(x)/b**2 - (-A*c + B*b)*log(b/c + x**2)/(2*
b**2)
```

Giac [A] time = 1.20614, size = 96, normalized size = 1.96

$$\frac{(Bb - Ac)\log\left(x^2\right)}{2\,b^2} - \frac{\left(Bbc - Ac^2\right)\log\left(\left|cx^2 + b\right|\right)}{2\,b^2c} - \frac{Bbx^2 - Acx^2 + Ab}{2\,b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*(B*b - A*c)*log(x^2)/b^2 - 1/2*(B*b*c - A*c^2)*log(abs(c*x^2 + b))/(b^2
*c) - 1/2*(B*b*x^2 - A*c*x^2 + A*b)/(b^2*x^2)
```

3.54 $\int \frac{A+Bx^2}{x^2\left(bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=61

$$-\frac{bB-Ac}{b^2x}-\frac{\sqrt{c}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{5/2}}-\frac{A}{3bx^3}$$

```
[Out] -A/(3*b*x^3) - (b*B - A*c)/(b^2*x) - (Sqrt[c]*(b*B - A*c)*ArcTan[(Sqrt[c]*x
)/Sqrt[b]])/b^(5/2)
```

Rubi [A] time = 0.0558791, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1584, 453, 325, 205}

$$-\frac{bB-Ac}{b^2x}-\frac{\sqrt{c}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{5/2}}-\frac{A}{3bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^2*(b*x^2 + c*x^4)),x]
```

```
[Out] -A/(3*b*x^3) - (b*B - A*c)/(b^2*x) - (Sqrt[c]*(b*B - A*c)*ArcTan[(Sqrt[c]*x
)/Sqrt[b]])/b^(5/2)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{x^2\left(bx^2+cx^4\right)}\,dx &= \int \frac{A+Bx^2}{x^4\left(b+cx^2\right)}\,dx\\ &= -\frac{A}{3bx^3}-\frac{(-3bB+3Ac)\int \frac{1}{x^2\left(b+cx^2\right)}\,dx}{3b}\\ &= -\frac{A}{3bx^3}-\frac{bB-Ac}{b^2x}-\frac{(c(bB-Ac))\int \frac{1}{b+cx^2}\,dx}{b^2}\\ &= -\frac{A}{3bx^3}-\frac{bB-Ac}{b^2x}-\frac{\sqrt{c}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.0525597, size = 60, normalized size = 0.98

$$\frac{Ac-bB}{b^2x}-\frac{\sqrt{c}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{5/2}}-\frac{A}{3bx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^2*(b*x^2 + c*x^4)),x]
```

```
[Out] -A/(3*b*x^3) + (-(b*B) + A*c)/(b^2*x) - (Sqrt[c]*(b*B - A*c)*ArcTan[(Sqrt[c
]*x)/Sqrt[b]])/b^(5/2)
```

Maple [A] time = 0.006, size = 72, normalized size = 1.2

$$-\frac{A}{3\,bx^{3}}+\frac{Ac}{b^{2}x}-\frac{B}{bx}+\frac{Ac^{2}}{b^{2}}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}-\frac{cB}{b}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^2/(c*x^4+b*x^2),x)
```

```
[Out] -1/3*A/b/x^3+1/b^2/x*A*c-1/b/x*B+c^2/b^2/(b*c)^(1/2)*arctan(x*c/(b*c)^(1/2)
)*A-c/b/(b*c)^(1/2)*arctan(x*c/(b*c)^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.86797, size = 296, normalized size = 4.85

$$\left[-\frac{3\,(Bb-Ac)x^{3}\sqrt{-\frac{c}{b}}\log\left(\frac{cx^{2}+2\,bx\sqrt{-\frac{c}{b}}-b}{cx^{2}+b}\right)+6\,(Bb-Ac)x^{2}+2\,Ab}{6\,b^{2}x^{3}},-\frac{3\,(Bb-Ac)x^{3}\sqrt{\frac{c}{b}}\arctan\left(x\sqrt{\frac{c}{b}}\right)+3\,(Bb-Ac)x^{2}+A\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(B*b - A*c)*x^3*sqrt(-c/b)*log((c*x^2 + 2*b*x*sqrt(-c/b) - b)/(c*x
^2 + b)) + 6*(B*b - A*c)*x^2 + 2*A*b)/(b^2*x^3), -1/3*(3*(B*b - A*c)*x^3*sq
rt(c/b)*arctan(x*sqrt(c/b)) + 3*(B*b - A*c)*x^2 + A*b)/(b^2*x^3)]
```

Sympy [B] time = 0.56351, size = 129, normalized size = 2.11

$$\frac{\sqrt{-\frac{c}{b^5}}\,(-Ac+Bb)\log\left(-\frac{b^3\sqrt{-\frac{c}{b^5}}(-Ac+Bb)}{-Ac^2+Bbc}+x\right)}{2}-\frac{\sqrt{-\frac{c}{b^5}}\,(-Ac+Bb)\log\left(\frac{b^3\sqrt{-\frac{c}{b^5}}(-Ac+Bb)}{-Ac^2+Bbc}+x\right)}{2}-\frac{Ab+x^2\,(-3Ac+3Bb)}{3b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**2/(c*x**4+b*x**2),x)
```

```
[Out] sqrt(-c/b**5)*(-A*c + B*b)*log(-b**3*sqrt(-c/b**5)*(-A*c + B*b)/(-A*c**2 +
B*b*c) + x)/2 - sqrt(-c/b**5)*(-A*c + B*b)*log(b**3*sqrt(-c/b**5)*(-A*c + B
*b)/(-A*c**2 + B*b*c) + x)/2 - (A*b + x**2*(-3*A*c + 3*B*b))/(3*b**2*x**3)
```

Giac [A] time = 1.27166, size = 77, normalized size = 1.26

$$-\frac{\left(Bbc-Ac^2\right)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}b^2}-\frac{3\,Bbx^2-3\,Acx^2+Ab}{3\,b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -(B*b*c - A*c^2)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^2) - 1/3*(3*B*b*x^2 - 3
*A*c*x^2 + A*b)/(b^2*x^3)
```

3.55 $\int \frac{A+Bx^2}{x^3\left(bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=70

$$-\frac{bB-Ac}{2b^2x^2}+\frac{c(bB-Ac)\log\left(b+cx^2\right)}{2b^3}-\frac{c\log(x)(bB-Ac)}{b^3}-\frac{A}{4bx^4}$$

```
[Out] -A/(4*b*x^4) - (b*B - A*c)/(2*b^2*x^2) - (c*(b*B - A*c)*Log[x])/b^3 + (c*(b
*B - A*c)*Log[b + c*x^2])/(2*b^3)
```

Rubi [A] time = 0.0722856, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 446, 77}

$$-\frac{bB-Ac}{2b^2x^2}+\frac{c(bB-Ac)\log\left(b+cx^2\right)}{2b^3}-\frac{c\log(x)(bB-Ac)}{b^3}-\frac{A}{4bx^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^3*(b*x^2 + c*x^4)),x]
```

```
[Out] -A/(4*b*x^4) - (b*B - A*c)/(2*b^2*x^2) - (c*(b*B - A*c)*Log[x])/b^3 + (c*(b
*B - A*c)*Log[b + c*x^2])/(2*b^3)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
```

```
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
&& ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{x^3\left(bx^2+cx^4\right)}\,dx &= \int \frac{A+Bx^2}{x^5\left(b+cx^2\right)}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{x^3(b+cx)}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{A}{bx^3}+\frac{bB-Ac}{b^2x^2}-\frac{c(bB-Ac)}{b^3x}+\frac{c^2(bB-Ac)}{b^3(b+cx)}\right)dx,x,x^2\right)\\ &= -\frac{A}{4bx^4}-\frac{bB-Ac}{2b^2x^2}-\frac{c(bB-Ac)\log(x)}{b^3}+\frac{c(bB-Ac)\log\left(b+cx^2\right)}{2b^3}\end{aligned}$$

Mathematica [A] time = 0.028756, size = 70, normalized size = 1.

$$\frac{-b\left(Ab-2Acx^2+2bBx^2\right)+4cx^4\log(x)(Ac-bB)+2cx^4(bB-Ac)\log\left(b+cx^2\right)}{4b^3x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^3*(b*x^2 + c*x^4)),x]
```

```
[Out] (-(b*(A*b + 2*b*B*x^2 - 2*A*c*x^2)) + 4*c*(-(b*B) + A*c)*x^4*Log[x] + 2*c*(
b*B - A*c)*x^4*Log[b + c*x^2])/(4*b^3*x^4)
```

Maple [A] time = 0.007, size = 81, normalized size = 1.2

$$-\frac{A}{4\,bx^4}+\frac{Ac}{2\,b^2x^2}-\frac{B}{2\,bx^2}+\frac{A\ln(x)\,c^2}{b^3}-\frac{Bc\ln(x)}{b^2}-\frac{c^2\ln\left(cx^2+b\right)A}{2\,b^3}+\frac{c\ln\left(cx^2+b\right)B}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^3/(c*x^4+b*x^2),x)
```

```
[Out] -1/4*A/b/x^4+1/2/b^2/x^2*A*c-1/2/b/x^2*B+1/b^3*c^2*ln(x)*A-1/b^2*c*ln(x)*B-
1/2*c^2/b^3*ln(c*x^2+b)*A+1/2*c/b^2*ln(c*x^2+b)*B
```

Maxima [A] time = 2.59029, size = 95, normalized size = 1.36

$$\frac{\left(Bbc - Ac^2\right)\log\left(cx^2 + b\right)}{2\,b^3} - \frac{\left(Bbc - Ac^2\right)\log\left(x^2\right)}{2\,b^3} - \frac{2\,(Bb - Ac)x^2 + Ab}{4\,b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/2*(B*b*c - A*c^2)*log(c*x^2 + b)/b^3 - 1/2*(B*b*c - A*c^2)*log(x^2)/b^3 -
 1/4*(2*(B*b - A*c)*x^2 + A*b)/(b^2*x^4)
```

Fricas [A] time = 0.792531, size = 158, normalized size = 2.26

$$\frac{2\left(Bbc - Ac^2\right)x^4\log\left(cx^2 + b\right) - 4\left(Bbc - Ac^2\right)x^4\log\left(x\right) - Ab^2 - 2\left(Bb^2 - Abc\right)x^2}{4\,b^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(B*b*c - A*c^2)*x^4*log(c*x^2 + b) - 4*(B*b*c - A*c^2)*x^4*log(x) -
A*b^2 - 2*(B*b^2 - A*b*c)*x^2)/(b^3*x^4)
```

Sympy [A] time = 0.997032, size = 61, normalized size = 0.87

$$-\frac{Ab + x^2\left(-2Ac + 2Bb\right)}{4b^2x^4} - \frac{c\left(-Ac + Bb\right)\log\left(x\right)}{b^3} + \frac{c\left(-Ac + Bb\right)\log\left(\frac{b}{c} + x^2\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**3/(c*x**4+b*x**2),x)
```

```
[Out] -(A*b + x**2*(-2*A*c + 2*B*b))/(4*b**2*x**4) - c*(-A*c + B*b)*log(x)/b**3 +
 c*(-A*c + B*b)*log(b/c + x**2)/(2*b**3)
```

Giac [A] time = 1.14446, size = 135, normalized size = 1.93

$$-\frac{\left(Bbc - Ac^2\right)\log\left(x^2\right)}{2\,b^3} + \frac{\left(Bbc^2 - Ac^3\right)\log\left(\left|cx^2 + b\right|\right)}{2\,b^3c} + \frac{3\,Bbcx^4 - 3\,Ac^2x^4 - 2\,Bb^2x^2 + 2\,Abcx^2 - Ab^2}{4\,b^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -1/2*(B*b*c - A*c^2)*log(x^2)/b^3 + 1/2*(B*b*c^2 - A*c^3)*log(abs(c*x^2 + b
))/(b^3*c) + 1/4*(3*B*b*c*x^4 - 3*A*c^2*x^4 - 2*B*b^2*x^2 + 2*A*b*c*x^2 - A
*b^2)/(b^3*x^4)
```

3.56 $\int \frac{A+Bx^2}{x^4\left(bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=78

$$\frac{c^{3/2}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{7/2}}-\frac{bB-Ac}{3b^2x^3}+\frac{c(bB-Ac)}{b^3x}-\frac{A}{5bx^5}$$

```
[Out] -A/(5*b*x^5) - (b*B - A*c)/(3*b^2*x^3) + (c*(b*B - A*c))/(b^3*x) + (c^(3/2)
*(b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/b^(7/2)
```

Rubi [A] time = 0.0681362, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1584, 453, 325, 205}

$$\frac{c^{3/2}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{7/2}}-\frac{bB-Ac}{3b^2x^3}+\frac{c(bB-Ac)}{b^3x}-\frac{A}{5bx^5}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^4*(b*x^2 + c*x^4)),x]
```

```
[Out] -A/(5*b*x^5) - (b*B - A*c)/(3*b^2*x^3) + (c*(b*B - A*c))/(b^3*x) + (c^(3/2)
*(b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/b^(7/2)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^2}{x^4\left(bx^2+cx^4\right)}\,dx &= \int \frac{A+Bx^2}{x^6\left(b+cx^2\right)}\,dx\\
&= -\frac{A}{5bx^5}-\frac{(-5bB+5Ac)\int \frac{1}{x^4\left(b+cx^2\right)}\,dx}{5b}\\
&= -\frac{A}{5bx^5}-\frac{bB-Ac}{3b^2x^3}-\frac{(c(bB-Ac))\int \frac{1}{x^2\left(b+cx^2\right)}\,dx}{b^2}\\
&= -\frac{A}{5bx^5}-\frac{bB-Ac}{3b^2x^3}+\frac{c(bB-Ac)}{b^3x}+\frac{\left(c^2(bB-Ac)\right)\int \frac{1}{b+cx^2}\,dx}{b^3}\\
&= -\frac{A}{5bx^5}-\frac{bB-Ac}{3b^2x^3}+\frac{c(bB-Ac)}{b^3x}+\frac{c^{3/2}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{7/2}}
\end{aligned}$$

Mathematica [A] time = 0.0534885, size = 78, normalized size = 1.

$$\frac{c^{3/2}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{7/2}}+\frac{Ac-bB}{3b^2x^3}+\frac{c(bB-Ac)}{b^3x}-\frac{A}{5bx^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^4*(b*x^2 + c*x^4)),x]
```

```
[Out] -A/(5*b*x^5) + (-(b*B) + A*c)/(3*b^2*x^3) + (c*(b*B - A*c))/(b^3*x) + (c^(3
/2)*(b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/b^(7/2)
```

Maple [A] time = 0.006, size = 96, normalized size = 1.2

$$-\frac{A}{5\,b x^{5}}+\frac{A c}{3\,b^{2} x^{3}}-\frac{B}{3\,b x^{3}}-\frac{A c^{2}}{b^{3} x}+\frac{c B}{b^{2} x}-\frac{A c^{3}}{b^{3}}\arctan\left(c x\frac{1}{\sqrt{b c}}\right)\frac{1}{\sqrt{b c}}+\frac{c^{2} B}{b^{2}}\arctan\left(c x\frac{1}{\sqrt{b c}}\right)\frac{1}{\sqrt{b c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^4/(c*x^4+b*x^2),x)
```

```
[Out] -1/5*A/b/x^5+1/3/b^2/x^3*A*c-1/3/b/x^3*B-1/b^3*c^2/x*A+1/b^2*c/x*B-c^3/b^3/
(b*c)^(1/2)*arctan(x*c/(b*c)^(1/2))*A+c^2/b^2/(b*c)^(1/2)*arctan(x*c/(b*c)^
(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.834914, size = 398, normalized size = 5.1

$$\left[-\frac{15\left(Bbc-Ac^{2}\right)x^{5}\sqrt{-\frac{c}{b}}\log\left(\frac{cx^{2}-2\,bx\sqrt{-\frac{c}{b}}-b}{cx^{2}+b}\right)-30\left(Bbc-Ac^{2}\right)x^{4}+6\,Ab^{2}+10\left(Bb^{2}-Abc\right)x^{2}}{30\,b^{3}x^{5}},\ \frac{15\left(Bbc-Ac^{2}\right)x^{5}\sqrt{\frac{c}{b}}\,\mathrm{arcta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [-1/30*(15*(B*b*c - A*c^2)*x^5*sqrt(-c/b)*log((c*x^2 - 2*b*x*sqrt(-c/b) - b
)/(c*x^2 + b)) - 30*(B*b*c - A*c^2)*x^4 + 6*A*b^2 + 10*(B*b^2 - A*b*c)*x^2)
/(b^3*x^5), 1/15*(15*(B*b*c - A*c^2)*x^5*sqrt(c/b)*arctan(x*sqrt(c/b)) + 15
```

```
*(B*b*c - A*c^2)*x^4 - 3*A*b^2 - 5*(B*b^2 - A*b*c)*x^2)/(b^3*x^5)]
```

Sympy [B] time = 0.677921, size = 163, normalized size = 2.09

$$-\frac{\sqrt{-\frac{c^3}{b^7}}(-Ac+Bb)\log\left(-\frac{b^4\sqrt{-\frac{c^3}{b^7}}(-Ac+Bb)}{-Ac^3+Bbc^2}+x\right)}{2}+\frac{\sqrt{-\frac{c^3}{b^7}}(-Ac+Bb)\log\left(\frac{b^4\sqrt{-\frac{c^3}{b^7}}(-Ac+Bb)}{-Ac^3+Bbc^2}+x\right)}{2}+\frac{-3Ab^2+x^4\left(-15Ac^2+\right.}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**4/(c*x**4+b*x**2),x)
```

```
[Out] -sqrt(-c**3/b**7)*(-A*c + B*b)*log(-b**4*sqrt(-c**3/b**7)*(-A*c + B*b)/(-A*
c**3 + B*b*c**2) + x)/2 + sqrt(-c**3/b**7)*(-A*c + B*b)*log(b**4*sqrt(-c**3
/b**7)*(-A*c + B*b)/(-A*c**3 + B*b*c**2) + x)/2 + (-3*A*b**2 + x**4*(-15*A*
c**2 + 15*B*b*c) + x**2*(5*A*b*c - 5*B*b**2))/(15*b**3*x**5)
```

Giac [A] time = 1.25137, size = 109, normalized size = 1.4

$$\frac{\left(Bbc^2-Ac^3\right)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}b^3}+\frac{15\,Bbcx^4-15\,Ac^2x^4-5\,Bb^2x^2+5\,Abcx^2-3\,Ab^2}{15\,b^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] (B*b*c^2 - A*c^3)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^3) + 1/15*(15*B*b*c*x^
4 - 15*A*c^2*x^4 - 5*B*b^2*x^2 + 5*A*b*c*x^2 - 3*A*b^2)/(b^3*x^5)
```

3.57 $\int \frac{A+Bx^2}{x^5\left(bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=92

$$-\frac{c^2(bB-Ac)\log\left(b+cx^2\right)}{2b^4}+\frac{c^2\log(x)(bB-Ac)}{b^4}+\frac{c(bB-Ac)}{2b^3x^2}-\frac{bB-Ac}{4b^2x^4}-\frac{A}{6bx^6}$$

```
[Out] -A/(6*b*x^6) - (b*B - A*c)/(4*b^2*x^4) + (c*(b*B - A*c))/(2*b^3*x^2) + (c^2
*(b*B - A*c)*Log[x])/b^4 - (c^2*(b*B - A*c)*Log[b + c*x^2])/(2*b^4)
```

Rubi [A] time = 0.0897576, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 446, 77}

$$-\frac{c^2(bB-Ac)\log\left(b+cx^2\right)}{2b^4}+\frac{c^2\log(x)(bB-Ac)}{b^4}+\frac{c(bB-Ac)}{2b^3x^2}-\frac{bB-Ac}{4b^2x^4}-\frac{A}{6bx^6}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^5*(b*x^2 + c*x^4)),x]
```

```
[Out] -A/(6*b*x^6) - (b*B - A*c)/(4*b^2*x^4) + (c*(b*B - A*c))/(2*b^3*x^2) + (c^2
*(b*B - A*c)*Log[x])/b^4 - (c^2*(b*B - A*c)*Log[b + c*x^2])/(2*b^4)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
```

```
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^2}{x^5\left(bx^2+cx^4\right)}\,dx &= \int \frac{A+Bx^2}{x^7\left(b+cx^2\right)}\,dx\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{x^4(b+cx)}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{A}{bx^4}+\frac{bB-Ac}{b^2x^3}-\frac{c(bB-Ac)}{b^3x^2}+\frac{c^2(bB-Ac)}{b^4x}-\frac{c^3(bB-Ac)}{b^4(b+cx)}\right)dx,x,x^2\right)\\
&= -\frac{A}{6bx^6}-\frac{bB-Ac}{4b^2x^4}+\frac{c(bB-Ac)}{2b^3x^2}+\frac{c^2(bB-Ac)\log(x)}{b^4}-\frac{c^2(bB-Ac)\log\left(b+cx^2\right)}{2b^4}
\end{aligned}
$$

Mathematica [A] time = 0.0398739, size = 96, normalized size = 1.04

$$
\frac{\left(Ac^3-bBc^2\right)\log\left(b+cx^2\right)}{2b^4}+\frac{\log(x)\left(bBc^2-Ac^3\right)}{b^4}+\frac{c(bB-Ac)}{2b^3x^2}+\frac{Ac-bB}{4b^2x^4}-\frac{A}{6bx^6}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^5*(b*x^2 + c*x^4)),x]
```

```
[Out] -A/(6*b*x^6) + (-(b*B) + A*c)/(4*b^2*x^4) + (c*(b*B - A*c))/(2*b^3*x^2) + (
(b*B*c^2 - A*c^3)*Log[x])/b^4 + ((-(b*B*c^2) + A*c^3)*Log[b + c*x^2])/(2*b^
4)
```

Maple [A] time = 0.006, size = 107, normalized size = 1.2

$$
-\frac{A}{6\,bx^6}+\frac{Ac}{4\,x^4b^2}-\frac{B}{4\,bx^4}-\frac{Ac^2}{2\,b^3x^2}+\frac{cB}{2\,b^2x^2}-\frac{A\ln(x)\,c^3}{b^4}+\frac{Bc^2\ln(x)}{b^3}+\frac{c^3\ln\left(cx^2+b\right)A}{2\,b^4}-\frac{c^2\ln\left(cx^2+b\right)B}{2\,b^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^5/(c*x^4+b*x^2),x)
```

```
[Out] -1/6*A/b/x^6+1/4/b^2/x^4*A*c-1/4/b/x^4*B-1/2/b^3*c^2/x^2*A+1/2/b^2*c/x^2*B-
1/b^4*c^3*ln(x)*A+1/b^3*c^2*ln(x)*B+1/2*c^3/b^4*ln(c*x^2+b)*A-1/2*c^2/b^3*l
n(c*x^2+b)*B
```

Maxima [A] time = 1.05247, size = 130, normalized size = 1.41

$$-\frac{\left(Bbc^2 - Ac^3\right)\log\left(cx^2 + b\right)}{2\,b^4} + \frac{\left(Bbc^2 - Ac^3\right)\log\left(x^2\right)}{2\,b^4} + \frac{6\left(Bbc - Ac^2\right)x^4 - 2\,Ab^2 - 3\left(Bb^2 - Abc\right)x^2}{12\,b^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -1/2*(B*b*c^2 - A*c^3)*log(c*x^2 + b)/b^4 + 1/2*(B*b*c^2 - A*c^3)*log(x^2)/
b^4 + 1/12*(6*(B*b*c - A*c^2)*x^4 - 2*A*b^2 - 3*(B*b^2 - A*b*c)*x^2)/(b^3*x
^6)
```

Fricas [A] time = 0.721874, size = 211, normalized size = 2.29

$$-\frac{6\left(Bbc^2 - Ac^3\right)x^6\log\left(cx^2 + b\right) - 12\left(Bbc^2 - Ac^3\right)x^6\log\left(x\right) - 6\left(Bb^2c - Abc^2\right)x^4 + 2\,Ab^3 + 3\left(Bb^3 - Ab^2c\right)x^2}{12\,b^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] -1/12*(6*(B*b*c^2 - A*c^3)*x^6*log(c*x^2 + b) - 12*(B*b*c^2 - A*c^3)*x^6*lo
g(x) - 6*(B*b^2*c - A*b*c^2)*x^4 + 2*A*b^3 + 3*(B*b^3 - A*b^2*c)*x^2)/(b^4*
x^6)
```

Sympy [A] time = 1.16355, size = 88, normalized size = 0.96

$$\frac{-2Ab^2 + x^4\left(-6Ac^2 + 6Bbc\right) + x^2\left(3Abc - 3Bb^2\right)}{12b^3x^6} + \frac{c^2\left(-Ac + Bb\right)\log\left(x\right)}{b^4} - \frac{c^2\left(-Ac + Bb\right)\log\left(\frac{b}{c} + x^2\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**5/(c*x**4+b*x**2),x)
```

```
[Out] (-2*A*b**2 + x**4*(-6*A*c**2 + 6*B*b*c) + x**2*(3*A*b*c - 3*B*b**2))/(12*b*
*3*x**6) + c**2*(-A*c + B*b)*log(x)/b**4 - c**2*(-A*c + B*b)*log(b/c + x**2
)/(2*b**4)
```

Giac [A] time = 1.28656, size = 170, normalized size = 1.85

$$\frac{\left(Bbc^2 - Ac^3\right)\log\left(x^2\right)}{2\,b^4} - \frac{\left(Bbc^3 - Ac^4\right)\log\left(\left|cx^2 + b\right|\right)}{2\,b^4c} - \frac{11\,Bbc^2x^6 - 11\,Ac^3x^6 - 6\,Bb^2cx^4 + 6\,Abc^2x^4 + 3\,Bb^3x^2 - 3\,Ab^2c}{12\,b^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*(B*b*c^2 - A*c^3)*log(x^2)/b^4 - 1/2*(B*b*c^3 - A*c^4)*log(abs(c*x^2 +
b))/(b^4*c) - 1/12*(11*B*b*c^2*x^6 - 11*A*c^3*x^6 - 6*B*b^2*c*x^4 + 6*A*b*c
^2*x^4 + 3*B*b^3*x^2 - 3*A*b^2*c*x^2 + 2*A*b^3)/(b^4*x^6)
```

3.58 $\int \frac{x^{12}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=133

$$-\frac{b^3x(bB-Ac)}{2c^5\left(b+cx^2\right)}-\frac{b^2x(4bB-3Ac)}{c^5}+\frac{b^{5/2}(9bB-7Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{11/2}}-\frac{x^5(2bB-Ac)}{5c^3}+\frac{bx^3(3bB-2Ac)}{3c^4}+\frac{Bx^7}{7c^2}$$

```
[Out] -((b^2*(4*b*B - 3*A*c)*x)/c^5) + (b*(3*b*B - 2*A*c)*x^3)/(3*c^4) - ((2*b*B
- A*c)*x^5)/(5*c^3) + (B*x^7)/(7*c^2) - (b^3*(b*B - A*c)*x)/(2*c^5*(b + c*x
^2)) + (b^(5/2)*(9*b*B - 7*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*c^(11/2))
```

Rubi [A] time = 0.166286, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1584, 455, 1810, 205}

$$-\frac{b^3x(bB-Ac)}{2c^5\left(b+cx^2\right)}-\frac{b^2x(4bB-3Ac)}{c^5}+\frac{b^{5/2}(9bB-7Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{11/2}}-\frac{x^5(2bB-Ac)}{5c^3}+\frac{bx^3(3bB-2Ac)}{3c^4}+\frac{Bx^7}{7c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^12*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -((b^2*(4*b*B - 3*A*c)*x)/c^5) + (b*(3*b*B - 2*A*c)*x^3)/(3*c^4) - ((2*b*B
- A*c)*x^5)/(5*c^3) + (B*x^7)/(7*c^2) - (b^3*(b*B - A*c)*x)/(2*c^5*(b + c*x
^2)) + (b^(5/2)*(9*b*B - 7*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*c^(11/2))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
```

```
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^{12}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx = \int \frac{x^8\left(A+Bx^2\right)}{\left(b+cx^2\right)^2}\,dx$$

$$= -\frac{b^3(bB-Ac)x}{2c^5\left(b+cx^2\right)} - \frac{\int \frac{-b^3(bB-Ac)+2b^2c(bB-Ac)x^2-2bc^2(bB-Ac)x^4+2c^3(bB-Ac)x^6-2Bc^4x^8}{b+cx^2}\,dx}{2c^5}$$

$$= -\frac{b^3(bB-Ac)x}{2c^5\left(b+cx^2\right)} - \frac{\int \Big(2b^2(4bB-3Ac)-2bc(3bB-2Ac)x^2+2c^2(2bB-Ac)x^4-2Bc^3x^6+\frac{-9b^4B+}{b+c}$$

$$= -\frac{b^2(4bB-3Ac)x}{c^5} + \frac{b(3bB-2Ac)x^3}{3c^4} - \frac{(2bB-Ac)x^5}{5c^3} + \frac{Bx^7}{7c^2} - \frac{b^3(bB-Ac)x}{2c^5\left(b+cx^2\right)} + \frac{\Big(b^3(9bB-7Ac)\Big)}{2c^5}$$

$$= -\frac{b^2(4bB-3Ac)x}{c^5} + \frac{b(3bB-2Ac)x^3}{3c^4} - \frac{(2bB-Ac)x^5}{5c^3} + \frac{Bx^7}{7c^2} - \frac{b^3(bB-Ac)x}{2c^5\left(b+cx^2\right)} + \frac{b^{5/2}(9bB-7Ac)}{2c^{11/}}$$

Mathematica [A] time = 0.103565, size = 134, normalized size = 1.01

$$\frac{x\left(Ab^3c-b^4B\right)}{2c^5\left(b+cx^2\right)} - \frac{b^2x(4bB-3Ac)}{c^5} + \frac{b^{5/2}(9bB-7Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{11/2}} + \frac{x^5(Ac-2bB)}{5c^3} + \frac{bx^3(3bB-2Ac)}{3c^4} + \frac{Bx^7}{7c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^12*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -((b^2*(4*b*B - 3*A*c)*x)/c^5) + (b*(3*b*B - 2*A*c)*x^3)/(3*c^4) + ((-2*b*B
 + A*c)*x^5)/(5*c^3) + (B*x^7)/(7*c^2) + ((-(b^4*B) + A*b^3*c)*x)/(2*c^5*(b
 + c*x^2)) + (b^(5/2)*(9*b*B - 7*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*c^(11
/2))
```

Maple [A] time = 0.01, size = 155, normalized size = 1.2

$$\frac{Bx^7}{7c^2}+\frac{Ax^5}{5c^2}-\frac{2Bx^5b}{5c^3}-\frac{2Abx^3}{3c^3}+\frac{Bx^3b^2}{c^4}+3\frac{Ab^2x}{c^4}-4\frac{Bb^3x}{c^5}+\frac{Ab^3x}{2c^4\left(cx^2+b\right)}-\frac{b^4xB}{2c^5\left(cx^2+b\right)}-\frac{7Ab^3}{2c^4}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(B*x^2+A)/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/7*B*x^7/c^2+1/5/c^2*A*x^5-2/5/c^3*B*x^5*b-2/3/c^3*A*x^3*b+1/c^4*B*x^3*b^2
+3/c^4*A*b^2*x-4/c^5*B*b^3*x+1/2*b^3/c^4*x/(c*x^2+b)*A-1/2*b^4/c^5*x/(c*x^2
+b)*B-7/2*b^3/c^4/(b*c)^(1/2)*arctan(x*c/(b*c)^(1/2))*A+9/2*b^4/c^5/(b*c)^(
1/2)*arctan(x*c/(b*c)^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.888588, size = 755, normalized size = 5.68

$$\left[\frac{60\,Bc^4x^9-12\left(9\,Bbc^3-7\,Ac^4\right)x^7+28\left(9\,Bb^2c^2-7\,Abc^3\right)x^5-140\left(9\,Bb^3c-7\,Ab^2c^2\right)x^3-105\left(9\,Bb^4-7\,Ab^3c+\left(9\,Bb^3c\right.\right.}{420\left(c^6x^2+bc^5\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [1/420*(60*B*c^4*x^9 - 12*(9*B*b*c^3 - 7*A*c^4)*x^7 + 28*(9*B*b^2*c^2 - 7*A
*b*c^3)*x^5 - 140*(9*B*b^3*c - 7*A*b^2*c^2)*x^3 - 105*(9*B*b^4 - 7*A*b^3*c
+ (9*B*b^3*c - 7*A*b^2*c^2)*x^2)*sqrt(-b/c)*log((c*x^2 - 2*c*x*sqrt(-b/c) -
 b)/(c*x^2 + b)) - 210*(9*B*b^4 - 7*A*b^3*c)*x)/(c^6*x^2 + b*c^5), 1/210*(3
0*B*c^4*x^9 - 6*(9*B*b*c^3 - 7*A*c^4)*x^7 + 14*(9*B*b^2*c^2 - 7*A*b*c^3)*x^
5 - 70*(9*B*b^3*c - 7*A*b^2*c^2)*x^3 + 105*(9*B*b^4 - 7*A*b^3*c + (9*B*b^3*
c - 7*A*b^2*c^2)*x^2)*sqrt(b/c)*arctan(c*x*sqrt(b/c)/b) - 105*(9*B*b^4 - 7*
A*b^3*c)*x)/(c^6*x^2 + b*c^5)]
```

Sympy [A] time = 0.926563, size = 233, normalized size = 1.75

$$\frac{Bx^7}{7c^2} - \frac{x\left(-Ab^3c + Bb^4\right)}{2bc^5 + 2c^6x^2} - \frac{\sqrt{-\frac{b^5}{c^{11}}}\left(-7Ac + 9Bb\right)\log\left(-\frac{c^5\sqrt{-\frac{b^5}{c^{11}}}(-7Ac+9Bb)}{-7Ab^2c+9Bb^3} + x\right)}{4} + \frac{\sqrt{-\frac{b^5}{c^{11}}}\left(-7Ac + 9Bb\right)\log\left(\frac{c^5\sqrt{-\frac{b^5}{c^{11}}}(-7Ac+\ldots}{-7Ab^2c+9Bb\ldots}\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] B*x**7/(7*c**2) - x*(-A*b**3*c + B*b**4)/(2*b*c**5 + 2*c**6*x**2) - sqrt(-b
**5/c**11)*(-7*A*c + 9*B*b)*log(-c**5*sqrt(-b**5/c**11)*(-7*A*c + 9*B*b)/(-
7*A*b**2*c + 9*B*b**3) + x)/4 + sqrt(-b**5/c**11)*(-7*A*c + 9*B*b)*log(c**5
*sqrt(-b**5/c**11)*(-7*A*c + 9*B*b)/(-7*A*b**2*c + 9*B*b**3) + x)/4 - x**5*
(-A*c + 2*B*b)/(5*c**3) + x**3*(-2*A*b*c + 3*B*b**2)/(3*c**4) - x*(-3*A*b**
2*c + 4*B*b**3)/c**5
```

Giac [A] time = 1.33625, size = 188, normalized size = 1.41

$$\frac{\left(9\,Bb^4 - 7\,Ab^3c\right)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}c^5} - \frac{Bb^4x - Ab^3cx}{2\left(cx^2 + b\right)c^5} + \frac{15\,Bc^{12}x^7 - 42\,Bbc^{11}x^5 + 21\,Ac^{12}x^5 + 105\,Bb^2c^{10}x^3 - 70\,Abc^{11}x^3 - \ldots}{105\,c^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(9*B*b^4 - 7*A*b^3*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^5) - 1/2*(B*b^
4*x - A*b^3*c*x)/((c*x^2 + b)*c^5) + 1/105*(15*B*c^12*x^7 - 42*B*b*c^11*x^5
 + 21*A*c^12*x^5 + 105*B*b^2*c^10*x^3 - 70*A*b*c^11*x^3 - 420*B*b^3*c^9*x +
 315*A*b^2*c^10*x)/c^14
```

3.59 $\int \frac{x^{11}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=105

$$-\frac{b^3(bB-Ac)}{2c^5(b+cx^2)}-\frac{b^2(4bB-3Ac)\log(b+cx^2)}{2c^5}-\frac{x^4(2bB-Ac)}{4c^3}+\frac{bx^2(3bB-2Ac)}{2c^4}+\frac{Bx^6}{6c^2}$$

```
[Out] (b*(3*b*B - 2*A*c)*x^2)/(2*c^4) - ((2*b*B - A*c)*x^4)/(4*c^3) + (B*x^6)/(6*
c^2) - (b^3*(b*B - A*c))/(2*c^5*(b + c*x^2)) - (b^2*(4*b*B - 3*A*c)*Log[b +
 c*x^2])/(2*c^5)
```

Rubi [A] time = 0.135018, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 446, 77}

$$-\frac{b^3(bB-Ac)}{2c^5(b+cx^2)}-\frac{b^2(4bB-3Ac)\log(b+cx^2)}{2c^5}-\frac{x^4(2bB-Ac)}{4c^3}+\frac{bx^2(3bB-2Ac)}{2c^4}+\frac{Bx^6}{6c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (b*(3*b*B - 2*A*c)*x^2)/(2*c^4) - ((2*b*B - A*c)*x^4)/(4*c^3) + (B*x^6)/(6*
c^2) - (b^3*(b*B - A*c))/(2*c^5*(b + c*x^2)) - (b^2*(4*b*B - 3*A*c)*Log[b +
 c*x^2])/(2*c^5)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx &= \int \frac{x^7\left(A+Bx^2\right)}{\left(b+cx^2\right)^2}\,dx\\ &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^3(A+Bx)}{(b+cx)^2}\,dx, x, x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{b(3bB-2Ac)}{c^4}+\frac{(-2bB+Ac)x}{c^3}+\frac{Bx^2}{c^2}+\frac{b^3(bB-Ac)}{c^4(b+cx)^2}-\frac{b^2(4bB-3Ac)}{c^4(b+cx)}\right)dx, x, x^2\right)\\ &= \frac{b(3bB-2Ac)x^2}{2c^4}-\frac{(2bB-Ac)x^4}{4c^3}+\frac{Bx^6}{6c^2}-\frac{b^3(bB-Ac)}{2c^5\left(b+cx^2\right)}-\frac{b^2(4bB-3Ac)\log\left(b+cx^2\right)}{2c^5}\end{aligned}$$

Mathematica [A] time = 0.0659772, size = 93, normalized size = 0.89

$$\frac{\frac{6b^3(Ac-bB)}{b+cx^2}+6b^2(3Ac-4bB)\log\left(b+cx^2\right)+3c^2x^4(Ac-2bB)+6bcx^2(3bB-2Ac)+2Bc^3x^6}{12c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (6*b*c*(3*b*B - 2*A*c)*x^2 + 3*c^2*(-2*b*B + A*c)*x^4 + 2*B*c^3*x^6 + (6*b^
3*(-(b*B) + A*c))/(b + c*x^2) + 6*b^2*(-4*b*B + 3*A*c)*Log[b + c*x^2])/(12*
c^5)
```

Maple [A] time = 0.01, size = 122, normalized size = 1.2

$$\frac{Bx^6}{6\,c^2}+\frac{Ax^4}{4\,c^2}-\frac{Bx^4b}{2\,c^3}-\frac{Abx^2}{c^3}+\frac{3\,Bx^2b^2}{2\,c^4}+\frac{3\,b^2\ln\left(cx^2+b\right)A}{2\,c^4}-2\,\frac{b^3\ln\left(cx^2+b\right)B}{c^5}+\frac{b^3A}{2\,c^4\left(cx^2+b\right)}-\frac{Bb^4}{2\,c^5\left(cx^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(B*x^2+A)/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/6*B*x^6/c^2+1/4/c^2*A*x^4-1/2/c^3*B*x^4*b-1/c^3*A*x^2*b+3/2/c^4*B*x^2*b^2
+3/2*b^2/c^4*ln(c*x^2+b)*A-2*b^3/c^5*ln(c*x^2+b)*B+1/2*b^3/c^4/(c*x^2+b)*A-
1/2*b^4/c^5/(c*x^2+b)*B
```

Maxima [A] time = 1.11953, size = 144, normalized size = 1.37

$$-\frac{Bb^4 - Ab^3c}{2\left(c^6x^2 + bc^5\right)} + \frac{2\,Bc^2x^6 - 3\left(2\,Bbc - Ac^2\right)x^4 + 6\left(3\,Bb^2 - 2\,Abc\right)x^2}{12\,c^4} - \frac{\left(4\,Bb^3 - 3\,Ab^2c\right)\log\left(cx^2 + b\right)}{2\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(B*b^4 - A*b^3*c)/(c^6*x^2 + b*c^5) + 1/12*(2*B*c^2*x^6 - 3*(2*B*b*c -
 A*c^2)*x^4 + 6*(3*B*b^2 - 2*A*b*c)*x^2)/c^4 - 1/2*(4*B*b^3 - 3*A*b^2*c)*lo
g(c*x^2 + b)/c^5
```

Fricas [A] time = 0.644268, size = 309, normalized size = 2.94

$$\frac{2\,Bc^4x^8 - \left(4\,Bbc^3 - 3\,Ac^4\right)x^6 - 6\,Bb^4 + 6\,Ab^3c + 3\left(4\,Bb^2c^2 - 3\,Abc^3\right)x^4 + 6\left(3\,Bb^3c - 2\,Ab^2c^2\right)x^2 - 6\left(4\,Bb^4 - 3\,Ab^3c + \right.}{12\left(c^6x^2 + bc^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(2*B*c^4*x^8 - (4*B*b*c^3 - 3*A*c^4)*x^6 - 6*B*b^4 + 6*A*b^3*c + 3*(4*
B*b^2*c^2 - 3*A*b*c^3)*x^4 + 6*(3*B*b^3*c - 2*A*b^2*c^2)*x^2 - 6*(4*B*b^4 -
 3*A*b^3*c + (4*B*b^3*c - 3*A*b^2*c^2)*x^2)*log(c*x^2 + b))/(c^6*x^2 + b*c^
5)
```

Sympy [A] time = 0.87239, size = 102, normalized size = 0.97

$$\frac{Bx^6}{6c^2} - \frac{b^2\left(-3Ac + 4Bb\right)\log\left(b + cx^2\right)}{2c^5} - \frac{-Ab^3c + Bb^4}{2bc^5 + 2c^6x^2} - \frac{x^4\left(-Ac + 2Bb\right)}{4c^3} + \frac{x^2\left(-2Abc + 3Bb^2\right)}{2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] B*x**6/(6*c**2) - b**2*(-3*A*c + 4*B*b)*log(b + c*x**2)/(2*c**5) - (-A*b**3
*c + B*b**4)/(2*b*c**5 + 2*c**6*x**2) - x**4*(-A*c + 2*B*b)/(4*c**3) + x**2
*(-2*A*b*c + 3*B*b**2)/(2*c**4)
```

Giac [A] time = 1.21849, size = 182, normalized size = 1.73

$$-\frac{\left(4\,Bb^3 - 3\,Ab^2c\right)\log\left(\left|cx^2 + b\right|\right)}{2\,c^5} + \frac{2\,Bc^4x^6 - 6\,Bbc^3x^4 + 3\,Ac^4x^4 + 18\,Bb^2c^2x^2 - 12\,Abc^3x^2}{12\,c^6} + \frac{4\,Bb^3cx^2 - 3\,Ab^2c^2x^2 + 3}{2\left(cx^2 + b\right)c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*B*b^3 - 3*A*b^2*c)*log(abs(c*x^2 + b))/c^5 + 1/12*(2*B*c^4*x^6 - 6*
B*b*c^3*x^4 + 3*A*c^4*x^4 + 18*B*b^2*c^2*x^2 - 12*A*b*c^3*x^2)/c^6 + 1/2*(4
*B*b^3*c*x^2 - 3*A*b^2*c^2*x^2 + 3*B*b^4 - 2*A*b^3*c)/((c*x^2 + b)*c^5)
```

3.60 $\int \frac{x^{10}(A+Bx^2)}{(bx^2+cx^4)^2} dx$

Optimal. Leaf size=110

$$\frac{b^2x(bB-Ac)}{2c^4(b+cx^2)}-\frac{b^{3/2}(7bB-5Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{9/2}}-\frac{x^3(2bB-Ac)}{3c^3}+\frac{bx(3bB-2Ac)}{c^4}+\frac{Bx^5}{5c^2}$$

```
[Out] (b*(3*b*B - 2*A*c)*x)/c^4 - ((2*b*B - A*c)*x^3)/(3*c^3) + (B*x^5)/(5*c^2) +
 (b^2*(b*B - A*c)*x)/(2*c^4*(b + c*x^2)) - (b^(3/2)*(7*b*B - 5*A*c)*ArcTan[
(Sqrt[c]*x)/Sqrt[b]])/(2*c^(9/2))
```

Rubi [A] time = 0.118471, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1584, 455, 1810, 205}

$$\frac{b^2x(bB-Ac)}{2c^4(b+cx^2)}-\frac{b^{3/2}(7bB-5Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{9/2}}-\frac{x^3(2bB-Ac)}{3c^3}+\frac{bx(3bB-2Ac)}{c^4}+\frac{Bx^5}{5c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^10*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (b*(3*b*B - 2*A*c)*x)/c^4 - ((2*b*B - A*c)*x^3)/(3*c^3) + (B*x^5)/(5*c^2) +
 (b^2*(b*B - A*c)*x)/(2*c^4*(b + c*x^2)) - (b^(3/2)*(7*b*B - 5*A*c)*ArcTan[
(Sqrt[c]*x)/Sqrt[b]])/(2*c^(9/2))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
```

```
1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{10}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx &= \int \frac{x^6\left(A+Bx^2\right)}{\left(b+cx^2\right)^2}\,dx\\
&= \frac{b^2(bB-Ac)x}{2c^4\left(b+cx^2\right)} - \frac{\int \frac{b^2(bB-Ac)-2bc(bB-Ac)x^2+2c^2(bB-Ac)x^4-2Bc^3x^6}{b+cx^2}\,dx}{2c^4}\\
&= \frac{b^2(bB-Ac)x}{2c^4\left(b+cx^2\right)} - \frac{\int \left(-2b(3bB-2Ac)+2c(2bB-Ac)x^2-2Bc^2x^4+\frac{7b^3B-5Ab^2c}{b+cx^2}\right)dx}{2c^4}\\
&= \frac{b(3bB-2Ac)x}{c^4} - \frac{(2bB-Ac)x^3}{3c^3} + \frac{Bx^5}{5c^2} + \frac{b^2(bB-Ac)x}{2c^4\left(b+cx^2\right)} - \frac{\left(b^2(7bB-5Ac)\right)\int \frac{1}{b+cx^2}\,dx}{2c^4}\\
&= \frac{b(3bB-2Ac)x}{c^4} - \frac{(2bB-Ac)x^3}{3c^3} + \frac{Bx^5}{5c^2} + \frac{b^2(bB-Ac)x}{2c^4\left(b+cx^2\right)} - \frac{b^{3/2}(7bB-5Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{9/2}}
\end{aligned}
$$

Mathematica [A] time = 0.08294, size = 111, normalized size = 1.01

$$
-\frac{x\left(Ab^2c-b^3B\right)}{2c^4\left(b+cx^2\right)} - \frac{b^{3/2}(7bB-5Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{9/2}} + \frac{x^3(Ac-2bB)}{3c^3} + \frac{bx(3bB-2Ac)}{c^4} + \frac{Bx^5}{5c^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^10*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (b*(3*b*B - 2*A*c)*x)/c^4 + ((-2*b*B + A*c)*x^3)/(3*c^3) + (B*x^5)/(5*c^2)
- ((-(b^3*B) + A*b^2*c)*x)/(2*c^4*(b + c*x^2)) - (b^(3/2)*(7*b*B - 5*A*c)*A
rcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*c^(9/2))
```

Maple [A] time = 0.01, size = 132, normalized size = 1.2

$$\frac{Bx^5}{5c^2}+\frac{Ax^3}{3c^2}-\frac{2Bx^3b}{3c^3}-2\frac{Abx}{c^3}+3\frac{Bb^2x}{c^4}-\frac{Ab^2x}{2c^3\left(cx^2+b\right)}+\frac{Bb^3x}{2c^4\left(cx^2+b\right)}+\frac{5Ab^2}{2c^3}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}-\frac{7Bb^3}{2c^4}\mathrm{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(B*x^2+A)/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/5*B*x^5/c^2+1/3/c^2*A*x^3-2/3/c^3*B*x^3*b-2/c^3*A*b*x+3/c^4*B*b^2*x-1/2*b
^2/c^3*x/(c*x^2+b)*A+1/2*b^3/c^4*x/(c*x^2+b)*B+5/2*b^2/c^3/(b*c)^(1/2)*arct
an(x*c/(b*c)^(1/2))*A-7/2*b^3/c^4/(b*c)^(1/2)*arctan(x*c/(b*c)^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.832423, size = 637, normalized size = 5.79

$$\left[\frac{12Bc^3x^7-4\left(7Bbc^2-5Ac^3\right)x^5+20\left(7Bb^2c-5Abc^2\right)x^3-15\left(7Bb^3-5Ab^2c+\left(7Bb^2c-5Abc^2\right)x^2\right)\sqrt{-\frac{b}{c}}\log\left(\frac{cx^2+2c}{cx}\right.}{60\left(c^5x^2+bc^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(12*B*c^3*x^7 - 4*(7*B*b*c^2 - 5*A*c^3)*x^5 + 20*(7*B*b^2*c - 5*A*b*c
^2)*x^3 - 15*(7*B*b^3 - 5*A*b^2*c + (7*B*b^2*c - 5*A*b*c^2)*x^2)*sqrt(-b/c)
*log((c*x^2 + 2*c*x*sqrt(-b/c) - b)/(c*x^2 + b)) + 30*(7*B*b^3 - 5*A*b^2*c)
*x)/(c^5*x^2 + b*c^4), 1/30*(6*B*c^3*x^7 - 2*(7*B*b*c^2 - 5*A*c^3)*x^5 + 10
*(7*B*b^2*c - 5*A*b*c^2)*x^3 - 15*(7*B*b^3 - 5*A*b^2*c + (7*B*b^2*c - 5*A*b
*c^2)*x^2)*sqrt(b/c)*arctan(c*x*sqrt(b/c)/b) + 15*(7*B*b^3 - 5*A*b^2*c)*x)/
(c^5*x^2 + b*c^4)]
```

Sympy [A] time = 0.862162, size = 206, normalized size = 1.87

$$\frac{Bx^5}{5c^2}+\frac{x\left(-Ab^2c+Bb^3\right)}{2bc^4+2c^5x^2}+\frac{\sqrt{-\frac{b^3}{c^9}}\,(-5Ac+7Bb)\log\left(-\frac{c^4\sqrt{-\frac{b^3}{c^9}}(-5Ac+7Bb)}{-5Abc+7Bb^2}+x\right)}{4}-\frac{\sqrt{-\frac{b^3}{c^9}}\,(-5Ac+7Bb)\log\left(\frac{c^4\sqrt{-\frac{b^3}{c^9}}(-5Ac+7Bb)}{-5Abc+7Bb^2}\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] B*x**5/(5*c**2) + x*(-A*b**2*c + B*b**3)/(2*b*c**4 + 2*c**5*x**2) + sqrt(-b
**3/c**9)*(-5*A*c + 7*B*b)*log(-c**4*sqrt(-b**3/c**9)*(-5*A*c + 7*B*b)/(-5*
A*b*c + 7*B*b**2) + x)/4 - sqrt(-b**3/c**9)*(-5*A*c + 7*B*b)*log(c**4*sqrt(
-b**3/c**9)*(-5*A*c + 7*B*b)/(-5*A*b*c + 7*B*b**2) + x)/4 - x**3*(-A*c + 2*
B*b)/(3*c**3) + x*(-2*A*b*c + 3*B*b**2)/c**4
```

Giac [A] time = 1.20538, size = 155, normalized size = 1.41

$$-\frac{\left(7\,Bb^3-5\,Ab^2c\right)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}c^4}+\frac{Bb^3x-Ab^2cx}{2\left(cx^2+b\right)c^4}+\frac{3\,Bc^8x^5-10\,Bbc^7x^3+5\,Ac^8x^3+45\,Bb^2c^6x-30\,Abc^7x}{15\,c^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(7*B*b^3 - 5*A*b^2*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^4) + 1/2*(B*b
^3*x - A*b^2*c*x)/((c*x^2 + b)*c^4) + 1/15*(3*B*c^8*x^5 - 10*B*b*c^7*x^3 +
5*A*c^8*x^3 + 45*B*b^2*c^6*x - 30*A*b*c^7*x)/c^10
```

3.61 $\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^2} dx$

Optimal. Leaf size=83

$$\frac{b^2(bB-Ac)}{2c^4(b+cx^2)}-\frac{x^2(2bB-Ac)}{2c^3}+\frac{b(3bB-2Ac)\log(b+cx^2)}{2c^4}+\frac{Bx^4}{4c^2}$$

```
[Out] -((2*b*B - A*c)*x^2)/(2*c^3) + (B*x^4)/(4*c^2) + (b^2*(b*B - A*c))/(2*c^4*(
b + c*x^2)) + (b*(3*b*B - 2*A*c)*Log[b + c*x^2])/(2*c^4)
```

Rubi [A] time = 0.0984574, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 446, 77}

$$\frac{b^2(bB-Ac)}{2c^4(b+cx^2)}-\frac{x^2(2bB-Ac)}{2c^3}+\frac{b(3bB-2Ac)\log(b+cx^2)}{2c^4}+\frac{Bx^4}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^9*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -((2*b*B - A*c)*x^2)/(2*c^3) + (B*x^4)/(4*c^2) + (b^2*(b*B - A*c))/(2*c^4*(
b + c*x^2)) + (b*(3*b*B - 2*A*c)*Log[b + c*x^2])/(2*c^4)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{x^9\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx &= \int \frac{x^5\left(A+Bx^2\right)}{\left(b+cx^2\right)^2}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2(A+Bx)}{(b+cx)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{-2bB+Ac}{c^3}+\frac{Bx}{c^2}-\frac{b^2(bB-Ac)}{c^3(b+cx)^2}+\frac{b(3bB-2Ac)}{c^3(b+cx)}\right)dx,x,x^2\right)\\ &= -\frac{(2bB-Ac)x^2}{2c^3}+\frac{Bx^4}{4c^2}+\frac{b^2(bB-Ac)}{2c^4\left(b+cx^2\right)}+\frac{b(3bB-2Ac)\log\left(b+cx^2\right)}{2c^4}\end{aligned}$$

Mathematica [A] time = 0.0524287, size = 72, normalized size = 0.87

$$\frac{\frac{2b^2(bB-Ac)}{b+cx^2}+2cx^2(Ac-2bB)+2b(3bB-2Ac)\log\left(b+cx^2\right)+Bc^2x^4}{4c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^9*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*c*(-2*b*B + A*c)*x^2 + B*c^2*x^4 + (2*b^2*(b*B - A*c))/(b + c*x^2) + 2*b
*(3*b*B - 2*A*c)*Log[b + c*x^2])/(4*c^4)
```

Maple [A] time = 0.01, size = 98, normalized size = 1.2

$$\frac{Bx^4}{4c^2}+\frac{Ax^2}{2c^2}-\frac{Bx^2b}{c^3}-\frac{b\ln\left(cx^2+b\right)A}{c^3}+\frac{3b^2\ln\left(cx^2+b\right)B}{2c^4}-\frac{b^2A}{2c^3\left(cx^2+b\right)}+\frac{Bb^3}{2c^4\left(cx^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(B*x^2+A)/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/4*B*x^4/c^2+1/2/c^2*A*x^2-1/c^3*B*x^2*b-b/c^3*ln(c*x^2+b)*A+3/2*b^2/c^4*l
n(c*x^2+b)*B-1/2*b^2/c^3/(c*x^2+b)*A+1/2*b^3/c^4/(c*x^2+b)*B
```

Maxima [A] time = 1.10743, size = 111, normalized size = 1.34

$$\frac{Bb^3 - Ab^2c}{2\left(c^5x^2 + bc^4\right)} + \frac{Bcx^4 - 2\,(2\,Bb - Ac)x^2}{4\,c^3} + \frac{\left(3\,Bb^2 - 2\,Abc\right)\log\left(cx^2 + b\right)}{2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*b^3 - A*b^2*c)/(c^5*x^2 + b*c^4) + 1/4*(B*c*x^4 - 2*(2*B*b - A*c)*x^
2)/c^3 + 1/2*(3*B*b^2 - 2*A*b*c)*log(c*x^2 + b)/c^4
```

Fricas [A] time = 0.753855, size = 251, normalized size = 3.02

$$\frac{Bc^3x^6 - \left(3\,Bbc^2 - 2\,Ac^3\right)x^4 + 2\,Bb^3 - 2\,Ab^2c - 2\left(2\,Bb^2c - Abc^2\right)x^2 + 2\left(3\,Bb^3 - 2\,Ab^2c + \left(3\,Bb^2c - 2\,Abc^2\right)x^2\right)\log\left(cx^2\right.}{4\left(c^5x^2 + bc^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(B*c^3*x^6 - (3*B*b*c^2 - 2*A*c^3)*x^4 + 2*B*b^3 - 2*A*b^2*c - 2*(2*B*b
^2*c - A*b*c^2)*x^2 + 2*(3*B*b^3 - 2*A*b^2*c + (3*B*b^2*c - 2*A*b*c^2)*x^2)
*log(c*x^2 + b))/(c^5*x^2 + b*c^4)
```

Sympy [A] time = 0.812214, size = 78, normalized size = 0.94

$$\frac{Bx^4}{4c^2} + \frac{b\,(-2Ac + 3Bb)\log\left(b + cx^2\right)}{2c^4} + \frac{-Ab^2c + Bb^3}{2bc^4 + 2c^5x^2} - \frac{x^2\,(-Ac + 2Bb)}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] B*x**4/(4*c**2) + b*(-2*A*c + 3*B*b)*log(b + c*x**2)/(2*c**4) + (-A*b**2*c
+ B*b**3)/(2*b*c**4 + 2*c**5*x**2) - x**2*(-A*c + 2*B*b)/(2*c**3)
```

Giac [A] time = 1.23714, size = 143, normalized size = 1.72

$$\frac{\left(3\,Bb^2 - 2\,Abc\right)\log\left(\left|cx^2 + b\right|\right)}{2\,c^4} + \frac{Bc^2x^4 - 4\,Bbcx^2 + 2\,Ac^2x^2}{4\,c^4} - \frac{3\,Bb^2cx^2 - 2\,Abc^2x^2 + 2\,Bb^3 - Ab^2c}{2\left(cx^2 + b\right)c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*B*b^2 - 2*A*b*c)*log(abs(c*x^2 + b))/c^4 + 1/4*(B*c^2*x^4 - 4*B*b*c*
x^2 + 2*A*c^2*x^2)/c^4 - 1/2*(3*B*b^2*c*x^2 - 2*A*b*c^2*x^2 + 2*B*b^3 - A*b
^2*c)/((c*x^2 + b)*c^4)
```

3.62 $\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=89

$$-\frac{bx(bB-Ac)}{2c^3\left(b+cx^2\right)}-\frac{x(2bB-Ac)}{c^3}+\frac{\sqrt{b}(5bB-3Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{7/2}}+\frac{Bx^3}{3c^2}$$

```
[Out] -(((2*b*B - A*c)*x)/c^3) + (B*x^3)/(3*c^2) - (b*(b*B - A*c)*x)/(2*c^3*(b +
c*x^2)) + (Sqrt[b]*(5*b*B - 3*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*c^(7/2))
```

Rubi [A] time = 0.0885159, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1584, 455, 1153, 205}

$$-\frac{bx(bB-Ac)}{2c^3\left(b+cx^2\right)}-\frac{x(2bB-Ac)}{c^3}+\frac{\sqrt{b}(5bB-3Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{7/2}}+\frac{Bx^3}{3c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -(((2*b*B - A*c)*x)/c^3) + (B*x^3)/(3*c^2) - (b*(b*B - A*c)*x)/(2*c^3*(b +
c*x^2)) + (Sqrt[b]*(5*b*B - 3*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*c^(7/2))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
```

```
(IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^8\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx &= \int \frac{x^4\left(A+Bx^2\right)}{\left(b+cx^2\right)^2}\,dx \\
&= -\frac{b(bB-Ac)x}{2c^3\left(b+cx^2\right)}-\frac{\int \frac{-b(bB-Ac)+2c(bB-Ac)x^2-2Bc^2x^4}{b+cx^2}\,dx}{2c^3} \\
&= -\frac{b(bB-Ac)x}{2c^3\left(b+cx^2\right)}-\frac{\int \left(2(2bB-Ac)-2Bcx^2+\frac{-5b^2B+3Abc}{b+cx^2}\right)dx}{2c^3} \\
&= -\frac{(2bB-Ac)x}{c^3}+\frac{Bx^3}{3c^2}-\frac{b(bB-Ac)x}{2c^3\left(b+cx^2\right)}+\frac{(b(5bB-3Ac))\int \frac{1}{b+cx^2}\,dx}{2c^3} \\
&= -\frac{(2bB-Ac)x}{c^3}+\frac{Bx^3}{3c^2}-\frac{b(bB-Ac)x}{2c^3\left(b+cx^2\right)}+\frac{\sqrt{b}(5bB-3Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{7/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0710759, size = 89, normalized size = 1.

$$
\frac{x\left(Abc-b^2B\right)}{2c^3\left(b+cx^2\right)}+\frac{x(Ac-2bB)}{c^3}+\frac{\sqrt{b}(5bB-3Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{7/2}}+\frac{Bx^3}{3c^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-2*b*B + A*c)*x)/c^3 + (B*x^3)/(3*c^2) + ((-(b^2*B) + A*b*c)*x)/(2*c^3*(b
 + c*x^2)) + (Sqrt[b]*(5*b*B - 3*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*c^(7/
2))
```

Maple [A] time = 0.009, size = 105, normalized size = 1.2

$$\frac{Bx^3}{3\,c^2}+\frac{Ax}{c^2}-2\,\frac{Bbx}{c^3}+\frac{Abx}{2\,c^2\left(cx^2+b\right)}-\frac{Bb^2x}{2\,c^3\left(cx^2+b\right)}-\frac{3\,Ab}{2\,c^2}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}+\frac{5\,Bb^2}{2\,c^3}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(B*x^2+A)/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/3*B*x^3/c^2+1/c^2*A*x-2/c^3*B*b*x+1/2*b/c^2*x/(c*x^2+b)*A-1/2*b^2/c^3*x/(
c*x^2+b)*B-3/2*b/c^2/(b*c)^(1/2)*arctan(x*c/(b*c)^(1/2))*A+5/2*b^2/c^3/(b*c
)^(1/2)*arctan(x*c/(b*c)^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.778381, size = 513, normalized size = 5.76

$$\left[\frac{4\,Bc^2x^5-4\left(5\,Bbc-3\,Ac^2\right)x^3-3\left(5\,Bb^2-3\,Abc+\left(5\,Bbc-3\,Ac^2\right)x^2\right)\sqrt{-\frac{b}{c}}\log\left(\frac{cx^2-2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)-6\left(5\,Bb^2-3\,Abc\right)x}{12\left(c^4x^2+bc^3\right)},-\right.$$ 2

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*B*c^2*x^5 - 4*(5*B*b*c - 3*A*c^2)*x^3 - 3*(5*B*b^2 - 3*A*b*c + (5*
B*b*c - 3*A*c^2)*x^2)*sqrt(-b/c)*log((c*x^2 - 2*c*x*sqrt(-b/c) - b)/(c*x^2
+ b)) - 6*(5*B*b^2 - 3*A*b*c)*x)/(c^4*x^2 + b*c^3), 1/6*(2*B*c^2*x^5 - 2*(5
*B*b*c - 3*A*c^2)*x^3 + 3*(5*B*b^2 - 3*A*b*c + (5*B*b*c - 3*A*c^2)*x^2)*sqr
t(b/c)*arctan(c*x*sqrt(b/c)/b) - 3*(5*B*b^2 - 3*A*b*c)*x)/(c^4*x^2 + b*c^3)
]
```

Sympy [A] time = 0.785215, size = 128, normalized size = 1.44

$$\frac{Bx^3}{3c^2} - \frac{x\left(-Abc + Bb^2\right)}{2bc^3 + 2c^4x^2} - \frac{\sqrt{-\frac{b}{c^7}}\,(-3Ac + 5Bb)\log\left(-c^3\sqrt{-\frac{b}{c^7}} + x\right)}{4} + \frac{\sqrt{-\frac{b}{c^7}}\,(-3Ac + 5Bb)\log\left(c^3\sqrt{-\frac{b}{c^7}} + x\right)}{4} - \frac{x\,(-Ac + 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] B*x**3/(3*c**2) - x*(-A*b*c + B*b**2)/(2*b*c**3 + 2*c**4*x**2) - sqrt(-b/c*
*7)*(-3*A*c + 5*B*b)*log(-c**3*sqrt(-b/c**7) + x)/4 + sqrt(-b/c**7)*(-3*A*c
 + 5*B*b)*log(c**3*sqrt(-b/c**7) + x)/4 - x*(-A*c + 2*B*b)/c**3
```

Giac [A] time = 1.27138, size = 119, normalized size = 1.34

$$\frac{\left(5\,Bb^2 - 3\,Abc\right)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}c^3} - \frac{Bb^2x - Abcx}{2\left(cx^2 + b\right)c^3} + \frac{Bc^4x^3 - 6\,Bbc^3x + 3\,Ac^4x}{3\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*B*b^2 - 3*A*b*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^3) - 1/2*(B*b^2*
x - A*b*c*x)/((c*x^2 + b)*c^3) + 1/3*(B*c^4*x^3 - 6*B*b*c^3*x + 3*A*c^4*x)/
c^6
```

3.63 $\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^2} dx$

Optimal. Leaf size=61

$$-\frac{b(bB-Ac)}{2c^3(b+cx^2)}-\frac{(2bB-Ac)\log(b+cx^2)}{2c^3}+\frac{Bx^2}{2c^2}$$

```
[Out] (B*x^2)/(2*c^2) - (b*(b*B - A*c))/(2*c^3*(b + c*x^2)) - ((2*b*B - A*c)*Log[
b + c*x^2])/(2*c^3)
```

Rubi [A] time = 0.0697961, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 446, 77}

$$-\frac{b(bB-Ac)}{2c^3(b+cx^2)}-\frac{(2bB-Ac)\log(b+cx^2)}{2c^3}+\frac{Bx^2}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (B*x^2)/(2*c^2) - (b*(b*B - A*c))/(2*c^3*(b + c*x^2)) - ((2*b*B - A*c)*Log[
b + c*x^2])/(2*c^3)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx &= \int \frac{x^3\left(A+Bx^2\right)}{\left(b+cx^2\right)^2}\,dx \\
&= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{x(A+Bx)}{(b+cx)^2}\,dx, x, x^2\right) \\
&= \frac{1}{2}\,\mathrm{Subst}\left(\int \left(\frac{B}{c^2}+\frac{b(bB-Ac)}{c^2(b+cx)^2}+\frac{-2bB+Ac}{c^2(b+cx)}\right)dx, x, x^2\right) \\
&= \frac{Bx^2}{2c^2}-\frac{b(bB-Ac)}{2c^3\left(b+cx^2\right)}-\frac{(2bB-Ac)\log\left(b+cx^2\right)}{2c^3}
\end{aligned}$$

Mathematica [A] time = 0.0360475, size = 50, normalized size = 0.82

$$\frac{\frac{b(Ac-bB)}{b+cx^2}+(Ac-2bB)\log\left(b+cx^2\right)+Bcx^2}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (B*c*x^2 + (b*(-(b*B) + A*c))/(b + c*x^2) + (-2*b*B + A*c)*Log[b + c*x^2])/
(2*c^3)
```

Maple [A] time = 0.008, size = 74, normalized size = 1.2

$$\frac{Bx^2}{2\,c^2}+\frac{\ln\left(cx^2+b\right)A}{2\,c^2}-\frac{\ln\left(cx^2+b\right)Bb}{c^3}+\frac{Ab}{2\,c^2\left(cx^2+b\right)}-\frac{Bb^2}{2\,c^3\left(cx^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(B*x^2+A)/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/2*B*x^2/c^2+1/2/c^2*ln(c*x^2+b)*A-1/c^3*ln(c*x^2+b)*B*b+1/2/c^2*b/(c*x^2+
b)*A-1/2/c^3*b^2/(c*x^2+b)*B
```

Maxima [A] time = 1.26542, size = 81, normalized size = 1.33

$$\frac{Bx^2}{2\,c^2} - \frac{Bb^2 - Abc}{2\left(c^4x^2 + bc^3\right)} - \frac{(2\,Bb - Ac)\log\left(cx^2 + b\right)}{2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*B*x^2/c^2 - 1/2*(B*b^2 - A*b*c)/(c^4*x^2 + b*c^3) - 1/2*(2*B*b - A*c)*l
og(c*x^2 + b)/c^3
```

Fricas [A] time = 0.689542, size = 165, normalized size = 2.7

$$\frac{Bc^2x^4 + Bbcx^2 - Bb^2 + Abc - \left(2\,Bb^2 - Abc + \left(2\,Bbc - Ac^2\right)x^2\right)\log\left(cx^2 + b\right)}{2\left(c^4x^2 + bc^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(B*c^2*x^4 + B*b*c*x^2 - B*b^2 + A*b*c - (2*B*b^2 - A*b*c + (2*B*b*c -
A*c^2)*x^2)*log(c*x^2 + b))/(c^4*x^2 + b*c^3)
```

Sympy [A] time = 0.715123, size = 56, normalized size = 0.92

$$\frac{Bx^2}{2c^2} - \frac{-Abc + Bb^2}{2bc^3 + 2c^4x^2} - \frac{(-Ac + 2Bb)\log\left(b + cx^2\right)}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] B*x**2/(2*c**2) - (-A*b*c + B*b**2)/(2*b*c**3 + 2*c**4*x**2) - (-A*c + 2*B*
b)*log(b + c*x**2)/(2*c**3)
```

Giac [A] time = 1.29789, size = 95, normalized size = 1.56

$$\frac{Bx^2}{2\,c^2} - \frac{(2\,Bb - Ac)\log\left(\left|cx^2 + b\right|\right)}{2\,c^3} + \frac{2\,Bbcx^2 - Ac^2x^2 + Bb^2}{2\left(cx^2 + b\right)c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*B*x^2/c^2 - 1/2*(2*B*b - A*c)*log(abs(c*x^2 + b))/c^3 + 1/2*(2*B*b*c*x^
2 - A*c^2*x^2 + B*b^2)/((c*x^2 + b)*c^3)
```

3.64 $\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=68

$$\frac{x(bB-Ac)}{2c^2(b+cx^2)}-\frac{(3bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2\sqrt{b}c^{5/2}}+\frac{Bx}{c^2}$$

```
[Out] (B*x)/c^2 + ((b*B - A*c)*x)/(2*c^2*(b + c*x^2)) - ((3*b*B - A*c)*ArcTan[(Sq
rt[c]*x)/Sqrt[b]])/(2*Sqrt[b]*c^(5/2))
```

Rubi [A] time = 0.0610082, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1584, 455, 388, 205}

$$\frac{x(bB-Ac)}{2c^2(b+cx^2)}-\frac{(3bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2\sqrt{b}c^{5/2}}+\frac{Bx}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (B*x)/c^2 + ((b*B - A*c)*x)/(2*c^2*(b + c*x^2)) - ((3*b*B - A*c)*ArcTan[(Sq
rt[c]*x)/Sqrt[b]])/(2*Sqrt[b]*c^(5/2))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
```

```
(IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^6\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx &= \int \frac{x^2\left(A+Bx^2\right)}{\left(b+cx^2\right)^2}\,dx \\
&= \frac{(bB-Ac)x}{2c^2\left(b+cx^2\right)} - \frac{\int \frac{bB-Ac-2Bcx^2}{b+cx^2}\,dx}{2c^2} \\
&= \frac{Bx}{c^2} + \frac{(bB-Ac)x}{2c^2\left(b+cx^2\right)} - \frac{(3bB-Ac)\int \frac{1}{b+cx^2}\,dx}{2c^2} \\
&= \frac{Bx}{c^2} + \frac{(bB-Ac)x}{2c^2\left(b+cx^2\right)} - \frac{(3bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2\sqrt{b}c^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0508732, size = 68, normalized size = 1.

$$
-\frac{x(Ac-bB)}{2c^2\left(b+cx^2\right)} - \frac{(3bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2\sqrt{b}c^{5/2}} + \frac{Bx}{c^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (B*x)/c^2 - ((-(b*B) + A*c)*x)/(2*c^2*(b + c*x^2)) - ((3*b*B - A*c)*ArcTan[
(Sqrt[c]*x)/Sqrt[b]])/(2*Sqrt[b]*c^(5/2))
```

Maple [A] time = 0.008, size = 82, normalized size = 1.2

$$\frac{Bx}{c^2}-\frac{xA}{2c\left(cx^2+b\right)}+\frac{Bbx}{2c^2\left(cx^2+b\right)}+\frac{A}{2c}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}-\frac{3Bb}{2c^2}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(B*x^2+A)/(c*x^4+b*x^2)^2,x)
```

```
[Out] B*x/c^2-1/2/c*x/(c*x^2+b)*A+1/2/c^2*x/(c*x^2+b)*B*b+1/2/c/(b*c)^(1/2)*arcta
n(x*c/(b*c)^(1/2))*A-3/2/c^2/(b*c)^(1/2)*arctan(x*c/(b*c)^(1/2))*B*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.83867, size = 433, normalized size = 6.37

$$\left[\frac{4Bbc^2x^3+\left(3Bb^2-Abc+\left(3Bbc-Ac^2\right)x^2\right)\sqrt{-bc}\log\left(\frac{cx^2-2\sqrt{-bc}x-b}{cx^2+b}\right)+2\left(3Bb^2c-Abc^2\right)x}{4\left(bc^4x^2+b^2c^3\right)},\frac{2Bbc^2x^3-\left(3Bb^2-Abc+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*B*b*c^2*x^3 + (3*B*b^2 - A*b*c + (3*B*b*c - A*c^2)*x^2)*sqrt(-b*c)*
log((c*x^2 - 2*sqrt(-b*c)*x - b)/(c*x^2 + b)) + 2*(3*B*b^2*c - A*b*c^2)*x)/
(b*c^4*x^2 + b^2*c^3), 1/2*(2*B*b*c^2*x^3 - (3*B*b^2 - A*b*c + (3*B*b*c - A
*c^2)*x^2)*sqrt(b*c)*arctan(sqrt(b*c)*x/b) + (3*B*b^2*c - A*b*c^2)*x)/(b*c^
```

```
4*x^2 + b^2*c^3)]
```

Sympy [A] time = 0.667197, size = 114, normalized size = 1.68

$$\frac{Bx}{c^2}+\frac{x(-Ac+Bb)}{2bc^2+2c^3x^2}+\frac{\sqrt{-\frac{1}{bc^5}}(-Ac+3Bb)\log\left(-bc^2\sqrt{-\frac{1}{bc^5}}+x\right)}{4}-\frac{\sqrt{-\frac{1}{bc^5}}(-Ac+3Bb)\log\left(bc^2\sqrt{-\frac{1}{bc^5}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] B*x/c**2 + x*(-A*c + B*b)/(2*b*c**2 + 2*c**3*x**2) + sqrt(-1/(b*c**5))*(-A*
c + 3*B*b)*log(-b*c**2*sqrt(-1/(b*c**5)) + x)/4 - sqrt(-1/(b*c**5))*(-A*c +
 3*B*b)*log(b*c**2*sqrt(-1/(b*c**5)) + x)/4
```

Giac [A] time = 1.28015, size = 80, normalized size = 1.18

$$\frac{Bx}{c^2}-\frac{(3\,Bb-Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}c^2}+\frac{Bbx-Acx}{2\left(cx^2+b\right)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] B*x/c^2 - 1/2*(3*B*b - A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^2) + 1/2*(B*
b*x - A*c*x)/((c*x^2 + b)*c^2)
```

3.65 $\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=41

$$\frac{bB-Ac}{2c^2(b+cx^2)}+\frac{B\log(b+cx^2)}{2c^2}$$

```
[Out] (b*B - A*c)/(2*c^2*(b + c*x^2)) + (B*Log[b + c*x^2])/(2*c^2)
```

Rubi [A] time = 0.0467391, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 444, 43}

$$\frac{bB-Ac}{2c^2(b+cx^2)}+\frac{B\log(b+cx^2)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (b*B - A*c)/(2*c^2*(b + c*x^2)) + (B*Log[b + c*x^2])/(2*c^2)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx &= \int \frac{x\left(A+Bx^2\right)}{\left(b+cx^2\right)^2}\,dx \\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{A+Bx}{(b+cx)^2}\,dx,x,x^2\right) \\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{-bB+Ac}{c(b+cx)^2}+\frac{B}{c(b+cx)}\right)dx,x,x^2\right) \\
&= \frac{bB-Ac}{2c^2\left(b+cx^2\right)}+\frac{B\log\left(b+cx^2\right)}{2c^2}
\end{aligned}
$$

Mathematica [A] time = 0.0122716, size = 41, normalized size = 1.

$$\frac{bB-Ac}{2c^2\left(b+cx^2\right)}+\frac{B\log\left(b+cx^2\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (b*B - A*c)/(2*c^2*(b + c*x^2)) + (B*Log[b + c*x^2])/(2*c^2)
```

Maple [A] time = 0.007, size = 47, normalized size = 1.2

$$\frac{B\ln\left(cx^2+b\right)}{2\,c^2}-\frac{A}{2\,c\left(cx^2+b\right)}+\frac{Bb}{2\,c^2\left(cx^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/2*B*ln(c*x^2+b)/c^2-1/2/c/(c*x^2+b)*A+1/2/c^2/(c*x^2+b)*B*b
```

Maxima [A] time = 2.76769, size = 54, normalized size = 1.32

$$\frac{Bb-Ac}{2\left(c^3x^2+bc^2\right)}+\frac{B\log\left(cx^2+b\right)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*b - A*c)/(c^3*x^2 + b*c^2) + 1/2*B*log(c*x^2 + b)/c^2
```

Fricas [A] time = 0.789347, size = 92, normalized size = 2.24

$$\frac{Bb-Ac+\left(Bcx^2+Bb\right)\log\left(cx^2+b\right)}{2\left(c^3x^2+bc^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(B*b - A*c + (B*c*x^2 + B*b)*log(c*x^2 + b))/(c^3*x^2 + b*c^2)
```

Sympy [A] time = 0.505632, size = 36, normalized size = 0.88

$$\frac{B\log\left(b+cx^2\right)}{2c^2}+\frac{-Ac+Bb}{2bc^2+2c^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] B*log(b + c*x**2)/(2*c**2) + (-A*c + B*b)/(2*b*c**2 + 2*c**3*x**2)
```

Giac [A] time = 1.26088, size = 50, normalized size = 1.22

$$\frac{B \log\left(\left|cx^2 + b\right|\right)}{2\,c^2} - \frac{Bx^2 + A}{2\left(cx^2 + b\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*B*log(abs(c*x^2 + b))/c^2 - 1/2*(B*x^2 + A)/((c*x^2 + b)*c)
```

3.66 $\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=63

$$\frac{(Ac+bB)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{3/2}c^{3/2}}-\frac{x(bB-Ac)}{2bc\left(b+cx^2\right)}$$

```
[Out] -((b*B - A*c)*x)/(2*b*c*(b + c*x^2)) + ((b*B + A*c)*ArcTan[(Sqrt[c]*x)/Sqrt
[b]])/(2*b^(3/2)*c^(3/2))
```

Rubi [A] time = 0.0327217, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 385, 205}

$$\frac{(Ac+bB)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{3/2}c^{3/2}}-\frac{x(bB-Ac)}{2bc\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -((b*B - A*c)*x)/(2*b*c*(b + c*x^2)) + ((b*B + A*c)*ArcTan[(Sqrt[c]*x)/Sqrt
[b]])/(2*b^(3/2)*c^(3/2))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx &= \int \frac{A+Bx^2}{\left(b+cx^2\right)^2}\,dx\\ &= -\frac{(bB-Ac)x}{2bc\left(b+cx^2\right)}+\frac{(bB+Ac)\int \frac{1}{b+cx^2}\,dx}{2bc}\\ &= -\frac{(bB-Ac)x}{2bc\left(b+cx^2\right)}+\frac{(bB+Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{3/2}c^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0458863, size = 63, normalized size = 1.

$$\frac{(Ac+bB)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{3/2}c^{3/2}}-\frac{x(bB-Ac)}{2bc\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -((b*B - A*c)*x)/(2*b*c*(b + c*x^2)) + ((b*B + A*c)*ArcTan[(Sqrt[c]*x)/Sqrt
[b]])/(2*b^(3/2)*c^(3/2))
```

Maple [A] time = 0.008, size = 68, normalized size = 1.1

$$\frac{(Ac-Bb)\,x}{2\,bc\left(cx^2+b\right)}+\frac{A}{2\,b}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}+\frac{B}{2\,c}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^2+A)/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/2*(A*c-B*b)/b/c*x/(c*x^2+b)+1/2/b/(b*c)^(1/2)*arctan(x*c/(b*c)^(1/2))*A+1
/2/c/(b*c)^(1/2)*arctan(x*c/(b*c)^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.710259, size = 381, normalized size = 6.05

$$\left[-\frac{\left(Bb^2+Abc+\left(Bbc+Ac^2\right)x^2\right)\sqrt{-bc}\log\left(\frac{cx^2-2\sqrt{-bc}x-b}{cx^2+b}\right)+2\left(Bb^2c-Abc^2\right)x}{4\left(b^2c^3x^2+b^3c^2\right)},\frac{\left(Bb^2+Abc+\left(Bbc+Ac^2\right)x^2\right)\sqrt{bc}\arctan}{2\left(b^2c^3x^2+b^3c^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*((B*b^2 + A*b*c + (B*b*c + A*c^2)*x^2)*sqrt(-b*c)*log((c*x^2 - 2*sqrt
(-b*c)*x - b)/(c*x^2 + b)) + 2*(B*b^2*c - A*b*c^2)*x)/(b^2*c^3*x^2 + b^3*c^
2), 1/2*((B*b^2 + A*b*c + (B*b*c + A*c^2)*x^2)*sqrt(b*c)*arctan(sqrt(b*c)*x
/b) - (B*b^2*c - A*b*c^2)*x)/(b^2*c^3*x^2 + b^3*c^2)]
```

Sympy [B] time = 0.541704, size = 112, normalized size = 1.78

$$-\frac{x\left(-Ac+Bb\right)}{2b^2c+2bc^2x^2}-\frac{\sqrt{-\frac{1}{b^3c^3}}\left(Ac+Bb\right)\log\left(-b^2c\sqrt{-\frac{1}{b^3c^3}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{b^3c^3}}\left(Ac+Bb\right)\log\left(b^2c\sqrt{-\frac{1}{b^3c^3}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] -x*(-A*c + B*b)/(2*b**2*c + 2*b*c**2*x**2) - sqrt(-1/(b**3*c**3))*(A*c + B*
b)*log(-b**2*c*sqrt(-1/(b**3*c**3)) + x)/4 + sqrt(-1/(b**3*c**3))*(A*c + B*
b)*log(b**2*c*sqrt(-1/(b**3*c**3)) + x)/4
```

Giac [A] time = 1.16941, size = 77, normalized size = 1.22

$$\frac{(Bb + Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}bc} - \frac{Bbx - Acx}{2\left(cx^2 + b\right)bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(B*b + A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b*c) - 1/2*(B*b*x - A*c*x)
/((c*x^2 + b)*b*c)
```

3.67 $\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^2} dx$

Optimal. Leaf size=51

$$-\frac{A\log\left(b+cx^2\right)}{2b^2}+\frac{A\log(x)}{b^2}-\frac{bB-Ac}{2bc\left(b+cx^2\right)}$$

```
[Out] -(b*B - A*c)/(2*b*c*(b + c*x^2)) + (A*Log[x])/b^2 - (A*Log[b + c*x^2])/(2*b
^2)
```

Rubi [A] time = 0.0547747, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 446, 77}

$$-\frac{A\log\left(b+cx^2\right)}{2b^2}+\frac{A\log(x)}{b^2}-\frac{bB-Ac}{2bc\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -(b*B - A*c)/(2*b*c*(b + c*x^2)) + (A*Log[x])/b^2 - (A*Log[b + c*x^2])/(2*b
^2)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx &= \int \frac{A+Bx^2}{x\left(b+cx^2\right)^2}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{x(b+cx)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{A}{b^2x}+\frac{bB-Ac}{b(b+cx)^2}-\frac{Ac}{b^2(b+cx)}\right)dx,x,x^2\right)\\ &= -\frac{bB-Ac}{2bc\left(b+cx^2\right)}+\frac{A\log(x)}{b^2}-\frac{A\log\left(b+cx^2\right)}{2b^2}\end{aligned}$$

Mathematica [A] time = 0.0285103, size = 46, normalized size = 0.9

$$\frac{\frac{b(Ac-bB)}{c\left(b+cx^2\right)}-A\log\left(b+cx^2\right)+2A\log(x)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((b*(-(b*B) + A*c))/(c*(b + c*x^2)) + 2*A*Log[x] - A*Log[b + c*x^2])/(2*b^2
)
```

Maple [A] time = 0.012, size = 53, normalized size = 1.

$$\frac{A\ln(x)}{b^2}-\frac{A\ln\left(cx^2+b\right)}{2\,b^2}+\frac{A}{2\,b\left(cx^2+b\right)}-\frac{B}{2\,c\left(cx^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)/(c*x^4+b*x^2)^2,x)
```

```
[Out] A*ln(x)/b^2-1/2*A*ln(c*x^2+b)/b^2+1/2/b/(c*x^2+b)*A-1/2/c/(c*x^2+b)*B
```

Maxima [A] time = 1.12098, size = 69, normalized size = 1.35

$$-\frac{Bb - Ac}{2\left(bc^2x^2 + b^2c\right)} - \frac{A\log\left(cx^2 + b\right)}{2\,b^2} + \frac{A\log\left(x^2\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(B*b - A*c)/(b*c^2*x^2 + b^2*c) - 1/2*A*log(c*x^2 + b)/b^2 + 1/2*A*log
(x^2)/b^2
```

Fricas [A] time = 0.791719, size = 151, normalized size = 2.96

$$-\frac{Bb^2 - Abc + \left(Ac^2x^2 + Abc\right)\log\left(cx^2 + b\right) - 2\left(Ac^2x^2 + Abc\right)\log\left(x\right)}{2\left(b^2c^2x^2 + b^3c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(B*b^2 - A*b*c + (A*c^2*x^2 + A*b*c)*log(c*x^2 + b) - 2*(A*c^2*x^2 + A
*b*c)*log(x))/(b^2*c^2*x^2 + b^3*c)
```

Sympy [A] time = 0.574344, size = 46, normalized size = 0.9

$$\frac{A\log\left(x\right)}{b^2} - \frac{A\log\left(\frac{b}{c} + x^2\right)}{2b^2} - \frac{-Ac + Bb}{2b^2c + 2bc^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] A*log(x)/b**2 - A*log(b/c + x**2)/(2*b**2) - (-A*c + B*b)/(2*b**2*c + 2*b*c
**2*x**2)
```

Giac [A] time = 1.59705, size = 70, normalized size = 1.37

$$-\frac{A\log\left(\left|cx^2+b\right|\right)}{2\,b^2}+\frac{A\log\left(\left|x\right|\right)}{b^2}-\frac{Bb^2-Abc}{2\left(cx^2+b\right)b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*A*log(abs(c*x^2 + b))/b^2 + A*log(abs(x))/b^2 - 1/2*(B*b^2 - A*b*c)/((
c*x^2 + b)*b^2*c)
```

3.68 $\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^2} dx$

Optimal. Leaf size=70

$$\frac{x(bB-Ac)}{2b^2(b+cx^2)}+\frac{(bB-3Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{5/2}\sqrt{c}}-\frac{A}{b^2x}$$

```
[Out] -(A/(b^2*x)) + ((b*B - A*c)*x)/(2*b^2*(b + c*x^2)) + ((b*B - 3*A*c)*ArcTan[
(Sqrt[c]*x)/Sqrt[b]])/(2*b^(5/2)*Sqrt[c])
```

Rubi [A] time = 0.0743364, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1584, 456, 453, 205}

$$\frac{x(bB-Ac)}{2b^2(b+cx^2)}+\frac{(bB-3Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{5/2}\sqrt{c}}-\frac{A}{b^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -(A/(b^2*x)) + ((b*B - A*c)*x)/(2*b^2*(b + c*x^2)) + ((b*B - 3*A*c)*ArcTan[
(Sqrt[c]*x)/Sqrt[b]])/(2*b^(5/2)*Sqrt[c])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
```

```
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx &= \int \frac{A+Bx^2}{x^2\left(b+cx^2\right)^2}\,dx\\
&= \frac{(bB-Ac)x}{2b^2\left(b+cx^2\right)}-\frac{1}{2}\int \frac{-\frac{2A}{b}-\frac{(bB-Ac)x^2}{b^2}}{x^2\left(b+cx^2\right)}\,dx\\
&= -\frac{A}{b^2x}+\frac{(bB-Ac)x}{2b^2\left(b+cx^2\right)}+\frac{(bB-3Ac)\int \frac{1}{b+cx^2}\,dx}{2b^2}\\
&= -\frac{A}{b^2x}+\frac{(bB-Ac)x}{2b^2\left(b+cx^2\right)}+\frac{(bB-3Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{5/2}\sqrt{c}}
\end{aligned}$$

Mathematica [A] time = 0.032931, size = 70, normalized size = 1.

$$\frac{x(bB-Ac)}{2b^2\left(b+cx^2\right)}+\frac{(bB-3Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{5/2}\sqrt{c}}-\frac{A}{b^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -(A/(b^2*x)) + ((b*B - A*c)*x)/(2*b^2*(b + c*x^2)) + ((b*B - 3*A*c)*ArcTan[
(Sqrt[c]*x)/Sqrt[b]])/(2*b^(5/2)*Sqrt[c])
```

Maple [A] time = 0.01, size = 85, normalized size = 1.2

$$-\frac{A}{b^2x}-\frac{Acx}{2\,b^2\left(cx^2+b\right)}+\frac{Bx}{2\,b\left(cx^2+b\right)}-\frac{3\,Ac}{2\,b^2}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}+\frac{B}{2\,b}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)/(c*x^4+b*x^2)^2,x)
```

```
[Out] -A/b^2/x-1/2/b^2*x/(c*x^2+b)*A*c+1/2/b*x/(c*x^2+b)*B-3/2/b^2/(b*c)^(1/2)*ar
ctan(x*c/(b*c)^(1/2))*A*c+1/2/b/(b*c)^(1/2)*arctan(x*c/(b*c)^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.927101, size = 447, normalized size = 6.39

$$\left[-\frac{4\,Ab^2c-2\left(Bb^2c-3\,Abc^2\right)x^2-\left(\left(Bbc-3\,Ac^2\right)x^3+\left(Bb^2-3\,Abc\right)x\right)\sqrt{-bc}\log\left(\frac{cx^2+2\,\sqrt{-bc}x-b}{cx^2+b}\right)}{4\left(b^3c^2x^3+b^4cx\right)},-\frac{2\,Ab^2c-\left(Bb^2c-3\,A\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*A*b^2*c - 2*(B*b^2*c - 3*A*b*c^2)*x^2 - ((B*b*c - 3*A*c^2)*x^3 + (
B*b^2 - 3*A*b*c)*x)*sqrt(-b*c)*log((c*x^2 + 2*sqrt(-b*c)*x - b)/(c*x^2 + b)
```

```
))/(b^3*c^2*x^3 + b^4*c*x), -1/2*(2*A*b^2*c - (B*b^2*c - 3*A*b*c^2)*x^2 - (
(B*b*c - 3*A*c^2)*x^3 + (B*b^2 - 3*A*b*c)*x)*sqrt(b*c)*arctan(sqrt(b*c)*x/b
))/(b^3*c^2*x^3 + b^4*c*x)]
```

Sympy [A] time = 0.653096, size = 114, normalized size = 1.63

$$-\frac{\sqrt{-\frac{1}{b^5c}}\,(-3Ac+Bb)\log\left(-b^3\sqrt{-\frac{1}{b^5c}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{b^5c}}\,(-3Ac+Bb)\log\left(b^3\sqrt{-\frac{1}{b^5c}}+x\right)}{4}+\frac{-2Ab+x^2\,(-3Ac+Bb)}{2b^3x+2b^2cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] -sqrt(-1/(b**5*c))*(-3*A*c + B*b)*log(-b**3*sqrt(-1/(b**5*c)) + x)/4 + sqrt
(-1/(b**5*c))*(-3*A*c + B*b)*log(b**3*sqrt(-1/(b**5*c)) + x)/4 + (-2*A*b +
x**2*(-3*A*c + B*b))/(2*b**3*x + 2*b**2*c*x**3)
```

Giac [A] time = 1.45723, size = 84, normalized size = 1.2

$$\frac{(Bb-3\,Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}b^2}+\frac{Bbx^2-3\,Acx^2-2\,Ab}{2\left(cx^3+bx\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(B*b - 3*A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^2) + 1/2*(B*b*x^2 - 3*
A*c*x^2 - 2*A*b)/((c*x^3 + b*x)*b^2)
```

3.69 $\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^2} \, dx$

Optimal. Leaf size=73

$$\frac{bB-Ac}{2b^2(b+cx^2)}-\frac{(bB-2Ac)\log(b+cx^2)}{2b^3}+\frac{\log(x)(bB-2Ac)}{b^3}-\frac{A}{2b^2x^2}$$

```
[Out] -A/(2*b^2*x^2) + (b*B - A*c)/(2*b^2*(b + c*x^2)) + ((b*B - 2*A*c)*Log[x])/b
^3 - ((b*B - 2*A*c)*Log[b + c*x^2])/(2*b^3)
```

Rubi [A] time = 0.0796694, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1584, 446, 77}

$$\frac{bB-Ac}{2b^2(b+cx^2)}-\frac{(bB-2Ac)\log(b+cx^2)}{2b^3}+\frac{\log(x)(bB-2Ac)}{b^3}-\frac{A}{2b^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -A/(2*b^2*x^2) + (b*B - A*c)/(2*b^2*(b + c*x^2)) + ((b*B - 2*A*c)*Log[x])/b
^3 - ((b*B - 2*A*c)*Log[b + c*x^2])/(2*b^3)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx &= \int \frac{A+Bx^2}{x^3\left(b+cx^2\right)^2}\,dx\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{x^2(b+cx)^2}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{A}{b^2x^2}+\frac{bB-2Ac}{b^3x}-\frac{c(bB-Ac)}{b^2(b+cx)^2}-\frac{c(bB-2Ac)}{b^3(b+cx)}\right)dx, x, x^2\right)\\
&= -\frac{A}{2b^2x^2}+\frac{bB-Ac}{2b^2\left(b+cx^2\right)}+\frac{(bB-2Ac)\log(x)}{b^3}-\frac{(bB-2Ac)\log\left(b+cx^2\right)}{2b^3}
\end{aligned}$$

Mathematica [A] time = 0.0515702, size = 64, normalized size = 0.88

$$\frac{\frac{b(bB-Ac)}{b+cx^2}+(2Ac-bB)\log\left(b+cx^2\right)+2\log(x)(bB-2Ac)-\frac{Ab}{x^2}}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (-((A*b)/x^2) + (b*(b*B - A*c))/(b + c*x^2) + 2*(b*B - 2*A*c)*Log[x] + (-(b
*B) + 2*A*c)*Log[b + c*x^2])/(2*b^3)
```

Maple [A] time = 0.014, size = 86, normalized size = 1.2

$$-\frac{A}{2\,b^2x^2}-2\,\frac{A\ln(x)\,c}{b^3}+\frac{\ln(x)\,B}{b^2}+\frac{c\ln\left(cx^2+b\right)A}{b^3}-\frac{\ln\left(cx^2+b\right)B}{2\,b^2}-\frac{Ac}{2\,b^2\left(cx^2+b\right)}+\frac{B}{2\,b\left(cx^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)/(c*x^4+b*x^2)^2,x)
```

```
[Out] -1/2*A/b^2/x^2-2/b^3*ln(x)*A*c+1/b^2*ln(x)*B+1/b^3*c*ln(c*x^2+b)*A-1/2/b^2*
ln(c*x^2+b)*B-1/2/b^2*c/(c*x^2+b)*A+1/2/b/(c*x^2+b)*B
```

Maxima [A] time = 1.21241, size = 103, normalized size = 1.41

$$\frac{(Bb-2\,Ac)x^2-Ab}{2\left(b^2cx^4+b^3x^2\right)}-\frac{(Bb-2\,Ac)\log\left(cx^2+b\right)}{2\,b^3}+\frac{(Bb-2\,Ac)\log\left(x^2\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((B*b - 2*A*c)*x^2 - A*b)/(b^2*c*x^4 + b^3*x^2) - 1/2*(B*b - 2*A*c)*log
(c*x^2 + b)/b^3 + 1/2*(B*b - 2*A*c)*log(x^2)/b^3
```

Fricas [A] time = 0.866581, size = 248, normalized size = 3.4

$$-\frac{Ab^2-\left(Bb^2-2\,Abc\right)x^2+\left(\left(Bbc-2\,Ac^2\right)x^4+\left(Bb^2-2\,Abc\right)x^2\right)\log\left(cx^2+b\right)-2\left(\left(Bbc-2\,Ac^2\right)x^4+\left(Bb^2-2\,Abc\right)x^2\right)}{2\left(b^3cx^4+b^4x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(A*b^2 - (B*b^2 - 2*A*b*c)*x^2 + ((B*b*c - 2*A*c^2)*x^4 + (B*b^2 - 2*A
*b*c)*x^2)*log(c*x^2 + b) - 2*((B*b*c - 2*A*c^2)*x^4 + (B*b^2 - 2*A*b*c)*x^
2)*log(x))/(b^3*c*x^4 + b^4*x^2)
```

Sympy [A] time = 1.02135, size = 70, normalized size = 0.96

$$\frac{-Ab+x^2\left(-2Ac+Bb\right)}{2b^3x^2+2b^2cx^4}+\frac{\left(-2Ac+Bb\right)\log\left(x\right)}{b^3}-\frac{\left(-2Ac+Bb\right)\log\left(\frac{b}{c}+x^2\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] (-A*b + x**2*(-2*A*c + B*b))/(2*b**3*x**2 + 2*b**2*c*x**4) + (-2*A*c + B*b)
*log(x)/b**3 - (-2*A*c + B*b)*log(b/c + x**2)/(2*b**3)
```

Giac [A] time = 1.328, size = 108, normalized size = 1.48

$$\frac{(Bb-2\,Ac)\log\left(|x|\right)}{b^3}+\frac{Bbx^2-2\,Acx^2-Ab}{2\left(cx^4+bx^2\right)b^2}-\frac{\left(Bbc-2\,Ac^2\right)\log\left(\left|cx^2+b\right|\right)}{2\,b^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] (B*b - 2*A*c)*log(abs(x))/b^3 + 1/2*(B*b*x^2 - 2*A*c*x^2 - A*b)/((c*x^4 + b
*x^2)*b^2) - 1/2*(B*b*c - 2*A*c^2)*log(abs(c*x^2 + b))/(b^3*c)
```

3.70 $\int \frac{A+Bx^2}{\left(bx^2+cx^4\right)^2} \, dx$

Optimal. Leaf size=90

$$-\frac{cx(bB-Ac)}{2b^3\left(b+cx^2\right)}-\frac{bB-2Ac}{b^3x}-\frac{\sqrt{c}(3bB-5Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{7/2}}-\frac{A}{3b^2x^3}$$

```
[Out] -A/(3*b^2*x^3) - (b*B - 2*A*c)/(b^3*x) - (c*(b*B - A*c)*x)/(2*b^3*(b + c*x^
2)) - (Sqrt[c]*(3*b*B - 5*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*b^(7/2))
```

Rubi [A] time = 0.117439, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {1593, 456, 1261, 205}

$$-\frac{cx(bB-Ac)}{2b^3\left(b+cx^2\right)}-\frac{bB-2Ac}{b^3x}-\frac{\sqrt{c}(3bB-5Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{7/2}}-\frac{A}{3b^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -A/(3*b^2*x^3) - (b*B - 2*A*c)/(b^3*x) - (c*(b*B - A*c)*x)/(2*b^3*(b + c*x^
2)) - (Sqrt[c]*(3*b*B - 5*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*b^(7/2))
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
```

```
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^2}{\left(bx^2+cx^4\right)^2}\,dx &= \int \frac{A+Bx^2}{x^4\left(b+cx^2\right)^2}\,dx\\
&= -\frac{c(bB-Ac)x}{2b^3\left(b+cx^2\right)}-\frac{1}{2}c\int \frac{-\frac{2A}{bc}-\frac{2(bB-Ac)x^2}{b^2c}+\frac{(bB-Ac)x^4}{b^3}}{x^4\left(b+cx^2\right)}\,dx\\
&= -\frac{c(bB-Ac)x}{2b^3\left(b+cx^2\right)}-\frac{1}{2}c\int \left(-\frac{2A}{b^2cx^4}-\frac{2(bB-2Ac)}{b^3cx^2}+\frac{3bB-5Ac}{b^3\left(b+cx^2\right)}\right)dx\\
&= -\frac{A}{3b^2x^3}-\frac{bB-2Ac}{b^3x}-\frac{c(bB-Ac)x}{2b^3\left(b+cx^2\right)}-\frac{(c(3bB-5Ac))\int \frac{1}{b+cx^2}\,dx}{2b^3}\\
&= -\frac{A}{3b^2x^3}-\frac{bB-2Ac}{b^3x}-\frac{c(bB-Ac)x}{2b^3\left(b+cx^2\right)}-\frac{\sqrt{c}(3bB-5Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{7/2}}
\end{aligned}$$

Mathematica [A] time = 0.0715265, size = 90, normalized size = 1.

$$-\frac{cx(bB-Ac)}{2b^3\left(b+cx^2\right)}+\frac{2Ac-bB}{b^3x}-\frac{\sqrt{c}(3bB-5Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{7/2}}-\frac{A}{3b^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -A/(3*b^2*x^3) + (-(b*B) + 2*A*c)/(b^3*x) - (c*(b*B - A*c)*x)/(2*b^3*(b + c
*x^2)) - (Sqrt[c]*(3*b*B - 5*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*b^(7/2))
```

Maple [A] time = 0.013, size = 110, normalized size = 1.2

$$-\frac{A}{3\,b^2x^3}+2\,\frac{Ac}{b^3x}-\frac{B}{b^2x}+\frac{Ac^2x}{2\,b^3\left(cx^2+b\right)}-\frac{Bcx}{2\,b^2\left(cx^2+b\right)}+\frac{5\,Ac^2}{2\,b^3}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}-\frac{3\,Bc}{2\,b^2}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(c*x^4+b*x^2)^2,x)
```

```
[Out] -1/3*A/b^2/x^3+2/b^3/x*A*c-1/b^2/x*B+1/2/b^3*c^2*x/(c*x^2+b)*A-1/2/b^2*c*x/
(c*x^2+b)*B+5/2/b^3*c^2/(b*c)^(1/2)*arctan(x*c/(b*c)^(1/2))*A-3/2/b^2*c/(b*
c)^(1/2)*arctan(x*c/(b*c)^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.846885, size = 532, normalized size = 5.91

$$\left[-\frac{6\left(3\,Bbc-5\,Ac^2\right)x^4+4\,Ab^2+4\left(3\,Bb^2-5\,Abc\right)x^2+3\left(\left(3\,Bbc-5\,Ac^2\right)x^5+\left(3\,Bb^2-5\,Abc\right)x^3\right)\sqrt{-\frac{c}{b}}\log\left(\frac{cx^2+2\,bx\sqrt{-\frac{c}{b}}\cdots}{cx^2+b}\right.}{12\left(b^3cx^5+b^4x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/12*(6*(3*B*b*c - 5*A*c^2)*x^4 + 4*A*b^2 + 4*(3*B*b^2 - 5*A*b*c)*x^2 + 3
*((3*B*b*c - 5*A*c^2)*x^5 + (3*B*b^2 - 5*A*b*c)*x^3)*sqrt(-c/b)*log((c*x^2
+ 2*b*x*sqrt(-c/b) - b)/(c*x^2 + b)))/(b^3*c*x^5 + b^4*x^3), -1/6*(3*(3*B*b
*c - 5*A*c^2)*x^4 + 2*A*b^2 + 2*(3*B*b^2 - 5*A*b*c)*x^2 + 3*((3*B*b*c - 5*A
*c^2)*x^5 + (3*B*b^2 - 5*A*b*c)*x^3)*sqrt(c/b)*arctan(x*sqrt(c/b)))/(b^3*c*
x^5 + b^4*x^3)]
```

Sympy [B] time = 0.827283, size = 184, normalized size = 2.04

$$\frac{\sqrt{-\frac{c}{b^7}}\,(-5Ac+3Bb)\log\left(-\frac{b^4\sqrt{-\frac{c}{b^7}}(-5Ac+3Bb)}{-5Ac^2+3Bbc}+x\right)}{4}-\frac{\sqrt{-\frac{c}{b^7}}\,(-5Ac+3Bb)\log\left(\frac{b^4\sqrt{-\frac{c}{b^7}}(-5Ac+3Bb)}{-5Ac^2+3Bbc}+x\right)}{4}-\frac{2Ab^2+x^4\left(-15Ac^2\right.}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] sqrt(-c/b**7)*(-5*A*c + 3*B*b)*log(-b**4*sqrt(-c/b**7)*(-5*A*c + 3*B*b)/(-5
*A*c**2 + 3*B*b*c) + x)/4 - sqrt(-c/b**7)*(-5*A*c + 3*B*b)*log(b**4*sqrt(-c
/b**7)*(-5*A*c + 3*B*b)/(-5*A*c**2 + 3*B*b*c) + x)/4 - (2*A*b**2 + x**4*(-1
5*A*c**2 + 9*B*b*c) + x**2*(-10*A*b*c + 6*B*b**2))/(6*b**4*x**3 + 6*b**3*c*
x**5)
```

Giac [A] time = 1.15127, size = 115, normalized size = 1.28

$$-\frac{\left(3\,Bbc-5\,Ac^2\right)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}b^3}-\frac{Bbcx-Ac^2x}{2\left(cx^2+b\right)b^3}-\frac{3\,Bbx^2-6\,Acx^2+Ab}{3\,b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*B*b*c - 5*A*c^2)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^3) - 1/2*(B*b*c
*x - A*c^2*x)/((c*x^2 + b)*b^3) - 1/3*(3*B*b*x^2 - 6*A*c*x^2 + A*b)/(b^3*x^
3)
```

3.71 $\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)^2} \, dx$

Optimal. Leaf size=97

$$-\frac{c(bB-Ac)}{2b^3\left(b+cx^2\right)}-\frac{bB-2Ac}{2b^3x^2}+\frac{c(2bB-3Ac)\log\left(b+cx^2\right)}{2b^4}-\frac{c\log(x)(2bB-3Ac)}{b^4}-\frac{A}{4b^2x^4}$$

```
[Out] -A/(4*b^2*x^4) - (b*B - 2*A*c)/(2*b^3*x^2) - (c*(b*B - A*c))/(2*b^3*(b + c*
x^2)) - (c*(2*b*B - 3*A*c)*Log[x])/b^4 + (c*(2*b*B - 3*A*c)*Log[b + c*x^2])
/(2*b^4)
```

Rubi [A] time = 0.108416, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 446, 77}

$$-\frac{c(bB-Ac)}{2b^3\left(b+cx^2\right)}-\frac{bB-2Ac}{2b^3x^2}+\frac{c(2bB-3Ac)\log\left(b+cx^2\right)}{2b^4}-\frac{c\log(x)(2bB-3Ac)}{b^4}-\frac{A}{4b^2x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x*(b*x^2 + c*x^4)^2),x]
```

```
[Out] -A/(4*b^2*x^4) - (b*B - 2*A*c)/(2*b^3*x^2) - (c*(b*B - A*c))/(2*b^3*(b + c*
x^2)) - (c*(2*b*B - 3*A*c)*Log[x])/b^4 + (c*(2*b*B - 3*A*c)*Log[b + c*x^2])
/(2*b^4)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)^2}\,dx &= \int \frac{A+Bx^2}{x^5\left(b+cx^2\right)^2}\,dx \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{x^3(b+cx)^2}\,dx, x, x^2\right) \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{A}{b^2x^3}+\frac{bB-2Ac}{b^3x^2}-\frac{c(2bB-3Ac)}{b^4x}+\frac{c^2(bB-Ac)}{b^3(b+cx)^2}+\frac{c^2(2bB-3Ac)}{b^4(b+cx)}\right)dx, x, x^2\right) \\
&= -\frac{A}{4b^2x^4}-\frac{bB-2Ac}{2b^3x^2}-\frac{c(bB-Ac)}{2b^3\left(b+cx^2\right)}-\frac{c(2bB-3Ac)\log(x)}{b^4}+\frac{c(2bB-3Ac)\log\left(b+cx^2\right)}{2b^4}
\end{aligned}$$

Mathematica [A] time = 0.0968646, size = 85, normalized size = 0.88

$$-\frac{\frac{Ab^2}{x^4}+\frac{2bc(bB-Ac)}{b+cx^2}+\frac{2b(bB-2Ac)}{x^2}+2c(3Ac-2bB)\log\left(b+cx^2\right)-4c\log(x)(3Ac-2bB)}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x*(b*x^2 + c*x^4)^2),x]
```

```
[Out] -((A*b^2)/x^4 + (2*b*(b*B - 2*A*c))/x^2 + (2*b*c*(b*B - A*c))/(b + c*x^2) -
 4*c*(-2*b*B + 3*A*c)*Log[x] + 2*c*(-2*b*B + 3*A*c)*Log[b + c*x^2])/(4*b^4)
```

Maple [A] time = 0.013, size = 114, normalized size = 1.2

$$-\frac{A}{4x^4b^2}+\frac{Ac}{b^3x^2}-\frac{B}{2b^2x^2}+3\frac{A\ln(x)c^2}{b^4}-2\frac{Bc\ln(x)}{b^3}-\frac{3c^2\ln\left(cx^2+b\right)A}{2b^4}+\frac{c\ln\left(cx^2+b\right)B}{b^3}+\frac{Ac^2}{2b^3\left(cx^2+b\right)}-\frac{Bc}{2b^2\left(cx^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x/(c*x^4+b*x^2)^2,x)
```

```
[Out] -1/4*A/x^4/b^2+1/b^3/x^2*A*c-1/2/b^2/x^2*B+3*c^2/b^4*ln(x)*A-2*c/b^3*ln(x)*
B-3/2/b^4*c^2*ln(c*x^2+b)*A+1/b^3*c*ln(c*x^2+b)*B+1/2/b^3*c^2/(c*x^2+b)*A-1
/2/b^2*c/(c*x^2+b)*B
```

Maxima [A] time = 1.17452, size = 143, normalized size = 1.47

$$-\frac{2\left(2\,Bbc-3\,Ac^{2}\right)x^{4}+Ab^{2}+\left(2\,Bb^{2}-3\,Abc\right)x^{2}}{4\left(b^{3}cx^{6}+b^{4}x^{4}\right)}+\frac{\left(2\,Bbc-3\,Ac^{2}\right)\log\left(cx^{2}+b\right)}{2\,b^{4}}-\frac{\left(2\,Bbc-3\,Ac^{2}\right)\log\left(x^{2}\right)}{2\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*(2*B*b*c - 3*A*c^2)*x^4 + A*b^2 + (2*B*b^2 - 3*A*b*c)*x^2)/(b^3*c*x
^6 + b^4*x^4) + 1/2*(2*B*b*c - 3*A*c^2)*log(c*x^2 + b)/b^4 - 1/2*(2*B*b*c -
 3*A*c^2)*log(x^2)/b^4
```

Fricas [A] time = 0.614585, size = 327, normalized size = 3.37

$$-\frac{2\left(2\,Bb^{2}c-3\,Abc^{2}\right)x^{4}+Ab^{3}+\left(2\,Bb^{3}-3\,Ab^{2}c\right)x^{2}-2\left(\left(2\,Bbc^{2}-3\,Ac^{3}\right)x^{6}+\left(2\,Bb^{2}c-3\,Abc^{2}\right)x^{4}\right)\log\left(cx^{2}+b\right)+4\left(\left(\right.\right.}{4\left(b^{4}cx^{6}+b^{5}x^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(2*B*b^2*c - 3*A*b*c^2)*x^4 + A*b^3 + (2*B*b^3 - 3*A*b^2*c)*x^2 - 2
*((2*B*b*c^2 - 3*A*c^3)*x^6 + (2*B*b^2*c - 3*A*b*c^2)*x^4)*log(c*x^2 + b) +
 4*((2*B*b*c^2 - 3*A*c^3)*x^6 + (2*B*b^2*c - 3*A*b*c^2)*x^4)*log(x))/(b^4*c
*x^6 + b^5*x^4)
```

Sympy [A] time = 1.29994, size = 100, normalized size = 1.03

$$-\frac{Ab^{2}+x^{4}\left(-6Ac^{2}+4Bbc\right)+x^{2}\left(-3Abc+2Bb^{2}\right)}{4b^{4}x^{4}+4b^{3}cx^{6}}-\frac{c\left(-3Ac+2Bb\right)\log\left(x\right)}{b^{4}}+\frac{c\left(-3Ac+2Bb\right)\log\left(\frac{b}{c}+x^{2}\right)}{2b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x/(c*x**4+b*x**2)**2,x)
```

```
[Out] -(A*b**2 + x**4*(-6*A*c**2 + 4*B*b*c) + x**2*(-3*A*b*c + 2*B*b**2))/(4*b**4
*x**4 + 4*b**3*c*x**6) - c*(-3*A*c + 2*B*b)*log(x)/b**4 + c*(-3*A*c + 2*B*b
)*log(b/c + x**2)/(2*b**4)
```

Giac [A] time = 1.14966, size = 203, normalized size = 2.09

$$-\frac{\left(2\,Bbc-3\,Ac^{2}\right)\log\left(x^{2}\right)}{2\,b^{4}}+\frac{\left(2\,Bbc^{2}-3\,Ac^{3}\right)\log\left(\left|cx^{2}+b\right|\right)}{2\,b^{4}c}-\frac{2\,Bbc^{2}x^{2}-3\,Ac^{3}x^{2}+3\,Bb^{2}c-4\,Abc^{2}}{2\left(cx^{2}+b\right)b^{4}}+\frac{6\,Bbcx^{4}-9\,Ac^{2}x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*B*b*c - 3*A*c^2)*log(x^2)/b^4 + 1/2*(2*B*b*c^2 - 3*A*c^3)*log(abs(c
*x^2 + b))/(b^4*c) - 1/2*(2*B*b*c^2*x^2 - 3*A*c^3*x^2 + 3*B*b^2*c - 4*A*b*c
^2)/((c*x^2 + b)*b^4) + 1/4*(6*B*b*c*x^4 - 9*A*c^2*x^4 - 2*B*b^2*x^2 + 4*A*
b*c*x^2 - A*b^2)/(b^4*x^4)
```

3.72 $\int \frac{A+Bx^2}{x^2(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=111

$$\frac{c^2x(bB-Ac)}{2b^4(b+cx^2)}+\frac{c^{3/2}(5bB-7Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{9/2}}-\frac{bB-2Ac}{3b^3x^3}+\frac{c(2bB-3Ac)}{b^4x}-\frac{A}{5b^2x^5}$$

```
[Out] -A/(5*b^2*x^5) - (b*B - 2*A*c)/(3*b^3*x^3) + (c*(2*b*B - 3*A*c))/(b^4*x) +
(c^2*(b*B - A*c)*x)/(2*b^4*(b + c*x^2)) + (c^(3/2)*(5*b*B - 7*A*c)*ArcTan[(
Sqrt[c]*x)/Sqrt[b]])/(2*b^(9/2))
```

Rubi [A] time = 0.198159, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1584, 456, 1802, 205}

$$\frac{c^2x(bB-Ac)}{2b^4(b+cx^2)}+\frac{c^{3/2}(5bB-7Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{9/2}}-\frac{bB-2Ac}{3b^3x^3}+\frac{c(2bB-3Ac)}{b^4x}-\frac{A}{5b^2x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^2*(b*x^2 + c*x^4)^2),x]
```

```
[Out] -A/(5*b^2*x^5) - (b*B - 2*A*c)/(3*b^3*x^3) + (c*(2*b*B - 3*A*c))/(b^4*x) +
(c^2*(b*B - A*c)*x)/(2*b^4*(b + c*x^2)) + (c^(3/2)*(5*b*B - 7*A*c)*ArcTan[(
Sqrt[c]*x)/Sqrt[b]])/(2*b^(9/2))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
```

```
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^2}{x^2\left(bx^2+cx^4\right)^2}\,dx &= \int \frac{A+Bx^2}{x^6\left(b+cx^2\right)^2}\,dx \\
&= \frac{c^2(bB-Ac)x}{2b^4\left(b+cx^2\right)}-\frac{1}{2}c^2\int \frac{-\frac{2A}{bc^2}-\frac{2(bB-Ac)x^2}{b^2c^2}+\frac{2(bB-Ac)x^4}{b^3c}-\frac{(bB-Ac)x^6}{b^4}}{x^6\left(b+cx^2\right)}\,dx \\
&= \frac{c^2(bB-Ac)x}{2b^4\left(b+cx^2\right)}-\frac{1}{2}c^2\int \left(-\frac{2A}{b^2c^2x^6}-\frac{2(bB-2Ac)}{b^3c^2x^4}+\frac{2(2bB-3Ac)}{b^4cx^2}+\frac{-5bB+7Ac}{b^4\left(b+cx^2\right)}\right)dx \\
&= -\frac{A}{5b^2x^5}-\frac{bB-2Ac}{3b^3x^3}+\frac{c(2bB-3Ac)}{b^4x}+\frac{c^2(bB-Ac)x}{2b^4\left(b+cx^2\right)}+\frac{\left(c^2(5bB-7Ac)\right)\int \frac{1}{b+cx^2}\,dx}{2b^4} \\
&= -\frac{A}{5b^2x^5}-\frac{bB-2Ac}{3b^3x^3}+\frac{c(2bB-3Ac)}{b^4x}+\frac{c^2(bB-Ac)x}{2b^4\left(b+cx^2\right)}+\frac{c^{3/2}(5bB-7Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{9/2}}
\end{aligned}$$

Mathematica [A] time = 0.0899056, size = 112, normalized size = 1.01

$$\frac{c^2x(bB-Ac)}{2b^4\left(b+cx^2\right)}+\frac{c^{3/2}(5bB-7Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{9/2}}+\frac{2Ac-bB}{3b^3x^3}+\frac{c(2bB-3Ac)}{b^4x}-\frac{A}{5b^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^2*(b*x^2 + c*x^4)^2),x]
```

```
[Out] -A/(5*b^2*x^5) + (-(b*B) + 2*A*c)/(3*b^3*x^3) + (c*(2*b*B - 3*A*c))/(b^4*x)
 + (c^2*(b*B - A*c)*x)/(2*b^4*(b + c*x^2)) + (c^(3/2)*(5*b*B - 7*A*c)*ArcTa
n[(Sqrt[c]*x)/Sqrt[b]])/(2*b^(9/2))
```

Maple [A] time = 0.013, size = 136, normalized size = 1.2

$$-\frac{A}{5\,b^2x^5}+\frac{2\,Ac}{3\,b^3x^3}-\frac{B}{3\,b^2x^3}-3\,\frac{Ac^2}{b^4x}+2\,\frac{Bc}{b^3x}-\frac{Ac^3x}{2\,b^4\left(cx^2+b\right)}+\frac{Bc^2x}{2\,b^3\left(cx^2+b\right)}-\frac{7\,Ac^3}{2\,b^4}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}+\frac{5\,Bc^2}{2\,b^3}\mathrm{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^2/(c*x^4+b*x^2)^2,x)
```

```
[Out] -1/5*A/b^2/x^5+2/3/b^3/x^3*A*c-1/3/b^2/x^3*B-3*c^2/b^4/x*A+2*c/b^3/x*B-1/2/
b^4*c^3*x/(c*x^2+b)*A+1/2/b^3*c^2*x/(c*x^2+b)*B-7/2/b^4*c^3/(b*c)^(1/2)*arc
tan(x*c/(b*c)^(1/2))*A+5/2/b^3*c^2/(b*c)^(1/2)*arctan(x*c/(b*c)^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.847408, size = 653, normalized size = 5.88

$$\left[\frac{30\left(5\,Bbc^2-7\,Ac^3\right)x^6+20\left(5\,Bb^2c-7\,Abc^2\right)x^4-12\,Ab^3-4\left(5\,Bb^3-7\,Ab^2c\right)x^2-15\left(\left(5\,Bbc^2-7\,Ac^3\right)x^7+\left(5\,Bb^2c-\right.\right.}{60\left(b^4cx^7+b^5x^5\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(30*(5*B*b*c^2 - 7*A*c^3)*x^6 + 20*(5*B*b^2*c - 7*A*b*c^2)*x^4 - 12*A
*b^3 - 4*(5*B*b^3 - 7*A*b^2*c)*x^2 - 15*((5*B*b*c^2 - 7*A*c^3)*x^7 + (5*B*b
^2*c - 7*A*b*c^2)*x^5)*sqrt(-c/b)*log((c*x^2 - 2*b*x*sqrt(-c/b) - b)/(c*x^2
 + b)))/(b^4*c*x^7 + b^5*x^5), 1/30*(15*(5*B*b*c^2 - 7*A*c^3)*x^6 + 10*(5*B
*b^2*c - 7*A*b*c^2)*x^4 - 6*A*b^3 - 2*(5*B*b^3 - 7*A*b^2*c)*x^2 + 15*((5*B*
b*c^2 - 7*A*c^3)*x^7 + (5*B*b^2*c - 7*A*b*c^2)*x^5)*sqrt(c/b)*arctan(x*sqrt
(c/b)))/(b^4*c*x^7 + b^5*x^5)]
```

Sympy [B] time = 1.02174, size = 218, normalized size = 1.96

$$-\frac{\sqrt{-\frac{c^3}{b^9}}\left(-7Ac+5Bb\right)\log\left(-\frac{b^5\sqrt{-\frac{c^3}{b^9}}(-7Ac+5Bb)}{-7Ac^3+5Bbc^2}+x\right)}{4}+\frac{\sqrt{-\frac{c^3}{b^9}}\left(-7Ac+5Bb\right)\log\left(\frac{b^5\sqrt{-\frac{c^3}{b^9}}(-7Ac+5Bb)}{-7Ac^3+5Bbc^2}+x\right)}{4}+\frac{-6Ab^3+x^6\left(-10\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**2/(c*x**4+b*x**2)**2,x)
```

```
[Out] -sqrt(-c**3/b**9)*(-7*A*c + 5*B*b)*log(-b**5*sqrt(-c**3/b**9)*(-7*A*c + 5*B
*b)/(-7*A*c**3 + 5*B*b*c**2) + x)/4 + sqrt(-c**3/b**9)*(-7*A*c + 5*B*b)*log
(b**5*sqrt(-c**3/b**9)*(-7*A*c + 5*B*b)/(-7*A*c**3 + 5*B*b*c**2) + x)/4 + (
-6*A*b**3 + x**6*(-105*A*c**3 + 75*B*b*c**2) + x**4*(-70*A*b*c**2 + 50*B*b*
*2*c) + x**2*(14*A*b**2*c - 10*B*b**3))/(30*b**5*x**5 + 30*b**4*c*x**7)
```

Giac [A] time = 1.1418, size = 151, normalized size = 1.36

$$\frac{\left(5\,Bbc^2-7\,Ac^3\right)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}b^4}+\frac{Bbc^2x-Ac^3x}{2\left(cx^2+b\right)b^4}+\frac{30\,Bbcx^4-45\,Ac^2x^4-5\,Bb^2x^2+10\,Abcx^2-3\,Ab^2}{15\,b^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*B*b*c^2 - 7*A*c^3)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^4) + 1/2*(B*b*
c^2*x - A*c^3*x)/((c*x^2 + b)*b^4) + 1/15*(30*B*b*c*x^4 - 45*A*c^2*x^4 - 5*
B*b^2*x^2 + 10*A*b*c*x^2 - 3*A*b^2)/(b^4*x^5)
```

3.73 $\int \frac{x^{14}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=140

$$\frac{b^2x(17bB-13Ac)}{8c^5\left(b+cx^2\right)}-\frac{b^3x(bB-Ac)}{4c^5\left(b+cx^2\right)^2}-\frac{7b^{3/2}(9bB-5Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8c^{11/2}}-\frac{x^3(3bB-Ac)}{3c^4}+\frac{3bx(2bB-Ac)}{c^5}+\frac{Bx^5}{5c^3}$$

```
[Out] (3*b*(2*b*B - A*c)*x)/c^5 - ((3*b*B - A*c)*x^3)/(3*c^4) + (B*x^5)/(5*c^3) -
 (b^3*(b*B - A*c)*x)/(4*c^5*(b + c*x^2)^2) + (b^2*(17*b*B - 13*A*c)*x)/(8*c
^5*(b + c*x^2)) - (7*b^(3/2)*(9*b*B - 5*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(
8*c^(11/2))
```

Rubi [A] time = 0.234623, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1584, 455, 1814, 1810, 205}

$$\frac{b^2x(17bB-13Ac)}{8c^5\left(b+cx^2\right)}-\frac{b^3x(bB-Ac)}{4c^5\left(b+cx^2\right)^2}-\frac{7b^{3/2}(9bB-5Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8c^{11/2}}-\frac{x^3(3bB-Ac)}{3c^4}+\frac{3bx(2bB-Ac)}{c^5}+\frac{Bx^5}{5c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^14*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (3*b*(2*b*B - A*c)*x)/c^5 - ((3*b*B - A*c)*x^3)/(3*c^4) + (B*x^5)/(5*c^3) -
 (b^3*(b*B - A*c)*x)/(4*c^5*(b + c*x^2)^2) + (b^2*(17*b*B - 13*A*c)*x)/(8*c
^5*(b + c*x^2)) - (7*b^(3/2)*(9*b*B - 5*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(
8*c^(11/2))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
```

```
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^{14}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{x^8\left(A+Bx^2\right)}{\left(b+cx^2\right)^3}\,dx$$

$$= -\frac{b^3(bB-Ac)x}{4c^5\left(b+cx^2\right)^2} - \frac{\int \frac{-b^3(bB-Ac)+4b^2c(bB-Ac)x^2-4bc^2(bB-Ac)x^4+4c^3(bB-Ac)x^6-4Bc^4x^8}{\left(b+cx^2\right)^2}\,dx}{4c^5}$$

$$= -\frac{b^3(bB-Ac)x}{4c^5\left(b+cx^2\right)^2} + \frac{b^2(17bB-13Ac)x}{8c^5\left(b+cx^2\right)} + \frac{\int \frac{-b^3(15bB-11Ac)+8b^2c(3bB-2Ac)x^2-8bc^2(2bB-Ac)x^4+8bBc^3x^6}{b+cx^2}\,dx}{8bc^5}$$

$$= -\frac{b^3(bB-Ac)x}{4c^5\left(b+cx^2\right)^2} + \frac{b^2(17bB-13Ac)x}{8c^5\left(b+cx^2\right)} + \frac{\int \Big(24b^2(2bB-Ac) - 8bc(3bB-Ac)x^2 + 8bBc^2x^4 - \frac{7(9b}{}}{8bc^5}$$

$$= \frac{3b(2bB-Ac)x}{c^5} - \frac{(3bB-Ac)x^3}{3c^4} + \frac{Bx^5}{5c^3} - \frac{b^3(bB-Ac)x}{4c^5\left(b+cx^2\right)^2} + \frac{b^2(17bB-13Ac)x}{8c^5\left(b+cx^2\right)} - \frac{\left(7b^2(9bB-5Ac\right.}{8c^5}$$

$$= \frac{3b(2bB-Ac)x}{c^5} - \frac{(3bB-Ac)x^3}{3c^4} + \frac{Bx^5}{5c^3} - \frac{b^3(bB-Ac)x}{4c^5\left(b+cx^2\right)^2} + \frac{b^2(17bB-13Ac)x}{8c^5\left(b+cx^2\right)} - \frac{7b^{3/2}(9bB-5Ac}{8c^{11}}$$

Mathematica [A] time = 0.104349, size = 133, normalized size = 0.95

$$\frac{x\left(7b^2c^2x^2\left(72Bx^2-125A\right)-525b^3c\left(A-3Bx^2\right)-8bc^3x^4\left(35A+9Bx^2\right)+8c^4x^6\left(5A+3Bx^2\right)+945b^4B\right)}{120c^5\left(b+cx^2\right)^2} - \frac{7b^{3/2}(9bB}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^14*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (x*(945*b^4*B - 525*b^3*c*(A - 3*B*x^2) + 8*c^4*x^6*(5*A + 3*B*x^2) - 8*b*c
^3*x^4*(35*A + 9*B*x^2) + 7*b^2*c^2*x^2*(-125*A + 72*B*x^2)))/(120*c^5*(b +
 c*x^2)^2) - (7*b^(3/2)*(9*b*B - 5*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*c^(
11/2))
```

Maple [A] time = 0.012, size = 174, normalized size = 1.2

$$\frac{Bx^5}{5c^3} + \frac{Ax^3}{3c^3} - \frac{Bx^3b}{c^4} - 3\frac{Abx}{c^4} + 6\frac{Bb^2x}{c^5} - \frac{13Ab^2x^3}{8c^3\left(cx^2+b\right)^2} + \frac{17Bb^3x^3}{8c^4\left(cx^2+b\right)^2} - \frac{11Ab^3x}{8c^4\left(cx^2+b\right)^2} + \frac{15Bb^4x}{8c^5\left(cx^2+b\right)^2} + \frac{35Ab^2}{8c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14*(B*x^2+A)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 1/5*B*x^5/c^3+1/3/c^3*A*x^3-1/c^4*B*x^3*b-3/c^4*A*b*x+6/c^5*B*b^2*x-13/8*b^
2/c^3/(c*x^2+b)^2*A*x^3+17/8*b^3/c^4/(c*x^2+b)^2*B*x^3-11/8*b^3/c^4/(c*x^2+
b)^2*A*x+15/8*b^4/c^5/(c*x^2+b)^2*B*x+35/8*b^2/c^4/(b*c)^(1/2)*arctan(x*c/(
b*c)^(1/2))*A-63/8*b^3/c^5/(b*c)^(1/2)*arctan(x*c/(b*c)^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.875004, size = 892, normalized size = 6.37

$$\left[\frac{48\,Bc^4x^9 - 16\left(9\,Bbc^3 - 5\,Ac^4\right)x^7 + 112\left(9\,Bb^2c^2 - 5\,Abc^3\right)x^5 + 350\left(9\,Bb^3c - 5\,Ab^2c^2\right)x^3 - 105\left(9\,Bb^4 - 5\,Ab^3c + \left(9\,Bb^2\right.\right.}{240\left(c^7x^4 + 2\,bc^6x^2 + b^2c^5\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [1/240*(48*B*c^4*x^9 - 16*(9*B*b*c^3 - 5*A*c^4)*x^7 + 112*(9*B*b^2*c^2 - 5*
A*b*c^3)*x^5 + 350*(9*B*b^3*c - 5*A*b^2*c^2)*x^3 - 105*(9*B*b^4 - 5*A*b^3*c
 + (9*B*b^2*c^2 - 5*A*b*c^3)*x^4 + 2*(9*B*b^3*c - 5*A*b^2*c^2)*x^2)*sqrt(-b
/c)*log((c*x^2 + 2*c*x*sqrt(-b/c) - b)/(c*x^2 + b)) + 210*(9*B*b^4 - 5*A*b^
3*c)*x)/(c^7*x^4 + 2*b*c^6*x^2 + b^2*c^5), 1/120*(24*B*c^4*x^9 - 8*(9*B*b*c
^3 - 5*A*c^4)*x^7 + 56*(9*B*b^2*c^2 - 5*A*b*c^3)*x^5 + 175*(9*B*b^3*c - 5*A
*b^2*c^2)*x^3 - 105*(9*B*b^4 - 5*A*b^3*c + (9*B*b^2*c^2 - 5*A*b*c^3)*x^4 +
2*(9*B*b^3*c - 5*A*b^2*c^2)*x^2)*sqrt(b/c)*arctan(c*x*sqrt(b/c)/b) + 105*(9
```

```
*B*b^4 - 5*A*b^3*c)*x)/(c^7*x^4 + 2*b*c^6*x^2 + b^2*c^5)]
```

Sympy [A] time = 1.63422, size = 250, normalized size = 1.79

$$\frac{Bx^5}{5c^3} + \frac{7\sqrt{-\frac{b^3}{c^{11}}}\,(-5Ac+9Bb)\log\left(-\frac{7c^5\sqrt{-\frac{b^3}{c^{11}}}(-5Ac+9Bb)}{-35Abc+63Bb^2}+x\right)}{16} - \frac{7\sqrt{-\frac{b^3}{c^{11}}}\,(-5Ac+9Bb)\log\left(\frac{7c^5\sqrt{-\frac{b^3}{c^{11}}}(-5Ac+9Bb)}{-35Abc+63Bb^2}+x\right)}{16} + \frac{x^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] B*x**5/(5*c**3) + 7*sqrt(-b**3/c**11)*(-5*A*c + 9*B*b)*log(-7*c**5*sqrt(-b*
*3/c**11)*(-5*A*c + 9*B*b)/(-35*A*b*c + 63*B*b**2) + x)/16 - 7*sqrt(-b**3/c
**11)*(-5*A*c + 9*B*b)*log(7*c**5*sqrt(-b**3/c**11)*(-5*A*c + 9*B*b)/(-35*A
*b*c + 63*B*b**2) + x)/16 + (x**3*(-13*A*b**2*c**2 + 17*B*b**3*c) + x*(-11*
A*b**3*c + 15*B*b**4))/(8*b**2*c**5 + 16*b*c**6*x**2 + 8*c**7*x**4) - x**3*
(-A*c + 3*B*b)/(3*c**4) + x*(-3*A*b*c + 6*B*b**2)/c**5
```

Giac [A] time = 1.15251, size = 186, normalized size = 1.33

$$-\frac{7\left(9\,Bb^3-5\,Ab^2c\right)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}c^5} + \frac{17\,Bb^3cx^3-13\,Ab^2c^2x^3+15\,Bb^4x-11\,Ab^3cx}{8\left(cx^2+b\right)^2c^5} + \frac{3\,Bc^{12}x^5-15\,Bbc^{11}x^3+5\,Ac^{12}x^3}{15\,c^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -7/8*(9*B*b^3 - 5*A*b^2*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^5) + 1/8*(17*
B*b^3*c*x^3 - 13*A*b^2*c^2*x^3 + 15*B*b^4*x - 11*A*b^3*c*x)/((c*x^2 + b)^2*
c^5) + 1/15*(3*B*c^12*x^5 - 15*B*b*c^11*x^3 + 5*A*c^12*x^3 + 90*B*b^2*c^10*
x - 45*A*b*c^11*x)/c^15
```

3.74 $\int \frac{x^{13}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=111

$$\frac{b^2(4bB-3Ac)}{2c^5\left(b+cx^2\right)}-\frac{b^3(bB-Ac)}{4c^5\left(b+cx^2\right)^2}-\frac{x^2(3bB-Ac)}{2c^4}+\frac{3b(2bB-Ac)\log\left(b+cx^2\right)}{2c^5}+\frac{Bx^4}{4c^3}$$

```
[Out] -((3*b*B - A*c)*x^2)/(2*c^4) + (B*x^4)/(4*c^3) - (b^3*(b*B - A*c))/(4*c^5*(
b + c*x^2)^2) + (b^2*(4*b*B - 3*A*c))/(2*c^5*(b + c*x^2)) + (3*b*(2*b*B - A
*c)*Log[b + c*x^2])/(2*c^5)
```

Rubi [A] time = 0.136048, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 446, 77}

$$\frac{b^2(4bB-3Ac)}{2c^5\left(b+cx^2\right)}-\frac{b^3(bB-Ac)}{4c^5\left(b+cx^2\right)^2}-\frac{x^2(3bB-Ac)}{2c^4}+\frac{3b(2bB-Ac)\log\left(b+cx^2\right)}{2c^5}+\frac{Bx^4}{4c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^13*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -((3*b*B - A*c)*x^2)/(2*c^4) + (B*x^4)/(4*c^3) - (b^3*(b*B - A*c))/(4*c^5*(
b + c*x^2)^2) + (b^2*(4*b*B - 3*A*c))/(2*c^5*(b + c*x^2)) + (3*b*(2*b*B - A
*c)*Log[b + c*x^2])/(2*c^5)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{13}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx &= \int \frac{x^7\left(A+Bx^2\right)}{\left(b+cx^2\right)^3}\,dx \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^3(A+Bx)}{(b+cx)^3}\,dx,x,x^2\right) \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{-3bB+Ac}{c^4}+\frac{Bx}{c^3}+\frac{b^3(bB-Ac)}{c^4(b+cx)^3}-\frac{b^2(4bB-3Ac)}{c^4(b+cx)^2}+\frac{3b(2bB-Ac)}{c^4(b+cx)}\right)dx,x,x^2\right) \\
&= -\frac{(3bB-Ac)x^2}{2c^4}+\frac{Bx^4}{4c^3}-\frac{b^3(bB-Ac)}{4c^5\left(b+cx^2\right)^2}+\frac{b^2(4bB-3Ac)}{2c^5\left(b+cx^2\right)}+\frac{3b(2bB-Ac)\log\left(b+cx^2\right)}{2c^5}
\end{aligned}$$

Mathematica [A] time = 0.0628342, size = 94, normalized size = 0.85

$$\frac{\frac{2b^2(4bB-3Ac)}{b+cx^2}+\frac{b^3(Ac-bB)}{\left(b+cx^2\right)^2}+2cx^2(Ac-3bB)+6b(2bB-Ac)\log\left(b+cx^2\right)+Bc^2x^4}{4c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^13*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*c*(-3*b*B + A*c)*x^2 + B*c^2*x^4 + (b^3*(-(b*B) + A*c))/(b + c*x^2)^2 +
(2*b^2*(4*b*B - 3*A*c))/(b + c*x^2) + 6*b*(2*b*B - A*c)*Log[b + c*x^2])/(4*
c^5)
```

Maple [A] time = 0.011, size = 134, normalized size = 1.2

$$\frac{Bx^4}{4\,c^3}-\frac{3\,Bx^2b}{2\,c^4}+\frac{Ax^2}{2\,c^3}-\frac{3\,b\ln\left(cx^2+b\right)A}{2\,c^4}+3\,\frac{b^2\ln\left(cx^2+b\right)B}{c^5}-\frac{3\,Ab^2}{2\,c^4\left(cx^2+b\right)}+2\,\frac{Bb^3}{c^5\left(cx^2+b\right)}+\frac{b^3A}{4\,c^4\left(cx^2+b\right)^2}-\frac{}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13*(B*x^2+A)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 1/4*B*x^4/c^3-3/2/c^4*B*x^2*b+1/2/c^3*A*x^2-3/2*b/c^4*ln(c*x^2+b)*A+3*b^2/c
^5*ln(c*x^2+b)*B-3/2*b^2/c^4/(c*x^2+b)*A+2*b^3/c^5/(c*x^2+b)*B+1/4*b^3/c^4/
(c*x^2+b)^2*A-1/4*b^4/c^5/(c*x^2+b)^2*B
```

Maxima [A] time = 1.153, size = 157, normalized size = 1.41

$$\frac{7\,Bb^4 - 5\,Ab^3c + 2\left(4\,Bb^3c - 3\,Ab^2c^2\right)x^2}{4\left(c^7x^4 + 2\,bc^6x^2 + b^2c^5\right)} + \frac{Bcx^4 - 2\,(3\,Bb - Ac)x^2}{4\,c^4} + \frac{3\left(2\,Bb^2 - Abc\right)\log\left(cx^2 + b\right)}{2\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(7*B*b^4 - 5*A*b^3*c + 2*(4*B*b^3*c - 3*A*b^2*c^2)*x^2)/(c^7*x^4 + 2*b*
c^6*x^2 + b^2*c^5) + 1/4*(B*c*x^4 - 2*(3*B*b - A*c)*x^2)/c^4 + 3/2*(2*B*b^2
 - A*b*c)*log(c*x^2 + b)/c^5
```

Fricas [A] time = 1.06674, size = 360, normalized size = 3.24

$$\frac{Bc^4x^8 - 2\left(2\,Bbc^3 - Ac^4\right)x^6 + 7\,Bb^4 - 5\,Ab^3c - \left(11\,Bb^2c^2 - 4\,Abc^3\right)x^4 + 2\left(Bb^3c - 2\,Ab^2c^2\right)x^2 + 6\left(2\,Bb^4 - Ab^3c + \left(2\,Bb^2\right.\right.}{4\left(c^7x^4 + 2\,bc^6x^2 + b^2c^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(B*c^4*x^8 - 2*(2*B*b*c^3 - A*c^4)*x^6 + 7*B*b^4 - 5*A*b^3*c - (11*B*b^
2*c^2 - 4*A*b*c^3)*x^4 + 2*(B*b^3*c - 2*A*b^2*c^2)*x^2 + 6*(2*B*b^4 - A*b^3
*c + (2*B*b^2*c^2 - A*b*c^3)*x^4 + 2*(2*B*b^3*c - A*b^2*c^2)*x^2)*log(c*x^2
 + b))/(c^7*x^4 + 2*b*c^6*x^2 + b^2*c^5)
```

Sympy [A] time = 1.64791, size = 116, normalized size = 1.05

$$\frac{Bx^4}{4c^3}+\frac{3b\left(-Ac+2Bb\right)\log\left(b+cx^2\right)}{2c^5}+\frac{-5Ab^3c+7Bb^4+x^2\left(-6Ab^2c^2+8Bb^3c\right)}{4b^2c^5+8bc^6x^2+4c^7x^4}-\frac{x^2\left(-Ac+3Bb\right)}{2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] B*x**4/(4*c**3) + 3*b*(-A*c + 2*B*b)*log(b + c*x**2)/(2*c**5) + (-5*A*b**3*
c + 7*B*b**4 + x**2*(-6*A*b**2*c**2 + 8*B*b**3*c))/(4*b**2*c**5 + 8*b*c**6*
x**2 + 4*c**7*x**4) - x**2*(-A*c + 3*B*b)/(2*c**4)
```

Giac [A] time = 1.13653, size = 178, normalized size = 1.6

$$\frac{3\left(2\,Bb^2-Abc\right)\log\left(\left|cx^2+b\right|\right)}{2\,c^5}+\frac{Bc^3x^4-6\,Bbc^2x^2+2\,Ac^3x^2}{4\,c^6}-\frac{18\,Bb^2c^2x^4-9\,Abc^3x^4+28\,Bb^3cx^2-12\,Ab^2c^2x^2+11\,B}{4\left(cx^2+b\right)^2c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 3/2*(2*B*b^2 - A*b*c)*log(abs(c*x^2 + b))/c^5 + 1/4*(B*c^3*x^4 - 6*B*b*c^2*
x^2 + 2*A*c^3*x^2)/c^6 - 1/4*(18*B*b^2*c^2*x^4 - 9*A*b*c^3*x^4 + 28*B*b^3*c
*x^2 - 12*A*b^2*c^2*x^2 + 11*B*b^4 - 4*A*b^3*c)/((c*x^2 + b)^2*c^5)
```

3.75 $\int \frac{x^{12}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=118

$$\frac{b^2x(bB-Ac)}{4c^4(b+cx^2)^2}-\frac{bx(13bB-9Ac)}{8c^4(b+cx^2)}-\frac{x(3bB-Ac)}{c^4}+\frac{5\sqrt{b}(7bB-3Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8c^{9/2}}+\frac{Bx^3}{3c^3}$$

```
[Out] -(((3*b*B - A*c)*x)/c^4) + (B*x^3)/(3*c^3) + (b^2*(b*B - A*c)*x)/(4*c^4*(b
+ c*x^2)^2) - (b*(13*b*B - 9*A*c)*x)/(8*c^4*(b + c*x^2)) + (5*Sqrt[b]*(7*b*
B - 3*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*c^(9/2))
```

Rubi [A] time = 0.162, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1584, 455, 1814, 1153, 205}

$$\frac{b^2x(bB-Ac)}{4c^4(b+cx^2)^2}-\frac{bx(13bB-9Ac)}{8c^4(b+cx^2)}-\frac{x(3bB-Ac)}{c^4}+\frac{5\sqrt{b}(7bB-3Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8c^{9/2}}+\frac{Bx^3}{3c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^12*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -(((3*b*B - A*c)*x)/c^4) + (B*x^3)/(3*c^3) + (b^2*(b*B - A*c)*x)/(4*c^4*(b
+ c*x^2)^2) - (b*(13*b*B - 9*A*c)*x)/(8*c^4*(b + c*x^2)) + (5*Sqrt[b]*(7*b*
B - 3*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*c^(9/2))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
```

```
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^{12}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{x^6\left(A+Bx^2\right)}{\left(b+cx^2\right)^3}\,dx$$

$$= \frac{b^2(bB-Ac)x}{4c^4\left(b+cx^2\right)^2} - \frac{\int \frac{b^2(bB-Ac)-4bc(bB-Ac)x^2+4c^2(bB-Ac)x^4-4Bc^3x^6}{\left(b+cx^2\right)^2}\,dx}{4c^4}$$

$$= \frac{b^2(bB-Ac)x}{4c^4\left(b+cx^2\right)^2} - \frac{b(13bB-9Ac)x}{8c^4\left(b+cx^2\right)} + \frac{\int \frac{b^2(11bB-7Ac)-8bc(2bB-Ac)x^2+8bBc^2x^4}{b+cx^2}\,dx}{8bc^4}$$

$$= \frac{b^2(bB-Ac)x}{4c^4\left(b+cx^2\right)^2} - \frac{b(13bB-9Ac)x}{8c^4\left(b+cx^2\right)} + \frac{\int \left(-8b(3bB-Ac)+8bBcx^2+\frac{5\left(7b^3B-3Ab^2c\right)}{b+cx^2}\right)dx}{8bc^4}$$

$$= -\frac{(3bB-Ac)x}{c^4} + \frac{Bx^3}{3c^3} + \frac{b^2(bB-Ac)x}{4c^4\left(b+cx^2\right)^2} - \frac{b(13bB-9Ac)x}{8c^4\left(b+cx^2\right)} + \frac{(5b(7bB-3Ac))\int \frac{1}{b+cx^2}\,dx}{8c^4}$$

$$= -\frac{(3bB-Ac)x}{c^4} + \frac{Bx^3}{3c^3} + \frac{b^2(bB-Ac)x}{4c^4\left(b+cx^2\right)^2} - \frac{b(13bB-9Ac)x}{8c^4\left(b+cx^2\right)} + \frac{5\sqrt{b}(7bB-3Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8c^{9/2}}$$

Mathematica [A] time = 0.0847434, size = 113, normalized size = 0.96

$$\frac{5b^2cx\left(9A-35Bx^2\right)+bc^2x^3\left(75A-56Bx^2\right)+8c^3x^5\left(3A+Bx^2\right)-105b^3Bx}{24c^4\left(b+cx^2\right)^2} + \frac{5\sqrt{b}(7bB-3Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^12*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (-105*b^3*B*x + b*c^2*x^3*(75*A - 56*B*x^2) + 5*b^2*c*x*(9*A - 35*B*x^2) +
8*c^3*x^5*(3*A + B*x^2))/(24*c^4*(b + c*x^2)^2) + (5*Sqrt[b]*(7*b*B - 3*A*c
)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*c^(9/2))
```

Maple [A] time = 0.01, size = 147, normalized size = 1.3

$$\frac{Bx^3}{3\,c^3} + \frac{Ax}{c^3} - 3\,\frac{Bbx}{c^4} + \frac{9\,Abx^3}{8\,c^2\left(cx^2+b\right)^2} - \frac{13\,Bb^2x^3}{8\,c^3\left(cx^2+b\right)^2} + \frac{7\,Ab^2x}{8\,c^3\left(cx^2+b\right)^2} - \frac{11\,Bb^3x}{8\,c^4\left(cx^2+b\right)^2} - \frac{15\,Ab}{8\,c^3}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(B*x^2+A)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 1/3*B*x^3/c^3+1/c^3*A*x-3/c^4*B*b*x+9/8*b/c^2/(c*x^2+b)^2*A*x^3-13/8*b^2/c^
3/(c*x^2+b)^2*B*x^3+7/8*b^2/c^3/(c*x^2+b)^2*A*x-11/8*b^3/c^4/(c*x^2+b)^2*B*
x-15/8*b/c^3/(b*c)^(1/2)*arctan(x*c/(b*c)^(1/2))*A+35/8*b^2/c^4/(b*c)^(1/2)
*arctan(x*c/(b*c)^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.17634, size = 763, normalized size = 6.47

$$\left[\frac{16\,Bc^3x^7 - 16\left(7\,Bbc^2 - 3\,Ac^3\right)x^5 - 50\left(7\,Bb^2c - 3\,Abc^2\right)x^3 - 15\left(\left(7\,Bbc^2 - 3\,Ac^3\right)x^4 + 7\,Bb^3 - 3\,Ab^2c + 2\left(7\,Bb^2c - 3\,A\right.\right.}{48\left(c^6x^4 + 2\,bc^5x^2 + b^2c^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(16*B*c^3*x^7 - 16*(7*B*b*c^2 - 3*A*c^3)*x^5 - 50*(7*B*b^2*c - 3*A*b*
c^2)*x^3 - 15*((7*B*b*c^2 - 3*A*c^3)*x^4 + 7*B*b^3 - 3*A*b^2*c + 2*(7*B*b^2
*c - 3*A*b*c^2)*x^2)*sqrt(-b/c)*log((c*x^2 - 2*c*x*sqrt(-b/c) - b)/(c*x^2 +
 b)) - 30*(7*B*b^3 - 3*A*b^2*c)*x)/(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4), 1/24*
(8*B*c^3*x^7 - 8*(7*B*b*c^2 - 3*A*c^3)*x^5 - 25*(7*B*b^2*c - 3*A*b*c^2)*x^3
 + 15*((7*B*b*c^2 - 3*A*c^3)*x^4 + 7*B*b^3 - 3*A*b^2*c + 2*(7*B*b^2*c - 3*A
*b*c^2)*x^2)*sqrt(b/c)*arctan(c*x*sqrt(b/c)/b) - 15*(7*B*b^3 - 3*A*b^2*c)*x
)/(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4)]
```

Sympy [A] time = 1.5052, size = 212, normalized size = 1.8

$$\frac{Bx^3}{3c^3} - \frac{5\sqrt{-\frac{b}{c^9}}\,(-3Ac+7Bb)\log\left(-\frac{5c^4\sqrt{-\frac{b}{c^9}}(-3Ac+7Bb)}{-15Ac+35Bb}+x\right)}{16} + \frac{5\sqrt{-\frac{b}{c^9}}\,(-3Ac+7Bb)\log\left(\frac{5c^4\sqrt{-\frac{b}{c^9}}(-3Ac+7Bb)}{-15Ac+35Bb}+x\right)}{16} - \frac{x^3\left(-9A\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] B*x**3/(3*c**3) - 5*sqrt(-b/c**9)*(-3*A*c + 7*B*b)*log(-5*c**4*sqrt(-b/c**9
)*(-3*A*c + 7*B*b)/(-15*A*c + 35*B*b) + x)/16 + 5*sqrt(-b/c**9)*(-3*A*c + 7
*B*b)*log(5*c**4*sqrt(-b/c**9)*(-3*A*c + 7*B*b)/(-15*A*c + 35*B*b) + x)/16
- (x**3*(-9*A*b*c**2 + 13*B*b**2*c) + x*(-7*A*b**2*c + 11*B*b**3))/(8*b**2*
c**4 + 16*b*c**5*x**2 + 8*c**6*x**4) - x*(-A*c + 3*B*b)/c**4
```

Giac [A] time = 1.18259, size = 150, normalized size = 1.27

$$\frac{5\left(7\,Bb^2 - 3\,Abc\right)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}c^4} - \frac{13\,Bb^2cx^3 - 9\,Abc^2x^3 + 11\,Bb^3x - 7\,Ab^2cx}{8\left(cx^2+b\right)^2c^4} + \frac{Bc^6x^3 - 9\,Bbc^5x + 3\,Ac^6x}{3\,c^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 5/8*(7*B*b^2 - 3*A*b*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^4) - 1/8*(13*B*b
^2*c*x^3 - 9*A*b*c^2*x^3 + 11*B*b^3*x - 7*A*b^2*c*x)/((c*x^2 + b)^2*c^4) +
1/3*(B*c^6*x^3 - 9*B*b*c^5*x + 3*A*c^6*x)/c^9
```

3.76 $\int \frac{x^{11}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=89

$$\frac{b^2(bB-Ac)}{4c^4\left(b+cx^2\right)^2}-\frac{b(3bB-2Ac)}{2c^4\left(b+cx^2\right)}-\frac{(3bB-Ac)\log\left(b+cx^2\right)}{2c^4}+\frac{Bx^2}{2c^3}$$

```
[Out] (B*x^2)/(2*c^3) + (b^2*(b*B - A*c))/(4*c^4*(b + c*x^2)^2) - (b*(3*b*B - 2*A
*c))/(2*c^4*(b + c*x^2)) - ((3*b*B - A*c)*Log[b + c*x^2])/(2*c^4)
```

Rubi [A] time = 0.101731, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 446, 77}

$$\frac{b^2(bB-Ac)}{4c^4\left(b+cx^2\right)^2}-\frac{b(3bB-2Ac)}{2c^4\left(b+cx^2\right)}-\frac{(3bB-Ac)\log\left(b+cx^2\right)}{2c^4}+\frac{Bx^2}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (B*x^2)/(2*c^3) + (b^2*(b*B - A*c))/(4*c^4*(b + c*x^2)^2) - (b*(3*b*B - 2*A
*c))/(2*c^4*(b + c*x^2)) - ((3*b*B - A*c)*Log[b + c*x^2])/(2*c^4)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{11}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx &= \int \frac{x^5\left(A+Bx^2\right)}{\left(b+cx^2\right)^3}\,dx\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2(A+Bx)}{(b+cx)^3}\,dx,x,x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{B}{c^3}-\frac{b^2(bB-Ac)}{c^3(b+cx)^3}+\frac{b(3bB-2Ac)}{c^3(b+cx)^2}+\frac{-3bB+Ac}{c^3(b+cx)}\right)dx,x,x^2\right)\\
&= \frac{Bx^2}{2c^3}+\frac{b^2(bB-Ac)}{4c^4\left(b+cx^2\right)^2}-\frac{b(3bB-2Ac)}{2c^4\left(b+cx^2\right)}-\frac{(3bB-Ac)\log\left(b+cx^2\right)}{2c^4}
\end{aligned}$$

Mathematica [A] time = 0.0369192, size = 92, normalized size = 1.03

$$\frac{2Abc-3b^2B}{2c^4\left(b+cx^2\right)}+\frac{b^3B-Ab^2c}{4c^4\left(b+cx^2\right)^2}+\frac{(Ac-3bB)\log\left(b+cx^2\right)}{2c^4}+\frac{Bx^2}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (B*x^2)/(2*c^3) + (b^3*B - A*b^2*c)/(4*c^4*(b + c*x^2)^2) + (-3*b^2*B + 2*A
*b*c)/(2*c^4*(b + c*x^2)) + ((-3*b*B + A*c)*Log[b + c*x^2])/(2*c^4)
```

Maple [A] time = 0.009, size = 109, normalized size = 1.2

$$\frac{Bx^2}{2\,c^3}+\frac{\ln\left(cx^2+b\right)A}{2\,c^3}-\frac{3\,\ln\left(cx^2+b\right)Bb}{2\,c^4}+\frac{Ab}{c^3\left(cx^2+b\right)}-\frac{3\,Bb^2}{2\,c^4\left(cx^2+b\right)}-\frac{b^2A}{4\,c^3\left(cx^2+b\right)^2}+\frac{Bb^3}{4\,c^4\left(cx^2+b\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(B*x^2+A)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 1/2*B*x^2/c^3+1/2/c^3*ln(c*x^2+b)*A-3/2/c^4*ln(c*x^2+b)*B*b+1/c^3*b/(c*x^2+
b)*A-3/2/c^4*b^2/(c*x^2+b)*B-1/4/c^3*b^2/(c*x^2+b)^2*A+1/4/c^4*b^3/(c*x^2+b
)^2*B
```

Maxima [A] time = 1.14462, size = 127, normalized size = 1.43

$$-\frac{5\,Bb^3-3\,Ab^2c+2\left(3\,Bb^2c-2\,Abc^2\right)x^2}{4\left(c^6x^4+2\,bc^5x^2+b^2c^4\right)}+\frac{Bx^2}{2\,c^3}-\frac{(3\,Bb-Ac)\log\left(cx^2+b\right)}{2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(5*B*b^3 - 3*A*b^2*c + 2*(3*B*b^2*c - 2*A*b*c^2)*x^2)/(c^6*x^4 + 2*b*c
^5*x^2 + b^2*c^4) + 1/2*B*x^2/c^3 - 1/2*(3*B*b - A*c)*log(c*x^2 + b)/c^4
```

Fricas [A] time = 1.04805, size = 289, normalized size = 3.25

$$\frac{2\,Bc^3x^6+4\,Bbc^2x^4-5\,Bb^3+3\,Ab^2c-4\left(Bb^2c-Abc^2\right)x^2-2\left(\left(3\,Bbc^2-Ac^3\right)x^4+3\,Bb^3-Ab^2c+2\left(3\,Bb^2c-Abc^2\right)x^2\right)}{4\left(c^6x^4+2\,bc^5x^2+b^2c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*B*c^3*x^6 + 4*B*b*c^2*x^4 - 5*B*b^3 + 3*A*b^2*c - 4*(B*b^2*c - A*b*c
^2)*x^2 - 2*((3*B*b*c^2 - A*c^3)*x^4 + 3*B*b^3 - A*b^2*c + 2*(3*B*b^2*c - A
*b*c^2)*x^2)*log(c*x^2 + b))/(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4)
```

Sympy [A] time = 1.46166, size = 94, normalized size = 1.06

$$\frac{Bx^2}{2c^3}-\frac{-3Ab^2c+5Bb^3+x^2\left(-4Abc^2+6Bb^2c\right)}{4b^2c^4+8bc^5x^2+4c^6x^4}-\frac{(-Ac+3Bb)\log\left(b+cx^2\right)}{2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] B*x**2/(2*c**3) - (-3*A*b**2*c + 5*B*b**3 + x**2*(-4*A*b*c**2 + 6*B*b**2*c)
)/(4*b**2*c**4 + 8*b*c**5*x**2 + 4*c**6*x**4) - (-A*c + 3*B*b)*log(b + c*x*
*2)/(2*c**4)
```

Giac [A] time = 1.21136, size = 126, normalized size = 1.42

$$\frac{Bx^2}{2\,c^3} - \frac{(3\,Bb - Ac)\log\left(\left|cx^2 + b\right|\right)}{2\,c^4} + \frac{9\,Bbc^2x^4 - 3\,Ac^3x^4 + 12\,Bb^2cx^2 - 2\,Abc^2x^2 + 4\,Bb^3}{4\left(cx^2 + b\right)^2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*B*x^2/c^3 - 1/2*(3*B*b - A*c)*log(abs(c*x^2 + b))/c^4 + 1/4*(9*B*b*c^2*
x^4 - 3*A*c^3*x^4 + 12*B*b^2*c*x^2 - 2*A*b*c^2*x^2 + 4*B*b^3)/((c*x^2 + b)^
2*c^4)
```

3.77 $\int \frac{x^{10}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=95

$$\frac{x(9bB-5Ac)}{8c^3\left(b+cx^2\right)}-\frac{bx(bB-Ac)}{4c^3\left(b+cx^2\right)^2}-\frac{3(5bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8\sqrt{b}c^{7/2}}+\frac{Bx}{c^3}$$

```
[Out] (B*x)/c^3 - (b*(b*B - A*c)*x)/(4*c^3*(b + c*x^2)^2) + ((9*b*B - 5*A*c)*x)/(
8*c^3*(b + c*x^2)) - (3*(5*b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*Sqrt[
b]*c^(7/2))
```

Rubi [A] time = 0.0991359, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1584, 455, 1157, 388, 205}

$$\frac{x(9bB-5Ac)}{8c^3\left(b+cx^2\right)}-\frac{bx(bB-Ac)}{4c^3\left(b+cx^2\right)^2}-\frac{3(5bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8\sqrt{b}c^{7/2}}+\frac{Bx}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^10*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (B*x)/c^3 - (b*(b*B - A*c)*x)/(4*c^3*(b + c*x^2)^2) + ((9*b*B - 5*A*c)*x)/(
8*c^3*(b + c*x^2)) - (3*(5*b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*Sqrt[
b]*c^(7/2))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
```

```
1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^{10}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{x^4\left(A+Bx^2\right)}{\left(b+cx^2\right)^3}\,dx$$

$$= -\frac{b(bB-Ac)x}{4c^3\left(b+cx^2\right)^2} - \frac{\int \frac{-b(bB-Ac)+4c(bB-Ac)x^2-4Bc^2x^4}{\left(b+cx^2\right)^2}\,dx}{4c^3}$$

$$= -\frac{b(bB-Ac)x}{4c^3\left(b+cx^2\right)^2} + \frac{(9bB-5Ac)x}{8c^3\left(b+cx^2\right)} + \frac{\int \frac{-b(7bB-3Ac)+8bBcx^2}{b+cx^2}\,dx}{8bc^3}$$

$$= \frac{Bx}{c^3} - \frac{b(bB-Ac)x}{4c^3\left(b+cx^2\right)^2} + \frac{(9bB-5Ac)x}{8c^3\left(b+cx^2\right)} - \frac{(3(5bB-Ac))\int \frac{1}{b+cx^2}\,dx}{8c^3}$$

$$= \frac{Bx}{c^3} - \frac{b(bB-Ac)x}{4c^3\left(b+cx^2\right)^2} + \frac{(9bB-5Ac)x}{8c^3\left(b+cx^2\right)} - \frac{3(5bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8\sqrt{b}c^{7/2}}$$

Mathematica [A] time = 0.0707353, size = 92, normalized size = 0.97

$$\frac{x\left(b\left(25Bcx^2-3Ac\right)+c^2x^2\left(8Bx^2-5A\right)+15b^2B\right)}{8c^3\left(b+cx^2\right)^2} - \frac{3(5bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8\sqrt{b}c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^10*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (x*(15*b^2*B + c^2*x^2*(-5*A + 8*B*x^2) + b*(-3*A*c + 25*B*c*x^2)))/(8*c^3*
(b + c*x^2)^2) - (3*(5*b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*Sqrt[b]*c
^(7/2))
```

Maple [A] time = 0.009, size = 122, normalized size = 1.3

$$\frac{Bx}{c^3} - \frac{5\,Ax^3}{8\,c\left(cx^2+b\right)^2} + \frac{9\,Bx^3b}{8\,c^2\left(cx^2+b\right)^2} - \frac{3\,Abx}{8\,c^2\left(cx^2+b\right)^2} + \frac{7\,Bb^2x}{8\,c^3\left(cx^2+b\right)^2} + \frac{3\,A}{8\,c^2}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}} - \frac{15\,Bb}{8\,c^3}\arctan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(B*x^2+A)/(c*x^4+b*x^2)^3,x)
```

```
[Out] B*x/c^3-5/8/c/(c*x^2+b)^2*A*x^3+9/8/c^2/(c*x^2+b)^2*B*x^3*b-3/8/c^2/(c*x^2+
b)^2*A*b*x+7/8/c^3/(c*x^2+b)^2*B*b^2*x+3/8/c^2/(b*c)^(1/2)*arctan(x*c/(b*c)
^(1/2))*A-15/8/c^3/(b*c)^(1/2)*arctan(x*c/(b*c)^(1/2))*B*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.09504, size = 672, normalized size = 7.07

$$\left[\frac{16\,Bbc^3x^5 + 10\left(5\,Bb^2c^2 - Abc^3\right)x^3 + 3\left(\left(5\,Bbc^2 - Ac^3\right)x^4 + 5\,Bb^3 - Ab^2c + 2\left(5\,Bb^2c - Abc^2\right)x^2\right)\sqrt{-bc}\log\left(\frac{cx^2-2\sqrt{-bc}x-b}{cx^2+b}\right.}{16\left(bc^6x^4 + 2\,b^2c^5x^2 + b^3c^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*B*b*c^3*x^5 + 10*(5*B*b^2*c^2 - A*b*c^3)*x^3 + 3*((5*B*b*c^2 - A*
c^3)*x^4 + 5*B*b^3 - A*b^2*c + 2*(5*B*b^2*c - A*b*c^2)*x^2)*sqrt(-b*c)*log(
(c*x^2 - 2*sqrt(-b*c)*x - b)/(c*x^2 + b)) + 6*(5*B*b^3*c - A*b^2*c^2)*x)/(b
*c^6*x^4 + 2*b^2*c^5*x^2 + b^3*c^4), 1/8*(8*B*b*c^3*x^5 + 5*(5*B*b^2*c^2 -
A*b*c^3)*x^3 - 3*((5*B*b*c^2 - A*c^3)*x^4 + 5*B*b^3 - A*b^2*c + 2*(5*B*b^2*
c - A*b*c^2)*x^2)*sqrt(b*c)*arctan(sqrt(b*c)*x/b) + 3*(5*B*b^3*c - A*b^2*c^
2)*x)/(b*c^6*x^4 + 2*b^2*c^5*x^2 + b^3*c^4)]
```

Sympy [B] time = 1.23636, size = 194, normalized size = 2.04

$$\frac{Bx}{c^3} + \frac{3\sqrt{-\frac{1}{bc^7}}\,(-Ac+5Bb)\log\left(-\frac{3bc^3\sqrt{-\frac{1}{bc^7}}(-Ac+5Bb)}{-3Ac+15Bb}+x\right)}{16} - \frac{3\sqrt{-\frac{1}{bc^7}}\,(-Ac+5Bb)\log\left(\frac{3bc^3\sqrt{-\frac{1}{bc^7}}(-Ac+5Bb)}{-3Ac+15Bb}+x\right)}{16} + \frac{x^3(-5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] B*x/c**3 + 3*sqrt(-1/(b*c**7))*(-A*c + 5*B*b)*log(-3*b*c**3*sqrt(-1/(b*c**7
))*(-A*c + 5*B*b)/(-3*A*c + 15*B*b) + x)/16 - 3*sqrt(-1/(b*c**7))*(-A*c + 5
*B*b)*log(3*b*c**3*sqrt(-1/(b*c**7))*(-A*c + 5*B*b)/(-3*A*c + 15*B*b) + x)/
16 + (x**3*(-5*A*c**2 + 9*B*b*c) + x*(-3*A*b*c + 7*B*b**2))/(8*b**2*c**3 +
16*b*c**4*x**2 + 8*c**5*x**4)
```

Giac [A] time = 1.1146, size = 108, normalized size = 1.14

$$\frac{Bx}{c^3} - \frac{3\,(5\,Bb - Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}c^3} + \frac{9\,Bbcx^3 - 5\,Ac^2x^3 + 7\,Bb^2x - 3\,Abcx}{8\left(cx^2+b\right)^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] B*x/c^3 - 3/8*(5*B*b - A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^3) + 1/8*(9*
B*b*c*x^3 - 5*A*c^2*x^3 + 7*B*b^2*x - 3*A*b*c*x)/((c*x^2 + b)^2*c^3)
```

3.78 $\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=67

$$-\frac{b(bB-Ac)}{4c^3\left(b+cx^2\right)^2}+\frac{2bB-Ac}{2c^3\left(b+cx^2\right)}+\frac{B\log\left(b+cx^2\right)}{2c^3}$$

```
[Out] -(b*(b*B - A*c))/(4*c^3*(b + c*x^2)^2) + (2*b*B - A*c)/(2*c^3*(b + c*x^2))
+ (B*Log[b + c*x^2])/(2*c^3)
```

Rubi [A] time = 0.0758642, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 446, 77}

$$-\frac{b(bB-Ac)}{4c^3\left(b+cx^2\right)^2}+\frac{2bB-Ac}{2c^3\left(b+cx^2\right)}+\frac{B\log\left(b+cx^2\right)}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^9*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -(b*(b*B - A*c))/(4*c^3*(b + c*x^2)^2) + (2*b*B - A*c)/(2*c^3*(b + c*x^2))
+ (B*Log[b + c*x^2])/(2*c^3)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^9 \left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx &= \int \frac{x^3 \left(A+Bx^2\right)}{\left(b+cx^2\right)^3}\,dx\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x(A+Bx)}{(b+cx)^3}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{b(bB-Ac)}{c^2(b+cx)^3}+\frac{-2bB+Ac}{c^2(b+cx)^2}+\frac{B}{c^2(b+cx)}\right)dx, x, x^2\right)\\
&= -\frac{b(bB-Ac)}{4c^3\left(b+cx^2\right)^2}+\frac{2bB-Ac}{2c^3\left(b+cx^2\right)}+\frac{B\log\left(b+cx^2\right)}{2c^3}
\end{aligned}
$$

Mathematica [A] time = 0.0237069, size = 64, normalized size = 0.96

$$
\frac{-bc\left(A-4Bx^2\right)-2Ac^2x^2+3b^2B+2B\left(b+cx^2\right)^2\log\left(b+cx^2\right)}{4c^3\left(b+cx^2\right)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^9*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (3*b^2*B - 2*A*c^2*x^2 - b*c*(A - 4*B*x^2) + 2*B*(b + c*x^2)^2*Log[b + c*x^
2])/(4*c^3*(b + c*x^2)^2)
```

Maple [A] time = 0.007, size = 80, normalized size = 1.2

$$
\frac{B\ln\left(cx^2+b\right)}{2\,c^3}-\frac{A}{2\,c^2\left(cx^2+b\right)}+\frac{Bb}{c^3\left(cx^2+b\right)}+\frac{Ab}{4\,c^2\left(cx^2+b\right)^2}-\frac{Bb^2}{4\,c^3\left(cx^2+b\right)^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(B*x^2+A)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 1/2*B*ln(c*x^2+b)/c^3-1/2/c^2/(c*x^2+b)*A+1/c^3/(c*x^2+b)*B*b+1/4*b/c^2/(c*
x^2+b)^2*A-1/4*b^2/c^3/(c*x^2+b)^2*B
```

Maxima [A] time = 1.19388, size = 97, normalized size = 1.45

$$\frac{3\,Bb^2 - Abc + 2\left(2\,Bbc - Ac^2\right)x^2}{4\left(c^5x^4 + 2\,bc^4x^2 + b^2c^3\right)} + \frac{B\log\left(cx^2 + b\right)}{2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(3*B*b^2 - A*b*c + 2*(2*B*b*c - A*c^2)*x^2)/(c^5*x^4 + 2*b*c^4*x^2 + b^
2*c^3) + 1/2*B*log(c*x^2 + b)/c^3
```

Fricas [A] time = 1.31711, size = 184, normalized size = 2.75

$$\frac{3\,Bb^2 - Abc + 2\left(2\,Bbc - Ac^2\right)x^2 + 2\left(Bc^2x^4 + 2\,Bbcx^2 + Bb^2\right)\log\left(cx^2 + b\right)}{4\left(c^5x^4 + 2\,bc^4x^2 + b^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(3*B*b^2 - A*b*c + 2*(2*B*b*c - A*c^2)*x^2 + 2*(B*c^2*x^4 + 2*B*b*c*x^2
 + B*b^2)*log(c*x^2 + b))/(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3)
```

Sympy [A] time = 1.07545, size = 70, normalized size = 1.04

$$\frac{B\log\left(b + cx^2\right)}{2c^3} + \frac{-Abc + 3Bb^2 + x^2\left(-2Ac^2 + 4Bbc\right)}{4b^2c^3 + 8bc^4x^2 + 4c^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] B*log(b + c*x**2)/(2*c**3) + (-A*b*c + 3*B*b**2 + x**2*(-2*A*c**2 + 4*B*b*c
))/(4*b**2*c**3 + 8*b*c**4*x**2 + 4*c**5*x**4)
```

Giac [A] time = 1.15932, size = 74, normalized size = 1.1

$$\frac{B\log\left(\left|cx^2+b\right|\right)}{2\,c^3}-\frac{3\,Bcx^4+2\,Bbx^2+2\,Acx^2+Ab}{4\left(cx^2+b\right)^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*B*log(abs(c*x^2 + b))/c^3 - 1/4*(3*B*c*x^4 + 2*B*b*x^2 + 2*A*c*x^2 + A*
b)/((c*x^2 + b)^2*c^2)
```

3.79 $\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=90

$$\frac{(Ac+3bB)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{3/2}c^{5/2}}-\frac{x(5bB-Ac)}{8bc^2\left(b+cx^2\right)}+\frac{x(bB-Ac)}{4c^2\left(b+cx^2\right)^2}$$

```
[Out] ((b*B - A*c)*x)/(4*c^2*(b + c*x^2)^2) - ((5*b*B - A*c)*x)/(8*b*c^2*(b + c*x
^2)) + ((3*b*B + A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(3/2)*c^(5/2))
```

Rubi [A] time = 0.0757985, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1584, 455, 385, 205}

$$\frac{(Ac+3bB)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{3/2}c^{5/2}}-\frac{x(5bB-Ac)}{8bc^2\left(b+cx^2\right)}+\frac{x(bB-Ac)}{4c^2\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((b*B - A*c)*x)/(4*c^2*(b + c*x^2)^2) - ((5*b*B - A*c)*x)/(8*b*c^2*(b + c*x
^2)) + ((3*b*B + A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(3/2)*c^(5/2))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
```

```
(IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^8\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx &= \int \frac{x^2\left(A+Bx^2\right)}{\left(b+cx^2\right)^3}\,dx \\
&= \frac{(bB-Ac)x}{4c^2\left(b+cx^2\right)^2}-\frac{\int \frac{bB-Ac-4Bcx^2}{\left(b+cx^2\right)^2}\,dx}{4c^2} \\
&= \frac{(bB-Ac)x}{4c^2\left(b+cx^2\right)^2}-\frac{(5bB-Ac)x}{8bc^2\left(b+cx^2\right)}+\frac{(3bB+Ac)\int \frac{1}{b+cx^2}\,dx}{8bc^2} \\
&= \frac{(bB-Ac)x}{4c^2\left(b+cx^2\right)^2}-\frac{(5bB-Ac)x}{8bc^2\left(b+cx^2\right)}+\frac{(3bB+Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{3/2}c^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0833735, size = 83, normalized size = 0.92

$$
\frac{\frac{\sqrt{c}x\left(-bc\left(A+5Bx^2\right)+Ac^2x^2-3b^2B\right)}{b\left(b+cx^2\right)^2}+\frac{(Ac+3bB)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{3/2}}}{8c^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((Sqrt[c]*x*(-3*b^2*B + A*c^2*x^2 - b*c*(A + 5*B*x^2)))/(b*(b + c*x^2)^2) +
 ((3*b*B + A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/b^(3/2))/(8*c^(5/2))
```

Maple [A] time = 0.008, size = 89, normalized size = 1.

$$\frac{1}{\left(cx^2+b\right)^2}\left(\frac{(Ac-5\,Bb)\,x^3}{8\,bc}-\frac{(Ac+3\,Bb)\,x}{8\,c^2}\right)+\frac{A}{8\,bc}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}+\frac{3\,B}{8\,c^2}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(B*x^2+A)/(c*x^4+b*x^2)^3,x)
```

```
[Out] (1/8*(A*c-5*B*b)/b/c*x^3-1/8*(A*c+3*B*b)/c^2*x)/(c*x^2+b)^2+1/8/c/b/(b*c)^(
1/2)*arctan(x*c/(b*c)^(1/2))*A+3/8/c^2/(b*c)^(1/2)*arctan(x*c/(b*c)^(1/2))*
B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.09793, size = 624, normalized size = 6.93

$$\left[-\frac{2\left(5\,Bb^2c^2-Abc^3\right)x^3+\left(\left(3\,Bbc^2+Ac^3\right)x^4+3\,Bb^3+Ab^2c+2\left(3\,Bb^2c+Abc^2\right)x^2\right)\sqrt{-bc}\log\left(\frac{cx^2-2\,\sqrt{-bc}x-b}{cx^2+b}\right)+2\left(3\,Bb^3c\right.}{16\left(b^2c^5x^4+2\,b^3c^4x^2+b^4c^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(2*(5*B*b^2*c^2 - A*b*c^3)*x^3 + ((3*B*b*c^2 + A*c^3)*x^4 + 3*B*b^3
+ A*b^2*c + 2*(3*B*b^2*c + A*b*c^2)*x^2)*sqrt(-b*c)*log((c*x^2 - 2*sqrt(-b*
c)*x - b)/(c*x^2 + b)) + 2*(3*B*b^3*c + A*b^2*c^2)*x)/(b^2*c^5*x^4 + 2*b^3*
c^4*x^2 + b^4*c^3), -1/8*((5*B*b^2*c^2 - A*b*c^3)*x^3 - ((3*B*b*c^2 + A*c^3
)*x^4 + 3*B*b^3 + A*b^2*c + 2*(3*B*b^2*c + A*b*c^2)*x^2)*sqrt(b*c)*arctan(s
qrt(b*c)*x/b) + (3*B*b^3*c + A*b^2*c^2)*x)/(b^2*c^5*x^4 + 2*b^3*c^4*x^2 + b
^4*c^3)]
```

Sympy [A] time = 0.895191, size = 153, normalized size = 1.7

$$-\frac{\sqrt{-\frac{1}{b^3c^5}}\,(Ac+3Bb)\log\left(-b^2c^2\sqrt{-\frac{1}{b^3c^5}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{b^3c^5}}\,(Ac+3Bb)\log\left(b^2c^2\sqrt{-\frac{1}{b^3c^5}}+x\right)}{16}-\frac{x^3\left(-Ac^2+5Bbc\right)+x\left(Abc\right.}{8b^3c^2+16b^2c^3x^2+8bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] -sqrt(-1/(b**3*c**5))*(A*c + 3*B*b)*log(-b**2*c**2*sqrt(-1/(b**3*c**5)) + x
)/16 + sqrt(-1/(b**3*c**5))*(A*c + 3*B*b)*log(b**2*c**2*sqrt(-1/(b**3*c**5)
) + x)/16 - (x**3*(-A*c**2 + 5*B*b*c) + x*(A*b*c + 3*B*b**2))/(8*b**3*c**2
+ 16*b**2*c**3*x**2 + 8*b*c**4*x**4)
```

Giac [A] time = 1.16175, size = 105, normalized size = 1.17

$$\frac{(3\,Bb+Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}bc^2}-\frac{5\,Bbcx^3-Ac^2x^3+3\,Bb^2x+Abcx}{8\left(cx^2+b\right)^2bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*B*b + A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b*c^2) - 1/8*(5*B*b*c*x^
3 - A*c^2*x^3 + 3*B*b^2*x + A*b*c*x)/((c*x^2 + b)^2*b*c^2)
```

3.80 $\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^3} dx$

Optimal. Leaf size=32

$$\frac{(A+Bx^2)^2}{4(b+cx^2)^2(bB-Ac)}$$

```
[Out] (A + B*x^2)^2/(4*(b*B - A*c)*(b + c*x^2)^2)
```

Rubi [A] time = 0.0307202, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 444, 37}

$$\frac{(A+Bx^2)^2}{4(b+cx^2)^2(bB-Ac)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (A + B*x^2)^2/(4*(b*B - A*c)*(b + c*x^2)^2)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
```

```
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx &= \int \frac{x\left(A+Bx^2\right)}{\left(b+cx^2\right)^3}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{(b+cx)^3}\,dx, x, x^2\right)\\ &= \frac{\left(A+Bx^2\right)^2}{4(bB-Ac)\left(b+cx^2\right)^2}\end{aligned}$$

Mathematica [A] time = 0.0130782, size = 30, normalized size = 0.94

$$-\frac{c\left(A+2Bx^2\right)+bB}{4c^2\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -(b*B + c*(A + 2*B*x^2))/(4*c^2*(b + c*x^2)^2)
```

Maple [A] time = 0.007, size = 39, normalized size = 1.2

$$-\frac{B}{2\,c^2\left(cx^2+b\right)}-\frac{Ac-Bb}{4\,c^2\left(cx^2+b\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(B*x^2+A)/(c*x^4+b*x^2)^3,x)
```

```
[Out] -1/2*B/c^2/(c*x^2+b)-1/4*(A*c-B*b)/c^2/(c*x^2+b)^2
```

Maxima [A] time = 1.14028, size = 57, normalized size = 1.78

$$-\frac{2\,Bcx^2 + Bb + Ac}{4\left(c^4x^4 + 2\,bc^3x^2 + b^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*B*c*x^2 + B*b + A*c)/(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)
```

Fricas [A] time = 1.01252, size = 86, normalized size = 2.69

$$-\frac{2\,Bcx^2 + Bb + Ac}{4\left(c^4x^4 + 2\,bc^3x^2 + b^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*B*c*x^2 + B*b + A*c)/(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)
```

Sympy [A] time = 0.672229, size = 42, normalized size = 1.31

$$-\frac{Ac + Bb + 2Bcx^2}{4b^2c^2 + 8bc^3x^2 + 4c^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] -(A*c + B*b + 2*B*c*x**2)/(4*b**2*c**2 + 8*b*c**3*x**2 + 4*c**4*x**4)
```

Giac [A] time = 1.14302, size = 38, normalized size = 1.19

$$-\frac{2\,Bcx^2 + Bb + Ac}{4\left(cx^2 + b\right)^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*B*c*x^2 + B*b + A*c)/((c*x^2 + b)^2*c^2)
```

3.81 $\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=92

$$\frac{(3Ac+bB)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{5/2}c^{3/2}}+\frac{x(3Ac+bB)}{8b^2c\left(b+cx^2\right)}-\frac{x(bB-Ac)}{4bc\left(b+cx^2\right)^2}$$

```
[Out] -((b*B - A*c)*x)/(4*b*c*(b + c*x^2)^2) + ((b*B + 3*A*c)*x)/(8*b^2*c*(b + c*
x^2)) + ((b*B + 3*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(5/2)*c^(3/2))
```

Rubi [A] time = 0.0452505, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1584, 385, 199, 205}

$$\frac{(3Ac+bB)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{5/2}c^{3/2}}+\frac{x(3Ac+bB)}{8b^2c\left(b+cx^2\right)}-\frac{x(bB-Ac)}{4bc\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -((b*B - A*c)*x)/(4*b*c*(b + c*x^2)^2) + ((b*B + 3*A*c)*x)/(8*b^2*c*(b + c*
x^2)) + ((b*B + 3*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(5/2)*c^(3/2))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^6\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx &= \int \frac{A+Bx^2}{\left(b+cx^2\right)^3}\,dx \\
&= -\frac{(bB-Ac)x}{4bc\left(b+cx^2\right)^2}+\frac{(bB+3Ac)\int \frac{1}{\left(b+cx^2\right)^2}\,dx}{4bc} \\
&= -\frac{(bB-Ac)x}{4bc\left(b+cx^2\right)^2}+\frac{(bB+3Ac)x}{8b^2c\left(b+cx^2\right)}+\frac{(bB+3Ac)\int \frac{1}{b+cx^2}\,dx}{8b^2c} \\
&= -\frac{(bB-Ac)x}{4bc\left(b+cx^2\right)^2}+\frac{(bB+3Ac)x}{8b^2c\left(b+cx^2\right)}+\frac{(bB+3Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{5/2}c^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0588319, size = 84, normalized size = 0.91

$$
\frac{x\left(bc\left(5A+Bx^2\right)+3Ac^2x^2+b^2(-B)\right)}{8b^2c\left(b+cx^2\right)^2}+\frac{(3Ac+bB)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{5/2}c^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (x*(-(b^2*B) + 3*A*c^2*x^2 + b*c*(5*A + B*x^2)))/(8*b^2*c*(b + c*x^2)^2) +
((b*B + 3*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(5/2)*c^(3/2))
```

Maple [A] time = 0.007, size = 90, normalized size = 1.

$$\frac{1}{\left(cx^2+b\right)^2}\left(\frac{(3Ac+Bb)x^3}{8b^2}+\frac{(5Ac-Bb)x}{8bc}\right)+\frac{3A}{8b^2}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}+\frac{B}{8bc}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(B*x^2+A)/(c*x^4+b*x^2)^3,x)
```

```
[Out] (1/8*(3*A*c+B*b)/b^2*x^3+1/8*(5*A*c-B*b)/b/c*x)/(c*x^2+b)^2+3/8/b^2/(b*c)^(
1/2)*arctan(x*c/(b*c)^(1/2))*A+1/8/b/c/(b*c)^(1/2)*arctan(x*c/(b*c)^(1/2))*
B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.11134, size = 621, normalized size = 6.75

$$\left[\frac{2\left(Bb^2c^2+3Abc^3\right)x^3-\left(\left(Bbc^2+3Ac^3\right)x^4+Bb^3+3Ab^2c+2\left(Bb^2c+3Abc^2\right)x^2\right)\sqrt{-bc}\log\left(\frac{cx^2-2\sqrt{-bc}x-b}{cx^2+b}\right)-2\left(Bb^3c-5\right.}{16\left(b^3c^4x^4+2b^4c^3x^2+b^5c^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*(B*b^2*c^2 + 3*A*b*c^3)*x^3 - ((B*b*c^2 + 3*A*c^3)*x^4 + B*b^3 + 3
*A*b^2*c + 2*(B*b^2*c + 3*A*b*c^2)*x^2)*sqrt(-b*c)*log((c*x^2 - 2*sqrt(-b*c
)*x - b)/(c*x^2 + b)) - 2*(B*b^3*c - 5*A*b^2*c^2)*x)/(b^3*c^4*x^4 + 2*b^4*c
^3*x^2 + b^5*c^2), 1/8*((B*b^2*c^2 + 3*A*b*c^3)*x^3 + ((B*b*c^2 + 3*A*c^3)*
```

```
x^4 + B*b^3 + 3*A*b^2*c + 2*(B*b^2*c + 3*A*b*c^2)*x^2)*sqrt(b*c)*arctan(sqr
t(b*c)*x/b) - (B*b^3*c - 5*A*b^2*c^2)*x)/(b^3*c^4*x^4 + 2*b^4*c^3*x^2 + b^5
*c^2)]
```

Sympy [A] time = 0.728523, size = 150, normalized size = 1.63

$$-\frac{\sqrt{-\frac{1}{b^5c^3}}\,(3Ac+Bb)\log\left(-b^3c\sqrt{-\frac{1}{b^5c^3}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{b^5c^3}}\,(3Ac+Bb)\log\left(b^3c\sqrt{-\frac{1}{b^5c^3}}+x\right)}{16}+\frac{x^3\left(3Ac^2+Bbc\right)+x\left(5Abc-\right.}{8b^4c+16b^3c^2x^2+8b^2c^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] -sqrt(-1/(b**5*c**3))*(3*A*c + B*b)*log(-b**3*c*sqrt(-1/(b**5*c**3)) + x)/1
6 + sqrt(-1/(b**5*c**3))*(3*A*c + B*b)*log(b**3*c*sqrt(-1/(b**5*c**3)) + x)
/16 + (x**3*(3*A*c**2 + B*b*c) + x*(5*A*b*c - B*b**2))/(8*b**4*c + 16*b**3*
c**2*x**2 + 8*b**2*c**3*x**4)
```

Giac [A] time = 1.12735, size = 105, normalized size = 1.14

$$\frac{(Bb+3\,Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}b^2c}+\frac{Bbcx^3+3\,Ac^2x^3-Bb^2x+5\,Abcx}{8\left(cx^2+b\right)^2b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(B*b + 3*A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^2*c) + 1/8*(B*b*c*x^3
+ 3*A*c^2*x^3 - B*b^2*x + 5*A*b*c*x)/((c*x^2 + b)^2*b^2*c)
```

3.82 $\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=68

$$\frac{A}{2b^2(b+cx^2)}-\frac{A\log(b+cx^2)}{2b^3}+\frac{A\log(x)}{b^3}-\frac{bB-Ac}{4bc(b+cx^2)^2}$$

```
[Out] -(b*B - A*c)/(4*b*c*(b + c*x^2)^2) + A/(2*b^2*(b + c*x^2)) + (A*Log[x])/b^3
 - (A*Log[b + c*x^2])/(2*b^3)
```

Rubi [A] time = 0.0728484, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 446, 77}

$$\frac{A}{2b^2(b+cx^2)}-\frac{A\log(b+cx^2)}{2b^3}+\frac{A\log(x)}{b^3}-\frac{bB-Ac}{4bc(b+cx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -(b*B - A*c)/(4*b*c*(b + c*x^2)^2) + A/(2*b^2*(b + c*x^2)) + (A*Log[x])/b^3
 - (A*Log[b + c*x^2])/(2*b^3)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx &= \int \frac{A+Bx^2}{x\left(b+cx^2\right)^3}\,dx\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{x(b+cx)^3}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{A}{b^3x}+\frac{bB-Ac}{b(b+cx)^3}-\frac{Ac}{b^2(b+cx)^2}-\frac{Ac}{b^3(b+cx)}\right)dx, x, x^2\right)\\
&= -\frac{bB-Ac}{4bc\left(b+cx^2\right)^2}+\frac{A}{2b^2\left(b+cx^2\right)}+\frac{A\log(x)}{b^3}-\frac{A\log\left(b+cx^2\right)}{2b^3}
\end{aligned}
$$

Mathematica [A] time = 0.046391, size = 59, normalized size = 0.87

$$
\frac{\frac{b\left(3Abc+2Ac^2x^2+b^2(-B)\right)}{c\left(b+cx^2\right)^2}-2A\log\left(b+cx^2\right)+4A\log(x)}{4b^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((b*(-(b^2*B) + 3*A*b*c + 2*A*c^2*x^2))/(c*(b + c*x^2)^2) + 4*A*Log[x] - 2*
A*Log[b + c*x^2])/(4*b^3)
```

Maple [A] time = 0.011, size = 68, normalized size = 1.

$$
\frac{A\ln(x)}{b^3}-\frac{A\ln\left(cx^2+b\right)}{2\,b^3}+\frac{A}{2\,b^2\left(cx^2+b\right)}+\frac{A}{4\,b\left(cx^2+b\right)^2}-\frac{B}{4\,c\left(cx^2+b\right)^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)/(c*x^4+b*x^2)^3,x)
```

```
[Out] A*ln(x)/b^3-1/2*A*ln(c*x^2+b)/b^3+1/2*A/b^2/(c*x^2+b)+1/4/b/(c*x^2+b)^2*A-1
/4/c/(c*x^2+b)^2*B
```

Maxima [A] time = 1.18785, size = 104, normalized size = 1.53

$$\frac{2\,Ac^2x^2 - Bb^2 + 3\,Abc}{4\left(b^2c^3x^4 + 2\,b^3c^2x^2 + b^4c\right)} - \frac{A\log\left(cx^2 + b\right)}{2\,b^3} + \frac{A\log\left(x^2\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*A*c^2*x^2 - B*b^2 + 3*A*b*c)/(b^2*c^3*x^4 + 2*b^3*c^2*x^2 + b^4*c) -
 1/2*A*log(c*x^2 + b)/b^3 + 1/2*A*log(x^2)/b^3
```

Fricas [A] time = 1.00913, size = 250, normalized size = 3.68

$$\frac{2\,Abc^2x^2 - Bb^3 + 3\,Ab^2c - 2\left(Ac^3x^4 + 2\,Abc^2x^2 + Ab^2c\right)\log\left(cx^2 + b\right) + 4\left(Ac^3x^4 + 2\,Abc^2x^2 + Ab^2c\right)\log\left(x\right)}{4\left(b^3c^3x^4 + 2\,b^4c^2x^2 + b^5c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*A*b*c^2*x^2 - B*b^3 + 3*A*b^2*c - 2*(A*c^3*x^4 + 2*A*b*c^2*x^2 + A*b
^2*c)*log(c*x^2 + b) + 4*(A*c^3*x^4 + 2*A*b*c^2*x^2 + A*b^2*c)*log(x))/(b^3
*c^3*x^4 + 2*b^4*c^2*x^2 + b^5*c)
```

Sympy [A] time = 0.747113, size = 75, normalized size = 1.1

$$\frac{A\log\left(x\right)}{b^3} - \frac{A\log\left(\frac{b}{c} + x^2\right)}{2b^3} + \frac{3Abc + 2Ac^2x^2 - Bb^2}{4b^4c + 8b^3c^2x^2 + 4b^2c^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] A*log(x)/b**3 - A*log(b/c + x**2)/(2*b**3) + (3*A*b*c + 2*A*c**2*x**2 - B*b
**2)/(4*b**4*c + 8*b**3*c**2*x**2 + 4*b**2*c**3*x**4)
```

Giac [A] time = 1.15348, size = 103, normalized size = 1.51

$$\frac{A\log\left(x^2\right)}{2\,b^3} - \frac{A\log\left(\left|cx^2+b\right|\right)}{2\,b^3} + \frac{3\,Ac^3x^4 + 8\,Abc^2x^2 - Bb^3 + 6\,Ab^2c}{4\left(cx^2+b\right)^2 b^3 c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*A*log(x^2)/b^3 - 1/2*A*log(abs(c*x^2 + b))/b^3 + 1/4*(3*A*c^3*x^4 + 8*A
*b*c^2*x^2 - B*b^3 + 6*A*b^2*c)/((c*x^2 + b)^2*b^3*c)
```

3.83 $\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=96

$$\frac{x(3bB-7Ac)}{8b^3\left(b+cx^2\right)}+\frac{x(bB-Ac)}{4b^2\left(b+cx^2\right)^2}+\frac{3(bB-5Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{7/2}\sqrt{c}}-\frac{A}{b^3x}$$

```
[Out] -(A/(b^3*x)) + ((b*B - A*c)*x)/(4*b^2*(b + c*x^2)^2) + ((3*b*B - 7*A*c)*x)/
(8*b^3*(b + c*x^2)) + (3*(b*B - 5*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(7
/2)*Sqrt[c])
```

Rubi [A] time = 0.118209, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1584, 456, 453, 205}

$$\frac{x(3bB-7Ac)}{8b^3\left(b+cx^2\right)}+\frac{x(bB-Ac)}{4b^2\left(b+cx^2\right)^2}+\frac{3(bB-5Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{7/2}\sqrt{c}}-\frac{A}{b^3x}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -(A/(b^3*x)) + ((b*B - A*c)*x)/(4*b^2*(b + c*x^2)^2) + ((3*b*B - 7*A*c)*x)/
(8*b^3*(b + c*x^2)) + (3*(b*B - 5*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(7
/2)*Sqrt[c])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
```

```
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx &= \int \frac{A+Bx^2}{x^2\left(b+cx^2\right)^3}\,dx\\
&= \frac{(bB-Ac)x}{4b^2\left(b+cx^2\right)^2}-\frac{1}{4}\int \frac{-\frac{4A}{b}-\frac{3(bB-Ac)x^2}{b^2}}{x^2\left(b+cx^2\right)^2}\,dx\\
&= \frac{(bB-Ac)x}{4b^2\left(b+cx^2\right)^2}+\frac{(3bB-7Ac)x}{8b^3\left(b+cx^2\right)}+\frac{1}{8}\int \frac{\frac{8A}{b^2}+\frac{(3bB-7Ac)x^2}{b^3}}{x^2\left(b+cx^2\right)}\,dx\\
&= -\frac{A}{b^3x}+\frac{(bB-Ac)x}{4b^2\left(b+cx^2\right)^2}+\frac{(3bB-7Ac)x}{8b^3\left(b+cx^2\right)}+\frac{(3(bB-5Ac))\int \frac{1}{b+cx^2}\,dx}{8b^3}\\
&= -\frac{A}{b^3x}+\frac{(bB-Ac)x}{4b^2\left(b+cx^2\right)^2}+\frac{(3bB-7Ac)x}{8b^3\left(b+cx^2\right)}+\frac{3(bB-5Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{7/2}\sqrt{c}}
\end{aligned}$$

Mathematica [A] time = 0.0621907, size = 96, normalized size = 1.

$$\frac{x(3bB-7Ac)}{8b^3\left(b+cx^2\right)}+\frac{x(bB-Ac)}{4b^2\left(b+cx^2\right)^2}+\frac{3(bB-5Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{7/2}\sqrt{c}}-\frac{A}{b^3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -(A/(b^3*x)) + ((b*B - A*c)*x)/(4*b^2*(b + c*x^2)^2) + ((3*b*B - 7*A*c)*x)/
(8*b^3*(b + c*x^2)) + (3*(b*B - 5*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(7
/2)*Sqrt[c])
```

Maple [A] time = 0.013, size = 125, normalized size = 1.3

$$-\frac{A}{b^3x}-\frac{7\,Ax^3c^2}{8\,b^3\left(cx^2+b\right)^2}+\frac{3\,Bcx^3}{8\,b^2\left(cx^2+b\right)^2}-\frac{9\,Acx}{8\,b^2\left(cx^2+b\right)^2}+\frac{5\,Bx}{8\,b\left(cx^2+b\right)^2}-\frac{15\,Ac}{8\,b^3}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}+\frac{3\,B}{8\,b^2}\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^2+A)/(c*x^4+b*x^2)^3,x)
```

```
[Out] -A/b^3/x-7/8/b^3/(c*x^2+b)^2*A*x^3*c^2+3/8/b^2/(c*x^2+b)^2*B*x^3*c-9/8/b^2/
(c*x^2+b)^2*A*c*x+5/8/b/(c*x^2+b)^2*B*x-15/8/b^3/(b*c)^(1/2)*arctan(x*c/(b*
c)^(1/2))*A*c+3/8/b^2/(b*c)^(1/2)*arctan(x*c/(b*c)^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.02086, size = 686, normalized size = 7.15

$$\left[-\frac{16\,Ab^3c-6\left(Bb^2c^2-5\,Abc^3\right)x^4-10\left(Bb^3c-5\,Ab^2c^2\right)x^2-3\left(\left(Bbc^2-5\,Ac^3\right)x^5+2\left(Bb^2c-5\,Abc^2\right)x^3+\left(Bb^3-5\,Ab^2c\right.\right.}{16\left(b^4c^3x^5+2\,b^5c^2x^3+b^6cx\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(16*A*b^3*c - 6*(B*b^2*c^2 - 5*A*b*c^3)*x^4 - 10*(B*b^3*c - 5*A*b^2*
c^2)*x^2 - 3*((B*b*c^2 - 5*A*c^3)*x^5 + 2*(B*b^2*c - 5*A*b*c^2)*x^3 + (B*b^
3 - 5*A*b^2*c)*x)*sqrt(-b*c)*log((c*x^2 + 2*sqrt(-b*c)*x - b)/(c*x^2 + b)))
/(b^4*c^3*x^5 + 2*b^5*c^2*x^3 + b^6*c*x), -1/8*(8*A*b^3*c - 3*(B*b^2*c^2 -
5*A*b*c^3)*x^4 - 5*(B*b^3*c - 5*A*b^2*c^2)*x^2 - 3*((B*b*c^2 - 5*A*c^3)*x^5
 + 2*(B*b^2*c - 5*A*b*c^2)*x^3 + (B*b^3 - 5*A*b^2*c)*x)*sqrt(b*c)*arctan(sq
rt(b*c)*x/b))/(b^4*c^3*x^5 + 2*b^5*c^2*x^3 + b^6*c*x)]
```

Sympy [B] time = 0.900642, size = 194, normalized size = 2.02

$$-\frac{3\sqrt{-\frac{1}{b^7c}}\left(-5Ac+Bb\right)\log\left(-\frac{3b^4\sqrt{-\frac{1}{b^7c}}(-5Ac+Bb)}{-15Ac+3Bb}+x\right)}{16}+\frac{3\sqrt{-\frac{1}{b^7c}}\left(-5Ac+Bb\right)\log\left(\frac{3b^4\sqrt{-\frac{1}{b^7c}}(-5Ac+Bb)}{-15Ac+3Bb}+x\right)}{16}+\frac{-8Ab^2+x^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] -3*sqrt(-1/(b**7*c))*(-5*A*c + B*b)*log(-3*b**4*sqrt(-1/(b**7*c))*(-5*A*c +
 B*b)/(-15*A*c + 3*B*b) + x)/16 + 3*sqrt(-1/(b**7*c))*(-5*A*c + B*b)*log(3*
b**4*sqrt(-1/(b**7*c))*(-5*A*c + B*b)/(-15*A*c + 3*B*b) + x)/16 + (-8*A*b**
2 + x**4*(-15*A*c**2 + 3*B*b*c) + x**2*(-25*A*b*c + 5*B*b**2))/(8*b**5*x +
16*b**4*c*x**3 + 8*b**3*c**2*x**5)
```

Giac [A] time = 1.15514, size = 111, normalized size = 1.16

$$\frac{3\,(Bb-5\,Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}b^3}-\frac{A}{b^3x}+\frac{3\,Bbcx^3-7\,Ac^2x^3+5\,Bb^2x-9\,Abcx}{8\left(cx^2+b\right)^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 3/8*(B*b - 5*A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^3) - A/(b^3*x) + 1/8*(
3*B*b*c*x^3 - 7*A*c^2*x^3 + 5*B*b^2*x - 9*A*b*c*x)/((c*x^2 + b)^2*b^3)
```

3.84 $\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^3} dx$

Optimal. Leaf size=97

$$\frac{bB-2Ac}{2b^3\left(b+cx^2\right)}+\frac{bB-Ac}{4b^2\left(b+cx^2\right)^2}-\frac{(bB-3Ac)\log\left(b+cx^2\right)}{2b^4}+\frac{\log(x)(bB-3Ac)}{b^4}-\frac{A}{2b^3x^2}$$

```
[Out] -A/(2*b^3*x^2) + (b*B - A*c)/(4*b^2*(b + c*x^2)^2) + (b*B - 2*A*c)/(2*b^3*(
b + c*x^2)) + ((b*B - 3*A*c)*Log[x])/b^4 - ((b*B - 3*A*c)*Log[b + c*x^2])/(
2*b^4)
```

Rubi [A] time = 0.117311, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 446, 77}

$$\frac{bB-2Ac}{2b^3\left(b+cx^2\right)}+\frac{bB-Ac}{4b^2\left(b+cx^2\right)^2}-\frac{(bB-3Ac)\log\left(b+cx^2\right)}{2b^4}+\frac{\log(x)(bB-3Ac)}{b^4}-\frac{A}{2b^3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -A/(2*b^3*x^2) + (b*B - A*c)/(4*b^2*(b + c*x^2)^2) + (b*B - 2*A*c)/(2*b^3*(
b + c*x^2)) + ((b*B - 3*A*c)*Log[x])/b^4 - ((b*B - 3*A*c)*Log[b + c*x^2])/(
2*b^4)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx &= \int \frac{A+Bx^2}{x^3\left(b+cx^2\right)^3}\,dx\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{A+Bx}{x^2(b+cx)^3}\,dx,x,x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{A}{b^3x^2}+\frac{bB-3Ac}{b^4x}-\frac{c(bB-Ac)}{b^2(b+cx)^3}-\frac{c(bB-2Ac)}{b^3(b+cx)^2}-\frac{c(bB-3Ac)}{b^4(b+cx)}\right)dx,x,x^2\right)\\
&= -\frac{A}{2b^3x^2}+\frac{bB-Ac}{4b^2\left(b+cx^2\right)^2}+\frac{bB-2Ac}{2b^3\left(b+cx^2\right)}+\frac{(bB-3Ac)\log(x)}{b^4}-\frac{(bB-3Ac)\log\left(b+cx^2\right)}{2b^4}
\end{aligned}$$

Mathematica [A] time = 0.0595325, size = 86, normalized size = 0.89

$$\frac{\frac{b^2(bB-Ac)}{\left(b+cx^2\right)^2}+\frac{2b(bB-2Ac)}{b+cx^2}-2(bB-3Ac)\log\left(b+cx^2\right)+4\log(x)(bB-3Ac)-\frac{2Ab}{x^2}}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-2*A*b)/x^2 + (b^2*(b*B - A*c))/(b + c*x^2)^2 + (2*b*(b*B - 2*A*c))/(b +
c*x^2) + 4*(b*B - 3*A*c)*Log[x] - 2*(b*B - 3*A*c)*Log[b + c*x^2])/(4*b^4)
```

Maple [A] time = 0.015, size = 118, normalized size = 1.2

$$-\frac{A}{2\,b^3x^2}-3\,\frac{A\ln(x)\,c}{b^4}+\frac{\ln(x)\,B}{b^3}+\frac{3\,c\ln\left(cx^2+b\right)A}{2\,b^4}-\frac{\ln\left(cx^2+b\right)B}{2\,b^3}-\frac{Ac}{b^3\left(cx^2+b\right)}+\frac{B}{2\,b^2\left(cx^2+b\right)}-\frac{Ac}{4\,b^2\left(cx^2+b\right)^2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)/(c*x^4+b*x^2)^3,x)
```

```
[Out] -1/2*A/b^3/x^2-3/b^4*ln(x)*A*c+1/b^3*ln(x)*B+3/2/b^4*c*ln(c*x^2+b)*A-1/2/b^
3*ln(c*x^2+b)*B-1/b^3*c*A/(c*x^2+b)+1/2/b^2/(c*x^2+b)*B-1/4/b^2*c/(c*x^2+b)
^2*A+1/4/b/(c*x^2+b)^2*B
```

Maxima [A] time = 1.0992, size = 147, normalized size = 1.52

$$\frac{2\left(Bbc-3\,Ac^{2}\right)x^{4}-2\,Ab^{2}+3\left(Bb^{2}-3\,Abc\right)x^{2}}{4\left(b^{3}c^{2}x^{6}+2\,b^{4}cx^{4}+b^{5}x^{2}\right)}-\frac{(Bb-3\,Ac)\log\left(cx^{2}+b\right)}{2\,b^{4}}+\frac{(Bb-3\,Ac)\log\left(x^{2}\right)}{2\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(B*b*c - 3*A*c^2)*x^4 - 2*A*b^2 + 3*(B*b^2 - 3*A*b*c)*x^2)/(b^3*c^2*
x^6 + 2*b^4*c*x^4 + b^5*x^2) - 1/2*(B*b - 3*A*c)*log(c*x^2 + b)/b^4 + 1/2*(
B*b - 3*A*c)*log(x^2)/b^4
```

Fricas [B] time = 1.05911, size = 412, normalized size = 4.25

$$\frac{2\left(Bb^{2}c-3\,Abc^{2}\right)x^{4}-2\,Ab^{3}+3\left(Bb^{3}-3\,Ab^{2}c\right)x^{2}-2\left(\left(Bbc^{2}-3\,Ac^{3}\right)x^{6}+2\left(Bb^{2}c-3\,Abc^{2}\right)x^{4}+\left(Bb^{3}-3\,Ab^{2}c\right)x^{2}\right)\log}{4\left(b^{4}c^{2}x^{6}+2\,b^{5}cx^{4}+b^{6}x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(B*b^2*c - 3*A*b*c^2)*x^4 - 2*A*b^3 + 3*(B*b^3 - 3*A*b^2*c)*x^2 - 2*
((B*b*c^2 - 3*A*c^3)*x^6 + 2*(B*b^2*c - 3*A*b*c^2)*x^4 + (B*b^3 - 3*A*b^2*c
)*x^2)*log(c*x^2 + b) + 4*((B*b*c^2 - 3*A*c^3)*x^6 + 2*(B*b^2*c - 3*A*b*c^2
)*x^4 + (B*b^3 - 3*A*b^2*c)*x^2)*log(x))/(b^4*c^2*x^6 + 2*b^5*c*x^4 + b^6*x
^2)
```

Sympy [A] time = 1.36029, size = 107, normalized size = 1.1

$$\frac{-2Ab^2 + x^4\left(-6Ac^2 + 2Bbc\right) + x^2\left(-9Abc + 3Bb^2\right)}{4b^5x^2 + 8b^4cx^4 + 4b^3c^2x^6} + \frac{(-3Ac + Bb)\log(x)}{b^4} - \frac{(-3Ac + Bb)\log\left(\frac{b}{c} + x^2\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] (-2*A*b**2 + x**4*(-6*A*c**2 + 2*B*b*c) + x**2*(-9*A*b*c + 3*B*b**2))/(4*b*
*5*x**2 + 8*b**4*c*x**4 + 4*b**3*c**2*x**6) + (-3*A*c + B*b)*log(x)/b**4 -
(-3*A*c + B*b)*log(b/c + x**2)/(2*b**4)
```

Giac [A] time = 1.19155, size = 142, normalized size = 1.46

$$\frac{(Bb - 3\,Ac)\log(|x|)}{b^4} - \frac{\left(Bbc - 3\,Ac^2\right)\log\left(\left|cx^2 + b\right|\right)}{2\,b^4c} + \frac{2\left(Bb^2c - 3\,Abc^2\right)x^4 - 2\,Ab^3 + 3\left(Bb^3 - 3\,Ab^2c\right)x^2}{4\left(cx^2 + b\right)^2 b^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] (B*b - 3*A*c)*log(abs(x))/b^4 - 1/2*(B*b*c - 3*A*c^2)*log(abs(c*x^2 + b))/(
b^4*c) + 1/4*(2*(B*b^2*c - 3*A*b*c^2)*x^4 - 2*A*b^3 + 3*(B*b^3 - 3*A*b^2*c)
*x^2)/((c*x^2 + b)^2*b^4*x^2)
```

3.85 $\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=117

$$-\frac{cx(7bB-11Ac)}{8b^4(b+cx^2)}-\frac{cx(bB-Ac)}{4b^3(b+cx^2)^2}-\frac{bB-3Ac}{b^4x}-\frac{5\sqrt{c}(3bB-7Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{9/2}}-\frac{A}{3b^3x^3}$$

```
[Out] -A/(3*b^3*x^3) - (b*B - 3*A*c)/(b^4*x) - (c*(b*B - A*c)*x)/(4*b^3*(b + c*x^
2)^2) - (c*(7*b*B - 11*A*c)*x)/(8*b^4*(b + c*x^2)) - (5*Sqrt[c]*(3*b*B - 7*
A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(9/2))
```

Rubi [A] time = 0.176717, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1584, 456, 1259, 1261, 205}

$$-\frac{cx(7bB-11Ac)}{8b^4(b+cx^2)}-\frac{cx(bB-Ac)}{4b^3(b+cx^2)^2}-\frac{bB-3Ac}{b^4x}-\frac{5\sqrt{c}(3bB-7Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{9/2}}-\frac{A}{3b^3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -A/(3*b^3*x^3) - (b*B - 3*A*c)/(b^4*x) - (c*(b*B - A*c)*x)/(4*b^3*(b + c*x^
2)^2) - (c*(7*b*B - 11*A*c)*x)/(8*b^4*(b + c*x^2)) - (5*Sqrt[c]*(3*b*B - 7*
A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(9/2))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
```

```
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^2\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{A+Bx^2}{x^4\left(b+cx^2\right)^3}\,dx$$

$$= -\frac{c(bB-Ac)x}{4b^3\left(b+cx^2\right)^2} - \frac{1}{4}c\int \frac{-\frac{4A}{bc}-\frac{4(bB-Ac)x^2}{b^2c}+\frac{3(bB-Ac)x^4}{b^3}}{x^4\left(b+cx^2\right)^2}\,dx$$

$$= -\frac{c(bB-Ac)x}{4b^3\left(b+cx^2\right)^2} - \frac{c(7bB-11Ac)x}{8b^4\left(b+cx^2\right)} - \frac{\int \frac{-8Abc-8c(bB-2Ac)x^2+\frac{c^2(7bB-11Ac)x^4}{b}}{x^4\left(b+cx^2\right)}\,dx}{8b^3c}$$

$$= -\frac{c(bB-Ac)x}{4b^3\left(b+cx^2\right)^2} - \frac{c(7bB-11Ac)x}{8b^4\left(b+cx^2\right)} - \frac{\int \left(-\frac{8Ac}{x^4}-\frac{8c(bB-3Ac)}{bx^2}+\frac{5c^2(3bB-7Ac)}{b\left(b+cx^2\right)}\right)dx}{8b^3c}$$

$$= -\frac{A}{3b^3x^3} - \frac{bB-3Ac}{b^4x} - \frac{c(bB-Ac)x}{4b^3\left(b+cx^2\right)^2} - \frac{c(7bB-11Ac)x}{8b^4\left(b+cx^2\right)} - \frac{(5c(3bB-7Ac))\int \frac{1}{b+cx^2}\,dx}{8b^4}$$

$$= -\frac{A}{3b^3x^3} - \frac{bB-3Ac}{b^4x} - \frac{c(bB-Ac)x}{4b^3\left(b+cx^2\right)^2} - \frac{c(7bB-11Ac)x}{8b^4\left(b+cx^2\right)} - \frac{5\sqrt{c}(3bB-7Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{9/2}}$$

Mathematica [A] time = 0.0697808, size = 119, normalized size = 1.02

$$-\frac{x\left(7bBc-11Ac^2\right)}{8b^4\left(b+cx^2\right)} - \frac{cx(bB-Ac)}{4b^3\left(b+cx^2\right)^2} + \frac{3Ac-bB}{b^4x} - \frac{5\sqrt{c}(3bB-7Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{9/2}} - \frac{A}{3b^3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -A/(3*b^3*x^3) + (-(b*B) + 3*A*c)/(b^4*x) - (c*(b*B - A*c)*x)/(4*b^3*(b + c
*x^2)^2) - ((7*b*B*c - 11*A*c^2)*x)/(8*b^4*(b + c*x^2)) - (5*Sqrt[c]*(3*b*B
 - 7*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(9/2))
```

Maple [A] time = 0.013, size = 152, normalized size = 1.3

$$-\frac{A}{3\,b^3x^3} + 3\,\frac{Ac}{b^4x} - \frac{B}{b^3x} + \frac{11\,Ax^3c^3}{8\,b^4\left(cx^2+b\right)^2} - \frac{7\,Bc^2x^3}{8\,b^3\left(cx^2+b\right)^2} + \frac{13\,Ac^2x}{8\,b^3\left(cx^2+b\right)^2} - \frac{9\,Bcx}{8\,b^2\left(cx^2+b\right)^2} + \frac{35\,Ac^2}{8\,b^4}\arctan\left(cx\frac{1}{\sqrt{bc}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)/(c*x^4+b*x^2)^3,x)
```

```
[Out] -1/3*A/b^3/x^3+3/b^4/x*A*c-1/b^3/x*B+11/8/b^4*c^3/(c*x^2+b)^2*A*x^3-7/8/b^3
*c^2/(c*x^2+b)^2*B*x^3+13/8/b^3*c^2/(c*x^2+b)^2*A*x-9/8/b^2*c/(c*x^2+b)^2*B
*x+35/8/b^4*c^2/(b*c)^(1/2)*arctan(x*c/(b*c)^(1/2))*A-15/8/b^3*c/(b*c)^(1/2
)*arctan(x*c/(b*c)^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.04025, size = 782, normalized size = 6.68

$$\left[-\frac{30\left(3\,Bbc^2-7\,Ac^3\right)x^6+50\left(3\,Bb^2c-7\,Abc^2\right)x^4+16\,Ab^3+16\left(3\,Bb^3-7\,Ab^2c\right)x^2+15\left(\left(3\,Bbc^2-7\,Ac^3\right)x^7+2\left(3\,Bb^2c\right.\right.}{48\left(b^4c^2x^7+2\,b^5cx^5+b^6x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/48*(30*(3*B*b*c^2 - 7*A*c^3)*x^6 + 50*(3*B*b^2*c - 7*A*b*c^2)*x^4 + 16*
A*b^3 + 16*(3*B*b^3 - 7*A*b^2*c)*x^2 + 15*((3*B*b*c^2 - 7*A*c^3)*x^7 + 2*(3
*B*b^2*c - 7*A*b*c^2)*x^5 + (3*B*b^3 - 7*A*b^2*c)*x^3)*sqrt(-c/b)*log((c*x^
2 + 2*b*x*sqrt(-c/b) - b)/(c*x^2 + b)))/(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^
3), -1/24*(15*(3*B*b*c^2 - 7*A*c^3)*x^6 + 25*(3*B*b^2*c - 7*A*b*c^2)*x^4 +
8*A*b^3 + 8*(3*B*b^3 - 7*A*b^2*c)*x^2 + 15*((3*B*b*c^2 - 7*A*c^3)*x^7 + 2*(
3*B*b^2*c - 7*A*b*c^2)*x^5 + (3*B*b^3 - 7*A*b^2*c)*x^3)*sqrt(c/b)*arctan(x*
sqrt(c/b)))/(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^3)]
```

Sympy [B] time = 1.21224, size = 226, normalized size = 1.93

$$\frac{5\sqrt{-\frac{c}{b^9}}\,(-7Ac+3Bb)\log\left(-\frac{5b^5\sqrt{-\frac{c}{b^9}}(-7Ac+3Bb)}{-35Ac^2+15Bbc}+x\right)}{16}-\frac{5\sqrt{-\frac{c}{b^9}}\,(-7Ac+3Bb)\log\left(\frac{5b^5\sqrt{-\frac{c}{b^9}}(-7Ac+3Bb)}{-35Ac^2+15Bbc}+x\right)}{16}-\frac{8Ab^3+x^6\left(-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] 5*sqrt(-c/b**9)*(-7*A*c + 3*B*b)*log(-5*b**5*sqrt(-c/b**9)*(-7*A*c + 3*B*b)
/(-35*A*c**2 + 15*B*b*c) + x)/16 - 5*sqrt(-c/b**9)*(-7*A*c + 3*B*b)*log(5*b
**5*sqrt(-c/b**9)*(-7*A*c + 3*B*b)/(-35*A*c**2 + 15*B*b*c) + x)/16 - (8*A*b
**3 + x**6*(-105*A*c**3 + 45*B*b*c**2) + x**4*(-175*A*b*c**2 + 75*B*b**2*c)
 + x**2*(-56*A*b**2*c + 24*B*b**3))/(24*b**6*x**3 + 48*b**5*c*x**5 + 24*b**
4*c**2*x**7)
```

Giac [A] time = 1.17738, size = 146, normalized size = 1.25

$$-\frac{5\left(3\,Bbc-7\,Ac^2\right)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}b^4}-\frac{7\,Bbc^2x^3-11\,Ac^3x^3+9\,Bb^2cx-13\,Abc^2x}{8\left(cx^2+b\right)^2b^4}-\frac{3\,Bbx^2-9\,Acx^2+Ab}{3\,b^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -5/8*(3*B*b*c - 7*A*c^2)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^4) - 1/8*(7*B*b
*c^2*x^3 - 11*A*c^3*x^3 + 9*B*b^2*c*x - 13*A*b*c^2*x)/((c*x^2 + b)^2*b^4) -
 1/3*(3*B*b*x^2 - 9*A*c*x^2 + A*b)/(b^4*x^3)
```

3.86 $\int \frac{x\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=121

$$-\frac{c(2bB-3Ac)}{2b^4\left(b+cx^2\right)}-\frac{bB-3Ac}{2b^4x^2}-\frac{c(bB-Ac)}{4b^3\left(b+cx^2\right)^2}+\frac{3c(bB-2Ac)\log\left(b+cx^2\right)}{2b^5}-\frac{3c\log(x)(bB-2Ac)}{b^5}-\frac{A}{4b^3x^4}$$

```
[Out] -A/(4*b^3*x^4) - (b*B - 3*A*c)/(2*b^4*x^2) - (c*(b*B - A*c))/(4*b^3*(b + c*
x^2)^2) - (c*(2*b*B - 3*A*c))/(2*b^4*(b + c*x^2)) - (3*c*(b*B - 2*A*c)*Log[
x])/b^5 + (3*c*(b*B - 2*A*c)*Log[b + c*x^2])/(2*b^5)
```

Rubi [A] time = 0.1309, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1584, 446, 77}

$$-\frac{c(2bB-3Ac)}{2b^4\left(b+cx^2\right)}-\frac{bB-3Ac}{2b^4x^2}-\frac{c(bB-Ac)}{4b^3\left(b+cx^2\right)^2}+\frac{3c(bB-2Ac)\log\left(b+cx^2\right)}{2b^5}-\frac{3c\log(x)(bB-2Ac)}{b^5}-\frac{A}{4b^3x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -A/(4*b^3*x^4) - (b*B - 3*A*c)/(2*b^4*x^2) - (c*(b*B - A*c))/(4*b^3*(b + c*
x^2)^2) - (c*(2*b*B - 3*A*c))/(2*b^4*(b + c*x^2)) - (3*c*(b*B - 2*A*c)*Log[
x])/b^5 + (3*c*(b*B - 2*A*c)*Log[b + c*x^2])/(2*b^5)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \frac{x\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{A+Bx^2}{x^5\left(b+cx^2\right)^3}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^3(b+cx)^3}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{b^3x^3}+\frac{bB-3Ac}{b^4x^2}-\frac{3c(bB-2Ac)}{b^5x}+\frac{c^2(bB-Ac)}{b^3(b+cx)^3}+\frac{c^2(2bB-3Ac)}{b^4(b+cx)^2}+\frac{3c^2(bB-2Ac}{b^5(b+cx)}\right.\right.$$
$$= -\frac{A}{4b^3x^4}-\frac{bB-3Ac}{2b^4x^2}-\frac{c(bB-Ac)}{4b^3\left(b+cx^2\right)^2}-\frac{c(2bB-3Ac)}{2b^4\left(b+cx^2\right)}-\frac{3c(bB-2Ac)\log(x)}{b^5}+\frac{3c(bB-2Ac)\log}{2b^5}$$

Mathematica [A] time = 0.0769802, size = 108, normalized size = 0.89

$$\frac{\frac{b^2c(Ac-bB)}{\left(b+cx^2\right)^2}-\frac{Ab^2}{x^4}+\frac{2bc(3Ac-2bB)}{b+cx^2}-\frac{2b(bB-3Ac)}{x^2}+6c(bB-2Ac)\log\left(b+cx^2\right)+12c\log(x)(2Ac-bB)}{4b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (-((A*b^2)/x^4) - (2*b*(b*B - 3*A*c))/x^2 + (b^2*c*(-(b*B) + A*c))/(b + c*x
^2)^2 + (2*b*c*(-2*b*B + 3*A*c))/(b + c*x^2) + 12*c*(-(b*B) + 2*A*c)*Log[x]
 + 6*c*(b*B - 2*A*c)*Log[b + c*x^2])/(4*b^5)
```

Maple [A] time = 0.015, size = 150, normalized size = 1.2

$$-\frac{A}{4\,b^3x^4}+\frac{3\,Ac}{2\,b^4x^2}-\frac{B}{2\,b^3x^2}+6\,\frac{A\ln(x)\,c^2}{b^5}-3\,\frac{Bc\ln(x)}{b^4}-3\,\frac{c^2\ln\left(cx^2+b\right)A}{b^5}+\frac{3\,c\ln\left(cx^2+b\right)B}{2\,b^4}+\frac{3\,Ac^2}{2\,b^4\left(cx^2+b\right)}-\frac{}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)/(c*x^4+b*x^2)^3,x)
```

```
[Out] -1/4*A/b^3/x^4+3/2/b^4/x^2*A*c-1/2/b^3/x^2*B+6*c^2/b^5*ln(x)*A-3*c/b^4*ln(x
)*B-3/b^5*c^2*ln(c*x^2+b)*A+3/2/b^4*c*ln(c*x^2+b)*B+3/2/b^4*c^2*A/(c*x^2+b)
-1/b^3*c/(c*x^2+b)*B+1/4/b^3*c^2/(c*x^2+b)^2*A-1/4/b^2*c/(c*x^2+b)^2*B
```

Maxima [A] time = 1.1779, size = 185, normalized size = 1.53

$$-\frac{6\left(Bbc^2-2\,Ac^3\right)x^6+9\left(Bb^2c-2\,Abc^2\right)x^4+Ab^3+2\left(Bb^3-2\,Ab^2c\right)x^2}{4\left(b^4c^2x^8+2\,b^5cx^6+b^6x^4\right)}+\frac{3\left(Bbc-2\,Ac^2\right)\log\left(cx^2+b\right)}{2\,b^5}-\frac{3\left(Bbc-2\,Ac\right.}{2\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(6*(B*b*c^2 - 2*A*c^3)*x^6 + 9*(B*b^2*c - 2*A*b*c^2)*x^4 + A*b^3 + 2*(
B*b^3 - 2*A*b^2*c)*x^2)/(b^4*c^2*x^8 + 2*b^5*c*x^6 + b^6*x^4) + 3/2*(B*b*c
- 2*A*c^2)*log(c*x^2 + b)/b^5 - 3/2*(B*b*c - 2*A*c^2)*log(x^2)/b^5
```

Fricas [B] time = 1.10057, size = 474, normalized size = 3.92

$$-\frac{6\left(Bb^2c^2-2\,Abc^3\right)x^6+Ab^4+9\left(Bb^3c-2\,Ab^2c^2\right)x^4+2\left(Bb^4-2\,Ab^3c\right)x^2-6\left(\left(Bbc^3-2\,Ac^4\right)x^8+2\left(Bb^2c^2-2\,Abc^3\right)x^6\right.}{4\left(b^5c^2x^8+2\,b^6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(6*(B*b^2*c^2 - 2*A*b*c^3)*x^6 + A*b^4 + 9*(B*b^3*c - 2*A*b^2*c^2)*x^4
 + 2*(B*b^4 - 2*A*b^3*c)*x^2 - 6*((B*b*c^3 - 2*A*c^4)*x^8 + 2*(B*b^2*c^2 -
2*A*b*c^3)*x^6 + (B*b^3*c - 2*A*b^2*c^2)*x^4)*log(c*x^2 + b) + 12*((B*b*c^3
 - 2*A*c^4)*x^8 + 2*(B*b^2*c^2 - 2*A*b*c^3)*x^6 + (B*b^3*c - 2*A*b^2*c^2)*x
^4)*log(x))/(b^5*c^2*x^8 + 2*b^6*c*x^6 + b^7*x^4)
```

Sympy [A] time = 1.85757, size = 136, normalized size = 1.12

$$-\frac{Ab^3 + x^6\left(-12Ac^3 + 6Bbc^2\right) + x^4\left(-18Abc^2 + 9Bb^2c\right) + x^2\left(-4Ab^2c + 2Bb^3\right)}{4b^6x^4 + 8b^5cx^6 + 4b^4c^2x^8} - \frac{3c\left(-2Ac + Bb\right)\log\left(x\right)}{b^5} + \frac{3c\left(-2Ac + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] -(A*b**3 + x**6*(-12*A*c**3 + 6*B*b*c**2) + x**4*(-18*A*b*c**2 + 9*B*b**2*c
) + x**2*(-4*A*b**2*c + 2*B*b**3))/(4*b**6*x**4 + 8*b**5*c*x**6 + 4*b**4*c*
*2*x**8) - 3*c*(-2*A*c + B*b)*log(x)/b**5 + 3*c*(-2*A*c + B*b)*log(b/c + x*
*2)/(2*b**5)
```

Giac [A] time = 1.16258, size = 178, normalized size = 1.47

$$-\frac{3\left(Bbc - 2\,Ac^2\right)\log\left(|x|\right)}{b^5} + \frac{3\left(Bbc^2 - 2\,Ac^3\right)\log\left(\left|cx^2 + b\right|\right)}{2\,b^5c} - \frac{6\,Bbc^2x^6 - 12\,Ac^3x^6 + 9\,Bb^2cx^4 - 18\,Abc^2x^4 + 2\,Bb^3x^2 -}{4\left(cx^4 + bx^2\right)^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -3*(B*b*c - 2*A*c^2)*log(abs(x))/b^5 + 3/2*(B*b*c^2 - 2*A*c^3)*log(abs(c*x^
2 + b))/(b^5*c) - 1/4*(6*B*b*c^2*x^6 - 12*A*c^3*x^6 + 9*B*b^2*c*x^4 - 18*A*
b*c^2*x^4 + 2*B*b^3*x^2 - 4*A*b^2*c*x^2 + A*b^3)/((c*x^4 + b*x^2)^2*b^4)
```

3.87 $\int \frac{A+Bx^2}{(bx^2+cx^4)^3} dx$

Optimal. Leaf size=140

$$\frac{c^2x(11bB-15Ac)}{8b^5(b+cx^2)}+\frac{c^2x(bB-Ac)}{4b^4(b+cx^2)^2}+\frac{7c^{3/2}(5bB-9Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{11/2}}-\frac{bB-3Ac}{3b^4x^3}+\frac{3c(bB-2Ac)}{b^5x}-\frac{A}{5b^3x^5}$$

```
[Out] -A/(5*b^3*x^5) - (b*B - 3*A*c)/(3*b^4*x^3) + (3*c*(b*B - 2*A*c))/(b^5*x) +
(c^2*(b*B - A*c)*x)/(4*b^4*(b + c*x^2)^2) + (c^2*(11*b*B - 15*A*c)*x)/(8*b^
5*(b + c*x^2)) + (7*c^(3/2)*(5*b*B - 9*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8
*b^(11/2))
```

Rubi [A] time = 0.331962, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1593, 456, 1805, 1802, 205}

$$\frac{c^2x(11bB-15Ac)}{8b^5(b+cx^2)}+\frac{c^2x(bB-Ac)}{4b^4(b+cx^2)^2}+\frac{7c^{3/2}(5bB-9Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{11/2}}-\frac{bB-3Ac}{3b^4x^3}+\frac{3c(bB-2Ac)}{b^5x}-\frac{A}{5b^3x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -A/(5*b^3*x^5) - (b*B - 3*A*c)/(3*b^4*x^3) + (3*c*(b*B - 2*A*c))/(b^5*x) +
(c^2*(b*B - A*c)*x)/(4*b^4*(b + c*x^2)^2) + (c^2*(11*b*B - 15*A*c)*x)/(8*b^
5*(b + c*x^2)) + (7*c^(3/2)*(5*b*B - 9*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8
*b^(11/2))
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
```

```
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+Bx^2}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{A+Bx^2}{x^6\left(b+cx^2\right)^3}\,dx$$

$$= \frac{c^2(bB-Ac)x}{4b^4\left(b+cx^2\right)^2} - \frac{1}{4}c^2 \int \frac{-\frac{4A}{bc^2} - \frac{4(bB-Ac)x^2}{b^2c^2} + \frac{4(bB-Ac)x^4}{b^3c} - \frac{3(bB-Ac)x^6}{b^4}}{x^6\left(b+cx^2\right)^2}\,dx$$

$$= \frac{c^2(bB-Ac)x}{4b^4\left(b+cx^2\right)^2} + \frac{c^2(11bB-15Ac)x}{8b^5\left(b+cx^2\right)} + \frac{c^2\int \frac{\frac{8A}{bc^2}+\frac{8(bB-2Ac)x^2}{b^2c^2}-\frac{8(2bB-3Ac)x^4}{b^3c}+\frac{(11bB-15Ac)x^6}{b^4}}{x^6\left(b+cx^2\right)}\,dx}{8b}$$

$$= \frac{c^2(bB-Ac)x}{4b^4\left(b+cx^2\right)^2} + \frac{c^2(11bB-15Ac)x}{8b^5\left(b+cx^2\right)} + \frac{c^2\int \left(\frac{8A}{b^2c^2x^6} + \frac{8(bB-3Ac)}{b^3c^2x^4} - \frac{24(bB-2Ac)}{b^4cx^2} + \frac{7(5bB-9Ac)}{b^4\left(b+cx^2\right)}\right)dx}{8b}$$

$$= -\frac{A}{5b^3x^5} - \frac{bB-3Ac}{3b^4x^3} + \frac{3c(bB-2Ac)}{b^5x} + \frac{c^2(bB-Ac)x}{4b^4\left(b+cx^2\right)^2} + \frac{c^2(11bB-15Ac)x}{8b^5\left(b+cx^2\right)} + \frac{\left(7c^2(5bB-9Ac)\right)\int}{8b^5}$$

$$= -\frac{A}{5b^3x^5} - \frac{bB-3Ac}{3b^4x^3} + \frac{3c(bB-2Ac)}{b^5x} + \frac{c^2(bB-Ac)x}{4b^4\left(b+cx^2\right)^2} + \frac{c^2(11bB-15Ac)x}{8b^5\left(b+cx^2\right)} + \frac{7c^{3/2}(5bB-9Ac)\tan}{8b^{11/2}}$$

Mathematica [A] time = 0.0746567, size = 140, normalized size = 1.

$$\frac{c^2x(11bB-15Ac)}{8b^5\left(b+cx^2\right)} + \frac{c^2x(bB-Ac)}{4b^4\left(b+cx^2\right)^2} + \frac{7c^{3/2}(5bB-9Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{11/2}} - \frac{bB-3Ac}{3b^4x^3} + \frac{3c(bB-2Ac)}{b^5x} - \frac{A}{5b^3x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -A/(5*b^3*x^5) - (b*B - 3*A*c)/(3*b^4*x^3) + (3*c*(b*B - 2*A*c))/(b^5*x) +
(c^2*(b*B - A*c)*x)/(4*b^4*(b + c*x^2)^2) + (c^2*(11*b*B - 15*A*c)*x)/(8*b^
5*(b + c*x^2)) + (7*c^(3/2)*(5*b*B - 9*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8
*b^(11/2))
```

Maple [A] time = 0.014, size = 177, normalized size = 1.3

$$-\frac{A}{5x^5b^3} + \frac{Ac}{b^4x^3} - \frac{B}{3b^3x^3} - 6\frac{Ac^2}{b^5x} + 3\frac{Bc}{b^4x} - \frac{15c^4Ax^3}{8b^5\left(cx^2+b\right)^2} + \frac{11Bc^3x^3}{8b^4\left(cx^2+b\right)^2} - \frac{17Ac^3x}{8b^4\left(cx^2+b\right)^2} + \frac{13Bc^2x}{8b^3\left(cx^2+b\right)^2} - \frac{63A}{8b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(c*x^4+b*x^2)^3,x)
```

```
[Out] -1/5*A/x^5/b^3+1/b^4/x^3*A*c-1/3/b^3/x^3*B-6*c^2/b^5/x*A+3*c/b^4/x*B-15/8/b
^5*c^4/(c*x^2+b)^2*A*x^3+11/8/b^4*c^3/(c*x^2+b)^2*B*x^3-17/8/b^4*c^3/(c*x^2
+b)^2*A*x+13/8/b^3*c^2/(c*x^2+b)^2*B*x-63/8/b^5*c^3/(b*c)^(1/2)*arctan(x*c/
(b*c)^(1/2))*A+35/8/b^4*c^2/(b*c)^(1/2)*arctan(x*c/(b*c)^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.07862, size = 909, normalized size = 6.49

$$\left[\frac{210\left(5\,Bbc^3-9\,Ac^4\right)x^8+350\left(5\,Bb^2c^2-9\,Abc^3\right)x^6-48\,Ab^4+112\left(5\,Bb^3c-9\,Ab^2c^2\right)x^4-16\left(5\,Bb^4-9\,Ab^3c\right)x^2-105}{240\left(b^5c^2x^9+2\,b^6cx^7+b^7\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [1/240*(210*(5*B*b*c^3 - 9*A*c^4)*x^8 + 350*(5*B*b^2*c^2 - 9*A*b*c^3)*x^6 -
 48*A*b^4 + 112*(5*B*b^3*c - 9*A*b^2*c^2)*x^4 - 16*(5*B*b^4 - 9*A*b^3*c)*x^
2 - 105*((5*B*b*c^3 - 9*A*c^4)*x^9 + 2*(5*B*b^2*c^2 - 9*A*b*c^3)*x^7 + (5*B
*b^3*c - 9*A*b^2*c^2)*x^5)*sqrt(-c/b)*log((c*x^2 - 2*b*x*sqrt(-c/b) - b)/(c
*x^2 + b)))/(b^5*c^2*x^9 + 2*b^6*c*x^7 + b^7*x^5), 1/120*(105*(5*B*b*c^3 -
9*A*c^4)*x^8 + 175*(5*B*b^2*c^2 - 9*A*b*c^3)*x^6 - 24*A*b^4 + 56*(5*B*b^3*c
 - 9*A*b^2*c^2)*x^4 - 8*(5*B*b^4 - 9*A*b^3*c)*x^2 + 105*((5*B*b*c^3 - 9*A*c
^4)*x^9 + 2*(5*B*b^2*c^2 - 9*A*b*c^3)*x^7 + (5*B*b^3*c - 9*A*b^2*c^2)*x^5)*
```

```
sqrt(c/b)*arctan(x*sqrt(c/b)))/(b^5*c^2*x^9 + 2*b^6*c*x^7 + b^7*x^5)]
```

Sympy [A] time = 1.69213, size = 260, normalized size = 1.86

$$-\frac{7\sqrt{-\frac{c^3}{b^{11}}}(-9Ac+5Bb)\log\left(-\frac{7b^6\sqrt{-\frac{c^3}{b^{11}}}(-9Ac+5Bb)}{-63Ac^3+35Bbc^2}+x\right)}{16}+\frac{7\sqrt{-\frac{c^3}{b^{11}}}(-9Ac+5Bb)\log\left(\frac{7b^6\sqrt{-\frac{c^3}{b^{11}}}(-9Ac+5Bb)}{-63Ac^3+35Bbc^2}+x\right)}{16}+\frac{-24Ab^4 -}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] -7*sqrt(-c**3/b**11)*(-9*A*c + 5*B*b)*log(-7*b**6*sqrt(-c**3/b**11)*(-9*A*c
 + 5*B*b)/(-63*A*c**3 + 35*B*b*c**2) + x)/16 + 7*sqrt(-c**3/b**11)*(-9*A*c
+ 5*B*b)*log(7*b**6*sqrt(-c**3/b**11)*(-9*A*c + 5*B*b)/(-63*A*c**3 + 35*B*b
*c**2) + x)/16 + (-24*A*b**4 + x**8*(-945*A*c**4 + 525*B*b*c**3) + x**6*(-1
575*A*b*c**3 + 875*B*b**2*c**2) + x**4*(-504*A*b**2*c**2 + 280*B*b**3*c) +
x**2*(72*A*b**3*c - 40*B*b**4))/(120*b**7*x**5 + 240*b**6*c*x**7 + 120*b**5
*c**2*x**9)
```

Giac [A] time = 1.1575, size = 182, normalized size = 1.3

$$\frac{7\left(5\,Bbc^2-9\,Ac^3\right)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}b^5}+\frac{11\,Bbc^3x^3-15\,Ac^4x^3+13\,Bb^2c^2x-17\,Abc^3x}{8\left(cx^2+b\right)^2b^5}+\frac{45\,Bbcx^4-90\,Ac^2x^4-5\,Bb^2x^2+15\,}{15\,b^5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 7/8*(5*B*b*c^2 - 9*A*c^3)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^5) + 1/8*(11*B
*b*c^3*x^3 - 15*A*c^4*x^3 + 13*B*b^2*c^2*x - 17*A*b*c^3*x)/((c*x^2 + b)^2*b
^5) + 1/15*(45*B*b*c*x^4 - 90*A*c^2*x^4 - 5*B*b^2*x^2 + 15*A*b*c*x^2 - 3*A*
b^2)/(b^5*x^5)
```

3.88 $\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=148

$$\frac{c^2(3bB-4Ac)}{2b^5\left(b+cx^2\right)}+\frac{c^2(bB-Ac)}{4b^4\left(b+cx^2\right)^2}-\frac{c^2(3bB-5Ac)\log\left(b+cx^2\right)}{b^6}+\frac{2c^2\log(x)(3bB-5Ac)}{b^6}+\frac{3c(bB-2Ac)}{2b^5x^2}-\frac{bB-3Ac}{4b^4x^4}-$$

```
[Out] -A/(6*b^3*x^6) - (b*B - 3*A*c)/(4*b^4*x^4) + (3*c*(b*B - 2*A*c))/(2*b^5*x^2
) + (c^2*(b*B - A*c))/(4*b^4*(b + c*x^2)^2) + (c^2*(3*b*B - 4*A*c))/(2*b^5*
(b + c*x^2)) + (2*c^2*(3*b*B - 5*A*c)*Log[x])/b^6 - (c^2*(3*b*B - 5*A*c)*Lo
g[b + c*x^2])/b^6
```

Rubi [A] time = 0.172682, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 446, 77}

$$\frac{c^2(3bB-4Ac)}{2b^5\left(b+cx^2\right)}+\frac{c^2(bB-Ac)}{4b^4\left(b+cx^2\right)^2}-\frac{c^2(3bB-5Ac)\log\left(b+cx^2\right)}{b^6}+\frac{2c^2\log(x)(3bB-5Ac)}{b^6}+\frac{3c(bB-2Ac)}{2b^5x^2}-\frac{bB-3Ac}{4b^4x^4}-$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x*(b*x^2 + c*x^4)^3),x]
```

```
[Out] -A/(6*b^3*x^6) - (b*B - 3*A*c)/(4*b^4*x^4) + (3*c*(b*B - 2*A*c))/(2*b^5*x^2
) + (c^2*(b*B - A*c))/(4*b^4*(b + c*x^2)^2) + (c^2*(3*b*B - 4*A*c))/(2*b^5*
(b + c*x^2)) + (2*c^2*(3*b*B - 5*A*c)*Log[x])/b^6 - (c^2*(3*b*B - 5*A*c)*Lo
g[b + c*x^2])/b^6
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)^3}\,dx = \int \frac{A+Bx^2}{x^7\left(b+cx^2\right)^3}\,dx$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{A+Bx}{x^4(b+cx)^3}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{A}{b^3x^4} + \frac{bB-3Ac}{b^4x^3} - \frac{3c(bB-2Ac)}{b^5x^2} + \frac{2c^2(3bB-5Ac)}{b^6x} - \frac{c^3(bB-Ac)}{b^4(b+cx)^3} - \frac{c^3(3bB-4A}{b^5(b+cx)^2}\right.\right.$$

$$= -\frac{A}{6b^3x^6} - \frac{bB-3Ac}{4b^4x^4} + \frac{3c(bB-2Ac)}{2b^5x^2} + \frac{c^2(bB-Ac)}{4b^4\left(b+cx^2\right)^2} + \frac{c^2(3bB-4Ac)}{2b^5\left(b+cx^2\right)} + \frac{2c^2(3bB-5Ac)\log(x)}{b^6}$$

Mathematica [A] time = 0.120139, size = 135, normalized size = 0.91

$$\frac{\frac{3b^2c^2(bB-Ac)}{\left(b+cx^2\right)^2} - \frac{3b^2(bB-3Ac)}{x^4} - \frac{2Ab^3}{x^6} + \frac{6bc^2(3bB-4Ac)}{b+cx^2} + 12c^2(5Ac-3bB)\log\left(b+cx^2\right) + 24c^2\log(x)(3bB-5Ac) + \frac{18bc(bB-2Ac)}{x^2}}{12b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x*(b*x^2 + c*x^4)^3),x]
```

```
[Out] ((-2*A*b^3)/x^6 - (3*b^2*(b*B - 3*A*c))/x^4 + (18*b*c*(b*B - 2*A*c))/x^2 +
(3*b^2*c^2*(b*B - A*c))/(b + c*x^2)^2 + (6*b*c^2*(3*b*B - 4*A*c))/(b + c*x^
2) + 24*c^2*(3*b*B - 5*A*c)*Log[x] + 12*c^2*(-3*b*B + 5*A*c)*Log[b + c*x^2]
)/(12*b^6)
```

Maple [A] time = 0.017, size = 180, normalized size = 1.2

$$-\frac{A}{6\,b^3x^6}+\frac{3\,Ac}{4\,b^4x^4}-\frac{B}{4\,b^3x^4}-3\,\frac{Ac^2}{b^5x^2}+\frac{3\,Bc}{2\,b^4x^2}-10\,\frac{A\ln(x)\,c^3}{b^6}+6\,\frac{Bc^2\ln(x)}{b^5}+5\,\frac{c^3\ln\left(cx^2+b\right)A}{b^6}-3\,\frac{c^2\ln\left(cx^2+b\right)B}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x/(c*x^4+b*x^2)^3,x)
```

```
[Out] -1/6*A/b^3/x^6+3/4/b^4/x^4*A*c-1/4/b^3/x^4*B-3*c^2/b^5/x^2*A+3/2*c/b^4/x^2*
B-10*c^3/b^6*ln(x)*A+6*c^2/b^5*ln(x)*B+5/b^6*c^3*ln(c*x^2+b)*A-3/b^5*c^2*ln
(c*x^2+b)*B-2/b^5*c^3*A/(c*x^2+b)+3/2/b^4*c^2/(c*x^2+b)*B-1/4/b^4*c^3/(c*x^
2+b)^2*A+1/4/b^3*c^2/(c*x^2+b)^2*B
```

Maxima [A] time = 1.3717, size = 230, normalized size = 1.55

$$\frac{12\left(3\,Bbc^3-5\,Ac^4\right)x^8+18\left(3\,Bb^2c^2-5\,Abc^3\right)x^6-2\,Ab^4+4\left(3\,Bb^3c-5\,Ab^2c^2\right)x^4-\left(3\,Bb^4-5\,Ab^3c\right)x^2}{12\left(b^5c^2x^{10}+2\,b^6cx^8+b^7x^6\right)}-\frac{\left(3\,Bbc^2-5\,A\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(12*(3*B*b*c^3 - 5*A*c^4)*x^8 + 18*(3*B*b^2*c^2 - 5*A*b*c^3)*x^6 - 2*A
*b^4 + 4*(3*B*b^3*c - 5*A*b^2*c^2)*x^4 - (3*B*b^4 - 5*A*b^3*c)*x^2)/(b^5*c^
2*x^10 + 2*b^6*c*x^8 + b^7*x^6) - (3*B*b*c^2 - 5*A*c^3)*log(c*x^2 + b)/b^6
+ (3*B*b*c^2 - 5*A*c^3)*log(x^2)/b^6
```

Fricas [A] time = 1.01222, size = 560, normalized size = 3.78

$$\frac{12\left(3\,Bb^2c^3-5\,Abc^4\right)x^8+18\left(3\,Bb^3c^2-5\,Ab^2c^3\right)x^6-2\,Ab^5+4\left(3\,Bb^4c-5\,Ab^3c^2\right)x^4-\left(3\,Bb^5-5\,Ab^4c\right)x^2-12\left(\left(3\,Bbc\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(12*(3*B*b^2*c^3 - 5*A*b*c^4)*x^8 + 18*(3*B*b^3*c^2 - 5*A*b^2*c^3)*x^6
 - 2*A*b^5 + 4*(3*B*b^4*c - 5*A*b^3*c^2)*x^4 - (3*B*b^5 - 5*A*b^4*c)*x^2 -
12*((3*B*b*c^4 - 5*A*c^5)*x^10 + 2*(3*B*b^2*c^3 - 5*A*b*c^4)*x^8 + (3*B*b^3
*c^2 - 5*A*b^2*c^3)*x^6)*log(c*x^2 + b) + 24*((3*B*b*c^4 - 5*A*c^5)*x^10 +
2*(3*B*b^2*c^3 - 5*A*b*c^4)*x^8 + (3*B*b^3*c^2 - 5*A*b^2*c^3)*x^6)*log(x))/
(b^6*c^2*x^10 + 2*b^7*c*x^8 + b^8*x^6)
```

Sympy [A] time = 2.57034, size = 165, normalized size = 1.11

$$\frac{-2Ab^4 + x^8\left(-60Ac^4 + 36Bbc^3\right) + x^6\left(-90Abc^3 + 54Bb^2c^2\right) + x^4\left(-20Ab^2c^2 + 12Bb^3c\right) + x^2\left(5Ab^3c - 3Bb^4\right)}{12b^7x^6 + 24b^6cx^8 + 12b^5c^2x^{10}} + \frac{2c^2(-5A}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x/(c*x**4+b*x**2)**3,x)
```

```
[Out] (-2*A*b**4 + x**8*(-60*A*c**4 + 36*B*b*c**3) + x**6*(-90*A*b*c**3 + 54*B*b*
*2*c**2) + x**4*(-20*A*b**2*c**2 + 12*B*b**3*c) + x**2*(5*A*b**3*c - 3*B*b*
*4))/(12*b**7*x**6 + 24*b**6*c*x**8 + 12*b**5*c**2*x**10) + 2*c**2*(-5*A*c
+ 3*B*b)*log(x)/b**6 - c**2*(-5*A*c + 3*B*b)*log(b/c + x**2)/b**6
```

Giac [A] time = 1.12625, size = 271, normalized size = 1.83

$$\frac{\left(3\,Bbc^2 - 5\,Ac^3\right)\log\left(x^2\right)}{b^6} - \frac{\left(3\,Bbc^3 - 5\,Ac^4\right)\log\left(\left|cx^2 + b\right|\right)}{b^6c} + \frac{18\,Bbc^4x^4 - 30\,Ac^5x^4 + 42\,Bb^2c^3x^2 - 68\,Abc^4x^2 + 25\,Bb^3c}{4\left(cx^2 + b\right)^2b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] (3*B*b*c^2 - 5*A*c^3)*log(x^2)/b^6 - (3*B*b*c^3 - 5*A*c^4)*log(abs(c*x^2 +
b))/(b^6*c) + 1/4*(18*B*b*c^4*x^4 - 30*A*c^5*x^4 + 42*B*b^2*c^3*x^2 - 68*A*
b*c^4*x^2 + 25*B*b^3*c^2 - 39*A*b^2*c^3)/((c*x^2 + b)^2*b^6) - 1/12*(66*B*b
*c^2*x^6 - 110*A*c^3*x^6 - 18*B*b^2*c*x^4 + 36*A*b*c^2*x^4 + 3*B*b^3*x^2 -
9*A*b^2*c*x^2 + 2*A*b^3)/(b^6*x^6)
```

3.89 $\int x^7 \left(A+Bx^2\right) \sqrt{bx^2+cx^4} \, dx$

Optimal. Leaf size=218

$$-\frac{7b^2\left(bx^2+cx^4\right)^{3/2}(3bB-4Ac)}{384c^4}+\frac{7b^3\left(b+2cx^2\right)\sqrt{bx^2+cx^4}(3bB-4Ac)}{1024c^5}-\frac{7b^5(3bB-4Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{1024c^{11/2}}-\frac{x^4\left(b\right.}{}$$

```
[Out] (7*b^3*(3*b*B - 4*A*c)*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(1024*c^5) - (7*b
^2*(3*b*B - 4*A*c)*(b*x^2 + c*x^4)^(3/2))/(384*c^4) + (7*b*(3*b*B - 4*A*c)*
x^2*(b*x^2 + c*x^4)^(3/2))/(320*c^3) - ((3*b*B - 4*A*c)*x^4*(b*x^2 + c*x^4)
^(3/2))/(40*c^2) + (B*x^6*(b*x^2 + c*x^4)^(3/2))/(12*c) - (7*b^5*(3*b*B - 4
*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(1024*c^(11/2))
```

Rubi [A] time = 0.381084, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {2034, 794, 670, 640, 612, 620, 206}

$$-\frac{7b^2\left(bx^2+cx^4\right)^{3/2}(3bB-4Ac)}{384c^4}+\frac{7b^3\left(b+2cx^2\right)\sqrt{bx^2+cx^4}(3bB-4Ac)}{1024c^5}-\frac{7b^5(3bB-4Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{1024c^{11/2}}-\frac{x^4\left(b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (7*b^3*(3*b*B - 4*A*c)*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(1024*c^5) - (7*b
^2*(3*b*B - 4*A*c)*(b*x^2 + c*x^4)^(3/2))/(384*c^4) + (7*b*(3*b*B - 4*A*c)*
x^2*(b*x^2 + c*x^4)^(3/2))/(320*c^3) - ((3*b*B - 4*A*c)*x^4*(b*x^2 + c*x^4)
^(3/2))/(40*c^2) + (B*x^6*(b*x^2 + c*x^4)^(3/2))/(12*c) - (7*b^5*(3*b*B - 4
*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(1024*c^(11/2))
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^7\left(A+Bx^2\right)\sqrt{bx^2+cx^4}\,dx=\frac{1}{2}\text{Subst}\left(\int x^3(A+Bx)\sqrt{bx+cx^2}\,dx,x,x^2\right)$$
$$=\frac{Bx^6\left(bx^2+cx^4\right)^{3/2}}{12c}+\frac{\left(3(-bB+Ac)+\frac{3}{2}(-bB+2Ac)\right)\text{Subst}\left(\int x^3\sqrt{bx+cx^2}\,dx,x,x^2\right.}{12c}$$
$$=-\frac{(3bB-4Ac)x^4\left(bx^2+cx^4\right)^{3/2}}{40c^2}+\frac{Bx^6\left(bx^2+cx^4\right)^{3/2}}{12c}+\frac{(7b(3bB-4Ac))\text{Subst}\left(\int x^2\right.}{80c^2}$$
$$=\frac{7b(3bB-4Ac)x^2\left(bx^2+cx^4\right)^{3/2}}{320c^3}-\frac{(3bB-4Ac)x^4\left(bx^2+cx^4\right)^{3/2}}{40c^2}+\frac{Bx^6\left(bx^2+cx^4\right)^{3/2}}{12c}$$
$$=-\frac{7b^2(3bB-4Ac)\left(bx^2+cx^4\right)^{3/2}}{384c^4}+\frac{7b(3bB-4Ac)x^2\left(bx^2+cx^4\right)^{3/2}}{320c^3}-\frac{(3bB-4Ac)x^4\left(\right.}{40c^2}$$
$$=\frac{7b^3(3bB-4Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{1024c^5}-\frac{7b^2(3bB-4Ac)\left(bx^2+cx^4\right)^{3/2}}{384c^4}+\frac{7b(3bB-4}{}$$
$$=\frac{7b^3(3bB-4Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{1024c^5}-\frac{7b^2(3bB-4Ac)\left(bx^2+cx^4\right)^{3/2}}{384c^4}+\frac{7b(3bB-4}{}$$
$$=\frac{7b^3(3bB-4Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{1024c^5}-\frac{7b^2(3bB-4Ac)\left(bx^2+cx^4\right)^{3/2}}{384c^4}+\frac{7b(3bB-4}{}$$

Mathematica [A] time = 0.293925, size = 193, normalized size = 0.89

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(\sqrt{c}x\sqrt{\frac{cx^2}{b}+1}\left(-16b^2c^3x^4\left(14A+9Bx^2\right)+56b^3c^2x^2\left(5A+3Bx^2\right)-210b^4c\left(2A+Bx^2\right)+64bc^4x^6\left(3A+\right.\right.\right.}{15360c^{11/2}x\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(Sqrt[c]*x*Sqrt[1 + (c*x^2)/b]*(315*b^5*B - 210*b^4*
c*(2*A + B*x^2) + 64*b*c^4*x^6*(3*A + 2*B*x^2) + 56*b^3*c^2*x^2*(5*A + 3*B*
x^2) + 256*c^5*x^8*(6*A + 5*B*x^2) - 16*b^2*c^3*x^4*(14*A + 9*B*x^2)) - 105
*b^(9/2)*(3*b*B - 4*A*c)*ArcSinh[(Sqrt[c]*x)/Sqrt[b]]))/(15360*c^(11/2)*x*S
qrt[1 + (c*x^2)/b])
```

Maple [A] time = 0.028, size = 290, normalized size = 1.3

$$\frac{1}{15360\,x}\sqrt{cx^4+bx^2}\left(1280\,B\left(cx^2+b\right)^{3/2}c^{9/2}x^9+1536\,A\left(cx^2+b\right)^{3/2}c^{9/2}x^7-1152\,B\left(cx^2+b\right)^{3/2}c^{7/2}x^7b-1344\,A\left(cx^2+b\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/15360*(c*x^4+b*x^2)^(1/2)*(1280*B*(c*x^2+b)^(3/2)*c^(9/2)*x^9+1536*A*(c*x
^2+b)^(3/2)*c^(9/2)*x^7-1152*B*(c*x^2+b)^(3/2)*c^(7/2)*x^7*b-1344*A*(c*x^2+
b)^(3/2)*c^(7/2)*x^5*b+1008*B*(c*x^2+b)^(3/2)*c^(5/2)*x^5*b^2+1120*A*(c*x^2
+b)^(3/2)*c^(5/2)*x^3*b^2-840*B*(c*x^2+b)^(3/2)*c^(3/2)*x^3*b^3-840*A*(c*x^
2+b)^(3/2)*c^(3/2)*x*b^3+630*B*(c*x^2+b)^(3/2)*c^(1/2)*x*b^4+420*A*(c*x^2+b
)^(1/2)*c^(3/2)*x*b^4-315*B*(c*x^2+b)^(1/2)*c^(1/2)*x*b^5+420*A*ln(x*c^(1/2
)+(c*x^2+b)^(1/2))*b^5*c-315*B*ln(x*c^(1/2)+(c*x^2+b)^(1/2))*b^6)/x/(c*x^2+
b)^(1/2)/c^(11/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.69133, size = 844, normalized size = 3.87

$$\left[-\frac{105\left(3\,Bb^6-4\,Ab^5c\right)\sqrt{c}\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\left(1280\,Bc^6x^{10}+128\left(Bbc^5+12\,Ac^6\right)x^8+315\,Bb^5c-420\right.}{30720\,c^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/30720*(105*(3*B*b^6 - 4*A*b^5*c)*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^
4 + b*x^2)*sqrt(c)) - 2*(1280*B*c^6*x^10 + 128*(B*b*c^5 + 12*A*c^6)*x^8 + 3
15*B*b^5*c - 420*A*b^4*c^2 - 48*(3*B*b^2*c^4 - 4*A*b*c^5)*x^6 + 56*(3*B*b^3
*c^3 - 4*A*b^2*c^4)*x^4 - 70*(3*B*b^4*c^2 - 4*A*b^3*c^3)*x^2)*sqrt(c*x^4 +
b*x^2))/c^6, 1/15360*(105*(3*B*b^6 - 4*A*b^5*c)*sqrt(-c)*arctan(sqrt(c*x^4
+ b*x^2)*sqrt(-c)/(c*x^2 + b)) + (1280*B*c^6*x^10 + 128*(B*b*c^5 + 12*A*c^6
)*x^8 + 315*B*b^5*c - 420*A*b^4*c^2 - 48*(3*B*b^2*c^4 - 4*A*b*c^5)*x^6 + 56
*(3*B*b^3*c^3 - 4*A*b^2*c^4)*x^4 - 70*(3*B*b^4*c^2 - 4*A*b^3*c^3)*x^2)*sqrt
(c*x^4 + b*x^2))/c^6]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^7\sqrt{x^2\left(b+cx^2\right)}\left(A+Bx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(B*x**2+A)*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**7*sqrt(x**2*(b + c*x**2))*(A + B*x**2), x)
```

Giac [A] time = 1.17702, size = 331, normalized size = 1.52

$$\frac{1}{15360}\left(2\left(4\left(2\left(8\left(10\,Bx^2\mathrm{sgn}\left(x\right)+\frac{Bbc^9\mathrm{sgn}\left(x\right)+12\,Ac^{10}\mathrm{sgn}\left(x\right)}{c^{10}}\right)x^2-\frac{3\left(3\,Bb^2c^8\mathrm{sgn}\left(x\right)-4\,Abc^9\mathrm{sgn}\left(x\right)\right)}{c^{10}}\right)x^2+\frac{7\left(3\,Bb\right.}{}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/15360*(2*(4*(2*(8*(10*B*x^2*sgn(x) + (B*b*c^9*sgn(x) + 12*A*c^10*sgn(x))/
c^10)*x^2 - 3*(3*B*b^2*c^8*sgn(x) - 4*A*b*c^9*sgn(x))/c^10)*x^2 + 7*(3*B*b^
3*c^7*sgn(x) - 4*A*b^2*c^8*sgn(x))/c^10)*x^2 - 35*(3*B*b^4*c^6*sgn(x) - 4*A
*b^3*c^7*sgn(x))/c^10)*x^2 + 105*(3*B*b^5*c^5*sgn(x) - 4*A*b^4*c^6*sgn(x))/
c^10)*sqrt(c*x^2 + b)*x + 7/1024*(3*B*b^6*sgn(x) - 4*A*b^5*c*sgn(x))*log(ab
s(-sqrt(c)*x + sqrt(c*x^2 + b)))/c^(11/2) - 7/2048*(3*B*b^6*log(abs(b)) - 4
*A*b^5*c*log(abs(b)))*sgn(x)/c^(11/2)
```

3.90 $\int x^5 \left(A+Bx^2\right) \sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=181

$$-\frac{b^2\left(b+2cx^2\right)\sqrt{bx^2+cx^4}(7bB-10Ac)}{256c^4}+\frac{b^4(7bB-10Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{256c^{9/2}}+\frac{b\left(bx^2+cx^4\right)^{3/2}(7bB-10Ac)}{96c^3}-\frac{x^2\left(bx^2\right.}{}$$

```
[Out] -(b^2*(7*b*B - 10*A*c)*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(256*c^4) + (b*(7
*b*B - 10*A*c)*(b*x^2 + c*x^4)^(3/2))/(96*c^3) - ((7*b*B - 10*A*c)*x^2*(b*x
^2 + c*x^4)^(3/2))/(80*c^2) + (B*x^4*(b*x^2 + c*x^4)^(3/2))/(10*c) + (b^4*(
7*b*B - 10*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(256*c^(9/2))
```

Rubi [A] time = 0.334355, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {2034, 794, 670, 640, 612, 620, 206}

$$-\frac{b^2\left(b+2cx^2\right)\sqrt{bx^2+cx^4}(7bB-10Ac)}{256c^4}+\frac{b^4(7bB-10Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{256c^{9/2}}+\frac{b\left(bx^2+cx^4\right)^{3/2}(7bB-10Ac)}{96c^3}-\frac{x^2\left(bx^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```


```
[Out] -(b^2*(7*b*B - 10*A*c)*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(256*c^4) + (b*(7
*b*B - 10*A*c)*(b*x^2 + c*x^4)^(3/2))/(96*c^3) - ((7*b*B - 10*A*c)*x^2*(b*x
^2 + c*x^4)^(3/2))/(80*c^2) + (B*x^4*(b*x^2 + c*x^4)^(3/2))/(10*c) + (b^4*(
7*b*B - 10*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(256*c^(9/2))
```


Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] && !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
```

```
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^5 \left(A+Bx^2\right) \sqrt{bx^2+cx^4}\,dx = \frac{1}{2}\text{Subst}\left(\int x^2(A+Bx)\sqrt{bx+cx^2}\,dx, x, x^2\right)$$
$$= \frac{Bx^4\left(bx^2+cx^4\right)^{3/2}}{10c} + \frac{\left(2(-bB+Ac)+\frac{3}{2}(-bB+2Ac)\right)\text{Subst}\left(\int x^2\sqrt{bx+cx^2}\,dx, x, x^2\right)}{10c}$$
$$= -\frac{(7bB-10Ac)x^2\left(bx^2+cx^4\right)^{3/2}}{80c^2} + \frac{Bx^4\left(bx^2+cx^4\right)^{3/2}}{10c} + \frac{(b(7bB-10Ac))\text{Subst}\left(\int x\sqrt{b\right.}{32c^2}$$
$$= \frac{b(7bB-10Ac)\left(bx^2+cx^4\right)^{3/2}}{96c^3} - \frac{(7bB-10Ac)x^2\left(bx^2+cx^4\right)^{3/2}}{80c^2} + \frac{Bx^4\left(bx^2+cx^4\right)^{3/2}}{10c} - $$
$$= -\frac{b^2(7bB-10Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{256c^4} + \frac{b(7bB-10Ac)\left(bx^2+cx^4\right)^{3/2}}{96c^3} - \frac{(7bB-10A}{}$$
$$= -\frac{b^2(7bB-10Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{256c^4} + \frac{b(7bB-10Ac)\left(bx^2+cx^4\right)^{3/2}}{96c^3} - \frac{(7bB-10A}{}$$
$$= -\frac{b^2(7bB-10Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{256c^4} + \frac{b(7bB-10Ac)\left(bx^2+cx^4\right)^{3/2}}{96c^3} - \frac{(7bB-10A}{}$$

Mathematica [A] time = 0.272944, size = 173, normalized size = 0.96

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(\sqrt{c}x\sqrt{\frac{cx^2}{b}+1}\left(-4b^2c^2x^2\left(25A+14Bx^2\right)+10b^3c\left(15A+7Bx^2\right)+16bc^3x^4\left(5A+3Bx^2\right)+96c^4x^6\left(5A+4B\right.\right.\right.}{3840c^{9/2}x\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(Sqrt[c]*x*Sqrt[1 + (c*x^2)/b]*(-105*b^4*B + 16*b*c^
3*x^4*(5*A + 3*B*x^2) + 96*c^4*x^6*(5*A + 4*B*x^2) + 10*b^3*c*(15*A + 7*B*x
^2) - 4*b^2*c^2*x^2*(25*A + 14*B*x^2)) + 15*b^(7/2)*(7*b*B - 10*A*c)*ArcSin
h[(Sqrt[c]*x)/Sqrt[b]]))/(3840*c^(9/2)*x*Sqrt[1 + (c*x^2)/b])
```

Maple [A] time = 0.008, size = 248, normalized size = 1.4

$$\frac{1}{3840\,x}\sqrt{cx^4+bx^2}\left(384\,Bc^{7/2}\left(cx^2+b\right)^{3/2}x^7+480\,Ac^{7/2}\left(cx^2+b\right)^{3/2}x^5-336\,Bc^{5/2}\left(cx^2+b\right)^{3/2}x^5b-400\,Ac^{5/2}\left(cx^2+b\right)^{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/3840*(c*x^4+b*x^2)^(1/2)*(384*B*c^(7/2)*(c*x^2+b)^(3/2)*x^7+480*A*c^(7/2)
*(c*x^2+b)^(3/2)*x^5-336*B*c^(5/2)*(c*x^2+b)^(3/2)*x^5*b-400*A*c^(5/2)*(c*x
^2+b)^(3/2)*x^3*b+280*B*c^(3/2)*(c*x^2+b)^(3/2)*x^3*b^2+300*A*c^(3/2)*(c*x^
2+b)^(3/2)*x*b^2-210*B*c^(1/2)*(c*x^2+b)^(3/2)*x*b^3-150*A*c^(3/2)*(c*x^2+b
)^(1/2)*x*b^3+105*B*c^(1/2)*(c*x^2+b)^(1/2)*x*b^4-150*A*ln(x*c^(1/2)+(c*x^2
+b)^(1/2))*b^4*c+105*B*ln(x*c^(1/2)+(c*x^2+b)^(1/2))*b^5)/x/(c*x^2+b)^(1/2)
/c^(9/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.3207, size = 737, normalized size = 4.07

$$\left[-\frac{15\left(7\,Bb^5-10\,Ab^4c\right)\sqrt{c}\log\left(-2\,cx^2-b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\left(384\,Bc^5x^8+48\left(Bbc^4+10\,Ac^5\right)x^6-105\,Bb^4c+150\,\right.}{7680\,c^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/7680*(15*(7*B*b^5 - 10*A*b^4*c)*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4
 + b*x^2)*sqrt(c)) - 2*(384*B*c^5*x^8 + 48*(B*b*c^4 + 10*A*c^5)*x^6 - 105*B
*b^4*c + 150*A*b^3*c^2 - 8*(7*B*b^2*c^3 - 10*A*b*c^4)*x^4 + 10*(7*B*b^3*c^2
 - 10*A*b^2*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^5, -1/3840*(15*(7*B*b^5 - 10*A
*b^4*c)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) - (384*B*
c^5*x^8 + 48*(B*b*c^4 + 10*A*c^5)*x^6 - 105*B*b^4*c + 150*A*b^3*c^2 - 8*(7*
```

```
B*b^2*c^3 - 10*A*b*c^4)*x^4 + 10*(7*B*b^3*c^2 - 10*A*b^2*c^3)*x^2)*sqrt(c*x
^4 + b*x^2))/c^5]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^5\sqrt{x^2\left(b+cx^2\right)}\left(A+Bx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**5*sqrt(x**2*(b + c*x**2))*(A + B*x**2), x)
```

Giac [A] time = 1.15415, size = 285, normalized size = 1.57

$$\frac{1}{3840}\left(2\left(4\left(6\left(8\,Bx^2\mathrm{sgn}\left(x\right)+\frac{Bbc^7\mathrm{sgn}\left(x\right)+10\,Ac^8\mathrm{sgn}\left(x\right)}{c^8}\right)x^2-\frac{7\,Bb^2c^6\mathrm{sgn}\left(x\right)-10\,Abc^7\mathrm{sgn}\left(x\right)}{c^8}\right)x^2+\frac{5\left(7\,Bb^3c^5\mathrm{sgn}\left(x\right.\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3840*(2*(4*(6*(8*B*x^2*sgn(x) + (B*b*c^7*sgn(x) + 10*A*c^8*sgn(x))/c^8)*x
^2 - (7*B*b^2*c^6*sgn(x) - 10*A*b*c^7*sgn(x))/c^8)*x^2 + 5*(7*B*b^3*c^5*sgn
(x) - 10*A*b^2*c^6*sgn(x))/c^8)*x^2 - 15*(7*B*b^4*c^4*sgn(x) - 10*A*b^3*c^5
*sgn(x))/c^8)*sqrt(c*x^2 + b)*x - 1/256*(7*B*b^5*sgn(x) - 10*A*b^4*c*sgn(x)
)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/c^(9/2) + 1/512*(7*B*b^5*log(abs(b
)) - 10*A*b^4*c*log(abs(b)))*sgn(x)/c^(9/2)
```

3.91 $\int x^3 \left(A+Bx^2\right) \sqrt{bx^2+cx^4}\, dx$

Optimal. Leaf size=125

$$-\frac{b^3(5bB-8Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{7/2}}+\frac{b\left(b+2cx^2\right)\sqrt{bx^2+cx^4}(5bB-8Ac)}{128c^3}-\frac{\left(bx^2+cx^4\right)^{3/2}\left(-8Ac+5bB-6Bcx^2\right)}{48c^2}$$

```
[Out] (b*(5*b*B - 8*A*c)*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(128*c^3) - ((5*b*B -
 8*A*c - 6*B*c*x^2)*(b*x^2 + c*x^4)^(3/2))/(48*c^2) - (b^3*(5*b*B - 8*A*c)*
ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(128*c^(7/2))
```

Rubi [A] time = 0.198895, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2034, 779, 612, 620, 206}

$$-\frac{b^3(5bB-8Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{7/2}}+\frac{b\left(b+2cx^2\right)\sqrt{bx^2+cx^4}(5bB-8Ac)}{128c^3}-\frac{\left(bx^2+cx^4\right)^{3/2}\left(-8Ac+5bB-6Bcx^2\right)}{48c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```


```
[Out] (b*(5*b*B - 8*A*c)*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(128*c^3) - ((5*b*B -
 8*A*c - 6*B*c*x^2)*(b*x^2 + c*x^4)^(3/2))/(48*c^2) - (b^3*(5*b*B - 8*A*c)*
ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(128*c^(7/2))
```


Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
```

```
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^3\left(A+Bx^2\right)\sqrt{bx^2+cx^4}\,dx=\frac{1}{2}\operatorname{Subst}\left(\int x(A+Bx)\sqrt{bx+cx^2}\,dx,x,x^2\right)$$

$$=-\frac{\left(5bB-8Ac-6Bcx^2\right)\left(bx^2+cx^4\right)^{3/2}}{48c^2}+\frac{(b(5bB-8Ac))\operatorname{Subst}\left(\int\sqrt{bx+cx^2}\,dx,x,x^2\right)}{32c^2}$$

$$=\frac{b(5bB-8Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{128c^3}-\frac{\left(5bB-8Ac-6Bcx^2\right)\left(bx^2+cx^4\right)^{3/2}}{48c^2}-\frac{\left(b^3(5bB\right.}{}$$

$$=\frac{b(5bB-8Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{128c^3}-\frac{\left(5bB-8Ac-6Bcx^2\right)\left(bx^2+cx^4\right)^{3/2}}{48c^2}-\frac{\left(b^3(5bB\right.}{}$$

$$=\frac{b(5bB-8Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{128c^3}-\frac{\left(5bB-8Ac-6Bcx^2\right)\left(bx^2+cx^4\right)^{3/2}}{48c^2}-\frac{b^3(5bB-}{}$$

Mathematica [A] time = 0.200793, size = 151, normalized size = 1.21

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(\sqrt{c}x\sqrt{\frac{cx^2}{b}+1}\left(-2b^2c\left(12A+5Bx^2\right)+8bc^2x^2\left(2A+Bx^2\right)+16c^3x^4\left(4A+3Bx^2\right)+15b^3B\right)-3b^{5/2}(5bB-8\right.}{384c^{7/2}x\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(Sqrt[c]*x*Sqrt[1 + (c*x^2)/b]*(15*b^3*B + 8*b*c^2*x
^2*(2*A + B*x^2) + 16*c^3*x^4*(4*A + 3*B*x^2) - 2*b^2*c*(12*A + 5*B*x^2)) -
 3*b^(5/2)*(5*b*B - 8*A*c)*ArcSinh[(Sqrt[c]*x)/Sqrt[b]]))/(384*c^(7/2)*x*Sq
rt[1 + (c*x^2)/b])
```

Maple [A] time = 0.009, size = 206, normalized size = 1.7

$$\frac{1}{384\,x}\sqrt{cx^4+bx^2}\left(48\,Bc^{5/2}\left(cx^2+b\right)^{3/2}x^5+64\,Ac^{5/2}\left(cx^2+b\right)^{3/2}x^3-40\,Bc^{3/2}\left(cx^2+b\right)^{3/2}x^3b-48\,Ac^{3/2}\left(cx^2+b\right)^{3/2}xb\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/384*(c*x^4+b*x^2)^(1/2)*(48*B*c^(5/2)*(c*x^2+b)^(3/2)*x^5+64*A*c^(5/2)*(c
*x^2+b)^(3/2)*x^3-40*B*c^(3/2)*(c*x^2+b)^(3/2)*x^3*b-48*A*c^(3/2)*(c*x^2+b)
^(3/2)*x*b+30*B*c^(1/2)*(c*x^2+b)^(3/2)*x*b^2+24*A*c^(3/2)*(c*x^2+b)^(1/2)*
x*b^2-15*B*c^(1/2)*(c*x^2+b)^(1/2)*x*b^3+24*A*ln(x*c^(1/2)+(c*x^2+b)^(1/2))
*b^3*c-15*B*ln(x*c^(1/2)+(c*x^2+b)^(1/2))*b^4)/x/(c*x^2+b)^(1/2)/c^(7/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.22919, size = 609, normalized size = 4.87

$$\left[-\frac{3\left(5\,Bb^4-8\,Ab^3c\right)\sqrt{c}\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\left(48\,Bc^4x^6+15\,Bb^3c-24\,Ab^2c^2+8\left(Bbc^3+8\,Ac^4\right)x^4-2\right.}{768\,c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/768*(3*(5*B*b^4 - 8*A*b^3*c)*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 +
b*x^2)*sqrt(c)) - 2*(48*B*c^4*x^6 + 15*B*b^3*c - 24*A*b^2*c^2 + 8*(B*b*c^3
+ 8*A*c^4)*x^4 - 2*(5*B*b^2*c^2 - 8*A*b*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^4,
 1/384*(3*(5*B*b^4 - 8*A*b^3*c)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c
)/(c*x^2 + b)) + (48*B*c^4*x^6 + 15*B*b^3*c - 24*A*b^2*c^2 + 8*(B*b*c^3 + 8
*A*c^4)*x^4 - 2*(5*B*b^2*c^2 - 8*A*b*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\sqrt{x^2\left(b+cx^2\right)}\left(A+Bx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(x**2*(b + c*x**2))*(A + B*x**2), x)
```

Giac [A] time = 1.22875, size = 239, normalized size = 1.91

$$\frac{1}{384}\left(2\left(4\left(6\,Bx^2\mathrm{sgn}\left(x\right)+\frac{Bbc^5\mathrm{sgn}\left(x\right)+8\,Ac^6\mathrm{sgn}\left(x\right)}{c^6}\right)x^2-\frac{5\,Bb^2c^4\mathrm{sgn}\left(x\right)-8\,Abc^5\mathrm{sgn}\left(x\right)}{c^6}\right)x^2+\frac{3\left(5\,Bb^3c^3\mathrm{sgn}\left(x\right)-8\,A\right.}{c^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*B*x^2*sgn(x) + (B*b*c^5*sgn(x) + 8*A*c^6*sgn(x))/c^6)*x^2 -
(5*B*b^2*c^4*sgn(x) - 8*A*b*c^5*sgn(x))/c^6)*x^2 + 3*(5*B*b^3*c^3*sgn(x) -
8*A*b^2*c^4*sgn(x))/c^6)*sqrt(c*x^2 + b)*x + 1/128*(5*B*b^4*sgn(x) - 8*A*b^
3*c*sgn(x))*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/c^(7/2) - 1/256*(5*B*b^4
*log(abs(b)) - 8*A*b^3*c*log(abs(b)))*sgn(x)/c^(7/2)
```

3.92 $\int x\left(A+Bx^2\right)\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=107

$$\frac{b^2(bB-2Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{5/2}}-\frac{\left(b+2cx^2\right)\sqrt{bx^2+cx^4}(bB-2Ac)}{16c^2}+\frac{B\left(bx^2+cx^4\right)^{3/2}}{6c}$$

```
[Out] -((b*B - 2*A*c)*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(16*c^2) + (B*(b*x^2 + c
*x^4)^(3/2))/(6*c) + (b^2*(b*B - 2*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 +
c*x^4]])/(16*c^(5/2))
```

Rubi [A] time = 0.153859, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2034, 640, 612, 620, 206}

$$\frac{b^2(bB-2Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{5/2}}-\frac{\left(b+2cx^2\right)\sqrt{bx^2+cx^4}(bB-2Ac)}{16c^2}+\frac{B\left(bx^2+cx^4\right)^{3/2}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```


```
[Out] -((b*B - 2*A*c)*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(16*c^2) + (B*(b*x^2 + c
*x^4)^(3/2))/(6*c) + (b^2*(b*B - 2*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 +
c*x^4]])/(16*c^(5/2))
```


Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
```

```
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x\left(A+Bx^2\right)\sqrt{bx^2+cx^4}\,dx=\frac{1}{2}\text{Subst}\left(\int(A+Bx)\sqrt{bx+cx^2}\,dx,x,x^2\right)$$

$$=\frac{B\left(bx^2+cx^4\right)^{3/2}}{6c}+\frac{(-bB+2Ac)\,\text{Subst}\left(\int\sqrt{bx+cx^2}\,dx,x,x^2\right)}{4c}$$

$$=-\frac{(bB-2Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{16c^2}+\frac{B\left(bx^2+cx^4\right)^{3/2}}{6c}+\frac{\left(b^2(bB-2Ac)\right)\text{Subst}\left(\int\frac{}{\sqrt{bx}}\right.}{32c^2}$$

$$=-\frac{(bB-2Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{16c^2}+\frac{B\left(bx^2+cx^4\right)^{3/2}}{6c}+\frac{\left(b^2(bB-2Ac)\right)\text{Subst}\left(\int\frac{1}{1-c}\right.}{16c^2}$$

$$=-\frac{(bB-2Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{16c^2}+\frac{B\left(bx^2+cx^4\right)^{3/2}}{6c}+\frac{b^2(bB-2Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+c}}\right.}{16c^{5/2}}$$

Mathematica [A] time = 0.179041, size = 129, normalized size = 1.21

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(\sqrt{c}x\sqrt{\frac{cx^2}{b}+1}\left(2bc\left(3A+Bx^2\right)+4c^2x^2\left(3A+2Bx^2\right)-3b^2B\right)+3b^{3/2}(bB-2Ac)\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)\right)}{48c^{5/2}x\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(Sqrt[c]*x*Sqrt[1 + (c*x^2)/b]*(-3*b^2*B + 2*b*c*(3*
A + B*x^2) + 4*c^2*x^2*(3*A + 2*B*x^2)) + 3*b^(3/2)*(b*B - 2*A*c)*ArcSinh[(
Sqrt[c]*x)/Sqrt[b]]))/(48*c^(5/2)*x*Sqrt[1 + (c*x^2)/b])
```

Maple [A] time = 0.009, size = 164, normalized size = 1.5

$$\frac{1}{48\,x}\sqrt{cx^4+bx^2}\left(8\,Bc^{3/2}\left(cx^2+b\right)^{3/2}x^3+12\,Ac^{3/2}\left(cx^2+b\right)^{3/2}x-6\,B\sqrt{c}\left(cx^2+b\right)^{3/2}xb-6\,Ac^{3/2}\sqrt{cx^2+b}xb+3\,B\sqrt{c}\sqrt{cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/48*(c*x^4+b*x^2)^(1/2)*(8*B*c^(3/2)*(c*x^2+b)^(3/2)*x^3+12*A*c^(3/2)*(c*x
^2+b)^(3/2)*x-6*B*c^(1/2)*(c*x^2+b)^(3/2)*x*b-6*A*c^(3/2)*(c*x^2+b)^(1/2)*x
*b+3*B*c^(1/2)*(c*x^2+b)^(1/2)*x*b^2-6*A*ln(x*c^(1/2)+(c*x^2+b)^(1/2))*b^2*
c+3*B*ln(x*c^(1/2)+(c*x^2+b)^(1/2))*b^3)/x/(c*x^2+b)^(1/2)/c^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.20916, size = 497, normalized size = 4.64

$$\left[-\frac{3\left(Bb^3-2\,Ab^2c\right)\sqrt{c}\log\left(-2\,cx^2-b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\left(8\,Bc^3x^4-3\,Bb^2c+6\,Abc^2+2\left(Bbc^2+6\,Ac^3\right)x^2\right)\sqrt{cx^4+bx}}{96\,c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(B*b^3 - 2*A*b^2*c)*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 + b*x
^2)*sqrt(c)) - 2*(8*B*c^3*x^4 - 3*B*b^2*c + 6*A*b*c^2 + 2*(B*b*c^2 + 6*A*c^
3)*x^2)*sqrt(c*x^4 + b*x^2))/c^3, -1/48*(3*(B*b^3 - 2*A*b^2*c)*sqrt(-c)*arc
tan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) - (8*B*c^3*x^4 - 3*B*b^2*c +
6*A*b*c^2 + 2*(B*b*c^2 + 6*A*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{x^2\left(b+cx^2\right)}\left(A+Bx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x*sqrt(x**2*(b + c*x**2))*(A + B*x**2), x)
```

Giac [A] time = 1.32685, size = 189, normalized size = 1.77

$$\frac{1}{48}\left(2\left(4\,Bx^2\mathrm{sgn}\left(x\right)+\frac{Bbc^3\mathrm{sgn}\left(x\right)+6\,Ac^4\mathrm{sgn}\left(x\right)}{c^4}\right)x^2-\frac{3\left(Bb^2c^2\mathrm{sgn}\left(x\right)-2\,Abc^3\mathrm{sgn}\left(x\right)\right)}{c^4}\right)\sqrt{cx^2+b}x-\frac{\left(Bb^3\mathrm{sgn}\left(x\right)-2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*B*x^2*sgn(x) + (B*b*c^3*sgn(x) + 6*A*c^4*sgn(x))/c^4)*x^2 - 3*(B
*b^2*c^2*sgn(x) - 2*A*b*c^3*sgn(x))/c^4)*sqrt(c*x^2 + b)*x - 1/16*(B*b^3*sg
n(x) - 2*A*b^2*c*sgn(x))*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/c^(5/2) + 1
/32*(B*b^3*log(abs(b)) - 2*A*b^2*c*log(abs(b)))*sgn(x)/c^(5/2)
```

3.93 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x}\,dx$

Optimal. Leaf size=100

$$-\frac{b(bB-4Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{3/2}}-\frac{\sqrt{bx^2+cx^4}(bB-4Ac)}{8c}+\frac{B\left(bx^2+cx^4\right)^{3/2}}{4cx^2}$$

```
[Out] -((b*B - 4*A*c)*Sqrt[b*x^2 + c*x^4])/(8*c) + (B*(b*x^2 + c*x^4)^(3/2))/(4*c
*x^2) - (b*(b*B - 4*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(8*c^(
3/2))
```

Rubi [A] time = 0.19702, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2034, 794, 664, 620, 206}

$$-\frac{b(bB-4Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{3/2}}-\frac{\sqrt{bx^2+cx^4}(bB-4Ac)}{8c}+\frac{B\left(bx^2+cx^4\right)^{3/2}}{4cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x,x]
```

```
[Out] -((b*B - 4*A*c)*Sqrt[b*x^2 + c*x^4])/(8*c) + (B*(b*x^2 + c*x^4)^(3/2))/(4*c
*x^2) - (b*(b*B - 4*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(8*c^(
3/2))
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
```

```
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(A+Bx^2\right)\sqrt{bx^2+cx^4}}{x}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x}\,dx,x,x^2\right)\\
&= \frac{B\left(bx^2+cx^4\right)^{3/2}}{4cx^2}+\frac{\left(bB-Ac+\frac{3}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x}\,dx,x,x^2\right)}{4c}\\
&= -\frac{(bB-4Ac)\sqrt{bx^2+cx^4}}{8c}+\frac{B\left(bx^2+cx^4\right)^{3/2}}{4cx^2}-\frac{(b(bB-4Ac))\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{16c}\\
&= -\frac{(bB-4Ac)\sqrt{bx^2+cx^4}}{8c}+\frac{B\left(bx^2+cx^4\right)^{3/2}}{4cx^2}-\frac{(b(bB-4Ac))\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{8c}\\
&= -\frac{(bB-4Ac)\sqrt{bx^2+cx^4}}{8c}+\frac{B\left(bx^2+cx^4\right)^{3/2}}{4cx^2}-\frac{b(bB-4Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.143771, size = 91, normalized size = 0.91

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(\sqrt{c}\left(4Ac+bB+2Bcx^2\right)-\frac{\sqrt{b}(bB-4Ac)\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{x\sqrt{\frac{cx^2}{b}+1}}\right)}{8c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x,x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(Sqrt[c]*(b*B + 4*A*c + 2*B*c*x^2) - (Sqrt[b]*(b*B -
 4*A*c)*ArcSinh[(Sqrt[c]*x)/Sqrt[b]])/(x*Sqrt[1 + (c*x^2)/b])))/(8*c^(3/2))
```

Maple [A] time = 0.006, size = 124, normalized size = 1.2

$$\frac{1}{8x}\sqrt{cx^4+bx^2}\left(2B\sqrt{c}\left(cx^2+b\right)^{3/2}x+4Ac^{3/2}\sqrt{cx^2+b}x-B\sqrt{c}\sqrt{cx^2+b}xb+4A\ln\left(x\sqrt{c}+\sqrt{cx^2+b}\right)bc-B\ln\left(x\sqrt{c}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x,x)
```

```
[Out] 1/8*(c*x^4+b*x^2)^(1/2)*(2*B*c^(1/2)*(c*x^2+b)^(3/2)*x+4*A*c^(3/2)*(c*x^2+b
)^(1/2)*x-B*c^(1/2)*(c*x^2+b)^(1/2)*x*b+4*A*ln(x*c^(1/2)+(c*x^2+b)^(1/2))*b
*c-B*ln(x*c^(1/2)+(c*x^2+b)^(1/2))*b^2)/c^(3/2)/(c*x^2+b)^(1/2)/x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.17224, size = 392, normalized size = 3.92

$$\left[-\frac{\left(Bb^2-4\,Abc\right)\sqrt{c}\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\left(2\,Bc^2x^2+Bbc+4\,Ac^2\right)\sqrt{cx^4+bx^2}}{16\,c^2},\frac{\left(Bb^2-4\,Abc\right)\sqrt{-c}\,\mathrm{arct}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [-1/16*((B*b^2 - 4*A*b*c)*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*
sqrt(c)) - 2*(2*B*c^2*x^2 + B*b*c + 4*A*c^2)*sqrt(c*x^4 + b*x^2))/c^2, 1/8*
((B*b^2 - 4*A*b*c)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)
) + (2*B*c^2*x^2 + B*b*c + 4*A*c^2)*sqrt(c*x^4 + b*x^2))/c^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2\left(b+cx^2\right)}\left(A+Bx^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x, x)
```

Giac [A] time = 1.34825, size = 139, normalized size = 1.39

$$\frac{1}{8}\left(2\,Bx^2\mathrm{sgn}\left(x\right)+\frac{Bbc\mathrm{sgn}\left(x\right)+4\,Ac^2\mathrm{sgn}\left(x\right)}{c^2}\right)\sqrt{cx^2+b}x+\frac{\left(Bb^2\mathrm{sgn}\left(x\right)-4\,Abc\mathrm{sgn}\left(x\right)\right)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)}{8\,c^{\frac{3}{2}}}-\frac{(Bb}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 1/8*(2*B*x^2*sgn(x) + (B*b*c*sgn(x) + 4*A*c^2*sgn(x))/c^2)*sqrt(c*x^2 + b)*
x + 1/8*(B*b^2*sgn(x) - 4*A*b*c*sgn(x))*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b
)))/c^(3/2) - 1/16*(B*b^2*log(abs(b)) - 4*A*b*c*log(abs(b)))*sgn(x)/c^(3/2)
```

3.94 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^3}\,dx$

Optimal. Leaf size=97

$$\frac{\sqrt{bx^2+cx^4}(2Ac+bB)}{2b}+\frac{(2Ac+bB)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{c}}-\frac{A\left(bx^2+cx^4\right)^{3/2}}{bx^4}$$

```
[Out] ((b*B + 2*A*c)*Sqrt[b*x^2 + c*x^4])/(2*b) - (A*(b*x^2 + c*x^4)^(3/2))/(b*x^
4) + ((b*B + 2*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(2*Sqrt[c])
```

Rubi [A] time = 0.213395, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2034, 792, 664, 620, 206}

$$\frac{\sqrt{bx^2+cx^4}(2Ac+bB)}{2b}+\frac{(2Ac+bB)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{c}}-\frac{A\left(bx^2+cx^4\right)^{3/2}}{bx^4}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^3,x]
```

```
[Out] ((b*B + 2*A*c)*Sqrt[b*x^2 + c*x^4])/(2*b) - (A*(b*x^2 + c*x^4)^(3/2))/(b*x^
4) + ((b*B + 2*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(2*Sqrt[c])
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
```

```
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(A+Bx^2\right)\sqrt{bx^2+cx^4}}{x^3}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^2}\,dx, x, x^2\right)$$

$$= -\frac{A\left(bx^2+cx^4\right)^{3/2}}{bx^4} + \frac{\left(-2(-bB+Ac)+\frac{3}{2}(-bB+2Ac)\right)\mathrm{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x}\,dx, x, x^2\right)}{b}$$

$$= \frac{(bB+2Ac)\sqrt{bx^2+cx^4}}{2b} - \frac{A\left(bx^2+cx^4\right)^{3/2}}{bx^4} + \frac{1}{4}(bB+2Ac)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x\right.$$

$$= \frac{(bB+2Ac)\sqrt{bx^2+cx^4}}{2b} - \frac{A\left(bx^2+cx^4\right)^{3/2}}{bx^4} + \frac{1}{2}(bB+2Ac)\,\mathrm{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{}{\sqrt{bx}}\right.$$

$$= \frac{(bB+2Ac)\sqrt{bx^2+cx^4}}{2b} - \frac{A\left(bx^2+cx^4\right)^{3/2}}{bx^4} + \frac{(bB+2Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{c}}$$

Mathematica [A] time = 0.142748, size = 78, normalized size = 0.8

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(\frac{x(2Ac+bB)\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{c}\sqrt{\frac{cx^2}{b}+1}}-2A+Bx^2\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^3,x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(-2*A + B*x^2 + ((b*B + 2*A*c)*x*ArcSinh[(Sqrt[c]*x)
/Sqrt[b]])/(Sqrt[b]*Sqrt[c]*Sqrt[1 + (c*x^2)/b])))/(2*x^2)
```

Maple [A] time = 0.008, size = 130, normalized size = 1.3

$$\frac{1}{2\,bx^2}\sqrt{cx^4+bx^2}\left(2\,Ac^{3/2}\sqrt{cx^2+b}x^2+B\sqrt{c}\sqrt{cx^2+b}x^2b-2\,A\sqrt{c}\left(cx^2+b\right)^{3/2}+2\,A\ln\left(x\sqrt{c}+\sqrt{cx^2+b}\right)xbc+B\ln\left(x\sqrt{c}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^3,x)
```

```
[Out] 1/2*(c*x^4+b*x^2)^(1/2)*(2*A*c^(3/2)*(c*x^2+b)^(1/2)*x^2+B*c^(1/2)*(c*x^2+b
)^(1/2)*x^2*b-2*A*c^(1/2)*(c*x^2+b)^(3/2)+2*A*ln(x*c^(1/2)+(c*x^2+b)^(1/2))
*x*b*c+B*ln(x*c^(1/2)+(c*x^2+b)^(1/2))*x*b^2)/x^2/(c*x^2+b)^(1/2)/b/c^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.18882, size = 363, normalized size = 3.74

$$\left[\frac{(Bb+2\,Ac)\sqrt{c}x^2\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)+2\,\sqrt{cx^4+bx^2}\left(Bcx^2-2\,Ac\right)}{4\,cx^2},-\frac{(Bb+2\,Ac)\sqrt{-c}x^2\arctan\left(\frac{\sqrt{cx^4+b}}{cx^2}\right.}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*((B*b + 2*A*c)*sqrt(c)*x^2*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sq
rt(c)) + 2*sqrt(c*x^4 + b*x^2)*(B*c*x^2 - 2*A*c))/(c*x^2), -1/2*((B*b + 2*A
*c)*sqrt(-c)*x^2*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) - sqrt(c*
x^4 + b*x^2)*(B*c*x^2 - 2*A*c))/(c*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x^2\left(b+cx^2\right)}\left(A+Bx^2\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**3, x)
```

Giac [A] time = 1.23202, size = 124, normalized size = 1.28

$$\frac{1}{2}\,\sqrt{cx^2+b}Bx\mathrm{sgn}\left(x\right)+\frac{2\,Ab\sqrt{c}\mathrm{sgn}\left(x\right)}{\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b}-\frac{\left(Bb\sqrt{c}\mathrm{sgn}\left(x\right)+2\,Ac^{\frac{3}{2}}\mathrm{sgn}\left(x\right)\right)\log\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^2 + b)*B*x*sgn(x) + 2*A*b*sqrt(c)*sgn(x)/((sqrt(c)*x - sqrt(c*
x^2 + b))^2 - b) - 1/4*(B*b*sqrt(c)*sgn(x) + 2*A*c^(3/2)*sgn(x))*log((sqrt(
c)*x - sqrt(c*x^2 + b))^2)/c
```

3.95 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^5}\,dx$

Optimal. Leaf size=80

$$-\frac{A\left(bx^2+cx^4\right)^{3/2}}{3bx^6}-\frac{B\sqrt{bx^2+cx^4}}{x^2}+B\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)$$

```
[Out] -((B*Sqrt[b*x^2 + c*x^4])/x^2) - (A*(b*x^2 + c*x^4)^(3/2))/(3*b*x^6) + B*Sq
rt[c]*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]]
```

Rubi [A] time = 0.195615, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2034, 792, 662, 620, 206}

$$-\frac{A\left(bx^2+cx^4\right)^{3/2}}{3bx^6}-\frac{B\sqrt{bx^2+cx^4}}{x^2}+B\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^5,x]
```

```
[Out] -((B*Sqrt[b*x^2 + c*x^4])/x^2) - (A*(b*x^2 + c*x^4)^(3/2))/(3*b*x^6) + B*Sq
rt[c]*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]]
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
```

```
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + p + 1)), x]
- Dist[(c*p)/(e^2*(m + p + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(A+Bx^2\right)\sqrt{bx^2+cx^4}}{x^5}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^3}\,dx,x,x^2\right)\\
&= -\frac{A\left(bx^2+cx^4\right)^{3/2}}{3bx^6}+\frac{1}{2}B\operatorname{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x^2}\,dx,x,x^2\right)\\
&= -\frac{B\sqrt{bx^2+cx^4}}{x^2}-\frac{A\left(bx^2+cx^4\right)^{3/2}}{3bx^6}+\frac{1}{2}(Bc)\operatorname{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)\\
&= -\frac{B\sqrt{bx^2+cx^4}}{x^2}-\frac{A\left(bx^2+cx^4\right)^{3/2}}{3bx^6}+(Bc)\operatorname{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x^2}{\sqrt{bx^2+cx^4}}\right)\\
&= -\frac{B\sqrt{bx^2+cx^4}}{x^2}-\frac{A\left(bx^2+cx^4\right)^{3/2}}{3bx^6}+B\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)
\end{aligned}$$

Mathematica [A] time = 0.122246, size = 86, normalized size = 1.08

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(-A\left(b+cx^2\right)+\frac{3\sqrt{b}B\sqrt{c}x^3\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{\sqrt{\frac{cx^2}{b}+1}}-3bBx^2\right)}{3bx^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^5,x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(-3*b*B*x^2 - A*(b + c*x^2) + (3*Sqrt[b]*B*Sqrt[c]*x
^3*ArcSinh[(Sqrt[c]*x)/Sqrt[b]])/Sqrt[1 + (c*x^2)/b]))/(3*b*x^4)
```

Maple [A] time = 0.01, size = 109, normalized size = 1.4

$$-\frac{1}{3\,bx^4}\sqrt{cx^4+bx^2}\left(-3\,Bc^{3/2}\sqrt{cx^2+b}x^4+3\,B\sqrt{c}\left(cx^2+b\right)^{3/2}x^2-3\,B\ln\left(x\sqrt{c}+\sqrt{cx^2+b}\right)x^3bc+A\sqrt{c}\left(cx^2+b\right)^{\frac{3}{2}}\right)\frac{1}{\sqrt{cx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^5,x)
```

```
[Out] -1/3*(c*x^4+b*x^2)^(1/2)*(-3*B*c^(3/2)*(c*x^2+b)^(1/2)*x^4+3*B*c^(1/2)*(c*x
^2+b)^(3/2)*x^2-3*B*ln(x*c^(1/2)+(c*x^2+b)^(1/2))*x^3*b*c+A*c^(1/2)*(c*x^2+
b)^(3/2))/x^4/(c*x^2+b)^(1/2)/b/c^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.09415, size = 363, normalized size = 4.54

$$\left[\frac{3\,Bb\sqrt{c}x^4\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\,\sqrt{cx^4+bx^2}\left((3\,Bb+Ac)x^2+Ab\right)}{6\,bx^4},-\frac{3\,Bb\sqrt{-c}x^4\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right)+\cdots}{3\,b\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/6*(3*B*b*sqrt(c)*x^4*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) -
 2*sqrt(c*x^4 + b*x^2)*((3*B*b + A*c)*x^2 + A*b))/(b*x^4), -1/3*(3*B*b*sqrt
(-c)*x^4*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) + sqrt(c*x^4 + b*
x^2)*((3*B*b + A*c)*x^2 + A*b))/(b*x^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x^2\left(b+cx^2\right)}\left(A+Bx^2\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**5, x)
```

Giac [B] time = 1.54875, size = 220, normalized size = 2.75

$$-\frac{1}{2}\,B\sqrt{c}\log\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2\right)\mathrm{sgn}\left(x\right)+\frac{2\left(3\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^4Bb\sqrt{c}\mathrm{sgn}\left(x\right)+3\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^4Ac^{\frac{3}{2}}\mathrm{sgn}\left(x\right)-6\left(\cdots\right.\right.}{3\left(\left(\sqrt{c}x-\sqrt{cx^2+\cdots}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] -1/2*B*sqrt(c)*log((sqrt(c)*x - sqrt(c*x^2 + b))^2)*sgn(x) + 2/3*(3*(sqrt(c
)*x - sqrt(c*x^2 + b))^4*B*b*sqrt(c)*sgn(x) + 3*(sqrt(c)*x - sqrt(c*x^2 + b
))^4*A*c^(3/2)*sgn(x) - 6*(sqrt(c)*x - sqrt(c*x^2 + b))^2*B*b^2*sqrt(c)*sgn
(x) + 3*B*b^3*sqrt(c)*sgn(x) + A*b^2*c^(3/2)*sgn(x))/((sqrt(c)*x - sqrt(c*x
^2 + b))^2 - b)^3
```

3.96 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^7}\,dx$

Optimal. Leaf size=61

$$-\frac{\left(bx^2+cx^4\right)^{3/2}(5bB-2Ac)}{15b^2x^6}-\frac{A\left(bx^2+cx^4\right)^{3/2}}{5bx^8}$$

```
[Out] -(A*(b*x^2 + c*x^4)^(3/2))/(5*b*x^8) - ((5*b*B - 2*A*c)*(b*x^2 + c*x^4)^(3/
2))/(15*b^2*x^6)
```

Rubi [A] time = 0.161813, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2034, 792, 650}

$$-\frac{\left(bx^2+cx^4\right)^{3/2}(5bB-2Ac)}{15b^2x^6}-\frac{A\left(bx^2+cx^4\right)^{3/2}}{5bx^8}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^7,x]
```

```
[Out] -(A*(b*x^2 + c*x^4)^(3/2))/(5*b*x^8) - ((5*b*B - 2*A*c)*(b*x^2 + c*x^4)^(3/
2))/(15*b^2*x^6)
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
```

```
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(A+Bx^2\right)\sqrt{bx^2+cx^4}}{x^7}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^4}\,dx, x, x^2\right)\\ &= -\frac{A\left(bx^2+cx^4\right)^{3/2}}{5bx^8}+\frac{\left(-4(-bB+Ac)+\frac{3}{2}(-bB+2Ac)\right)\operatorname{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x^3}\,dx, x, x^2\right)}{5b}\\ &= -\frac{A\left(bx^2+cx^4\right)^{3/2}}{5bx^8}-\frac{(5bB-2Ac)\left(bx^2+cx^4\right)^{3/2}}{15b^2x^6}\end{aligned}$$

Mathematica [A] time = 0.0176179, size = 44, normalized size = 0.72

$$-\frac{\left(x^2\left(b+cx^2\right)\right)^{3/2}\left(3Ab-2Acx^2+5bBx^2\right)}{15b^2x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^7,x]
```

```
[Out] -((x^2*(b + c*x^2))^(3/2)*(3*A*b + 5*b*B*x^2 - 2*A*c*x^2))/(15*b^2*x^8)
```

Maple [A] time = 0.006, size = 48, normalized size = 0.8

$$-\frac{\left(cx^2+b\right)\left(-2\,Ax^2c+5\,Bx^2b+3\,Ab\right)}{15\,b^2x^6}\sqrt{cx^4+bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^7,x)
```

```
[Out] -1/15*(c*x^2+b)*(-2*A*c*x^2+5*B*b*x^2+3*A*b)*(c*x^4+b*x^2)^(1/2)/b^2/x^6
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.919044, size = 131, normalized size = 2.15

$$-\frac{\left(\left(5\,Bbc-2\,Ac^{2}\right)x^{4}+3\,Ab^{2}+\left(5\,Bb^{2}+Abc\right)x^{2}\right)\sqrt{cx^{4}+bx^{2}}}{15\,b^{2}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] -1/15*((5*B*b*c - 2*A*c^2)*x^4 + 3*A*b^2 + (5*B*b^2 + A*b*c)*x^2)*sqrt(c*x^
4 + b*x^2)/(b^2*x^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^{2}\left(b+cx^{2}\right)}\left(A+Bx^{2}\right)}{x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**7,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**7, x)
```

Giac [B] time = 1.8069, size = 338, normalized size = 5.54

$$\frac{2\left(15\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^8Bc^{\frac{3}{2}}\operatorname{sgn}(x)-30\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^6Bbc^{\frac{3}{2}}\operatorname{sgn}(x)+30\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^6Ac^{\frac{5}{2}}\operatorname{sgn}(x)+20\left(\sqrt{c}x\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] 2/15*(15*(sqrt(c)*x - sqrt(c*x^2 + b))^8*B*c^(3/2)*sgn(x) - 30*(sqrt(c)*x -
 sqrt(c*x^2 + b))^6*B*b*c^(3/2)*sgn(x) + 30*(sqrt(c)*x - sqrt(c*x^2 + b))^6
*A*c^(5/2)*sgn(x) + 20*(sqrt(c)*x - sqrt(c*x^2 + b))^4*B*b^2*c^(3/2)*sgn(x)
 + 10*(sqrt(c)*x - sqrt(c*x^2 + b))^4*A*b*c^(5/2)*sgn(x) - 10*(sqrt(c)*x -
sqrt(c*x^2 + b))^2*B*b^3*c^(3/2)*sgn(x) + 10*(sqrt(c)*x - sqrt(c*x^2 + b))^
2*A*b^2*c^(5/2)*sgn(x) + 5*B*b^4*c^(3/2)*sgn(x) - 2*A*b^3*c^(5/2)*sgn(x))/(
(sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^5
```

3.97 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^9}\,dx$

Optimal. Leaf size=96

$$\frac{2c\left(bx^2+cx^4\right)^{3/2}(7bB-4Ac)}{105b^3x^6}-\frac{\left(bx^2+cx^4\right)^{3/2}(7bB-4Ac)}{35b^2x^8}-\frac{A\left(bx^2+cx^4\right)^{3/2}}{7bx^{10}}$$

```
[Out] -(A*(b*x^2 + c*x^4)^(3/2))/(7*b*x^10) - ((7*b*B - 4*A*c)*(b*x^2 + c*x^4)^(3
/2))/(35*b^2*x^8) + (2*c*(7*b*B - 4*A*c)*(b*x^2 + c*x^4)^(3/2))/(105*b^3*x^
6)
```

Rubi [A] time = 0.21089, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2034, 792, 658, 650}

$$\frac{2c\left(bx^2+cx^4\right)^{3/2}(7bB-4Ac)}{105b^3x^6}-\frac{\left(bx^2+cx^4\right)^{3/2}(7bB-4Ac)}{35b^2x^8}-\frac{A\left(bx^2+cx^4\right)^{3/2}}{7bx^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^9,x]
```

```
[Out] -(A*(b*x^2 + c*x^4)^(3/2))/(7*b*x^10) - ((7*b*B - 4*A*c)*(b*x^2 + c*x^4)^(3
/2))/(35*b^2*x^8) + (2*c*(7*b*B - 4*A*c)*(b*x^2 + c*x^4)^(3/2))/(105*b^3*x^
6)
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
```

```
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(A+Bx^2\right)\sqrt{bx^2+cx^4}}{x^9}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^5}\,dx,x,x^2\right)\\ &= -\frac{A\left(bx^2+cx^4\right)^{3/2}}{7bx^{10}}+\frac{\left(-5(-bB+Ac)+\frac{3}{2}(-bB+2Ac)\right)\operatorname{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x^4}\,dx,x,x^2\right)}{7b}\\ &= -\frac{A\left(bx^2+cx^4\right)^{3/2}}{7bx^{10}}-\frac{(7bB-4Ac)\left(bx^2+cx^4\right)^{3/2}}{35b^2x^8}-\frac{(c(7bB-4Ac))\operatorname{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x^3}\,dx,\right.}{35b^2}\\ &= -\frac{A\left(bx^2+cx^4\right)^{3/2}}{7bx^{10}}-\frac{(7bB-4Ac)\left(bx^2+cx^4\right)^{3/2}}{35b^2x^8}+\frac{2c(7bB-4Ac)\left(bx^2+cx^4\right)^{3/2}}{105b^3x^6}\end{aligned}$$

Mathematica [A] time = 0.0247352, size = 66, normalized size = 0.69

$$\frac{\left(x^2\left(b+cx^2\right)\right)^{3/2}\left(A\left(-15b^2+12bcx^2-8c^2x^4\right)+7bBx^2\left(2cx^2-3b\right)\right)}{105b^3x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^9,x]
```

```
[Out] ((x^2*(b + c*x^2))^(3/2)*(7*b*B*x^2*(-3*b + 2*c*x^2) + A*(-15*b^2 + 12*b*c*
x^2 - 8*c^2*x^4)))/(105*b^3*x^10)
```

Maple [A] time = 0.005, size = 70, normalized size = 0.7

$$-\frac{\left(cx^{2}+b\right)\left(8\,Ac^{2}x^{4}-14\,Bx^{4}bc-12\,Abcx^{2}+21\,Bx^{2}b^{2}+15\,Ab^{2}\right)}{105\,x^{8}b^{3}}\sqrt{cx^{4}+bx^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^9,x)
```

```
[Out] -1/105*(c*x^2+b)*(8*A*c^2*x^4-14*B*b*c*x^4-12*A*b*c*x^2+21*B*b^2*x^2+15*A*b
^2)*(c*x^4+b*x^2)^(1/2)/x^8/b^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^9,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.11355, size = 184, normalized size = 1.92

$$\frac{\left(2\left(7\,Bbc^{2}-4\,Ac^{3}\right)x^{6}-\left(7\,Bb^{2}c-4\,Abc^{2}\right)x^{4}-15\,Ab^{3}-3\left(7\,Bb^{3}+Ab^{2}c\right)x^{2}\right)\sqrt{cx^{4}+bx^{2}}}{105\,b^{3}x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^9,x, algorithm="fricas")
```

```
[Out] 1/105*(2*(7*B*b*c^2 - 4*A*c^3)*x^6 - (7*B*b^2*c - 4*A*b*c^2)*x^4 - 15*A*b^3
 - 3*(7*B*b^3 + A*b^2*c)*x^2)*sqrt(c*x^4 + b*x^2)/(b^3*x^8)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2\left(b+cx^2\right)}\left(A+Bx^2\right)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**9,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**9, x)
```

Giac [B] time = 2.31339, size = 419, normalized size = 4.36

$$4\left(105\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{10}Bc^{\frac{5}{2}}\mathrm{sgn}\left(x\right)-175\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{8}Bbc^{\frac{5}{2}}\mathrm{sgn}\left(x\right)+280\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{8}Ac^{\frac{7}{2}}\mathrm{sgn}\left(x\right)+70\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^9,x, algorithm="giac")
```

```
[Out] 4/105*(105*(sqrt(c)*x - sqrt(c*x^2 + b))^10*B*c^(5/2)*sgn(x) - 175*(sqrt(c)
*x - sqrt(c*x^2 + b))^8*B*b*c^(5/2)*sgn(x) + 280*(sqrt(c)*x - sqrt(c*x^2 +
b))^8*A*c^(7/2)*sgn(x) + 70*(sqrt(c)*x - sqrt(c*x^2 + b))^6*B*b^2*c^(5/2)*s
gn(x) + 140*(sqrt(c)*x - sqrt(c*x^2 + b))^6*A*b*c^(7/2)*sgn(x) - 42*(sqrt(c
)*x - sqrt(c*x^2 + b))^4*B*b^3*c^(5/2)*sgn(x) + 84*(sqrt(c)*x - sqrt(c*x^2
+ b))^4*A*b^2*c^(7/2)*sgn(x) + 49*(sqrt(c)*x - sqrt(c*x^2 + b))^2*B*b^4*c^(
5/2)*sgn(x) - 28*(sqrt(c)*x - sqrt(c*x^2 + b))^2*A*b^3*c^(7/2)*sgn(x) - 7*B
*b^5*c^(5/2)*sgn(x) + 4*A*b^4*c^(7/2)*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + b)
)^2 - b)^7
```

3.98 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{11}}\,dx$

Optimal. Leaf size=133

$$-\frac{8c^2\left(bx^2+cx^4\right)^{3/2}(3bB-2Ac)}{315b^4x^6}+\frac{4c\left(bx^2+cx^4\right)^{3/2}(3bB-2Ac)}{105b^3x^8}-\frac{\left(bx^2+cx^4\right)^{3/2}(3bB-2Ac)}{21b^2x^{10}}-\frac{A\left(bx^2+cx^4\right)^{3/2}}{9bx^{12}}$$

```
[Out] -(A*(b*x^2 + c*x^4)^(3/2))/(9*b*x^12) - ((3*b*B - 2*A*c)*(b*x^2 + c*x^4)^(3
/2))/(21*b^2*x^10) + (4*c*(3*b*B - 2*A*c)*(b*x^2 + c*x^4)^(3/2))/(105*b^3*x
^8) - (8*c^2*(3*b*B - 2*A*c)*(b*x^2 + c*x^4)^(3/2))/(315*b^4*x^6)
```

Rubi [A] time = 0.244999, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2034, 792, 658, 650}

$$-\frac{8c^2\left(bx^2+cx^4\right)^{3/2}(3bB-2Ac)}{315b^4x^6}+\frac{4c\left(bx^2+cx^4\right)^{3/2}(3bB-2Ac)}{105b^3x^8}-\frac{\left(bx^2+cx^4\right)^{3/2}(3bB-2Ac)}{21b^2x^{10}}-\frac{A\left(bx^2+cx^4\right)^{3/2}}{9bx^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^11,x]
```

```
[Out] -(A*(b*x^2 + c*x^4)^(3/2))/(9*b*x^12) - ((3*b*B - 2*A*c)*(b*x^2 + c*x^4)^(3
/2))/(21*b^2*x^10) + (4*c*(3*b*B - 2*A*c)*(b*x^2 + c*x^4)^(3/2))/(105*b^3*x
^8) - (8*c^2*(3*b*B - 2*A*c)*(b*x^2 + c*x^4)^(3/2))/(315*b^4*x^6)
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
```

```
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{\left(A+Bx^2\right)\sqrt{bx^2+cx^4}}{x^{11}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^6}\,dx,x,x^2\right)$$

$$= -\frac{A\left(bx^2+cx^4\right)^{3/2}}{9bx^{12}} + \frac{\left(-6(-bB+Ac)+\frac{3}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x^5}\,dx,x,x^2\right)}{9b}$$

$$= -\frac{A\left(bx^2+cx^4\right)^{3/2}}{9bx^{12}} - \frac{(3bB-2Ac)\left(bx^2+cx^4\right)^{3/2}}{21b^2x^{10}} - \frac{(2c(3bB-2Ac))\,\text{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x^4}\,dx\right.}{21b^2}$$

$$= -\frac{A\left(bx^2+cx^4\right)^{3/2}}{9bx^{12}} - \frac{(3bB-2Ac)\left(bx^2+cx^4\right)^{3/2}}{21b^2x^{10}} + \frac{4c(3bB-2Ac)\left(bx^2+cx^4\right)^{3/2}}{105b^3x^8} + \frac{(4c^2}{}$$

$$= -\frac{A\left(bx^2+cx^4\right)^{3/2}}{9bx^{12}} - \frac{(3bB-2Ac)\left(bx^2+cx^4\right)^{3/2}}{21b^2x^{10}} + \frac{4c(3bB-2Ac)\left(bx^2+cx^4\right)^{3/2}}{105b^3x^8} - \frac{8c^2(3}{}$$

Mathematica [A] time = 0.0294505, size = 88, normalized size = 0.66

$$-\frac{\left(x^2\left(b+cx^2\right)\right)^{3/2}\left(A\left(-30b^2cx^2+35b^3+24bc^2x^4-16c^3x^6\right)+3bBx^2\left(15b^2-12bcx^2+8c^2x^4\right)\right)}{315b^4x^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^11,x]
```

```
[Out] -((x^2*(b + c*x^2))^(3/2)*(3*b*B*x^2*(15*b^2 - 12*b*c*x^2 + 8*c^2*x^4) + A*
(35*b^3 - 30*b^2*c*x^2 + 24*b*c^2*x^4 - 16*c^3*x^6)))/(315*b^4*x^12)
```

Maple [A] time = 0.006, size = 94, normalized size = 0.7

$$-\frac{\left(cx^2+b\right)\left(-16\,Ac^3x^6+24\,Bx^6bc^2+24\,Abc^2x^4-36\,Bx^4b^2c-30\,Ab^2cx^2+45\,Bx^2b^3+35\,Ab^3\right)}{315\,x^{10}b^4}\sqrt{cx^4+bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^11,x)
```

```
[Out] -1/315*(c*x^2+b)*(-16*A*c^3*x^6+24*B*b*c^2*x^6+24*A*b*c^2*x^4-36*B*b^2*c*x^
4-30*A*b^2*c*x^2+45*B*b^3*x^2+35*A*b^3)*(c*x^4+b*x^2)^(1/2)/x^10/b^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^11,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.35793, size = 238, normalized size = 1.79

$$-\frac{\left(8\left(3\,Bbc^3-2\,Ac^4\right)x^8-4\left(3\,Bb^2c^2-2\,Abc^3\right)x^6+35\,Ab^4+3\left(3\,Bb^3c-2\,Ab^2c^2\right)x^4+5\left(9\,Bb^4+Ab^3c\right)x^2\right)\sqrt{cx^4+bx^2}}{315\,b^4x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^11,x, algorithm="fricas")
```

```
[Out] -1/315*(8*(3*B*b*c^3 - 2*A*c^4)*x^8 - 4*(3*B*b^2*c^2 - 2*A*b*c^3)*x^6 + 35*
A*b^4 + 3*(3*B*b^3*c - 2*A*b^2*c^2)*x^4 + 5*(9*B*b^4 + A*b^3*c)*x^2)*sqrt(c
*x^4 + b*x^2)/(b^4*x^10)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2\left(b+cx^2\right)}\left(A+Bx^2\right)}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**11,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**11, x)
```

Giac [B] time = 3.13908, size = 500, normalized size = 3.76

$$\frac{16\left(210\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{12}Bc^{\frac{7}{2}}\operatorname{sgn}(x)-315\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{10}Bbc^{\frac{7}{2}}\operatorname{sgn}(x)+630\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{10}Ac^{\frac{9}{2}}\operatorname{sgn}(x)+\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^11,x, algorithm="giac")
```

```
[Out] 16/315*(210*(sqrt(c)*x - sqrt(c*x^2 + b))^12*B*c^(7/2)*sgn(x) - 315*(sqrt(c
)*x - sqrt(c*x^2 + b))^10*B*b*c^(7/2)*sgn(x) + 630*(sqrt(c)*x - sqrt(c*x^2
+ b))^10*A*c^(9/2)*sgn(x) + 63*(sqrt(c)*x - sqrt(c*x^2 + b))^8*B*b^2*c^(7/2
)*sgn(x) + 378*(sqrt(c)*x - sqrt(c*x^2 + b))^8*A*b*c^(9/2)*sgn(x) - 42*(sqr
t(c)*x - sqrt(c*x^2 + b))^6*B*b^3*c^(7/2)*sgn(x) + 168*(sqrt(c)*x - sqrt(c*
x^2 + b))^6*A*b^2*c^(9/2)*sgn(x) + 108*(sqrt(c)*x - sqrt(c*x^2 + b))^4*B*b^
4*c^(7/2)*sgn(x) - 72*(sqrt(c)*x - sqrt(c*x^2 + b))^4*A*b^3*c^(9/2)*sgn(x)
- 27*(sqrt(c)*x - sqrt(c*x^2 + b))^2*B*b^5*c^(7/2)*sgn(x) + 18*(sqrt(c)*x -
```

```
 sqrt(c*x^2 + b))^2*A*b^4*c^(9/2)*sgn(x) + 3*B*b^6*c^(7/2)*sgn(x) - 2*A*b^5
*c^(9/2)*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^9
```

3.99 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{13}}\,dx$

Optimal. Leaf size=170

$$\frac{16c^3\left(bx^2+cx^4\right)^{3/2}(11bB-8Ac)}{3465b^5x^6}-\frac{8c^2\left(bx^2+cx^4\right)^{3/2}(11bB-8Ac)}{1155b^4x^8}+\frac{2c\left(bx^2+cx^4\right)^{3/2}(11bB-8Ac)}{231b^3x^{10}}-\frac{\left(bx^2+cx^4\right)^{3/2}(}{99b^2x}$$

```
[Out] -(A*(b*x^2 + c*x^4)^(3/2))/(11*b*x^14) - ((11*b*B - 8*A*c)*(b*x^2 + c*x^4)^
(3/2))/(99*b^2*x^12) + (2*c*(11*b*B - 8*A*c)*(b*x^2 + c*x^4)^(3/2))/(231*b^
3*x^10) - (8*c^2*(11*b*B - 8*A*c)*(b*x^2 + c*x^4)^(3/2))/(1155*b^4*x^8) + (
16*c^3*(11*b*B - 8*A*c)*(b*x^2 + c*x^4)^(3/2))/(3465*b^5*x^6)
```

Rubi [A] time = 0.300112, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2034, 792, 658, 650}

$$\frac{16c^3\left(bx^2+cx^4\right)^{3/2}(11bB-8Ac)}{3465b^5x^6}-\frac{8c^2\left(bx^2+cx^4\right)^{3/2}(11bB-8Ac)}{1155b^4x^8}+\frac{2c\left(bx^2+cx^4\right)^{3/2}(11bB-8Ac)}{231b^3x^{10}}-\frac{\left(bx^2+cx^4\right)^{3/2}(}{99b^2x}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^13,x]
```

```
[Out] -(A*(b*x^2 + c*x^4)^(3/2))/(11*b*x^14) - ((11*b*B - 8*A*c)*(b*x^2 + c*x^4)^
(3/2))/(99*b^2*x^12) + (2*c*(11*b*B - 8*A*c)*(b*x^2 + c*x^4)^(3/2))/(231*b^
3*x^10) - (8*c^2*(11*b*B - 8*A*c)*(b*x^2 + c*x^4)^(3/2))/(1155*b^4*x^8) + (
16*c^3*(11*b*B - 8*A*c)*(b*x^2 + c*x^4)^(3/2))/(3465*b^5*x^6)
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
```

```
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{\left(A+Bx^2\right)\sqrt{bx^2+cx^4}}{x^{13}}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^7}\,dx,x,x^2\right)$$

$$= -\frac{A\left(bx^2+cx^4\right)^{3/2}}{11bx^{14}}+\frac{\left(-7(-bB+Ac)+\frac{3}{2}(-bB+2Ac)\right)\mathrm{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x^6}\,dx,x,x^2\right)}{11b}$$

$$= -\frac{A\left(bx^2+cx^4\right)^{3/2}}{11bx^{14}}-\frac{(11bB-8Ac)\left(bx^2+cx^4\right)^{3/2}}{99b^2x^{12}}-\frac{(c(11bB-8Ac))\,\mathrm{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x^5}\,dx,\right.}{33b^2}$$

$$= -\frac{A\left(bx^2+cx^4\right)^{3/2}}{11bx^{14}}-\frac{(11bB-8Ac)\left(bx^2+cx^4\right)^{3/2}}{99b^2x^{12}}+\frac{2c(11bB-8Ac)\left(bx^2+cx^4\right)^{3/2}}{231b^3x^{10}}+\frac{\left(4c^2\right.}{}$$

$$= -\frac{A\left(bx^2+cx^4\right)^{3/2}}{11bx^{14}}-\frac{(11bB-8Ac)\left(bx^2+cx^4\right)^{3/2}}{99b^2x^{12}}+\frac{2c(11bB-8Ac)\left(bx^2+cx^4\right)^{3/2}}{231b^3x^{10}}-\frac{8c^2(}{}$$

$$= -\frac{A\left(bx^2+cx^4\right)^{3/2}}{11bx^{14}}-\frac{(11bB-8Ac)\left(bx^2+cx^4\right)^{3/2}}{99b^2x^{12}}+\frac{2c(11bB-8Ac)\left(bx^2+cx^4\right)^{3/2}}{231b^3x^{10}}-\frac{8c^2(}{}$$

Mathematica [A] time = 0.0684124, size = 94, normalized size = 0.55

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(x^2\left(\frac{cx^2}{b}+1\right)\left(-30b^2cx^2+35b^3+24bc^2x^4-16c^3x^6\right)(8Ac-11bB)-315Ab^3\left(b+cx^2\right)\right)}{3465b^4x^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^13,x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(-315*A*b^3*(b + c*x^2) + (-11*b*B + 8*A*c)*x^2*(1 +
 (c*x^2)/b)*(35*b^3 - 30*b^2*c*x^2 + 24*b*c^2*x^4 - 16*c^3*x^6)))/(3465*b^4
*x^12)
```

Maple [A] time = 0.006, size = 118, normalized size = 0.7

$$-\frac{\left(cx^2+b\right)\left(128\,Ac^4x^8-176\,Bbc^3x^8-192\,Abc^3x^6+264\,Bb^2c^2x^6+240\,Ab^2c^2x^4-330\,Bb^3cx^4-280\,Ab^3cx^2+385\,Bb^4x\right.}{3465\,x^{12}b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^13,x)
```

```
[Out] -1/3465*(c*x^2+b)*(128*A*c^4*x^8-176*B*b*c^3*x^8-192*A*b*c^3*x^6+264*B*b^2*
c^2*x^6+240*A*b^2*c^2*x^4-330*B*b^3*c*x^4-280*A*b^3*c*x^2+385*B*b^4*x^2+315
*A*b^4)*(c*x^4+b*x^2)^(1/2)/x^12/b^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^13,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.78683, size = 298, normalized size = 1.75

$$\frac{\left(16\left(11\,Bbc^4-8\,Ac^5\right)x^{10}-8\left(11\,Bb^2c^3-8\,Abc^4\right)x^8+6\left(11\,Bb^3c^2-8\,Ab^2c^3\right)x^6-315\,Ab^5-5\left(11\,Bb^4c-8\,Ab^3c^2\right)x^4-35}{3465\,b^5x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^13,x, algorithm="fricas")
```

```
[Out] 1/3465*(16*(11*B*b*c^4 - 8*A*c^5)*x^10 - 8*(11*B*b^2*c^3 - 8*A*b*c^4)*x^8 +
 6*(11*B*b^3*c^2 - 8*A*b^2*c^3)*x^6 - 315*A*b^5 - 5*(11*B*b^4*c - 8*A*b^3*c
^2)*x^4 - 35*(11*B*b^5 + A*b^4*c)*x^2)*sqrt(c*x^4 + b*x^2)/(b^5*x^12)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2\left(b+cx^2\right)}\left(A+Bx^2\right)}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**13,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**13, x)
```

Giac [B] time = 4.42151, size = 581, normalized size = 3.42

$$32\left(3465\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{14}Bc^{\frac{9}{2}}\mathrm{sgn}\left(x\right)-4851\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{12}Bbc^{\frac{9}{2}}\mathrm{sgn}\left(x\right)+11088\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{12}Ac^{\frac{11}{2}}\mathrm{sgn}\left(x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^13,x, algorithm="giac")
```

```
[Out] 32/3465*(3465*(sqrt(c)*x - sqrt(c*x^2 + b))^14*B*c^(9/2)*sgn(x) - 4851*(sqr
t(c)*x - sqrt(c*x^2 + b))^12*B*b*c^(9/2)*sgn(x) + 11088*(sqrt(c)*x - sqrt(c
*x^2 + b))^12*A*c^(11/2)*sgn(x) + 231*(sqrt(c)*x - sqrt(c*x^2 + b))^10*B*b^
```

```
2*c^(9/2)*sgn(x) + 7392*(sqrt(c)*x - sqrt(c*x^2 + b))^10*A*b*c^(11/2)*sgn(x
) - 165*(sqrt(c)*x - sqrt(c*x^2 + b))^8*B*b^3*c^(9/2)*sgn(x) + 2640*(sqrt(c
)*x - sqrt(c*x^2 + b))^8*A*b^2*c^(11/2)*sgn(x) + 1815*(sqrt(c)*x - sqrt(c*x
^2 + b))^6*B*b^4*c^(9/2)*sgn(x) - 1320*(sqrt(c)*x - sqrt(c*x^2 + b))^6*A*b^
3*c^(11/2)*sgn(x) - 605*(sqrt(c)*x - sqrt(c*x^2 + b))^4*B*b^5*c^(9/2)*sgn(x
) + 440*(sqrt(c)*x - sqrt(c*x^2 + b))^4*A*b^4*c^(11/2)*sgn(x) + 121*(sqrt(c
)*x - sqrt(c*x^2 + b))^2*B*b^6*c^(9/2)*sgn(x) - 88*(sqrt(c)*x - sqrt(c*x^2
+ b))^2*A*b^5*c^(11/2)*sgn(x) - 11*B*b^7*c^(9/2)*sgn(x) + 8*A*b^6*c^(11/2)*
sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^11
```

3.100 $\int x^4 \left(A + Bx^2\right) \sqrt{bx^2 + cx^4}\, dx$

Optimal. Leaf size=131

$$-\frac{8b^2\left(bx^2+cx^4\right)^{3/2}(2bB-3Ac)}{315c^4x^3}-\frac{x\left(bx^2+cx^4\right)^{3/2}(2bB-3Ac)}{21c^2}+\frac{4b\left(bx^2+cx^4\right)^{3/2}(2bB-3Ac)}{105c^3x}+\frac{Bx^3\left(bx^2+cx^4\right)^{3/2}}{9c}$$

```
[Out] (-8*b^2*(2*b*B - 3*A*c)*(b*x^2 + c*x^4)^(3/2))/(315*c^4*x^3) + (4*b*(2*b*B
- 3*A*c)*(b*x^2 + c*x^4)^(3/2))/(105*c^3*x) - ((2*b*B - 3*A*c)*x*(b*x^2 + c
*x^4)^(3/2))/(21*c^2) + (B*x^3*(b*x^2 + c*x^4)^(3/2))/(9*c)
```

Rubi [A] time = 0.219836, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2039, 2016, 2000}

$$-\frac{8b^2\left(bx^2+cx^4\right)^{3/2}(2bB-3Ac)}{315c^4x^3}-\frac{x\left(bx^2+cx^4\right)^{3/2}(2bB-3Ac)}{21c^2}+\frac{4b\left(bx^2+cx^4\right)^{3/2}(2bB-3Ac)}{105c^3x}+\frac{Bx^3\left(bx^2+cx^4\right)^{3/2}}{9c}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (-8*b^2*(2*b*B - 3*A*c)*(b*x^2 + c*x^4)^(3/2))/(315*c^4*x^3) + (4*b*(2*b*B
- 3*A*c)*(b*x^2 + c*x^4)^(3/2))/(105*c^3*x) - ((2*b*B - 3*A*c)*x*(b*x^2 + c
*x^4)^(3/2))/(21*c^2) + (B*x^3*(b*x^2 + c*x^4)^(3/2))/(9*c)
```

Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
```

```
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] && !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\begin{aligned}\int x^4\left(A+Bx^2\right)\sqrt{bx^2+cx^4}\,dx &= \frac{Bx^3\left(bx^2+cx^4\right)^{3/2}}{9c}-\frac{(6bB-9Ac)\int x^4\sqrt{bx^2+cx^4}\,dx}{9c}\\ &= -\frac{(2bB-3Ac)x\left(bx^2+cx^4\right)^{3/2}}{21c^2}+\frac{Bx^3\left(bx^2+cx^4\right)^{3/2}}{9c}+\frac{(4b(2bB-3Ac))\int x^2\sqrt{bx^2+cx^4}\cdots}{21c^2}\\ &= \frac{4b(2bB-3Ac)\left(bx^2+cx^4\right)^{3/2}}{105c^3x}-\frac{(2bB-3Ac)x\left(bx^2+cx^4\right)^{3/2}}{21c^2}+\frac{Bx^3\left(bx^2+cx^4\right)^{3/2}}{9c}-\cdots\\ &= -\frac{8b^2(2bB-3Ac)\left(bx^2+cx^4\right)^{3/2}}{315c^4x^3}+\frac{4b(2bB-3Ac)\left(bx^2+cx^4\right)^{3/2}}{105c^3x}-\frac{(2bB-3Ac)x\left(bx^2\cdots\right.}{21c^2}\end{aligned}$$

Mathematica [A] time = 0.0596196, size = 82, normalized size = 0.63

$$\frac{\left(x^2\left(b+cx^2\right)\right)^{3/2}\left(24b^2c\left(A+Bx^2\right)-6bc^2x^2\left(6A+5Bx^2\right)+5c^3x^4\left(9A+7Bx^2\right)-16b^3B\right)}{315c^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] ((x^2*(b + c*x^2))^(3/2)*(-16*b^3*B + 24*b^2*c*(A + B*x^2) - 6*b*c^2*x^2*(6
*A + 5*B*x^2) + 5*c^3*x^4*(9*A + 7*B*x^2)))/(315*c^4*x^3)
```

Maple [A] time = 0.006, size = 91, normalized size = 0.7

$$\frac{\left(cx^2+b\right)\left(35\,Bc^3x^6+45\,Ax^4c^3-30\,Bx^4bc^2-36\,Ax^2bc^2+24\,Bx^2b^2c+24\,Ab^2c-16\,Bb^3\right)}{315\,c^4x}\sqrt{cx^4+bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/315*(c*x^2+b)*(35*B*c^3*x^6+45*A*c^3*x^4-30*B*b*c^2*x^4-36*A*b*c^2*x^2+24
*B*b^2*c*x^2+24*A*b^2*c-16*B*b^3)*(c*x^4+b*x^2)^(1/2)/c^4/x
```

Maxima [A] time = 1.20731, size = 143, normalized size = 1.09

$$\frac{\left(15\,c^3x^6+3\,bc^2x^4-4\,b^2cx^2+8\,b^3\right)\sqrt{cx^2+b}A}{105\,c^3}+\frac{\left(35\,c^4x^8+5\,bc^3x^6-6\,b^2c^2x^4+8\,b^3cx^2-16\,b^4\right)\sqrt{cx^2+b}B}{315\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/105*(15*c^3*x^6 + 3*b*c^2*x^4 - 4*b^2*c*x^2 + 8*b^3)*sqrt(c*x^2 + b)*A/c^
3 + 1/315*(35*c^4*x^8 + 5*b*c^3*x^6 - 6*b^2*c^2*x^4 + 8*b^3*c*x^2 - 16*b^4)
*sqrt(c*x^2 + b)*B/c^4
```

Fricas [A] time = 1.11853, size = 230, normalized size = 1.76

$$\frac{\left(35\,Bc^4x^8+5\left(Bbc^3+9\,Ac^4\right)x^6-16\,Bb^4+24\,Ab^3c-3\left(2\,Bb^2c^2-3\,Abc^3\right)x^4+4\left(2\,Bb^3c-3\,Ab^2c^2\right)x^2\right)\sqrt{cx^4+bx^2}}{315\,c^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/315*(35*B*c^4*x^8 + 5*(B*b*c^3 + 9*A*c^4)*x^6 - 16*B*b^4 + 24*A*b^3*c - 3
*(2*B*b^2*c^2 - 3*A*b*c^3)*x^4 + 4*(2*B*b^3*c - 3*A*b^2*c^2)*x^2)*sqrt(c*x^
4 + b*x^2)/(c^4*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4\sqrt{x^2\left(b+cx^2\right)}\left(A+Bx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**4*sqrt(x**2*(b + c*x**2))*(A + B*x**2), x)
```

Giac [A] time = 1.16876, size = 180, normalized size = 1.37

$$\frac{\frac{3\left(15\left(cx^2+b\right)^{\frac{7}{2}}-42\left(cx^2+b\right)^{\frac{5}{2}}b+35\left(cx^2+b\right)^{\frac{3}{2}}b^2\right)A\mathrm{sgn}(x)}{c^2}+\frac{\left(35\left(cx^2+b\right)^{\frac{9}{2}}-135\left(cx^2+b\right)^{\frac{7}{2}}b+189\left(cx^2+b\right)^{\frac{5}{2}}b^2-105\left(cx^2+b\right)^{\frac{3}{2}}b^3\right)B\mathrm{sgn}(x)}{c^3}}{315\,c}+\frac{8\left(2\,Bb^{\frac{9}{2}}-3\,A\right.}{315}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/315*(3*(15*(c*x^2 + b)^(7/2) - 42*(c*x^2 + b)^(5/2)*b + 35*(c*x^2 + b)^(3
/2)*b^2)*A*sgn(x)/c^2 + (35*(c*x^2 + b)^(9/2) - 135*(c*x^2 + b)^(7/2)*b + 1
89*(c*x^2 + b)^(5/2)*b^2 - 105*(c*x^2 + b)^(3/2)*b^3)*B*sgn(x)/c^3)/c + 8/3
15*(2*B*b^(9/2) - 3*A*b^(7/2)*c)*sgn(x)/c^4
```

3.101 $\int x^2 \left(A+Bx^2\right) \sqrt{bx^2+cx^4} \, dx$

Optimal. Leaf size=94

$$-\frac{\left(bx^2+cx^4\right)^{3/2}(4bB-7Ac)}{35c^2x}+\frac{2b\left(bx^2+cx^4\right)^{3/2}(4bB-7Ac)}{105c^3x^3}+\frac{Bx\left(bx^2+cx^4\right)^{3/2}}{7c}$$

```
[Out] (2*b*(4*b*B - 7*A*c)*(b*x^2 + c*x^4)^(3/2))/(105*c^3*x^3) - ((4*b*B - 7*A*c
)*(b*x^2 + c*x^4)^(3/2))/(35*c^2*x) + (B*x*(b*x^2 + c*x^4)^(3/2))/(7*c)
```

Rubi [A] time = 0.165806, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2039, 2016, 2000}

$$-\frac{\left(bx^2+cx^4\right)^{3/2}(4bB-7Ac)}{35c^2x}+\frac{2b\left(bx^2+cx^4\right)^{3/2}(4bB-7Ac)}{105c^3x^3}+\frac{Bx\left(bx^2+cx^4\right)^{3/2}}{7c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*b*(4*b*B - 7*A*c)*(b*x^2 + c*x^4)^(3/2))/(105*c^3*x^3) - ((4*b*B - 7*A*c
)*(b*x^2 + c*x^4)^(3/2))/(35*c^2*x) + (B*x*(b*x^2 + c*x^4)^(3/2))/(7*c)
```

Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
```

```
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int x^2 \left(A+Bx^2\right) \sqrt{bx^2+cx^4}\,dx &= \frac{Bx\left(bx^2+cx^4\right)^{3/2}}{7c} - \frac{(4bB-7Ac)\int x^2\sqrt{bx^2+cx^4}\,dx}{7c} \\
&= -\frac{(4bB-7Ac)\left(bx^2+cx^4\right)^{3/2}}{35c^2x} + \frac{Bx\left(bx^2+cx^4\right)^{3/2}}{7c} + \frac{(2b(4bB-7Ac))\int\sqrt{bx^2+cx^4}\,dx}{35c^2} \\
&= \frac{2b(4bB-7Ac)\left(bx^2+cx^4\right)^{3/2}}{105c^3x^3} - \frac{(4bB-7Ac)\left(bx^2+cx^4\right)^{3/2}}{35c^2x} + \frac{Bx\left(bx^2+cx^4\right)^{3/2}}{7c}
\end{aligned}$$

Mathematica [A] time = 0.041685, size = 64, normalized size = 0.68

$$\frac{\left(x^2\left(b+cx^2\right)\right)^{3/2}\left(-2bc\left(7A+6Bx^2\right)+3c^2x^2\left(7A+5Bx^2\right)+8b^2B\right)}{105c^3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] ((x^2*(b + c*x^2))^(3/2)*(8*b^2*B + 3*c^2*x^2*(7*A + 5*B*x^2) - 2*b*c*(7*A
+ 6*B*x^2)))/(105*c^3*x^3)
```

Maple [A] time = 0.006, size = 67, normalized size = 0.7

$$-\frac{\left(cx^2+b\right)\left(-15\,Bc^2x^4-21\,Ax^2c^2+12\,Bx^2bc+14\,Abc-8\,Bb^2\right)}{105\,c^3x}\sqrt{cx^4+bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/105*(c*x^2+b)*(-15*B*c^2*x^4-21*A*c^2*x^2+12*B*b*c*x^2+14*A*b*c-8*B*b^2)
*(c*x^4+b*x^2)^(1/2)/c^3/x
```

Maxima [A] time = 1.10136, size = 112, normalized size = 1.19

$$\frac{\left(3\,c^2x^4+bcx^2-2\,b^2\right)\sqrt{cx^2+b}A}{15\,c^2}+\frac{\left(15\,c^3x^6+3\,bc^2x^4-4\,b^2cx^2+8\,b^3\right)\sqrt{cx^2+b}B}{105\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*c^2*x^4 + b*c*x^2 - 2*b^2)*sqrt(c*x^2 + b)*A/c^2 + 1/105*(15*c^3*x^
6 + 3*b*c^2*x^4 - 4*b^2*c*x^2 + 8*b^3)*sqrt(c*x^2 + b)*B/c^3
```

Fricas [A] time = 1.16213, size = 177, normalized size = 1.88

$$\frac{\left(15\,Bc^3x^6+3\left(Bbc^2+7\,Ac^3\right)x^4+8\,Bb^3-14\,Ab^2c-\left(4\,Bb^2c-7\,Abc^2\right)x^2\right)\sqrt{cx^4+bx^2}}{105\,c^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*B*c^3*x^6 + 3*(B*b*c^2 + 7*A*c^3)*x^4 + 8*B*b^3 - 14*A*b^2*c - (4
*B*b^2*c - 7*A*b*c^2)*x^2)*sqrt(c*x^4 + b*x^2)/(c^3*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{x^2\left(b+cx^2\right)}\left(A+Bx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(x**2*(b + c*x**2))*(A + B*x**2), x)
```

Giac [A] time = 1.16053, size = 142, normalized size = 1.51

$$\frac{\frac{7\left(3\left(cx^2+b\right)^{\frac{5}{2}}-5\left(cx^2+b\right)^{\frac{3}{2}}b\right)A\mathrm{sgn}(x)}{c}+\frac{\left(15\left(cx^2+b\right)^{\frac{7}{2}}-42\left(cx^2+b\right)^{\frac{5}{2}}b+35\left(cx^2+b\right)^{\frac{3}{2}}b^2\right)B\mathrm{sgn}(x)}{c^2}}{105\,c}-\frac{2\left(4\,Bb^{\frac{7}{2}}-7\,Ab^{\frac{5}{2}}c\right)\mathrm{sgn}\,(x)}{105\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/105*(7*(3*(c*x^2 + b)^(5/2) - 5*(c*x^2 + b)^(3/2)*b)*A*sgn(x)/c + (15*(c*
x^2 + b)^(7/2) - 42*(c*x^2 + b)^(5/2)*b + 35*(c*x^2 + b)^(3/2)*b^2)*B*sgn(x
)/c^2)/c - 2/105*(4*B*b^(7/2) - 7*A*b^(5/2)*c)*sgn(x)/c^3
```

3.102 $\int \left(A+Bx^2\right)\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=61

$$\frac{B\left(bx^2+cx^4\right)^{3/2}}{5cx}-\frac{\left(bx^2+cx^4\right)^{3/2}(2bB-5Ac)}{15c^2x^3}$$

```
[Out] -((2*b*B - 5*A*c)*(b*x^2 + c*x^4)^(3/2))/(15*c^2*x^3) + (B*(b*x^2 + c*x^4)^
(3/2))/(5*c*x)
```

Rubi [A] time = 0.018547, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1145, 2000}

$$\frac{B\left(bx^2+cx^4\right)^{3/2}}{5cx}-\frac{\left(bx^2+cx^4\right)^{3/2}(2bB-5Ac)}{15c^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```



```
[Out] -((2*b*B - 5*A*c)*(b*x^2 + c*x^4)^(3/2))/(15*c^2*x^3) + (B*(b*x^2 + c*x^4)^
(3/2))/(5*c*x)
```


Rule 1145

```
Int[((d_) + (e_.)*(x_)^2)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
Simp[(e*(b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 3)*x), x] - Dist[(b*e*(2*p + 1)
- c*d*(4*p + 3))/(c*(4*p + 3)), Int[(b*x^2 + c*x^4)^p, x], x] /; FreeQ[{b,
c, d, e, p}, x] &&  !IntegerQ[p] && NeQ[4*p + 3, 0] && NeQ[b*e*(2*p + 1) -
c*d*(4*p + 3), 0]
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\int \left(A+Bx^2\right)\sqrt{bx^2+cx^4}\,dx = \frac{B\left(bx^2+cx^4\right)^{3/2}}{5cx} - \frac{(2bB-5Ac)\int \sqrt{bx^2+cx^4}\,dx}{5c}$$
$$= -\frac{(2bB-5Ac)\left(bx^2+cx^4\right)^{3/2}}{15c^2x^3} + \frac{B\left(bx^2+cx^4\right)^{3/2}}{5cx}$$

Mathematica [A] time = 0.0253083, size = 41, normalized size = 0.67

$$\frac{\left(x^2\left(b+cx^2\right)\right)^{3/2}\left(5Ac-2bB+3Bcx^2\right)}{15c^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] ((x^2*(b + c*x^2))^(3/2)*(-2*b*B + 5*A*c + 3*B*c*x^2))/(15*c^2*x^3)
```

Maple [A] time = 0.005, size = 45, normalized size = 0.7

$$\frac{\left(cx^2+b\right)\left(3\,Bcx^2+5\,Ac-2\,Bb\right)}{15\,c^2x}\sqrt{cx^4+bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/15*(c*x^2+b)*(3*B*c*x^2+5*A*c-2*B*b)*(c*x^4+b*x^2)^(1/2)/c^2/x
```

Maxima [A] time = 1.20372, size = 69, normalized size = 1.13

$$\frac{\left(cx^2+b\right)^{\frac{3}{2}}A}{3\,c} + \frac{\left(3\,c^2x^4+bcx^2-2\,b^2\right)\sqrt{cx^2+b}B}{15\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(c*x^2 + b)^(3/2)*A/c + 1/15*(3*c^2*x^4 + b*c*x^2 - 2*b^2)*sqrt(c*x^2 +
 b)*B/c^2
```

Fricas [A] time = 0.961428, size = 124, normalized size = 2.03

$$\frac{\left(3\,Bc^2x^4 - 2\,Bb^2 + 5\,Abc + \left(Bbc + 5\,Ac^2\right)x^2\right)\sqrt{cx^4 + bx^2}}{15\,c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*B*c^2*x^4 - 2*B*b^2 + 5*A*b*c + (B*b*c + 5*A*c^2)*x^2)*sqrt(c*x^4 +
 b*x^2)/(c^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^2\left(b + cx^2\right)}\left(A + Bx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2), x)
```

Giac [A] time = 1.21547, size = 99, normalized size = 1.62

$$\frac{5\left(cx^2 + b\right)^{\frac{3}{2}}A\mathrm{sgn}\left(x\right) + \frac{\left(3\left(cx^2+b\right)^{\frac{5}{2}} - 5\left(cx^2+b\right)^{\frac{3}{2}}b\right)B\mathrm{sgn}(x)}{c}}{15\,c} + \frac{\left(2\,Bb^{\frac{5}{2}} - 5\,Ab^{\frac{3}{2}}c\right)\mathrm{sgn}\left(x\right)}{15\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/15*(5*(c*x^2 + b)^(3/2)*A*sgn(x) + (3*(c*x^2 + b)^(5/2) - 5*(c*x^2 + b)^(
3/2)*b)*B*sgn(x)/c)/c + 1/15*(2*B*b^(5/2) - 5*A*b^(3/2)*c)*sgn(x)/c^2
```

3.103 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^2}\,dx$

Optimal. Leaf size=78

$$\frac{A\sqrt{bx^2+cx^4}}{x} - A\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right) + \frac{B\left(bx^2+cx^4\right)^{3/2}}{3cx^3}$$

```
[Out] (A*Sqrt[b*x^2 + c*x^4])/x + (B*(b*x^2 + c*x^4)^(3/2))/(3*c*x^3) - A*Sqrt[b]
*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]]
```

Rubi [A] time = 0.145952, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2039, 2021, 2008, 206}

$$\frac{A\sqrt{bx^2+cx^4}}{x} - A\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right) + \frac{B\left(bx^2+cx^4\right)^{3/2}}{3cx^3}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^2,x]
```

```
[Out] (A*Sqrt[b*x^2 + c*x^4])/x + (B*(b*x^2 + c*x^4)^(3/2))/(3*c*x^3) - A*Sqrt[b]
*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]]
```

Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
```

```
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^2}\,dx &= \frac{B(bx^2+cx^4)^{3/2}}{3cx^3}+A\int \frac{\sqrt{bx^2+cx^4}}{x^2}\,dx\\
&= \frac{A\sqrt{bx^2+cx^4}}{x}+\frac{B(bx^2+cx^4)^{3/2}}{3cx^3}+(Ab)\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx\\
&= \frac{A\sqrt{bx^2+cx^4}}{x}+\frac{B(bx^2+cx^4)^{3/2}}{3cx^3}-(Ab)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2+cx^4}}\right)\\
&= \frac{A\sqrt{bx^2+cx^4}}{x}+\frac{B(bx^2+cx^4)^{3/2}}{3cx^3}-A\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0769255, size = 84, normalized size = 1.08

$$\frac{x\left((b+cx^2)(3Ac+bB+Bcx^2)-3A\sqrt{b}c\sqrt{b+cx^2}\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)\right)}{3c\sqrt{x^2(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^2,x]
```

```
[Out] (x*((b + c*x^2)*(b*B + 3*A*c + B*c*x^2) - 3*A*Sqrt[b]*c*Sqrt[b + c*x^2]*Arc
Tanh[Sqrt[b + c*x^2]/Sqrt[b]]))/(3*c*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.007, size = 85, normalized size = 1.1

$$-\frac{1}{3cx}\sqrt{cx^4+bx^2}\left(3A\ln\left(2\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)\sqrt{b}c-B\left(cx^2+b\right)^{\frac{3}{2}}-3A\sqrt{cx^2+b}c\right)\frac{1}{\sqrt{cx^2+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^2,x)
```

```
[Out] -1/3*(c*x^4+b*x^2)^(1/2)*(3*A*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*b^(1/2)*c
-B*(c*x^2+b)^(3/2)-3*A*(c*x^2+b)^(1/2)*c)/x/(c*x^2+b)^(1/2)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$A\int\frac{\sqrt{cx^2+b}}{x}\,dx+\frac{\left(cx^2+b\right)^{\frac{3}{2}}B}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] A*integrate(sqrt(c*x^2 + b)/x, x) + 1/3*(c*x^2 + b)^(3/2)*B/c
```

Fricas [A] time = 1.25291, size = 359, normalized size = 4.6

$$\left[\frac{3A\sqrt{b}cx\log\left(-\frac{cx^3+2bx-2\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)+2\sqrt{cx^4+bx^2}\left(Bcx^2+Bb+3Ac\right)}{6cx},\frac{3A\sqrt{-b}cx\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-b}}{cx^3+bx}\right)+\sqrt{cx^4+bx}}{3cx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/6*(3*A*sqrt(b)*c*x*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/
x^3) + 2*sqrt(c*x^4 + b*x^2)*(B*c*x^2 + B*b + 3*A*c))/(c*x), 1/3*(3*A*sqrt(
```

```
-b)*c*x*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + sqrt(c*x^4 + b
*x^2)*(B*c*x^2 + B*b + 3*A*c))/(c*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2\left(b+cx^2\right)}\left(A+Bx^2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**2, x)
```

Giac [A] time = 1.15775, size = 157, normalized size = 2.01

$$\frac{Ab\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)\operatorname{sgn}(x)}{\sqrt{-b}} - \frac{\left(3\,Abc\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right) + B\sqrt{-b}b^{\frac{3}{2}} + 3\,A\sqrt{-b}\sqrt{b}c\right)\operatorname{sgn}(x)}{3\sqrt{-b}c} + \frac{\left(cx^2+b\right)^{\frac{3}{2}}Bc^2\operatorname{sgn}(x) + 3\sqrt{cx^2}}{3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] A*b*arctan(sqrt(c*x^2 + b)/sqrt(-b))*sgn(x)/sqrt(-b) - 1/3*(3*A*b*c*arctan(
sqrt(b)/sqrt(-b)) + B*sqrt(-b)*b^(3/2) + 3*A*sqrt(-b)*sqrt(b)*c)*sgn(x)/(sq
rt(-b)*c) + 1/3*((c*x^2 + b)^(3/2)*B*c^2*sgn(x) + 3*sqrt(c*x^2 + b)*A*c^3*s
gn(x))/c^3
```

3.104 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^4} dx$

Optimal. Leaf size=100

$$\frac{\sqrt{bx^2+cx^4}(Ac+2bB)}{2bx}-\frac{(Ac+2bB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{b}}-\frac{A\left(bx^2+cx^4\right)^{3/2}}{2bx^5}$$

```
[Out] ((2*b*B + A*c)*Sqrt[b*x^2 + c*x^4])/(2*b*x) - (A*(b*x^2 + c*x^4)^(3/2))/(2*
b*x^5) - ((2*b*B + A*c)*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(2*Sqrt[b
])
```

Rubi [A] time = 0.16086, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2038, 2021, 2008, 206}

$$\frac{\sqrt{bx^2+cx^4}(Ac+2bB)}{2bx}-\frac{(Ac+2bB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{b}}-\frac{A\left(bx^2+cx^4\right)^{3/2}}{2bx^5}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^4,x]
```

```
[Out] ((2*b*B + A*c)*Sqrt[b*x^2 + c*x^4])/(2*b*x) - (A*(b*x^2 + c*x^4)^(3/2))/(2*
b*x^5) - ((2*b*B + A*c)*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(2*Sqrt[b
])
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(A+Bx^2\right)\sqrt{bx^2+cx^4}}{x^4}\,dx &= -\frac{A\left(bx^2+cx^4\right)^{3/2}}{2bx^5}-\frac{(-2bB-Ac)\int \frac{\sqrt{bx^2+cx^4}}{x^2}\,dx}{2b}\\
&= \frac{(2bB+Ac)\sqrt{bx^2+cx^4}}{2bx}-\frac{A\left(bx^2+cx^4\right)^{3/2}}{2bx^5}-\frac{1}{2}(-2bB-Ac)\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx\\
&= \frac{(2bB+Ac)\sqrt{bx^2+cx^4}}{2bx}-\frac{A\left(bx^2+cx^4\right)^{3/2}}{2bx^5}-\frac{1}{2}(2bB+Ac)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2}}\right.\\
&= \frac{(2bB+Ac)\sqrt{bx^2+cx^4}}{2bx}-\frac{A\left(bx^2+cx^4\right)^{3/2}}{2bx^5}-\frac{(2bB+Ac)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.04587, size = 94, normalized size = 0.94

$$-\frac{\sqrt{x^2\left(b+cx^2\right)}\left(\sqrt{b}\left(A-2Bx^2\right)\sqrt{b+cx^2}+x^2(Ac+2bB)\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)\right)}{2\sqrt{b}x^3\sqrt{b+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^4,x]
```

```
[Out] -(Sqrt[x^2*(b + c*x^2)]*(Sqrt[b]*(A - 2*B*x^2)*Sqrt[b + c*x^2] + (2*b*B + A
*c)*x^2*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]]))/(2*Sqrt[b]*x^3*Sqrt[b + c*x^2])
```

Maple [A] time = 0.01, size = 135, normalized size = 1.4

$$-\frac{1}{2\,b x^3}\sqrt{c x^4+b x^2}\left(A\sqrt{b}\ln\left(2\,\frac{\sqrt{b}\sqrt{c x^2+b}+b}{x}\right)x^2 c+2\,B b^{3/2}\ln\left(2\,\frac{\sqrt{b}\sqrt{c x^2+b}+b}{x}\right)x^2-A\sqrt{c x^2+b}x^2 c-2\,B\sqrt{c x^2+b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^4,x)
```

```
[Out] -1/2*(c*x^4+b*x^2)^(1/2)*(A*b^(1/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*x^2
*c+2*B*b^(3/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*x^2-A*(c*x^2+b)^(1/2)*x^
2*c-2*B*(c*x^2+b)^(1/2)*x^2*b+A*(c*x^2+b)^(3/2))/x^3/(c*x^2+b)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c x^4+b x^2}\left(B x^2+A\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^4, x)
```

Fricas [A] time = 1.04321, size = 375, normalized size = 3.75

$$\left[\frac{(2\,B b+A c)\sqrt{b}x^3\log\left(-\frac{c x^3+2\,b x-2\,\sqrt{c x^4+b x^2}\sqrt{b}}{x^3}\right)+2\,\sqrt{c x^4+b x^2}\left(2\,B b x^2-A b\right)}{4\,b x^3},\ \frac{(2\,B b+A c)\sqrt{-b}x^3\arctan\left(\frac{\sqrt{c x^4+b x^2}\sqrt{-b}}{c x^3+b x}\right)+}{2\,b x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/4*((2*B*b + A*c)*sqrt(b)*x^3*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)
*sqrt(b))/x^3) + 2*sqrt(c*x^4 + b*x^2)*(2*B*b*x^2 - A*b))/(b*x^3), 1/2*((2*
B*b + A*c)*sqrt(-b)*x^3*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x))
+ sqrt(c*x^4 + b*x^2)*(2*B*b*x^2 - A*b))/(b*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2\left(b+cx^2\right)}\left(A+Bx^2\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**4, x)
```

Giac [A] time = 1.24104, size = 103, normalized size = 1.03

$$\frac{2\sqrt{cx^2+b}Bc\,\mathrm{sgn}\left(x\right)+\frac{\left(2\,Bbc\mathrm{sgn}(x)+Ac^2\mathrm{sgn}(x)\right)\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}}-\frac{\sqrt{cx^2+b}Ac\mathrm{sgn}(x)}{x^2}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/2*(2*sqrt(c*x^2 + b)*B*c*sgn(x) + (2*B*b*c*sgn(x) + A*c^2*sgn(x))*arctan(
sqrt(c*x^2 + b)/sqrt(-b))/sqrt(-b) - sqrt(c*x^2 + b)*A*c*sgn(x)/x^2)/c
```

3.105 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^6}\,dx$

Optimal. Leaf size=103

$$-\frac{c(4bB-Ac)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8b^{3/2}}-\frac{\sqrt{bx^2+cx^4}(4bB-Ac)}{8bx^3}-\frac{A\left(bx^2+cx^4\right)^{3/2}}{4bx^7}$$

```
[Out] -((4*b*B - A*c)*Sqrt[b*x^2 + c*x^4])/(8*b*x^3) - (A*(b*x^2 + c*x^4)^(3/2))/
(4*b*x^7) - (c*(4*b*B - A*c)*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(8*b
^(3/2))
```

Rubi [A] time = 0.157621, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2038, 2020, 2008, 206}

$$-\frac{c(4bB-Ac)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8b^{3/2}}-\frac{\sqrt{bx^2+cx^4}(4bB-Ac)}{8bx^3}-\frac{A\left(bx^2+cx^4\right)^{3/2}}{4bx^7}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^6,x]
```

```
[Out] -((4*b*B - A*c)*Sqrt[b*x^2 + c*x^4])/(8*b*x^3) - (A*(b*x^2 + c*x^4)^(3/2))/
(4*b*x^7) - (c*(4*b*B - A*c)*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(8*b
^(3/2))
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(A+Bx^2\right)\sqrt{bx^2+cx^4}}{x^6}\,dx &= -\frac{A\left(bx^2+cx^4\right)^{3/2}}{4bx^7}-\frac{(-4bB+Ac)\int \frac{\sqrt{bx^2+cx^4}}{x^4}\,dx}{4b}\\
&= -\frac{(4bB-Ac)\sqrt{bx^2+cx^4}}{8bx^3}-\frac{A\left(bx^2+cx^4\right)^{3/2}}{4bx^7}+\frac{(c(4bB-Ac))\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{8b}\\
&= -\frac{(4bB-Ac)\sqrt{bx^2+cx^4}}{8bx^3}-\frac{A\left(bx^2+cx^4\right)^{3/2}}{4bx^7}-\frac{(c(4bB-Ac))\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2+cx^4}}\right)}{8b}\\
&= -\frac{(4bB-Ac)\sqrt{bx^2+cx^4}}{8bx^3}-\frac{A\left(bx^2+cx^4\right)^{3/2}}{4bx^7}-\frac{c(4bB-Ac)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8b^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.107429, size = 95, normalized size = 0.92

$$
-\frac{\left(b+cx^2\right)\left(2Ab+Acx^2+4bBx^2\right)+cx^4\sqrt{\frac{cx^2}{b}+1}(4bB-Ac)\tanh^{-1}\left(\sqrt{\frac{cx^2}{b}+1}\right)}{8bx^3\sqrt{x^2\left(b+cx^2\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^6,x]
```

```
[Out] -((b + c*x^2)*(2*A*b + 4*b*B*x^2 + A*c*x^2) + c*(4*b*B - A*c)*x^4*Sqrt[1 +
(c*x^2)/b]*ArcTanh[Sqrt[1 + (c*x^2)/b]])/(8*b*x^3*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.01, size = 174, normalized size = 1.7

$$\frac{1}{8\,x^5b^2}\sqrt{cx^4+bx^2}\left(A\sqrt{b}\ln\left(2\,\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)x^4c^2-4\,Bb^{3/2}\ln\left(2\,\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)x^4c-A\sqrt{cx^2+b}x^4c^2+4\,B\sqrt{cx^2-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^6,x)
```

```
[Out] 1/8*(c*x^4+b*x^2)^(1/2)*(A*b^(1/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*x^4*
c^2-4*B*b^(3/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*x^4*c-A*(c*x^2+b)^(1/2)
*x^4*c^2+4*B*(c*x^2+b)^(1/2)*x^4*b*c+A*(c*x^2+b)^(3/2)*x^2*c-4*B*(c*x^2+b)^
(3/2)*x^2*b-2*A*(c*x^2+b)^(3/2)*b)/x^5/(c*x^2+b)^(1/2)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{cx^4+bx^2}\left(Bx^2+A\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^6, x)
```

Fricas [A] time = 1.07663, size = 437, normalized size = 4.24

$$\left[-\frac{\left(4\,Bbc-Ac^2\right)\sqrt{b}x^5\log\left(-\frac{cx^3+2\,bx+2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)+2\,\sqrt{cx^4+bx^2}\left(2\,Ab^2+\left(4\,Bb^2+Abc\right)x^2\right)}{16\,b^2x^5},\frac{\left(4\,Bbc-Ac^2\right)\sqrt{-b}x^5\,\mathrm{arct}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] [-1/16*((4*B*b*c - A*c^2)*sqrt(b)*x^5*log(-(c*x^3 + 2*b*x + 2*sqrt(c*x^4 +
b*x^2)*sqrt(b))/x^3) + 2*sqrt(c*x^4 + b*x^2)*(2*A*b^2 + (4*B*b^2 + A*b*c)*x
^2))/(b^2*x^5), 1/8*((4*B*b*c - A*c^2)*sqrt(-b)*x^5*arctan(sqrt(c*x^4 + b*x
^2)*sqrt(-b)/(c*x^3 + b*x)) - sqrt(c*x^4 + b*x^2)*(2*A*b^2 + (4*B*b^2 + A*b
*c)*x^2))/(b^2*x^5)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2\left(b + cx^2\right)}\left(A + Bx^2\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**6,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**6, x)
```

Giac [A] time = 1.24954, size = 178, normalized size = 1.73

$$\frac{\frac{\left(4\,Bbc^2\text{sgn}(x) - Ac^3\text{sgn}(x)\right)\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b} - \frac{4\left(cx^2+b\right)^{\frac{3}{2}}Bbc^2\text{sgn}(x) - 4\sqrt{cx^2+b}Bb^2c^2\text{sgn}(x) + \left(cx^2+b\right)^{\frac{3}{2}}Ac^3\text{sgn}(x) + \sqrt{cx^2+b}Abc^3\text{sgn}(x)}{bc^2x^4}}{8\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/8*((4*B*b*c^2*sgn(x) - A*c^3*sgn(x))*arctan(sqrt(c*x^2 + b)/sqrt(-b))/(sq
rt(-b)*b) - (4*(c*x^2 + b)^(3/2)*B*b*c^2*sgn(x) - 4*sqrt(c*x^2 + b)*B*b^2*c
^2*sgn(x) + (c*x^2 + b)^(3/2)*A*c^3*sgn(x) + sqrt(c*x^2 + b)*A*b*c^3*sgn(x)
)/(b*c^2*x^4))/c
```

3.106 $\int x^5 \left(A+Bx^2\right) \left(bx^2+cx^4\right)^{3/2} \, dx$

Optimal. Leaf size=223

$$\frac{b^4 \left(b+2cx^2\right) \sqrt{bx^2+cx^4}(9bB-14Ac)}{2048c^5}-\frac{b^2 \left(b+2cx^2\right) \left(bx^2+cx^4\right)^{3/2} (9bB-14Ac)}{768c^4}-\frac{b^6(9bB-14Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2048c^{11/2}}$$

```
[Out] (b^4*(9*b*B - 14*A*c)*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(2048*c^5) - (b^2*
(9*b*B - 14*A*c)*(b + 2*c*x^2)*(b*x^2 + c*x^4)^(3/2))/(768*c^4) + (b*(9*b*B
 - 14*A*c)*(b*x^2 + c*x^4)^(5/2))/(240*c^3) - ((9*b*B - 14*A*c)*x^2*(b*x^2
+ c*x^4)^(5/2))/(168*c^2) + (B*x^4*(b*x^2 + c*x^4)^(5/2))/(14*c) - (b^6*(9*
b*B - 14*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(2048*c^(11/2))
```

Rubi [A] time = 0.403646, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {2034, 794, 670, 640, 612, 620, 206}

$$\frac{b^4 \left(b+2cx^2\right) \sqrt{bx^2+cx^4}(9bB-14Ac)}{2048c^5}-\frac{b^2 \left(b+2cx^2\right) \left(bx^2+cx^4\right)^{3/2} (9bB-14Ac)}{768c^4}-\frac{b^6(9bB-14Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2048c^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```


```
[Out] (b^4*(9*b*B - 14*A*c)*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(2048*c^5) - (b^2*
(9*b*B - 14*A*c)*(b + 2*c*x^2)*(b*x^2 + c*x^4)^(3/2))/(768*c^4) + (b*(9*b*B
 - 14*A*c)*(b*x^2 + c*x^4)^(5/2))/(240*c^3) - ((9*b*B - 14*A*c)*x^2*(b*x^2
+ c*x^4)^(5/2))/(168*c^2) + (B*x^4*(b*x^2 + c*x^4)^(5/2))/(14*c) - (b^6*(9*
b*B - 14*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(2048*c^(11/2))
```


Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^5\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}dx=\frac{1}{2}\text{Subst}\left(\int x^2(A+Bx)\left(bx+cx^2\right)^{3/2}dx,x,x^2\right)$$
$$=\frac{Bx^4\left(bx^2+cx^4\right)^{5/2}}{14c}+\frac{\left(2(-bB+Ac)+\frac{5}{2}(-bB+2Ac)\right)\text{Subst}\left(\int x^2\left(bx+cx^2\right)^{3/2}dx\right.}{14c}$$
$$=-\frac{(9bB-14Ac)x^2\left(bx^2+cx^4\right)^{5/2}}{168c^2}+\frac{Bx^4\left(bx^2+cx^4\right)^{5/2}}{14c}+\frac{(b(9bB-14Ac))\text{Subst}\left(\int\right.}{48}$$
$$=\frac{b(9bB-14Ac)\left(bx^2+cx^4\right)^{5/2}}{240c^3}-\frac{(9bB-14Ac)x^2\left(bx^2+cx^4\right)^{5/2}}{168c^2}+\frac{Bx^4\left(bx^2+cx^4\right)^{5/}}{14c}$$
$$=-\frac{b^2(9bB-14Ac)\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{768c^4}+\frac{b(9bB-14Ac)\left(bx^2+cx^4\right)^{5/2}}{240c^3}-(9bB$$
$$=\frac{b^4(9bB-14Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{2048c^5}-\frac{b^2(9bB-14Ac)\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{768c^4}$$
$$=\frac{b^4(9bB-14Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{2048c^5}-\frac{b^2(9bB-14Ac)\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{768c^4}$$
$$=\frac{b^4(9bB-14Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{2048c^5}-\frac{b^2(9bB-14Ac)\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{768c^4}$$

Mathematica [A] time = 0.324806, size = 215, normalized size = 0.96

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(\sqrt{c}x\sqrt{\frac{cx^2}{b}+1}\left(96b^2c^4x^6\left(7A+4Bx^2\right)-16b^3c^3x^4\left(49A+27Bx^2\right)+28b^4c^2x^2\left(35A+18Bx^2\right)-210b^5c\left(7\right.\right.\right.}{215040c^{11/2}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(Sqrt[c]*x*Sqrt[1 + (c*x^2)/b]*(945*b^6*B - 210*b^5*
c*(7*A + 3*B*x^2) + 96*b^2*c^4*x^6*(7*A + 4*B*x^2) + 2560*c^6*x^10*(7*A + 6
*B*x^2) + 28*b^4*c^2*x^2*(35*A + 18*B*x^2) - 16*b^3*c^3*x^4*(49*A + 27*B*x^
2) + 256*b*c^5*x^8*(91*A + 75*B*x^2)) - 105*b^(11/2)*(9*b*B - 14*A*c)*ArcSi
nh[(Sqrt[c]*x)/Sqrt[b]]))/(215040*c^(11/2)*x*Sqrt[1 + (c*x^2)/b])
```

Maple [A] time = 0.032, size = 328, normalized size = 1.5

$$\frac{1}{215040\,x^3}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(15360\,B\left(cx^2+b\right)^{5/2}c^{9/2}x^9+17920\,A\left(cx^2+b\right)^{5/2}c^{9/2}x^7-11520\,B\left(cx^2+b\right)^{5/2}c^{7/2}x^7b-12544\,A\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/215040*(c*x^4+b*x^2)^(3/2)*(15360*B*(c*x^2+b)^(5/2)*c^(9/2)*x^9+17920*A*(
c*x^2+b)^(5/2)*c^(9/2)*x^7-11520*B*(c*x^2+b)^(5/2)*c^(7/2)*x^7*b-12544*A*(c
*x^2+b)^(5/2)*c^(7/2)*x^5*b+8064*B*(c*x^2+b)^(5/2)*c^(5/2)*x^5*b^2+7840*A*(
c*x^2+b)^(5/2)*c^(5/2)*x^3*b^2-5040*B*(c*x^2+b)^(5/2)*c^(3/2)*x^3*b^3-3920*
A*(c*x^2+b)^(5/2)*c^(3/2)*x*b^3+2520*B*(c*x^2+b)^(5/2)*c^(1/2)*x*b^4+980*A*
(c*x^2+b)^(3/2)*c^(3/2)*x*b^4-630*B*(c*x^2+b)^(3/2)*c^(1/2)*x*b^5+1470*A*(c
*x^2+b)^(1/2)*c^(3/2)*x*b^5-945*B*(c*x^2+b)^(1/2)*c^(1/2)*x*b^6+1470*A*ln(x
*c^(1/2)+(c*x^2+b)^(1/2))*b^6*c-945*B*ln(x*c^(1/2)+(c*x^2+b)^(1/2))*b^7)/x^
3/(c*x^2+b)^(3/2)/c^(11/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.33344, size = 984, normalized size = 4.41

$$\left[-\frac{105\left(9\,Bb^7-14\,Ab^6c\right)\sqrt{c}\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\left(15360\,Bc^7x^{12}+1280\left(15\,Bbc^6+14\,Ac^7\right)x^{10}+128\left(3\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/430080*(105*(9*B*b^7 - 14*A*b^6*c)*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*
x^4 + b*x^2)*sqrt(c)) - 2*(15360*B*c^7*x^12 + 1280*(15*B*b*c^6 + 14*A*c^7)*
x^10 + 128*(3*B*b^2*c^5 + 182*A*b*c^6)*x^8 + 945*B*b^6*c - 1470*A*b^5*c^2 -
 48*(9*B*b^3*c^4 - 14*A*b^2*c^5)*x^6 + 56*(9*B*b^4*c^3 - 14*A*b^3*c^4)*x^4
- 70*(9*B*b^5*c^2 - 14*A*b^4*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^6, 1/215040*(
105*(9*B*b^7 - 14*A*b^6*c)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*
x^2 + b)) + (15360*B*c^7*x^12 + 1280*(15*B*b*c^6 + 14*A*c^7)*x^10 + 128*(3*
B*b^2*c^5 + 182*A*b*c^6)*x^8 + 945*B*b^6*c - 1470*A*b^5*c^2 - 48*(9*B*b^3*c
^4 - 14*A*b^2*c^5)*x^6 + 56*(9*B*b^4*c^3 - 14*A*b^3*c^4)*x^4 - 70*(9*B*b^5*
c^2 - 14*A*b^4*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^6]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \left(x^2 \left(b + cx^2\right)\right)^{\frac{3}{2}} \left(A + Bx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**5*(x**2*(b + c*x**2))**(3/2)*(A + B*x**2), x)
```

Giac [A] time = 1.16556, size = 378, normalized size = 1.7

$$\frac{1}{215040}\left(2\left(4\left(2\left(8\left(10\left(12\,Bcx^2\mathrm{sgn}\left(x\right)+\frac{15\,Bbc^{12}\mathrm{sgn}\left(x\right)+14\,Ac^{13}\mathrm{sgn}\left(x\right)}{c^{12}}\right)x^2+\frac{3\,Bb^2c^{11}\mathrm{sgn}\left(x\right)+182\,Abc^{12}\mathrm{sgn}\left(x\right)}{c^{12}}\right)x\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/215040*(2*(4*(2*(8*(10*(12*B*c*x^2*sgn(x) + (15*B*b*c^12*sgn(x) + 14*A*c^
13*sgn(x))/c^12)*x^2 + (3*B*b^2*c^11*sgn(x) + 182*A*b*c^12*sgn(x))/c^12)*x^
2 - 3*(9*B*b^3*c^10*sgn(x) - 14*A*b^2*c^11*sgn(x))/c^12)*x^2 + 7*(9*B*b^4*c
^9*sgn(x) - 14*A*b^3*c^10*sgn(x))/c^12)*x^2 - 35*(9*B*b^5*c^8*sgn(x) - 14*A
*b^4*c^9*sgn(x))/c^12)*x^2 + 105*(9*B*b^6*c^7*sgn(x) - 14*A*b^5*c^8*sgn(x))
/c^12)*sqrt(c*x^2 + b)*x + 1/2048*(9*B*b^7*sgn(x) - 14*A*b^6*c*sgn(x))*log(
abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/c^(11/2) - 1/4096*(9*B*b^7*log(abs(b)) -
 14*A*b^6*c*log(abs(b)))*sgn(x)/c^(11/2)
```

3.107 $\int x^3 \left(A+Bx^2\right) \left(bx^2+cx^4\right)^{3/2} \, dx$

Optimal. Leaf size=167

$$-\frac{b^3\left(b+2cx^2\right)\sqrt{bx^2+cx^4}(7bB-12Ac)}{1024c^4}+\frac{b^5(7bB-12Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{1024c^{9/2}}+\frac{b\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}(7bB-12Ac)}{384c^3}$$

```
[Out] -(b^3*(7*b*B - 12*A*c)*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(1024*c^4) + (b*(
7*b*B - 12*A*c)*(b + 2*c*x^2)*(b*x^2 + c*x^4)^(3/2))/(384*c^3) - ((7*b*B -
12*A*c - 10*B*c*x^2)*(b*x^2 + c*x^4)^(5/2))/(120*c^2) + (b^5*(7*b*B - 12*A*
c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(1024*c^(9/2))
```

Rubi [A] time = 0.247852, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2034, 779, 612, 620, 206}

$$-\frac{b^3\left(b+2cx^2\right)\sqrt{bx^2+cx^4}(7bB-12Ac)}{1024c^4}+\frac{b^5(7bB-12Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{1024c^{9/2}}+\frac{b\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}(7bB-12Ac)}{384c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(b^3*(7*b*B - 12*A*c)*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(1024*c^4) + (b*(
7*b*B - 12*A*c)*(b + 2*c*x^2)*(b*x^2 + c*x^4)^(3/2))/(384*c^3) - ((7*b*B -
12*A*c - 10*B*c*x^2)*(b*x^2 + c*x^4)^(5/2))/(120*c^2) + (b^5*(7*b*B - 12*A*
c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(1024*c^(9/2))
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
```

```
2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^3\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{1}{2}\text{Subst}\left(\int x(A+Bx)\left(bx+cx^2\right)^{3/2}\,dx,x,x^2\right)$$

$$=-\frac{\left(7bB-12Ac-10Bcx^2\right)\left(bx^2+cx^4\right)^{5/2}}{120c^2}+\frac{(b(7bB-12Ac))\,\text{Subst}\left(\int\left(bx+cx^2\right)^{3/2}\right.}{48c^2}$$

$$=\frac{b(7bB-12Ac)\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{384c^3}-\frac{\left(7bB-12Ac-10Bcx^2\right)\left(bx^2+cx^4\right)^{5/2}}{120c^2}$$

$$=-\frac{b^3(7bB-12Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{1024c^4}+\frac{b(7bB-12Ac)\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{384c^3}$$

$$=-\frac{b^3(7bB-12Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{1024c^4}+\frac{b(7bB-12Ac)\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{384c^3}$$

$$=-\frac{b^3(7bB-12Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{1024c^4}+\frac{b(7bB-12Ac)\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{384c^3}$$

Mathematica [A] time = 0.282804, size = 193, normalized size = 1.16

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(\sqrt{c}x\sqrt{\frac{cx^2}{b}+1}\left(48b^2c^3x^4\left(2A+Bx^2\right)-8b^3c^2x^2\left(15A+7Bx^2\right)+10b^4c\left(18A+7Bx^2\right)+64bc^4x^6\left(33A+26\right.\right.\right.}{15360c^{9/2}x\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(Sqrt[c]*x*Sqrt[1 + (c*x^2)/b]*(-105*b^5*B + 48*b^2*
c^3*x^4*(2*A + B*x^2) + 256*c^5*x^8*(6*A + 5*B*x^2) - 8*b^3*c^2*x^2*(15*A +
 7*B*x^2) + 10*b^4*c*(18*A + 7*B*x^2) + 64*b*c^4*x^6*(33*A + 26*B*x^2)) + 1
5*b^(9/2)*(7*b*B - 12*A*c)*ArcSinh[(Sqrt[c]*x)/Sqrt[b]]))/(15360*c^(9/2)*x*
Sqrt[1 + (c*x^2)/b])
```

Maple [A] time = 0.011, size = 286, normalized size = 1.7

$$\frac{1}{15360\,x^3}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(1280\,Bc^{7/2}\left(cx^2+b\right)^{5/2}x^7+1536\,Ac^{7/2}\left(cx^2+b\right)^{5/2}x^5-896\,Bc^{5/2}\left(cx^2+b\right)^{5/2}x^5b-960\,Ac^{5/2}\left(cx^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/15360*(c*x^4+b*x^2)^(3/2)*(1280*B*c^(7/2)*(c*x^2+b)^(5/2)*x^7+1536*A*c^(7
/2)*(c*x^2+b)^(5/2)*x^5-896*B*c^(5/2)*(c*x^2+b)^(5/2)*x^5*b-960*A*c^(5/2)*(
c*x^2+b)^(5/2)*x^3*b+560*B*c^(3/2)*(c*x^2+b)^(5/2)*x^3*b^2+480*A*c^(3/2)*(c
*x^2+b)^(5/2)*x*b^2-280*B*c^(1/2)*(c*x^2+b)^(5/2)*x*b^3-120*A*(c*x^2+b)^(3/
2)*c^(3/2)*x*b^3+70*B*(c*x^2+b)^(3/2)*c^(1/2)*x*b^4-180*A*(c*x^2+b)^(1/2)*c
^(3/2)*x*b^4+105*B*(c*x^2+b)^(1/2)*c^(1/2)*x*b^5-180*A*ln(x*c^(1/2)+(c*x^2+
b)^(1/2))*b^5*c+105*B*ln(x*c^(1/2)+(c*x^2+b)^(1/2))*b^6)/x^3/(c*x^2+b)^(3/2
)/c^(9/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.69933, size = 853, normalized size = 5.11

$$\left[-\frac{15\left(7Bb^6-12Ab^5c\right)\sqrt{c}\log\left(-2cx^2-b+2\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\left(1280Bc^6x^{10}+128\left(13Bbc^5+12Ac^6\right)x^8-105Bb^5c\right.}{30720c^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/30720*(15*(7*B*b^6 - 12*A*b^5*c)*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^
4 + b*x^2)*sqrt(c)) - 2*(1280*B*c^6*x^10 + 128*(13*B*b*c^5 + 12*A*c^6)*x^8
- 105*B*b^5*c + 180*A*b^4*c^2 + 48*(B*b^2*c^4 + 44*A*b*c^5)*x^6 - 8*(7*B*b^
3*c^3 - 12*A*b^2*c^4)*x^4 + 10*(7*B*b^4*c^2 - 12*A*b^3*c^3)*x^2)*sqrt(c*x^4
 + b*x^2))/c^5, -1/15360*(15*(7*B*b^6 - 12*A*b^5*c)*sqrt(-c)*arctan(sqrt(c*
x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) - (1280*B*c^6*x^10 + 128*(13*B*b*c^5 + 1
2*A*c^6)*x^8 - 105*B*b^5*c + 180*A*b^4*c^2 + 48*(B*b^2*c^4 + 44*A*b*c^5)*x^
6 - 8*(7*B*b^3*c^3 - 12*A*b^2*c^4)*x^4 + 10*(7*B*b^4*c^2 - 12*A*b^3*c^3)*x^
2)*sqrt(c*x^4 + b*x^2))/c^5]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}\left(A+Bx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**3*(x**2*(b + c*x**2))**(3/2)*(A + B*x**2), x)
```

Giac [A] time = 1.18638, size = 332, normalized size = 1.99

$$\frac{1}{15360}\left(2\left(4\left(2\left(8\left(10\,Bcx^{2}\mathrm{sgn}\,(x)+\frac{13\,Bbc^{10}\mathrm{sgn}\,(x)+12\,Ac^{11}\mathrm{sgn}\,(x)}{c^{10}}\right)x^{2}+\frac{3\left(Bb^{2}c^{9}\mathrm{sgn}\,(x)+44\,Abc^{10}\mathrm{sgn}\,(x)\right)}{c^{10}}\right)x^{2}-\frac{7\,B\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/15360*(2*(4*(2*(8*(10*B*c*x^2*sgn(x) + (13*B*b*c^10*sgn(x) + 12*A*c^11*sg
n(x))/c^10)*x^2 + 3*(B*b^2*c^9*sgn(x) + 44*A*b*c^10*sgn(x))/c^10)*x^2 - (7*
B*b^3*c^8*sgn(x) - 12*A*b^2*c^9*sgn(x))/c^10)*x^2 + 5*(7*B*b^4*c^7*sgn(x) -
 12*A*b^3*c^8*sgn(x))/c^10)*x^2 - 15*(7*B*b^5*c^6*sgn(x) - 12*A*b^4*c^7*sgn
(x))/c^10)*sqrt(c*x^2 + b)*x - 1/1024*(7*B*b^6*sgn(x) - 12*A*b^5*c*sgn(x))*
log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/c^(9/2) + 1/2048*(7*B*b^6*log(abs(b)
) - 12*A*b^5*c*log(abs(b)))*sgn(x)/c^(9/2)
```

3.108 $\int x\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=148

$$\frac{3b^2\left(b+2cx^2\right)\sqrt{bx^2+cx^4}(bB-2Ac)}{256c^3}-\frac{3b^4(bB-2Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{256c^{7/2}}-\frac{\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}(bB-2Ac)}{32c^2}+\frac{B(}{}$$

```
[Out] (3*b^2*(b*B - 2*A*c)*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(256*c^3) - ((b*B -
 2*A*c)*(b + 2*c*x^2)*(b*x^2 + c*x^4)^(3/2))/(32*c^2) + (B*(b*x^2 + c*x^4)^
(5/2))/(10*c) - (3*b^4*(b*B - 2*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x
^4]])/(256*c^(7/2))
```

Rubi [A] time = 0.191446, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2034, 640, 612, 620, 206}

$$\frac{3b^2\left(b+2cx^2\right)\sqrt{bx^2+cx^4}(bB-2Ac)}{256c^3}-\frac{3b^4(bB-2Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{256c^{7/2}}-\frac{\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}(bB-2Ac)}{32c^2}+\frac{B(}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (3*b^2*(b*B - 2*A*c)*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(256*c^3) - ((b*B -
 2*A*c)*(b + 2*c*x^2)*(b*x^2 + c*x^4)^(3/2))/(32*c^2) + (B*(b*x^2 + c*x^4)^
(5/2))/(10*c) - (3*b^4*(b*B - 2*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x
^4]])/(256*c^(7/2))
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{1}{2}\,\text{Subst}\left(\int(A+Bx)\left(bx+cx^2\right)^{3/2}\,dx,x,x^2\right)$$

$$=\frac{B\left(bx^2+cx^4\right)^{5/2}}{10c}+\frac{(-bB+2Ac)\,\text{Subst}\left(\int\left(bx+cx^2\right)^{3/2}\,dx,x,x^2\right)}{4c}$$

$$=-\frac{(bB-2Ac)\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{32c^2}+\frac{B\left(bx^2+cx^4\right)^{5/2}}{10c}+\frac{\left(3b^2(bB-2Ac)\right)\text{Subst}\left(\right.}{64c}$$

$$=\frac{3b^2(bB-2Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{256c^3}-\frac{(bB-2Ac)\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{32c^2}+\frac{B\left(bx\right.}{}$$

$$=\frac{3b^2(bB-2Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{256c^3}-\frac{(bB-2Ac)\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{32c^2}+\frac{B\left(bx\right.}{}$$

$$=\frac{3b^2(bB-2Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{256c^3}-\frac{(bB-2Ac)\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{32c^2}+\frac{B\left(bx\right.}{}$$

Mathematica [A] time = 0.23249, size = 171, normalized size = 1.16

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(\sqrt{c}x\sqrt{\frac{cx^2}{b}+1}\left(4b^2c^2x^2\left(5A+2Bx^2\right)-10b^3c\left(3A+Bx^2\right)+16bc^3x^4\left(15A+11Bx^2\right)+32c^4x^6\left(5A+4Bx^2\right.\right.\right.}{1280c^{7/2}x\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(Sqrt[c]*x*Sqrt[1 + (c*x^2)/b]*(15*b^4*B - 10*b^3*c*
(3*A + B*x^2) + 4*b^2*c^2*x^2*(5*A + 2*B*x^2) + 32*c^4*x^6*(5*A + 4*B*x^2)
+ 16*b*c^3*x^4*(15*A + 11*B*x^2)) - 15*b^(7/2)*(b*B - 2*A*c)*ArcSinh[(Sqrt[
c]*x)/Sqrt[b]]))/(1280*c^(7/2)*x*Sqrt[1 + (c*x^2)/b])
```

Maple [A] time = 0.01, size = 244, normalized size = 1.7

$$\frac{1}{1280\,x^3}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(128\,Bc^{5/2}\left(cx^2+b\right)^{5/2}x^5+160\,Ac^{5/2}\left(cx^2+b\right)^{5/2}x^3-80\,Bc^{3/2}\left(cx^2+b\right)^{5/2}x^3b-80\,Ac^{3/2}\left(cx^2+b\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/1280*(c*x^4+b*x^2)^(3/2)*(128*B*c^(5/2)*(c*x^2+b)^(5/2)*x^5+160*A*c^(5/2)
*(c*x^2+b)^(5/2)*x^3-80*B*c^(3/2)*(c*x^2+b)^(5/2)*x^3*b-80*A*c^(3/2)*(c*x^2
+b)^(5/2)*x*b+40*B*c^(1/2)*(c*x^2+b)^(5/2)*x*b^2+20*A*c^(3/2)*(c*x^2+b)^(3/
2)*x*b^2-10*B*c^(1/2)*(c*x^2+b)^(3/2)*x*b^3+30*A*c^(3/2)*(c*x^2+b)^(1/2)*x*
b^3-15*B*c^(1/2)*(c*x^2+b)^(1/2)*x*b^4+30*A*ln(x*c^(1/2)+(c*x^2+b)^(1/2))*b
^4*c-15*B*ln(x*c^(1/2)+(c*x^2+b)^(1/2))*b^5)/x^3/(c*x^2+b)^(3/2)/c^(7/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.60786, size = 717, normalized size = 4.84

$$\left[-\frac{15\left(Bb^5-2\,Ab^4c\right)\sqrt{c}\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\left(128\,Bc^5x^8+16\left(11\,Bbc^4+10\,Ac^5\right)x^6+15\,Bb^4c-30\,Ab^3c\right.}{2560\,c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2560*(15*(B*b^5 - 2*A*b^4*c)*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 +
b*x^2)*sqrt(c)) - 2*(128*B*c^5*x^8 + 16*(11*B*b*c^4 + 10*A*c^5)*x^6 + 15*B*
b^4*c - 30*A*b^3*c^2 + 8*(B*b^2*c^3 + 30*A*b*c^4)*x^4 - 10*(B*b^3*c^2 - 2*A
*b^2*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^4, 1/1280*(15*(B*b^5 - 2*A*b^4*c)*sqr
t(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) + (128*B*c^5*x^8 + 1
6*(11*B*b*c^4 + 10*A*c^5)*x^6 + 15*B*b^4*c - 30*A*b^3*c^2 + 8*(B*b^2*c^3 +
30*A*b*c^4)*x^4 - 10*(B*b^3*c^2 - 2*A*b^2*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^
4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}\left(A+Bx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x*(x**2*(b + c*x**2))**(3/2)*(A + B*x**2), x)
```

Giac [A] time = 1.16955, size = 279, normalized size = 1.89

$$\frac{1}{1280}\left(2\left(4\left(2\left(8\,Bcx^2\mathrm{sgn}\left(x\right)+\frac{11\,Bbc^8\mathrm{sgn}\left(x\right)+10\,Ac^9\mathrm{sgn}\left(x\right)}{c^8}\right)x^2+\frac{Bb^2c^7\mathrm{sgn}\left(x\right)+30\,Abc^8\mathrm{sgn}\left(x\right)}{c^8}\right)x^2-\frac{5\left(Bb^3c^6\mathrm{sgn}\left(x\right.\right.}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/1280*(2*(4*(2*(8*B*c*x^2*sgn(x) + (11*B*b*c^8*sgn(x) + 10*A*c^9*sgn(x))/c
^8)*x^2 + (B*b^2*c^7*sgn(x) + 30*A*b*c^8*sgn(x))/c^8)*x^2 - 5*(B*b^3*c^6*sg
n(x) - 2*A*b^2*c^7*sgn(x))/c^8)*x^2 + 15*(B*b^4*c^5*sgn(x) - 2*A*b^3*c^6*sg
n(x))/c^8)*sqrt(c*x^2 + b)*x + 3/256*(B*b^5*sgn(x) - 2*A*b^4*c*sgn(x))*log(
abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/c^(7/2) - 3/512*(B*b^5*log(abs(b)) - 2*A
*b^4*c*log(abs(b)))*sgn(x)/c^(7/2)
```

3.109 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x} \, dx$

Optimal. Leaf size=144

$$\frac{b^3(3bB-8Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{5/2}}-\frac{b(b+2cx^2)\sqrt{bx^2+cx^4}(3bB-8Ac)}{128c^2}-\frac{(bx^2+cx^4)^{3/2}(3bB-8Ac)}{48c}+\frac{B(bx^2+cx^4)^{5/2}}{8cx^2}$$

```
[Out] -(b*(3*b*B - 8*A*c)*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(128*c^2) - ((3*b*B
- 8*A*c)*(b*x^2 + c*x^4)^(3/2))/(48*c) + (B*(b*x^2 + c*x^4)^(5/2))/(8*c*x^2
) + (b^3*(3*b*B - 8*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(128*c
^(5/2))
```

Rubi [A] time = 0.26578, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2034, 794, 664, 612, 620, 206}

$$\frac{b^3(3bB-8Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{5/2}}-\frac{b(b+2cx^2)\sqrt{bx^2+cx^4}(3bB-8Ac)}{128c^2}-\frac{(bx^2+cx^4)^{3/2}(3bB-8Ac)}{48c}+\frac{B(bx^2+cx^4)^{5/2}}{8cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x,x]
```

```
[Out] -(b*(3*b*B - 8*A*c)*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(128*c^2) - ((3*b*B
- 8*A*c)*(b*x^2 + c*x^4)^(3/2))/(48*c) + (B*(b*x^2 + c*x^4)^(5/2))/(8*c*x^2
) + (b^3*(3*b*B - 8*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(128*c
^(5/2))
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x}dx = \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)(bx+cx^2)^{3/2}}{x}dx, x, x^2\right)$$

$$= \frac{B(bx^2+cx^4)^{5/2}}{8cx^2} + \frac{\left(bB - Ac + \frac{5}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{(bx+cx^2)^{3/2}}{x}dx, x, x^2\right)}{8c}$$

$$= -\frac{(3bB-8Ac)(bx^2+cx^4)^{3/2}}{48c} + \frac{B(bx^2+cx^4)^{5/2}}{8cx^2} - \frac{(b(3bB-8Ac))\text{Subst}\left(\int \sqrt{bx+cx^2}\right.}{32c}$$

$$= -\frac{b(3bB-8Ac)(b+2cx^2)\sqrt{bx^2+cx^4}}{128c^2} - \frac{(3bB-8Ac)(bx^2+cx^4)^{3/2}}{48c} + \frac{B(bx^2+cx^4)^{5/2}}{8cx^2}$$

$$= -\frac{b(3bB-8Ac)(b+2cx^2)\sqrt{bx^2+cx^4}}{128c^2} - \frac{(3bB-8Ac)(bx^2+cx^4)^{3/2}}{48c} + \frac{B(bx^2+cx^4)^{5/2}}{8cx^2}$$

$$= -\frac{b(3bB-8Ac)(b+2cx^2)\sqrt{bx^2+cx^4}}{128c^2} - \frac{(3bB-8Ac)(bx^2+cx^4)^{3/2}}{48c} + \frac{B(bx^2+cx^4)^{5/2}}{8cx^2}$$

Mathematica [A] time = 0.218564, size = 151, normalized size = 1.05

$$\frac{\sqrt{x^2(b+cx^2)}\left(\sqrt{c}x\sqrt{\frac{cx^2}{b}+1}\left(6b^2c(4A+Bx^2)+8bc^2x^2(14A+9Bx^2)+16c^3x^4(4A+3Bx^2)-9b^3B\right)+3b^{5/2}(3bB-8Ac\right.}{384c^{5/2}x\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x,x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(Sqrt[c]*x*Sqrt[1 + (c*x^2)/b]*(-9*b^3*B + 6*b^2*c*(
4*A + B*x^2) + 16*c^3*x^4*(4*A + 3*B*x^2) + 8*b*c^2*x^2*(14*A + 9*B*x^2)) +
 3*b^(5/2)*(3*b*B - 8*A*c)*ArcSinh[(Sqrt[c]*x)/Sqrt[b]]))/(384*c^(5/2)*x*Sq
rt[1 + (c*x^2)/b])
```

Maple [A] time = 0.009, size = 202, normalized size = 1.4

$$\frac{1}{384x^3}(cx^4+bx^2)^{\frac{3}{2}}\left(48Bc^{3/2}(cx^2+b)^{5/2}x^3+64Ac^{3/2}(cx^2+b)^{5/2}x-24B\sqrt{c}(cx^2+b)^{5/2}xb-16Ac^{3/2}(cx^2+b)^{3/2}xb+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x,x)
```

```
[Out] 1/384*(c*x^4+b*x^2)^(3/2)*(48*B*c^(3/2)*(c*x^2+b)^(5/2)*x^3+64*A*c^(3/2)*(c
*x^2+b)^(5/2)*x-24*B*c^(1/2)*(c*x^2+b)^(5/2)*x*b-16*A*c^(3/2)*(c*x^2+b)^(3/
2)*x*b+6*B*c^(1/2)*(c*x^2+b)^(3/2)*x*b^2-24*A*c^(3/2)*(c*x^2+b)^(1/2)*x*b^2
+9*B*c^(1/2)*(c*x^2+b)^(1/2)*x*b^3-24*A*ln(x*c^(1/2)+(c*x^2+b)^(1/2))*b^3*c
+9*B*ln(x*c^(1/2)+(c*x^2+b)^(1/2))*b^4)/x^3/(c*x^2+b)^(3/2)/c^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.2829, size = 616, normalized size = 4.28

$$\left[-\frac{3\left(3\,Bb^4-8\,Ab^3c\right)\sqrt{c}\log\left(-2\,cx^2-b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\left(48\,Bc^4x^6-9\,Bb^3c+24\,Ab^2c^2+8\left(9\,Bbc^3+8\,Ac^4\right)x^4+\right.}{768\,c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [-1/768*(3*(3*B*b^4 - 8*A*b^3*c)*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 +
b*x^2)*sqrt(c)) - 2*(48*B*c^4*x^6 - 9*B*b^3*c + 24*A*b^2*c^2 + 8*(9*B*b*c^3
 + 8*A*c^4)*x^4 + 2*(3*B*b^2*c^2 + 56*A*b*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^
3, -1/384*(3*(3*B*b^4 - 8*A*b^3*c)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt
(-c)/(c*x^2 + b)) - (48*B*c^4*x^6 - 9*B*b^3*c + 24*A*b^2*c^2 + 8*(9*B*b*c^3
 + 8*A*c^4)*x^4 + 2*(3*B*b^2*c^2 + 56*A*b*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^
3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}\left(A+Bx^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x, x)
```

Giac [A] time = 1.19221, size = 240, normalized size = 1.67

$$\frac{1}{384}\left(2\left(4\left(6\,Bcx^2\mathrm{sgn}\left(x\right)+\frac{9\,Bbc^6\mathrm{sgn}\left(x\right)+8\,Ac^7\mathrm{sgn}\left(x\right)}{c^6}\right)x^2+\frac{3\,Bb^2c^5\mathrm{sgn}\left(x\right)+56\,Abc^6\mathrm{sgn}\left(x\right)}{c^6}\right)x^2-\frac{3\left(3\,Bb^3c^4\mathrm{sgn}\left(x\right)-\right.}{c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*B*c*x^2*sgn(x) + (9*B*b*c^6*sgn(x) + 8*A*c^7*sgn(x))/c^6)*x^
2 + (3*B*b^2*c^5*sgn(x) + 56*A*b*c^6*sgn(x))/c^6)*x^2 - 3*(3*B*b^3*c^4*sgn(
x) - 8*A*b^2*c^5*sgn(x))/c^6)*sqrt(c*x^2 + b)*x - 1/128*(3*B*b^4*sgn(x) - 8
*A*b^3*c*sgn(x))*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/c^(5/2) + 1/256*(3*
B*b^4*log(abs(b)) - 8*A*b^3*c*log(abs(b)))*sgn(x)/c^(5/2)
```

3.110 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^3} \, dx$

Optimal. Leaf size=137

$$-\frac{b^2(bB-6Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{3/2}}+\frac{(bx^2+cx^4)^{3/2}(bB-6Ac)}{6b}+\frac{(b+2cx^2)\sqrt{bx^2+cx^4}(bB-6Ac)}{16c}+\frac{A(bx^2+cx^4)^{5/2}}{bx^4}$$

```
[Out] ((b*B - 6*A*c)*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(16*c) + ((b*B - 6*A*c)*(
b*x^2 + c*x^4)^(3/2))/(6*b) + (A*(b*x^2 + c*x^4)^(5/2))/(b*x^4) - (b^2*(b*B
 - 6*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(16*c^(3/2))
```

Rubi [A] time = 0.295905, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2034, 792, 664, 612, 620, 206}

$$-\frac{b^2(bB-6Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{3/2}}+\frac{(bx^2+cx^4)^{3/2}(bB-6Ac)}{6b}+\frac{(b+2cx^2)\sqrt{bx^2+cx^4}(bB-6Ac)}{16c}+\frac{A(bx^2+cx^4)^{5/2}}{bx^4}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^3,x]
```

```
[Out] ((b*B - 6*A*c)*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(16*c) + ((b*B - 6*A*c)*(
b*x^2 + c*x^4)^(3/2))/(6*b) + (A*(b*x^2 + c*x^4)^(5/2))/(b*x^4) - (b^2*(b*B
 - 6*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(16*c^(3/2))
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
```

```
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)(bx+cx^2)^{3/2}}{x^2}\,dx, x, x^2\right)$$

$$= \frac{A(bx^2+cx^4)^{5/2}}{bx^4} - \frac{\left(-2(-bB+Ac)+\frac{5}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{(bx+cx^2)^{3/2}}{x}\,dx, x, x^2\right)}{b}$$

$$= \frac{(bB-6Ac)(bx^2+cx^4)^{3/2}}{6b} + \frac{A(bx^2+cx^4)^{5/2}}{bx^4} - \frac{1}{4}(-bB+6Ac)\,\text{Subst}\left(\int \sqrt{bx+cx^2}\,d\right.$$

$$= \frac{(bB-6Ac)(b+2cx^2)\sqrt{bx^2+cx^4}}{16c} + \frac{(bB-6Ac)(bx^2+cx^4)^{3/2}}{6b} + \frac{A(bx^2+cx^4)^{5/2}}{bx^4} -$$

$$= \frac{(bB-6Ac)(b+2cx^2)\sqrt{bx^2+cx^4}}{16c} + \frac{(bB-6Ac)(bx^2+cx^4)^{3/2}}{6b} + \frac{A(bx^2+cx^4)^{5/2}}{bx^4} -$$

$$= \frac{(bB-6Ac)(b+2cx^2)\sqrt{bx^2+cx^4}}{16c} + \frac{(bB-6Ac)(bx^2+cx^4)^{3/2}}{6b} + \frac{A(bx^2+cx^4)^{5/2}}{bx^4} -$$

Mathematica [A] time = 0.118235, size = 130, normalized size = 0.95

$$\frac{\sqrt{x^2(b+cx^2)}\left(\sqrt{c}x\sqrt{\frac{cx^2}{b}+1}\left(2bc(15A+7Bx^2)+4c^2x^2(3A+2Bx^2)+3b^2B\right)-3b^{3/2}(bB-6Ac)\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)\right)}{48c^{3/2}x\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^3,x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(Sqrt[c]*x*Sqrt[1 + (c*x^2)/b]*(3*b^2*B + 4*c^2*x^2*
(3*A + 2*B*x^2) + 2*b*c*(15*A + 7*B*x^2)) - 3*b^(3/2)*(b*B - 6*A*c)*ArcSinh
[(Sqrt[c]*x)/Sqrt[b]]))/(48*c^(3/2)*x*Sqrt[1 + (c*x^2)/b])
```

Maple [A] time = 0.004, size = 162, normalized size = 1.2

$$\frac{1}{48\,x^3}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(8\,B\sqrt{c}\left(cx^2+b\right)^{5/2}x+12\,Ac^{3/2}\left(cx^2+b\right)^{3/2}x-2\,B\sqrt{c}\left(cx^2+b\right)^{3/2}xb+18\,Ac^{3/2}\sqrt{cx^2+b}xb-3\,B\sqrt{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^3,x)
```

```
[Out] 1/48*(c*x^4+b*x^2)^(3/2)*(8*B*c^(1/2)*(c*x^2+b)^(5/2)*x+12*A*c^(3/2)*(c*x^2
+b)^(3/2)*x-2*B*c^(1/2)*(c*x^2+b)^(3/2)*x*b+18*A*c^(3/2)*(c*x^2+b)^(1/2)*x*
b-3*B*c^(1/2)*(c*x^2+b)^(1/2)*x*b^2+18*A*ln(x*c^(1/2)+(c*x^2+b)^(1/2))*b^2*
c-3*B*ln(x*c^(1/2)+(c*x^2+b)^(1/2))*b^3)/x^3/(c*x^2+b)^(3/2)/c^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.3026, size = 504, normalized size = 3.68

$$\left[-\frac{3\left(Bb^3 - 6\,Ab^2c\right)\sqrt{c}\log\left(-2\,cx^2 - b - 2\,\sqrt{cx^4 + bx^2}\sqrt{c}\right) - 2\left(8\,Bc^3x^4 + 3\,Bb^2c + 30\,Abc^2 + 2\left(7\,Bbc^2 + 6\,Ac^3\right)x^2\right)\sqrt{cx^4 \cdots}}{96\,c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(B*b^3 - 6*A*b^2*c)*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x
^2)*sqrt(c)) - 2*(8*B*c^3*x^4 + 3*B*b^2*c + 30*A*b*c^2 + 2*(7*B*b*c^2 + 6*A
*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^2, 1/48*(3*(B*b^3 - 6*A*b^2*c)*sqrt(-c)*a
rctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) + (8*B*c^3*x^4 + 3*B*b^2*c
+ 30*A*b*c^2 + 2*(7*B*b*c^2 + 6*A*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(b + cx^2\right)\right)^{\frac{3}{2}}\left(A + Bx^2\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**3,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**3, x)
```

Giac [A] time = 1.18436, size = 192, normalized size = 1.4

$$\frac{1}{48}\left(2\left(4\,Bcx^2\mathrm{sgn}\left(x\right)+\frac{7\,Bbc^4\mathrm{sgn}\left(x\right)+6\,Ac^5\mathrm{sgn}\left(x\right)}{c^4}\right)x^2+\frac{3\left(Bb^2c^3\mathrm{sgn}\left(x\right)+10\,Abc^4\mathrm{sgn}\left(x\right)\right)}{c^4}\right)\sqrt{cx^2+b}x+\frac{\left(Bb^3\mathrm{sgn}\left(x\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*B*c*x^2*sgn(x) + (7*B*b*c^4*sgn(x) + 6*A*c^5*sgn(x))/c^4)*x^2 +
3*(B*b^2*c^3*sgn(x) + 10*A*b*c^4*sgn(x))/c^4)*sqrt(c*x^2 + b)*x + 1/16*(B*b
^3*sgn(x) - 6*A*b^2*c*sgn(x))*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/c^(3/2
) - 1/32*(B*b^3*log(abs(b)) - 6*A*b^2*c*log(abs(b)))*sgn(x)/c^(3/2)
```

3.111 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^5} \, dx$

Optimal. Leaf size=128

$$\frac{(bx^2+cx^4)^{3/2}(4Ac+bB)}{4bx^2}+\frac{3}{8}\sqrt{bx^2+cx^4}(4Ac+bB)+\frac{3b(4Ac+bB)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8\sqrt{c}}-\frac{A(bx^2+cx^4)^{5/2}}{bx^6}$$

```
[Out] (3*(b*B + 4*A*c)*Sqrt[b*x^2 + c*x^4])/8 + ((b*B + 4*A*c)*(b*x^2 + c*x^4)^(3
/2))/(4*b*x^2) - (A*(b*x^2 + c*x^4)^(5/2))/(b*x^6) + (3*b*(b*B + 4*A*c)*Arc
Tanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(8*Sqrt[c])
```

Rubi [A] time = 0.277322, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2034, 792, 664, 620, 206}

$$\frac{(bx^2+cx^4)^{3/2}(4Ac+bB)}{4bx^2}+\frac{3}{8}\sqrt{bx^2+cx^4}(4Ac+bB)+\frac{3b(4Ac+bB)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8\sqrt{c}}-\frac{A(bx^2+cx^4)^{5/2}}{bx^6}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^5,x]
```

```
[Out] (3*(b*B + 4*A*c)*Sqrt[b*x^2 + c*x^4])/8 + ((b*B + 4*A*c)*(b*x^2 + c*x^4)^(3
/2))/(4*b*x^2) - (A*(b*x^2 + c*x^4)^(5/2))/(b*x^6) + (3*b*(b*B + 4*A*c)*Arc
Tanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(8*Sqrt[c])
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
```

```
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^5}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)(bx+cx^2)^{3/2}}{x^3}\,dx, x, x^2\right)$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{bx^6} + \frac{\left(-3(-bB+Ac)+\frac{5}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{(bx+cx^2)^{3/2}}{x^2}\,dx, x, x^2\right)}{b}$$

$$= \frac{(bB+4Ac)(bx^2+cx^4)^{3/2}}{4bx^2} - \frac{A(bx^2+cx^4)^{5/2}}{bx^6} + \frac{1}{8}(3(bB+4Ac))\,\text{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x}\right.$$

$$= \frac{3}{8}(bB+4Ac)\sqrt{bx^2+cx^4} + \frac{(bB+4Ac)(bx^2+cx^4)^{3/2}}{4bx^2} - \frac{A(bx^2+cx^4)^{5/2}}{bx^6} + \frac{1}{16}(3b(bB+$$

$$= \frac{3}{8}(bB+4Ac)\sqrt{bx^2+cx^4} + \frac{(bB+4Ac)(bx^2+cx^4)^{3/2}}{4bx^2} - \frac{A(bx^2+cx^4)^{5/2}}{bx^6} + \frac{1}{8}(3b(bB+$$

$$= \frac{3}{8}(bB+4Ac)\sqrt{bx^2+cx^4} + \frac{(bB+4Ac)(bx^2+cx^4)^{3/2}}{4bx^2} - \frac{A(bx^2+cx^4)^{5/2}}{bx^6} + \frac{3b(bB+4A}{}$$

Mathematica [A] time = 0.174219, size = 96, normalized size = 0.75

$$\frac{\sqrt{x^2(b+cx^2)}\left(\frac{3\sqrt{b}x(4Ac+bB)\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{\sqrt{c}\sqrt{\frac{cx^2}{b}+1}} - 8Ab + 4Acx^2 + 5bBx^2 + 2Bcx^4\right)}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^5,x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(-8*A*b + 5*b*B*x^2 + 4*A*c*x^2 + 2*B*c*x^4 + (3*Sqr
t[b]*(b*B + 4*A*c)*x*ArcSinh[(Sqrt[c]*x)/Sqrt[b]])/(Sqrt[c]*Sqrt[1 + (c*x^2
)/b])))/(8*x^2)
```

Maple [A] time = 0.009, size = 174, normalized size = 1.4

$$\frac{1}{8\,bx^4}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(8\,Ac^{3/2}\left(cx^2+b\right)^{3/2}x^2+12\,Ac^{3/2}\sqrt{cx^2+b}x^2b+2\,B\sqrt{c}\left(cx^2+b\right)^{3/2}x^2b-8\,A\sqrt{c}\left(cx^2+b\right)^{5/2}+3\,B\sqrt{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^5,x)
```

```
[Out] 1/8*(c*x^4+b*x^2)^(3/2)*(8*A*c^(3/2)*(c*x^2+b)^(3/2)*x^2+12*A*c^(3/2)*(c*x^
2+b)^(1/2)*x^2*b+2*B*c^(1/2)*(c*x^2+b)^(3/2)*x^2*b-8*A*c^(1/2)*(c*x^2+b)^(5
/2)+3*B*c^(1/2)*(c*x^2+b)^(1/2)*x^2*b^2+12*A*ln(x*c^(1/2)+(c*x^2+b)^(1/2))*
x*b^2*c+3*B*ln(x*c^(1/2)+(c*x^2+b)^(1/2))*x*b^3)/x^4/(c*x^2+b)^(3/2)/b/c^(1
/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.11389, size = 467, normalized size = 3.65

$$\left[\frac{3\left(Bb^2+4\,Abc\right)\sqrt{c}x^2\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)+2\left(2\,Bc^2x^4-8\,Abc+\left(5\,Bbc+4\,Ac^2\right)x^2\right)\sqrt{cx^4+bx^2}}{16\,cx^2},-\frac{3\left(Bb\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/16*(3*(B*b^2 + 4*A*b*c)*sqrt(c)*x^2*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*
x^2)*sqrt(c)) + 2*(2*B*c^2*x^4 - 8*A*b*c + (5*B*b*c + 4*A*c^2)*x^2)*sqrt(c*
x^4 + b*x^2))/(c*x^2), -1/8*(3*(B*b^2 + 4*A*b*c)*sqrt(-c)*x^2*arctan(sqrt(c
*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) - (2*B*c^2*x^4 - 8*A*b*c + (5*B*b*c + 4
*A*c^2)*x^2)*sqrt(c*x^4 + b*x^2))/(c*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^{2}\left(b+cx^{2}\right)\right)^{\frac{3}{2}}\left(A+Bx^{2}\right)}{x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**5,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**5, x)
```

Giac [A] time = 1.23806, size = 170, normalized size = 1.33

$$\frac{2\,Ab^{2}\sqrt{c}\operatorname{sgn}\left(x\right)}{\left(\sqrt{c}x-\sqrt{cx^{2}+b}\right)^{2}-b}+\frac{1}{8}\left(2\,Bcx^{2}\operatorname{sgn}\left(x\right)+\frac{5\,Bbc^{2}\operatorname{sgn}\left(x\right)+4\,Ac^{3}\operatorname{sgn}\left(x\right)}{c^{2}}\right)\sqrt{cx^{2}+b}x-\frac{3\left(Bb^{2}\sqrt{c}\operatorname{sgn}\left(x\right)+4\,Abc^{\frac{3}{2}}\operatorname{sgn}\left(\right.\right.}{16\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] 2*A*b^2*sqrt(c)*sgn(x)/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b) + 1/8*(2*B*c*x
^2*sgn(x) + (5*B*b*c^2*sgn(x) + 4*A*c^3*sgn(x))/c^2)*sqrt(c*x^2 + b)*x - 3/
16*(B*b^2*sqrt(c)*sgn(x) + 4*A*b*c^(3/2)*sgn(x))*log((sqrt(c)*x - sqrt(c*x^
2 + b))^2)/c
```

3.112 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=136

$$-\frac{(bx^2+cx^4)^{3/2}(2Ac+3bB)}{3bx^4}+\frac{c\sqrt{bx^2+cx^4}(2Ac+3bB)}{2b}+\frac{1}{2}\sqrt{c}(2Ac+3bB)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)-\frac{A(bx^2+cx^4)^{5/2}}{3bx^8}$$

```
[Out] (c*(3*b*B + 2*A*c)*Sqrt[b*x^2 + c*x^4])/(2*b) - ((3*b*B + 2*A*c)*(b*x^2 + c
*x^4)^(3/2))/(3*b*x^4) - (A*(b*x^2 + c*x^4)^(5/2))/(3*b*x^8) + (Sqrt[c]*(3*
b*B + 2*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/2
```

Rubi [A] time = 0.273528, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2034, 792, 662, 664, 620, 206}

$$-\frac{(bx^2+cx^4)^{3/2}(2Ac+3bB)}{3bx^4}+\frac{c\sqrt{bx^2+cx^4}(2Ac+3bB)}{2b}+\frac{1}{2}\sqrt{c}(2Ac+3bB)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)-\frac{A(bx^2+cx^4)^{5/2}}{3bx^8}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^7,x]
```

```
[Out] (c*(3*b*B + 2*A*c)*Sqrt[b*x^2 + c*x^4])/(2*b) - ((3*b*B + 2*A*c)*(b*x^2 + c
*x^4)^(3/2))/(3*b*x^4) - (A*(b*x^2 + c*x^4)^(5/2))/(3*b*x^8) + (Sqrt[c]*(3*
b*B + 2*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/2
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
```

```
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + p + 1)), x]
- Dist[(c*p)/(e^2*(m + p + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}}{x^7}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^4}\,dx, x, x^2\right)$$

$$= -\frac{A\left(bx^2+cx^4\right)^{5/2}}{3bx^8} + \frac{\left(-4(-bB+Ac)+\frac{5}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{\left(bx+cx^2\right)^{3/2}}{x^3}\,dx, x, x^2\right)}{3b}$$

$$= -\frac{(3bB+2Ac)\left(bx^2+cx^4\right)^{3/2}}{3bx^4} - \frac{A\left(bx^2+cx^4\right)^{5/2}}{3bx^8} + \frac{\left(c\left(-4(-bB+Ac)+\frac{5}{2}(-bB+2Ac\right.\right.}{b}$$

$$= \frac{c(3bB+2Ac)\sqrt{bx^2+cx^4}}{2b} - \frac{(3bB+2Ac)\left(bx^2+cx^4\right)^{3/2}}{3bx^4} - \frac{A\left(bx^2+cx^4\right)^{5/2}}{3bx^8} + \frac{1}{4}(c(3bE$$

$$= \frac{c(3bB+2Ac)\sqrt{bx^2+cx^4}}{2b} - \frac{(3bB+2Ac)\left(bx^2+cx^4\right)^{3/2}}{3bx^4} - \frac{A\left(bx^2+cx^4\right)^{5/2}}{3bx^8} + \frac{1}{2}(c(3bE$$

$$= \frac{c(3bB+2Ac)\sqrt{bx^2+cx^4}}{2b} - \frac{(3bB+2Ac)\left(bx^2+cx^4\right)^{3/2}}{3bx^4} - \frac{A\left(bx^2+cx^4\right)^{5/2}}{3bx^8} + \frac{1}{2}\sqrt{c}(3b$$

Mathematica [C] time = 0.0505867, size = 98, normalized size = 0.72

$$-\frac{\sqrt{x^2\left(b+cx^2\right)}\left(bx^2(2Ac+3bB)\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};-\frac{cx^2}{b}\right)+A\left(b+cx^2\right)^2\sqrt{\frac{cx^2}{b}+1}\right)}{3bx^4\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^7,x]
```

```
[Out] -(Sqrt[x^2*(b + c*x^2)]*(A*(b + c*x^2)^2*Sqrt[1 + (c*x^2)/b] + b*(3*b*B + 2
*A*c)*x^2*Hypergeometric2F1[-3/2, -1/2, 1/2, -((c*x^2)/b)]))/(3*b*x^4*Sqrt[
1 + (c*x^2)/b])
```

Maple [A] time = 0.01, size = 219, normalized size = 1.6

$$\frac{1}{6\,b^2x^6}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(4\,Ac^{5/2}\left(cx^2+b\right)^{3/2}x^4+6\,Ac^{5/2}\sqrt{cx^2+b}x^4b+6\,Bc^{3/2}\left(cx^2+b\right)^{3/2}x^4b-4\,Ac^{3/2}\left(cx^2+b\right)^{5/2}x^2+9\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^7,x)
```

```
[Out] 1/6*(c*x^4+b*x^2)^(3/2)*(4*A*c^(5/2)*(c*x^2+b)^(3/2)*x^4+6*A*c^(5/2)*(c*x^2
+b)^(1/2)*x^4*b+6*B*c^(3/2)*(c*x^2+b)^(3/2)*x^4*b-4*A*c^(3/2)*(c*x^2+b)^(5/
2)*x^2+9*B*c^(3/2)*(c*x^2+b)^(1/2)*x^4*b^2-6*B*c^(1/2)*(c*x^2+b)^(5/2)*x^2*
b+6*A*ln(x*c^(1/2)+(c*x^2+b)^(1/2))*x^3*b^2*c^2+9*B*ln(x*c^(1/2)+(c*x^2+b)^
(1/2))*x^3*b^3*c-2*A*c^(1/2)*(c*x^2+b)^(5/2)*b)/x^6/(c*x^2+b)^(3/2)/b^2/c^(
1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.27299, size = 435, normalized size = 3.2

$$\left[\frac{3\,(3\,Bb+2\,Ac)\sqrt{c}x^4\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)+2\left(3\,Bcx^4-2\,(3\,Bb+4\,Ac)x^2-2\,Ab\right)\sqrt{cx^4+bx^2}}{12\,x^4},-\frac{3\,(3\,Bb+2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(3*B*b + 2*A*c)*sqrt(c)*x^4*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^
2)*sqrt(c)) + 2*(3*B*c*x^4 - 2*(3*B*b + 4*A*c)*x^2 - 2*A*b)*sqrt(c*x^4 + b*
x^2))/x^4, -1/6*(3*(3*B*b + 2*A*c)*sqrt(-c)*x^4*arctan(sqrt(c*x^4 + b*x^2)*
sqrt(-c)/(c*x^2 + b)) - (3*B*c*x^4 - 2*(3*B*b + 4*A*c)*x^2 - 2*A*b)*sqrt(c*
x^4 + b*x^2))/x^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}\left(A+Bx^2\right)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**7,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**7, x)
```

Giac [A] time = 1.45597, size = 304, normalized size = 2.24

$$\frac{1}{2}\sqrt{cx^2+b}Bcx\operatorname{sgn}(x)-\frac{1}{4}\left(3\,Bb\sqrt{c}\operatorname{sgn}(x)+2\,Ac^{\frac{3}{2}}\operatorname{sgn}(x)\right)\log\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2\right)+\frac{2\left(3\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^4Bb^2\sqrt{c}\mathrm{sg}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^2 + b)*B*c*x*sgn(x) - 1/4*(3*B*b*sqrt(c)*sgn(x) + 2*A*c^(3/2)*
sgn(x))*log((sqrt(c)*x - sqrt(c*x^2 + b))^2) + 2/3*(3*(sqrt(c)*x - sqrt(c*x
^2 + b))^4*B*b^2*sqrt(c)*sgn(x) + 6*(sqrt(c)*x - sqrt(c*x^2 + b))^4*A*b*c^(
3/2)*sgn(x) - 6*(sqrt(c)*x - sqrt(c*x^2 + b))^2*B*b^3*sqrt(c)*sgn(x) - 6*(s
qrt(c)*x - sqrt(c*x^2 + b))^2*A*b^2*c^(3/2)*sgn(x) + 3*B*b^4*sqrt(c)*sgn(x)
 + 4*A*b^3*c^(3/2)*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^3
```

3.113 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^9} dx$

Optimal. Leaf size=104

$$-\frac{A\left(bx^2+cx^4\right)^{5/2}}{5bx^{10}}+Bc^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)-\frac{B\left(bx^2+cx^4\right)^{3/2}}{3x^6}-\frac{Bc\sqrt{bx^2+cx^4}}{x^2}$$

```
[Out] -((B*c*Sqrt[b*x^2 + c*x^4])/x^2) - (B*(b*x^2 + c*x^4)^(3/2))/(3*x^6) - (A*(
b*x^2 + c*x^4)^(5/2))/(5*b*x^10) + B*c^(3/2)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x
^2 + c*x^4]]
```

Rubi [A] time = 0.251363, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2034, 792, 662, 620, 206}

$$-\frac{A\left(bx^2+cx^4\right)^{5/2}}{5bx^{10}}+Bc^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)-\frac{B\left(bx^2+cx^4\right)^{3/2}}{3x^6}-\frac{Bc\sqrt{bx^2+cx^4}}{x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^9,x]
```


```
[Out] -((B*c*Sqrt[b*x^2 + c*x^4])/x^2) - (B*(b*x^2 + c*x^4)^(3/2))/(3*x^6) - (A*(
b*x^2 + c*x^4)^(5/2))/(5*b*x^10) + B*c^(3/2)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x
^2 + c*x^4]]
```


Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
```

```
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + p + 1)), x]
- Dist[(c*p)/(e^2*(m + p + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^9}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(A+Bx)(bx+cx^2)^{3/2}}{x^5}\,dx, x, x^2\right)$$
$$= -\frac{A(bx^2+cx^4)^{5/2}}{5bx^{10}} + \frac{1}{2}B\,\text{Subst}\left(\int \frac{(bx+cx^2)^{3/2}}{x^4}\,dx, x, x^2\right)$$
$$= -\frac{B(bx^2+cx^4)^{3/2}}{3x^6} - \frac{A(bx^2+cx^4)^{5/2}}{5bx^{10}} + \frac{1}{2}(Bc)\,\text{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x^2}\,dx, x, x^2\right)$$
$$= -\frac{Bc\sqrt{bx^2+cx^4}}{x^2} - \frac{B(bx^2+cx^4)^{3/2}}{3x^6} - \frac{A(bx^2+cx^4)^{5/2}}{5bx^{10}} + \frac{1}{2}(Bc^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\right.$$
$$= -\frac{Bc\sqrt{bx^2+cx^4}}{x^2} - \frac{B(bx^2+cx^4)^{3/2}}{3x^6} - \frac{A(bx^2+cx^4)^{5/2}}{5bx^{10}} + (Bc^2)\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x\right.$$
$$= -\frac{Bc\sqrt{bx^2+cx^4}}{x^2} - \frac{B(bx^2+cx^4)^{3/2}}{3x^6} - \frac{A(bx^2+cx^4)^{5/2}}{5bx^{10}} + Bc^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)$$

Mathematica [C] time = 0.0469147, size = 94, normalized size = 0.9

$$-\frac{\sqrt{x^2(b+cx^2)}\left(3A(b+cx^2)^2\sqrt{\frac{cx^2}{b}+1}+5b^2Bx^2\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};-\frac{cx^2}{b}\right)\right)}{15bx^6\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^9,x]
```

```
[Out] -(Sqrt[x^2*(b + c*x^2)]*(3*A*(b + c*x^2)^2*Sqrt[1 + (c*x^2)/b] + 5*b^2*B*x^
2*Hypergeometric2F1[-3/2, -3/2, -1/2, -((c*x^2)/b)]))/(15*b*x^6*Sqrt[1 + (c
*x^2)/b])
```

Maple [A] time = 0.011, size = 153, normalized size = 1.5

$$-\frac{1}{15\,b^2x^8}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(-10\,Bc^{5/2}\left(cx^2+b\right)^{3/2}x^6+10\,Bc^{3/2}\left(cx^2+b\right)^{5/2}x^4-15\,Bc^{5/2}\sqrt{cx^2+b}x^6b-15\,B\ln\left(x\sqrt{c}+\sqrt{cx^2+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^9,x)
```

```
[Out] -1/15*(c*x^4+b*x^2)^(3/2)*(-10*B*c^(5/2)*(c*x^2+b)^(3/2)*x^6+10*B*c^(3/2)*(
c*x^2+b)^(5/2)*x^4-15*B*c^(5/2)*(c*x^2+b)^(1/2)*x^6*b-15*B*ln(x*c^(1/2)+(c*
x^2+b)^(1/2))*x^5*b^2*c^2+5*B*c^(1/2)*(c*x^2+b)^(5/2)*x^2*b+3*A*c^(1/2)*(c*
x^2+b)^(5/2)*b)/x^8/(c*x^2+b)^(3/2)/b^2/c^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.27494, size = 471, normalized size = 4.53

$$\left[\frac{15\,Bbc^{\frac{3}{2}}x^6\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\left(\left(20\,Bbc+3\,Ac^2\right)x^4+3\,Ab^2+\left(5\,Bb^2+6\,Abc\right)x^2\right)\sqrt{cx^4+bx^2}}{30\,bx^6},-\frac{15\,B\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] [1/30*(15*B*b*c^(3/2)*x^6*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c))
 - 2*((20*B*b*c + 3*A*c^2)*x^4 + 3*A*b^2 + (5*B*b^2 + 6*A*b*c)*x^2)*sqrt(c*
x^4 + b*x^2))/(b*x^6), -1/15*(15*B*b*sqrt(-c)*c*x^6*arctan(sqrt(c*x^4 + b*x
^2)*sqrt(-c)/(c*x^2 + b)) + ((20*B*b*c + 3*A*c^2)*x^4 + 3*A*b^2 + (5*B*b^2
+ 6*A*b*c)*x^2)*sqrt(c*x^4 + b*x^2))/(b*x^6)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}\left(A+Bx^2\right)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**9,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**9, x)
```

Giac [B] time = 1.89374, size = 343, normalized size = 3.3

$$-\frac{1}{2}\,Bc^{\frac{3}{2}}\log\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2\right)\operatorname{sgn}(x)+\frac{2\left(30\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^8Bbc^{\frac{3}{2}}\operatorname{sgn}(x)+15\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^8Ac^{\frac{5}{2}}\operatorname{sgn}(x)-90\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] -1/2*B*c^(3/2)*log((sqrt(c)*x - sqrt(c*x^2 + b))^2)*sgn(x) + 2/15*(30*(sqrt
(c)*x - sqrt(c*x^2 + b))^8*B*b*c^(3/2)*sgn(x) + 15*(sqrt(c)*x - sqrt(c*x^2
+ b))^8*A*c^(5/2)*sgn(x) - 90*(sqrt(c)*x - sqrt(c*x^2 + b))^6*B*b^2*c^(3/2)
*sgn(x) + 110*(sqrt(c)*x - sqrt(c*x^2 + b))^4*B*b^3*c^(3/2)*sgn(x) + 30*(sq
rt(c)*x - sqrt(c*x^2 + b))^4*A*b^2*c^(5/2)*sgn(x) - 70*(sqrt(c)*x - sqrt(c*
x^2 + b))^2*B*b^4*c^(3/2)*sgn(x) + 20*B*b^5*c^(3/2)*sgn(x) + 3*A*b^4*c^(5/2
)*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^5
```

3.114 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{11}} dx$

Optimal. Leaf size=61

$$-\frac{(bx^2+cx^4)^{5/2}(7bB-2Ac)}{35b^2x^{10}}-\frac{A(bx^2+cx^4)^{5/2}}{7bx^{12}}$$

```
[Out] -(A*(b*x^2 + c*x^4)^(5/2))/(7*b*x^12) - ((7*b*B - 2*A*c)*(b*x^2 + c*x^4)^(5
/2))/(35*b^2*x^10)
```

Rubi [A] time = 0.173004, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2034, 792, 650}

$$-\frac{(bx^2+cx^4)^{5/2}(7bB-2Ac)}{35b^2x^{10}}-\frac{A(bx^2+cx^4)^{5/2}}{7bx^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^11,x]
```

```
[Out] -(A*(b*x^2 + c*x^4)^(5/2))/(7*b*x^12) - ((7*b*B - 2*A*c)*(b*x^2 + c*x^4)^(5
/2))/(35*b^2*x^10)
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
```

```
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{11}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(A+Bx)(bx+cx^2)^{3/2}}{x^6}\,dx, x, x^2\right)\\ &= -\frac{A(bx^2+cx^4)^{5/2}}{7bx^{12}} + \frac{\left(-6(-bB+Ac)+\frac{5}{2}(-bB+2Ac)\right)\operatorname{Subst}\left(\int \frac{(bx+cx^2)^{3/2}}{x^5}\,dx, x, x^2\right)}{7b}\\ &= -\frac{A(bx^2+cx^4)^{5/2}}{7bx^{12}} - \frac{(7bB-2Ac)(bx^2+cx^4)^{5/2}}{35b^2x^{10}}\end{aligned}$$

Mathematica [A] time = 0.0238985, size = 44, normalized size = 0.72

$$-\frac{\left(x^2(b+cx^2)\right)^{5/2}(5Ab-2Acx^2+7bBx^2)}{35b^2x^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^11,x]
```

```
[Out] -((x^2*(b + c*x^2))^(5/2)*(5*A*b + 7*b*B*x^2 - 2*A*c*x^2))/(35*b^2*x^12)
```

Maple [A] time = 0.004, size = 48, normalized size = 0.8

$$-\frac{(cx^2+b)(-2Ax^2c+7Bx^2b+5Ab)}{35x^{10}b^2}(cx^4+bx^2)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^11,x)
```

```
[Out] -1/35*(c*x^2+b)*(-2*A*c*x^2+7*B*b*x^2+5*A*b)*(c*x^4+b*x^2)^(3/2)/x^10/b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^11,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.30742, size = 178, normalized size = 2.92

$$-\frac{\left(\left(7\,Bbc^{2}-2\,Ac^{3}\right)x^{6}+\left(14\,Bb^{2}c+Abc^{2}\right)x^{4}+5\,Ab^{3}+\left(7\,Bb^{3}+8\,Ab^{2}c\right)x^{2}\right)\sqrt{cx^{4}+bx^{2}}}{35\,b^{2}x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^11,x, algorithm="fricas")
```

```
[Out] -1/35*((7*B*b*c^2 - 2*A*c^3)*x^6 + (14*B*b^2*c + A*b*c^2)*x^4 + 5*A*b^3 + (
7*B*b^3 + 8*A*b^2*c)*x^2)*sqrt(c*x^4 + b*x^2)/(b^2*x^8)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^{2}\left(b+cx^{2}\right)\right)^{\frac{3}{2}}\left(A+Bx^{2}\right)}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**11,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**11, x)
```

Giac [B] time = 3.53941, size = 500, normalized size = 8.2

$$\frac{2\left(35\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{12}Bc^{\frac{5}{2}}\operatorname{sgn}\left(x\right)-70\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{10}Bbc^{\frac{5}{2}}\operatorname{sgn}\left(x\right)+70\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{10}Ac^{\frac{7}{2}}\operatorname{sgn}\left(x\right)+105\left(\sqrt{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^11,x, algorithm="giac")
```

```
[Out] 2/35*(35*(sqrt(c)*x - sqrt(c*x^2 + b))^12*B*c^(5/2)*sgn(x) - 70*(sqrt(c)*x
- sqrt(c*x^2 + b))^10*B*b*c^(5/2)*sgn(x) + 70*(sqrt(c)*x - sqrt(c*x^2 + b))
^10*A*c^(7/2)*sgn(x) + 105*(sqrt(c)*x - sqrt(c*x^2 + b))^8*B*b^2*c^(5/2)*sg
n(x) + 70*(sqrt(c)*x - sqrt(c*x^2 + b))^8*A*b*c^(7/2)*sgn(x) - 140*(sqrt(c)
*x - sqrt(c*x^2 + b))^6*B*b^3*c^(5/2)*sgn(x) + 140*(sqrt(c)*x - sqrt(c*x^2
+ b))^6*A*b^2*c^(7/2)*sgn(x) + 77*(sqrt(c)*x - sqrt(c*x^2 + b))^4*B*b^4*c^(
5/2)*sgn(x) + 28*(sqrt(c)*x - sqrt(c*x^2 + b))^4*A*b^3*c^(7/2)*sgn(x) - 14*
(sqrt(c)*x - sqrt(c*x^2 + b))^2*B*b^5*c^(5/2)*sgn(x) + 14*(sqrt(c)*x - sqrt
(c*x^2 + b))^2*A*b^4*c^(7/2)*sgn(x) + 7*B*b^6*c^(5/2)*sgn(x) - 2*A*b^5*c^(7
/2)*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^7
```

3.115 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{13}} \, dx$

Optimal. Leaf size=96

$$\frac{2c(bx^2+cx^4)^{5/2}(9bB-4Ac)}{315b^3x^{10}}-\frac{(bx^2+cx^4)^{5/2}(9bB-4Ac)}{63b^2x^{12}}-\frac{A(bx^2+cx^4)^{5/2}}{9bx^{14}}$$

```
[Out] -(A*(b*x^2 + c*x^4)^(5/2))/(9*b*x^14) - ((9*b*B - 4*A*c)*(b*x^2 + c*x^4)^(5
/2))/(63*b^2*x^12) + (2*c*(9*b*B - 4*A*c)*(b*x^2 + c*x^4)^(5/2))/(315*b^3*x
^10)
```

Rubi [A] time = 0.233431, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2034, 792, 658, 650}

$$\frac{2c(bx^2+cx^4)^{5/2}(9bB-4Ac)}{315b^3x^{10}}-\frac{(bx^2+cx^4)^{5/2}(9bB-4Ac)}{63b^2x^{12}}-\frac{A(bx^2+cx^4)^{5/2}}{9bx^{14}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^13,x]
```

```
[Out] -(A*(b*x^2 + c*x^4)^(5/2))/(9*b*x^14) - ((9*b*B - 4*A*c)*(b*x^2 + c*x^4)^(5
/2))/(63*b^2*x^12) + (2*c*(9*b*B - 4*A*c)*(b*x^2 + c*x^4)^(5/2))/(315*b^3*x
^10)
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
```

```
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}}{x^{13}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^7}\,dx,x,x^2\right)\\
&= -\frac{A\left(bx^2+cx^4\right)^{5/2}}{9bx^{14}}+\frac{\left(-7(-bB+Ac)+\frac{5}{2}(-bB+2Ac)\right)\operatorname{Subst}\left(\int \frac{\left(bx+cx^2\right)^{3/2}}{x^6}\,dx,x,x^2\right)}{9b}\\
&= -\frac{A\left(bx^2+cx^4\right)^{5/2}}{9bx^{14}}-\frac{(9bB-4Ac)\left(bx^2+cx^4\right)^{5/2}}{63b^2x^{12}}-\frac{(c(9bB-4Ac))\operatorname{Subst}\Bigg(\int \frac{\left(bx+cx^2\right)^{3/2}}{x^5}\,d}{63b^2}\\
&= -\frac{A\left(bx^2+cx^4\right)^{5/2}}{9bx^{14}}-\frac{(9bB-4Ac)\left(bx^2+cx^4\right)^{5/2}}{63b^2x^{12}}+\frac{2c(9bB-4Ac)\left(bx^2+cx^4\right)^{5/2}}{315b^3x^{10}}
\end{aligned}
$$

Mathematica [A] time = 0.0306484, size = 66, normalized size = 0.69

$$\frac{\left(x^2\left(b+cx^2\right)\right)^{5/2}\left(A\left(-35b^2+20bcx^2-8c^2x^4\right)+9bBx^2\left(2cx^2-5b\right)\right)}{315b^3x^{14}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^13,x]
```

```
[Out] ((x^2*(b + c*x^2))^(5/2)*(9*b*B*x^2*(-5*b + 2*c*x^2) + A*(-35*b^2 + 20*b*c*
x^2 - 8*c^2*x^4)))/(315*b^3*x^14)
```

Maple [A] time = 0.005, size = 70, normalized size = 0.7

$$-\frac{\left(cx^2+b\right)\left(8\,Ac^2x^4-18\,Bx^4bc-20\,Abcx^2+45\,Bx^2b^2+35\,Ab^2\right)}{315\,x^{12}b^3}\left(cx^4+bx^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^13,x)
```

```
[Out] -1/315*(c*x^2+b)*(8*A*c^2*x^4-18*B*b*c*x^4-20*A*b*c*x^2+45*B*b^2*x^2+35*A*b
^2)*(c*x^4+b*x^2)^(3/2)/x^12/b^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^13,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.33211, size = 236, normalized size = 2.46

$$\frac{\left(2\left(9\,Bbc^3-4\,Ac^4\right)x^8-\left(9\,Bb^2c^2-4\,Abc^3\right)x^6-35\,Ab^4-3\left(24\,Bb^3c+Ab^2c^2\right)x^4-5\left(9\,Bb^4+10\,Ab^3c\right)x^2\right)\sqrt{cx^4+bx^2}}{315\,b^3x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^13,x, algorithm="fricas")
```

[Out] 1/315*(2*(9*B*b*c^3 - 4*A*c^4)*x^8 - (9*B*b^2*c^2 - 4*A*b*c^3)*x^6 - 35*A*b
^4 - 3*(24*B*b^3*c + A*b^2*c^2)*x^4 - 5*(9*B*b^4 + 10*A*b^3*c)*x^2)*sqrt(c*
x^4 + b*x^2)/(b^3*x^10)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}\left(A+Bx^2\right)}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**13,x)

[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**13, x)

Giac [B] time = 4.78231, size = 581, normalized size = 6.05

$$\frac{4\left(315\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{14}Bc^{\frac{7}{2}}\operatorname{sgn}(x)-315\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{12}Bbc^{\frac{7}{2}}\operatorname{sgn}(x)+840\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{12}Ac^{\frac{9}{2}}\operatorname{sgn}(x)+315\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^13,x, algorithm="giac")

[Out] 4/315*(315*(sqrt(c)*x - sqrt(c*x^2 + b))^14*B*c^(7/2)*sgn(x) - 315*(sqrt(c)
*x - sqrt(c*x^2 + b))^12*B*b*c^(7/2)*sgn(x) + 840*(sqrt(c)*x - sqrt(c*x^2 +
 b))^12*A*c^(9/2)*sgn(x) + 315*(sqrt(c)*x - sqrt(c*x^2 + b))^10*B*b^2*c^(7/
2)*sgn(x) + 1260*(sqrt(c)*x - sqrt(c*x^2 + b))^10*A*b*c^(9/2)*sgn(x) - 819*
(sqrt(c)*x - sqrt(c*x^2 + b))^8*B*b^3*c^(7/2)*sgn(x) + 1764*(sqrt(c)*x - sq
rt(c*x^2 + b))^8*A*b^2*c^(9/2)*sgn(x) + 441*(sqrt(c)*x - sqrt(c*x^2 + b))^6
*B*b^4*c^(7/2)*sgn(x) + 504*(sqrt(c)*x - sqrt(c*x^2 + b))^6*A*b^3*c^(9/2)*s
gn(x) - 9*(sqrt(c)*x - sqrt(c*x^2 + b))^4*B*b^5*c^(7/2)*sgn(x) + 144*(sqrt(
c)*x - sqrt(c*x^2 + b))^4*A*b^4*c^(9/2)*sgn(x) + 81*(sqrt(c)*x - sqrt(c*x^2
 + b))^2*B*b^6*c^(7/2)*sgn(x) - 36*(sqrt(c)*x - sqrt(c*x^2 + b))^2*A*b^5*c^
(9/2)*sgn(x) - 9*B*b^7*c^(7/2)*sgn(x) + 4*A*b^6*c^(9/2)*sgn(x))/((sqrt(c)*x
 - sqrt(c*x^2 + b))^2 - b)^9

3.116 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{15}} dx$

Optimal. Leaf size=133

$$-\frac{8c^2\left(bx^2+cx^4\right)^{5/2}(11bB-6Ac)}{3465b^4x^{10}}+\frac{4c\left(bx^2+cx^4\right)^{5/2}(11bB-6Ac)}{693b^3x^{12}}-\frac{\left(bx^2+cx^4\right)^{5/2}(11bB-6Ac)}{99b^2x^{14}}-\frac{A\left(bx^2+cx^4\right)^{5/2}}{11bx^{16}}$$

```
[Out] -(A*(b*x^2 + c*x^4)^(5/2))/(11*b*x^16) - ((11*b*B - 6*A*c)*(b*x^2 + c*x^4)^
(5/2))/(99*b^2*x^14) + (4*c*(11*b*B - 6*A*c)*(b*x^2 + c*x^4)^(5/2))/(693*b^
3*x^12) - (8*c^2*(11*b*B - 6*A*c)*(b*x^2 + c*x^4)^(5/2))/(3465*b^4*x^10)
```

Rubi [A] time = 0.278839, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2034, 792, 658, 650}

$$-\frac{8c^2\left(bx^2+cx^4\right)^{5/2}(11bB-6Ac)}{3465b^4x^{10}}+\frac{4c\left(bx^2+cx^4\right)^{5/2}(11bB-6Ac)}{693b^3x^{12}}-\frac{\left(bx^2+cx^4\right)^{5/2}(11bB-6Ac)}{99b^2x^{14}}-\frac{A\left(bx^2+cx^4\right)^{5/2}}{11bx^{16}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^15,x]
```

```
[Out] -(A*(b*x^2 + c*x^4)^(5/2))/(11*b*x^16) - ((11*b*B - 6*A*c)*(b*x^2 + c*x^4)^
(5/2))/(99*b^2*x^14) + (4*c*(11*b*B - 6*A*c)*(b*x^2 + c*x^4)^(5/2))/(693*b^
3*x^12) - (8*c^2*(11*b*B - 6*A*c)*(b*x^2 + c*x^4)^(5/2))/(3465*b^4*x^10)
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
```

```
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}}{x^{15}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^8}\,dx,x,x^2\right)$$

$$= -\frac{A\left(bx^2+cx^4\right)^{5/2}}{11bx^{16}} + \frac{\left(-8(-bB+Ac)+\frac{5}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{\left(bx+cx^2\right)^{3/2}}{x^7}\,dx,x,x^2\right)}{11b}$$

$$= -\frac{A\left(bx^2+cx^4\right)^{5/2}}{11bx^{16}} - \frac{(11bB-6Ac)\left(bx^2+cx^4\right)^{5/2}}{99b^2x^{14}} - \frac{(2c(11bB-6Ac))\,\text{Subst}\left(\int \frac{\left(bx+cx^2\right)}{x^6}\right.}{99b^2}$$

$$= -\frac{A\left(bx^2+cx^4\right)^{5/2}}{11bx^{16}} - \frac{(11bB-6Ac)\left(bx^2+cx^4\right)^{5/2}}{99b^2x^{14}} + \frac{4c(11bB-6Ac)\left(bx^2+cx^4\right)^{5/2}}{693b^3x^{12}} + \ldots$$

$$= -\frac{A\left(bx^2+cx^4\right)^{5/2}}{11bx^{16}} - \frac{(11bB-6Ac)\left(bx^2+cx^4\right)^{5/2}}{99b^2x^{14}} + \frac{4c(11bB-6Ac)\left(bx^2+cx^4\right)^{5/2}}{693b^3x^{12}} - \ldots$$

Mathematica [A] time = 0.0344796, size = 89, normalized size = 0.67

$$-\frac{\left(x^2\left(b+cx^2\right)\right)^{5/2}\left(3A\left(-70b^2cx^2+105b^3+40bc^2x^4-16c^3x^6\right)+11bBx^2\left(35b^2-20bcx^2+8c^2x^4\right)\right)}{3465b^4x^{16}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^15,x]
```

```
[Out] -((x^2*(b + c*x^2))^(5/2)*(11*b*B*x^2*(35*b^2 - 20*b*c*x^2 + 8*c^2*x^4) + 3
*A*(105*b^3 - 70*b^2*c*x^2 + 40*b*c^2*x^4 - 16*c^3*x^6)))/(3465*b^4*x^16)
```

Maple [A] time = 0.006, size = 94, normalized size = 0.7

$$-\frac{\left(cx^2+b\right)\left(-48\,Ac^3x^6+88\,Bx^6bc^2+120\,Abc^2x^4-220\,Bx^4b^2c-210\,Ab^2cx^2+385\,Bx^2b^3+315\,Ab^3\right)}{3465\,x^{14}b^4}\left(cx^4+bx^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^15,x)
```

```
[Out] -1/3465*(c*x^2+b)*(-48*A*c^3*x^6+88*B*b*c^2*x^6+120*A*b*c^2*x^4-220*B*b^2*c
*x^4-210*A*b^2*c*x^2+385*B*b^3*x^2+315*A*b^3)*(c*x^4+b*x^2)^(3/2)/x^14/b^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^15,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.48329, size = 304, normalized size = 2.29

$$-\frac{\left(8\left(11\,Bbc^4-6\,Ac^5\right)x^{10}-4\left(11\,Bb^2c^3-6\,Abc^4\right)x^8+3\left(11\,Bb^3c^2-6\,Ab^2c^3\right)x^6+315\,Ab^5+5\left(110\,Bb^4c+3\,Ab^3c^2\right)x^4+}{3465\,b^4x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^15,x, algorithm="fricas")
```

```
[Out] -1/3465*(8*(11*B*b*c^4 - 6*A*c^5)*x^10 - 4*(11*B*b^2*c^3 - 6*A*b*c^4)*x^8 +
 3*(11*B*b^3*c^2 - 6*A*b^2*c^3)*x^6 + 315*A*b^5 + 5*(110*B*b^4*c + 3*A*b^3*
c^2)*x^4 + 35*(11*B*b^5 + 12*A*b^4*c)*x^2)*sqrt(c*x^4 + b*x^2)/(b^4*x^12)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}\left(A+Bx^2\right)}{x^{15}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**15,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**15, x)
```

Giac [B] time = 4.91959, size = 662, normalized size = 4.98

$$\frac{16\left(2310\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{16}Bc^{\frac{9}{2}}\operatorname{sgn}(x)-1155\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{14}Bbc^{\frac{9}{2}}\operatorname{sgn}(x)+6930\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{14}Ac^{\frac{11}{2}}\operatorname{sgn}(x)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^15,x, algorithm="giac")
```

```
[Out] 16/3465*(2310*(sqrt(c)*x - sqrt(c*x^2 + b))^16*B*c^(9/2)*sgn(x) - 1155*(sqr
t(c)*x - sqrt(c*x^2 + b))^14*B*b*c^(9/2)*sgn(x) + 6930*(sqrt(c)*x - sqrt(c*
x^2 + b))^14*A*c^(11/2)*sgn(x) + 231*(sqrt(c)*x - sqrt(c*x^2 + b))^12*B*b^2
*c^(9/2)*sgn(x) + 12474*(sqrt(c)*x - sqrt(c*x^2 + b))^12*A*b*c^(11/2)*sgn(x
) - 4851*(sqrt(c)*x - sqrt(c*x^2 + b))^10*B*b^3*c^(9/2)*sgn(x) + 15246*(sqr
t(c)*x - sqrt(c*x^2 + b))^10*A*b^2*c^(11/2)*sgn(x) + 2475*(sqrt(c)*x - sqrt
(c*x^2 + b))^8*B*b^4*c^(9/2)*sgn(x) + 4950*(sqrt(c)*x - sqrt(c*x^2 + b))^8*
A*b^3*c^(11/2)*sgn(x) + 495*(sqrt(c)*x - sqrt(c*x^2 + b))^6*B*b^5*c^(9/2)*s
```

```
gn(x) + 990*(sqrt(c)*x - sqrt(c*x^2 + b))^6*A*b^4*c^(11/2)*sgn(x) + 605*(sq
rt(c)*x - sqrt(c*x^2 + b))^4*B*b^6*c^(9/2)*sgn(x) - 330*(sqrt(c)*x - sqrt(c
*x^2 + b))^4*A*b^5*c^(11/2)*sgn(x) - 121*(sqrt(c)*x - sqrt(c*x^2 + b))^2*B*
b^7*c^(9/2)*sgn(x) + 66*(sqrt(c)*x - sqrt(c*x^2 + b))^2*A*b^6*c^(11/2)*sgn(
x) + 11*B*b^8*c^(9/2)*sgn(x) - 6*A*b^7*c^(11/2)*sgn(x))/((sqrt(c)*x - sqrt(
c*x^2 + b))^2 - b)^11
```

3.117 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{17}} \, dx$

Optimal. Leaf size=170

$$\frac{16c^3 \left(bx^2+cx^4\right)^{5/2} (13bB-8Ac)}{15015b^5x^{10}} - \frac{8c^2 \left(bx^2+cx^4\right)^{5/2} (13bB-8Ac)}{3003b^4x^{12}} + \frac{2c \left(bx^2+cx^4\right)^{5/2} (13bB-8Ac)}{429b^3x^{14}} - \frac{\left(bx^2+cx^4\right)^{5/2} (13}{143b^2x}$$

```
[Out] -(A*(b*x^2 + c*x^4)^(5/2))/(13*b*x^18) - ((13*b*B - 8*A*c)*(b*x^2 + c*x^4)^
(5/2))/(143*b^2*x^16) + (2*c*(13*b*B - 8*A*c)*(b*x^2 + c*x^4)^(5/2))/(429*b
^3*x^14) - (8*c^2*(13*b*B - 8*A*c)*(b*x^2 + c*x^4)^(5/2))/(3003*b^4*x^12) +
 (16*c^3*(13*b*B - 8*A*c)*(b*x^2 + c*x^4)^(5/2))/(15015*b^5*x^10)
```

Rubi [A] time = 0.321332, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2034, 792, 658, 650}

$$\frac{16c^3 \left(bx^2+cx^4\right)^{5/2} (13bB-8Ac)}{15015b^5x^{10}} - \frac{8c^2 \left(bx^2+cx^4\right)^{5/2} (13bB-8Ac)}{3003b^4x^{12}} + \frac{2c \left(bx^2+cx^4\right)^{5/2} (13bB-8Ac)}{429b^3x^{14}} - \frac{\left(bx^2+cx^4\right)^{5/2} (13}{143b^2x}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^17,x]
```

```
[Out] -(A*(b*x^2 + c*x^4)^(5/2))/(13*b*x^18) - ((13*b*B - 8*A*c)*(b*x^2 + c*x^4)^
(5/2))/(143*b^2*x^16) + (2*c*(13*b*B - 8*A*c)*(b*x^2 + c*x^4)^(5/2))/(429*b
^3*x^14) - (8*c^2*(13*b*B - 8*A*c)*(b*x^2 + c*x^4)^(5/2))/(3003*b^4*x^12) +
 (16*c^3*(13*b*B - 8*A*c)*(b*x^2 + c*x^4)^(5/2))/(15015*b^5*x^10)
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
```

```
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{17}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{(A+Bx)(bx+cx^2)^{3/2}}{x^9}\,dx, x, x^2\right)$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{13bx^{18}} + \frac{\left(-9(-bB+Ac)+\frac{5}{2}(-bB+2Ac)\right)\operatorname{Subst}\left(\int \frac{(bx+cx^2)^{3/2}}{x^8}\,dx, x, x^2\right)}{13b}$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{13bx^{18}} - \frac{(13bB-8Ac)(bx^2+cx^4)^{5/2}}{143b^2x^{16}} - \frac{(3c(13bB-8Ac))\operatorname{Subst}\left(\int \frac{(bx+cx^2)}{x^7}\right.}{143b^2}$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{13bx^{18}} - \frac{(13bB-8Ac)(bx^2+cx^4)^{5/2}}{143b^2x^{16}} + \frac{2c(13bB-8Ac)(bx^2+cx^4)^{5/2}}{429b^3x^{14}} + \cdots$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{13bx^{18}} - \frac{(13bB-8Ac)(bx^2+cx^4)^{5/2}}{143b^2x^{16}} + \frac{2c(13bB-8Ac)(bx^2+cx^4)^{5/2}}{429b^3x^{14}} - \frac{8\cdots}{\cdots}$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{13bx^{18}} - \frac{(13bB-8Ac)(bx^2+cx^4)^{5/2}}{143b^2x^{16}} + \frac{2c(13bB-8Ac)(bx^2+cx^4)^{5/2}}{429b^3x^{14}} - \frac{8\cdots}{\cdots}$$

Mathematica [A] time = 0.0704421, size = 95, normalized size = 0.56

$$\frac{\sqrt{x^2(b+cx^2)}\left(\left(\frac{cx^3}{b}+x\right)^2\left(-70b^2cx^2+105b^3+40bc^2x^4-16c^3x^6\right)(8Ac-13bB)-1155Ab^2\left(b+cx^2\right)^2\right)}{15015b^3x^{14}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^17,x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(-1155*A*b^2*(b + c*x^2)^2 + (-13*b*B + 8*A*c)*(x +
(c*x^3)/b)^2*(105*b^3 - 70*b^2*c*x^2 + 40*b*c^2*x^4 - 16*c^3*x^6)))/(15015*
b^3*x^14)
```

Maple [A] time = 0.006, size = 118, normalized size = 0.7

$$-\frac{(cx^2+b)\left(128\,Ac^4x^8-208\,Bbc^3x^8-320\,Abc^3x^6+520\,Bb^2c^2x^6+560\,Ab^2c^2x^4-910\,Bb^3cx^4-840\,Ab^3cx^2+1365\,Bb^4x\cdots\right.}{15015\,x^{16}b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^17,x)
```

```
[Out] -1/15015*(c*x^2+b)*(128*A*c^4*x^8-208*B*b*c^3*x^8-320*A*b*c^3*x^6+520*B*b^2
*c^2*x^6+560*A*b^2*c^2*x^4-910*B*b^3*c*x^4-840*A*b^3*c*x^2+1365*B*b^4*x^2+1
155*A*b^4)*(c*x^4+b*x^2)^(3/2)/x^16/b^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^17,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.81635, size = 356, normalized size = 2.09

$$\frac{\left(16\left(13\,Bbc^5-8\,Ac^6\right)x^{12}-8\left(13\,Bb^2c^4-8\,Abc^5\right)x^{10}+6\left(13\,Bb^3c^3-8\,Ab^2c^4\right)x^8-1155\,Ab^6-5\left(13\,Bb^4c^2-8\,Ab^3c^3\right)x^6}{15015\,b^5x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^17,x, algorithm="fricas")
```

```
[Out] 1/15015*(16*(13*B*b*c^5 - 8*A*c^6)*x^12 - 8*(13*B*b^2*c^4 - 8*A*b*c^5)*x^10
 + 6*(13*B*b^3*c^3 - 8*A*b^2*c^4)*x^8 - 1155*A*b^6 - 5*(13*B*b^4*c^2 - 8*A*
b^3*c^3)*x^6 - 35*(52*B*b^5*c + A*b^4*c^2)*x^4 - 105*(13*B*b^6 + 14*A*b^5*c
)*x^2)*sqrt(c*x^4 + b*x^2)/(b^5*x^14)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}\left(A+Bx^2\right)}{x^{17}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**17,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**17, x)
```

Giac [B] time = 5.15756, size = 743, normalized size = 4.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^17,x, algorithm="giac")
```

```
[Out] 32/15015*(15015*(sqrt(c)*x - sqrt(c*x^2 + b))^18*B*c^(11/2)*sgn(x) - 3003*(
sqrt(c)*x - sqrt(c*x^2 + b))^16*B*b*c^(11/2)*sgn(x) + 48048*(sqrt(c)*x - sq
rt(c*x^2 + b))^16*A*c^(13/2)*sgn(x) - 6006*(sqrt(c)*x - sqrt(c*x^2 + b))^14
*B*b^2*c^(11/2)*sgn(x) + 96096*(sqrt(c)*x - sqrt(c*x^2 + b))^14*A*b*c^(13/2
)*sgn(x) - 28314*(sqrt(c)*x - sqrt(c*x^2 + b))^12*B*b^3*c^(11/2)*sgn(x) + 1
09824*(sqrt(c)*x - sqrt(c*x^2 + b))^12*A*b^2*c^(13/2)*sgn(x) + 13728*(sqrt(
c)*x - sqrt(c*x^2 + b))^10*B*b^4*c^(11/2)*sgn(x) + 37752*(sqrt(c)*x - sqrt(
c*x^2 + b))^10*A*b^3*c^(13/2)*sgn(x) + 5720*(sqrt(c)*x - sqrt(c*x^2 + b))^8
*B*b^5*c^(11/2)*sgn(x) + 5720*(sqrt(c)*x - sqrt(c*x^2 + b))^8*A*b^4*c^(13/2
)*sgn(x) + 3718*(sqrt(c)*x - sqrt(c*x^2 + b))^6*B*b^6*c^(11/2)*sgn(x) - 228
8*(sqrt(c)*x - sqrt(c*x^2 + b))^6*A*b^5*c^(13/2)*sgn(x) - 1014*(sqrt(c)*x -
 sqrt(c*x^2 + b))^4*B*b^7*c^(11/2)*sgn(x) + 624*(sqrt(c)*x - sqrt(c*x^2 + b
))^4*A*b^6*c^(13/2)*sgn(x) + 169*(sqrt(c)*x - sqrt(c*x^2 + b))^2*B*b^8*c^(1
1/2)*sgn(x) - 104*(sqrt(c)*x - sqrt(c*x^2 + b))^2*A*b^7*c^(13/2)*sgn(x) - 1
3*B*b^9*c^(11/2)*sgn(x) + 8*A*b^8*c^(13/2)*sgn(x))/((sqrt(c)*x - sqrt(c*x^2
 + b))^2 - b)^13
```

3.118 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{19}}\,dx$

Optimal. Leaf size=207

$$-\frac{128c^4\left(bx^2+cx^4\right)^{5/2}(3bB-2Ac)}{45045b^6x^{10}}+\frac{64c^3\left(bx^2+cx^4\right)^{5/2}(3bB-2Ac)}{9009b^5x^{12}}-\frac{16c^2\left(bx^2+cx^4\right)^{5/2}(3bB-2Ac)}{1287b^4x^{14}}+\frac{8c\left(bx^2+cx\right.}{42}$$

```
[Out] -(A*(b*x^2 + c*x^4)^(5/2))/(15*b*x^20) - ((3*b*B - 2*A*c)*(b*x^2 + c*x^4)^(
5/2))/(39*b^2*x^18) + (8*c*(3*b*B - 2*A*c)*(b*x^2 + c*x^4)^(5/2))/(429*b^3*
x^16) - (16*c^2*(3*b*B - 2*A*c)*(b*x^2 + c*x^4)^(5/2))/(1287*b^4*x^14) + (6
4*c^3*(3*b*B - 2*A*c)*(b*x^2 + c*x^4)^(5/2))/(9009*b^5*x^12) - (128*c^4*(3*
b*B - 2*A*c)*(b*x^2 + c*x^4)^(5/2))/(45045*b^6*x^10)
```

Rubi [A] time = 0.351677, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2034, 792, 658, 650}

$$-\frac{128c^4\left(bx^2+cx^4\right)^{5/2}(3bB-2Ac)}{45045b^6x^{10}}+\frac{64c^3\left(bx^2+cx^4\right)^{5/2}(3bB-2Ac)}{9009b^5x^{12}}-\frac{16c^2\left(bx^2+cx^4\right)^{5/2}(3bB-2Ac)}{1287b^4x^{14}}+\frac{8c\left(bx^2+cx\right.}{42}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^19,x]
```

```
[Out] -(A*(b*x^2 + c*x^4)^(5/2))/(15*b*x^20) - ((3*b*B - 2*A*c)*(b*x^2 + c*x^4)^(
5/2))/(39*b^2*x^18) + (8*c*(3*b*B - 2*A*c)*(b*x^2 + c*x^4)^(5/2))/(429*b^3*
x^16) - (16*c^2*(3*b*B - 2*A*c)*(b*x^2 + c*x^4)^(5/2))/(1287*b^4*x^14) + (6
4*c^3*(3*b*B - 2*A*c)*(b*x^2 + c*x^4)^(5/2))/(9009*b^5*x^12) - (128*c^4*(3*
b*B - 2*A*c)*(b*x^2 + c*x^4)^(5/2))/(45045*b^6*x^10)
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] && !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{19}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)(bx+cx^2)^{3/2}}{x^{10}}\,dx, x, x^2\right)$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{15bx^{20}} + \frac{\left(-10(-bB+Ac)+\frac{5}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{(bx+cx^2)^{3/2}}{x^9}\,dx, x, x^2\right)}{15b}$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{15bx^{20}} - \frac{(3bB-2Ac)(bx^2+cx^4)^{5/2}}{39b^2x^{18}} - \frac{(4c(3bB-2Ac))\text{Subst}\left(\int \frac{(bx+cx^2)^{3}}{x^8}\right.}{39b^2}$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{15bx^{20}} - \frac{(3bB-2Ac)(bx^2+cx^4)^{5/2}}{39b^2x^{18}} + \frac{8c(3bB-2Ac)(bx^2+cx^4)^{5/2}}{429b^3x^{16}} + \frac{(8}{}$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{15bx^{20}} - \frac{(3bB-2Ac)(bx^2+cx^4)^{5/2}}{39b^2x^{18}} + \frac{8c(3bB-2Ac)(bx^2+cx^4)^{5/2}}{429b^3x^{16}} - \frac{16}{}$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{15bx^{20}} - \frac{(3bB-2Ac)(bx^2+cx^4)^{5/2}}{39b^2x^{18}} + \frac{8c(3bB-2Ac)(bx^2+cx^4)^{5/2}}{429b^3x^{16}} - \frac{16}{}$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{15bx^{20}} - \frac{(3bB-2Ac)(bx^2+cx^4)^{5/2}}{39b^2x^{18}} + \frac{8c(3bB-2Ac)(bx^2+cx^4)^{5/2}}{429b^3x^{16}} - \frac{16}{}$$

Mathematica [A] time = 0.0670279, size = 89, normalized size = 0.43

$$\frac{(x^2(b+cx^2))^{5/2}\left(-x^2(560b^2c^2x^4-840b^3cx^2+1155b^4-320bc^3x^6+128c^4x^8)(3bB-2Ac)-3003Ab^5\right)}{45045b^6x^{20}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^19,x]
```

```
[Out] ((x^2*(b + c*x^2))^(5/2)*(-3003*A*b^5 - (3*b*B - 2*A*c)*x^2*(1155*b^4 - 840
*b^3*c*x^2 + 560*b^2*c^2*x^4 - 320*b*c^3*x^6 + 128*c^4*x^8)))/(45045*b^6*x^
20)
```

Maple [A] time = 0.007, size = 142, normalized size = 0.7

$$-\frac{(cx^2+b)\left(-256\,Ac^5x^{10}+384\,Bbc^4x^{10}+640\,Abc^4x^8-960\,Bb^2c^3x^8-1120\,Ab^2c^3x^6+1680\,Bb^3c^2x^6+1680\,Ab^3c^2x^4-\right.}{45045\,x^{18}b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^19,x)
```

```
[Out] -1/45045*(c*x^2+b)*(-256*A*c^5*x^10+384*B*b*c^4*x^10+640*A*b*c^4*x^8-960*B*
b^2*c^3*x^8-1120*A*b^2*c^3*x^6+1680*B*b^3*c^2*x^6+1680*A*b^3*c^2*x^4-2520*B
*b^4*c*x^4-2310*A*b^4*c*x^2+3465*B*b^5*x^2+3003*A*b^5)*(c*x^4+b*x^2)^(3/2)/
x^18/b^6
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^19,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.55657, size = 409, normalized size = 1.98

$$-\frac{\left(128\left(3\,Bbc^{6}-2\,Ac^{7}\right)x^{14}-64\left(3\,Bb^{2}c^{5}-2\,Abc^{6}\right)x^{12}+48\left(3\,Bb^{3}c^{4}-2\,Ab^{2}c^{5}\right)x^{10}-40\left(3\,Bb^{4}c^{3}-2\,Ab^{3}c^{4}\right)x^{8}+3003\,Ab}{45045\,b^{6}x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^19,x, algorithm="fricas")
```

```
[Out] -1/45045*(128*(3*B*b*c^6 - 2*A*c^7)*x^14 - 64*(3*B*b^2*c^5 - 2*A*b*c^6)*x^1
2 + 48*(3*B*b^3*c^4 - 2*A*b^2*c^5)*x^10 - 40*(3*B*b^4*c^3 - 2*A*b^3*c^4)*x^
8 + 3003*A*b^7 + 35*(3*B*b^5*c^2 - 2*A*b^4*c^3)*x^6 + 63*(70*B*b^6*c + A*b^
5*c^2)*x^4 + 231*(15*B*b^7 + 16*A*b^6*c)*x^2)*sqrt(c*x^4 + b*x^2)/(b^6*x^16
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^{2}\left(b+cx^{2}\right)\right)^{\frac{3}{2}}\left(A+Bx^{2}\right)}{x^{19}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**19,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**19, x)
```

Giac [B] time = 6.31061, size = 786, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^19,x, algorithm="giac")
```

```
[Out] 256/45045*(18018*(sqrt(c)*x - sqrt(c*x^2 + b))^20*B*c^(13/2)*sgn(x) + 60060
*(sqrt(c)*x - sqrt(c*x^2 + b))^18*A*c^(15/2)*sgn(x) - 12870*(sqrt(c)*x - sq
rt(c*x^2 + b))^16*B*b^2*c^(13/2)*sgn(x) + 128700*(sqrt(c)*x - sqrt(c*x^2 +
b))^16*A*b*c^(15/2)*sgn(x) - 32175*(sqrt(c)*x - sqrt(c*x^2 + b))^14*B*b^3*c
^(13/2)*sgn(x) + 141570*(sqrt(c)*x - sqrt(c*x^2 + b))^14*A*b^2*c^(15/2)*sgn
(x) + 15015*(sqrt(c)*x - sqrt(c*x^2 + b))^12*B*b^4*c^(13/2)*sgn(x) + 50050*
(sqrt(c)*x - sqrt(c*x^2 + b))^12*A*b^3*c^(15/2)*sgn(x) + 9009*(sqrt(c)*x -
sqrt(c*x^2 + b))^10*B*b^5*c^(13/2)*sgn(x) + 6006*(sqrt(c)*x - sqrt(c*x^2 +
b))^10*A*b^4*c^(15/2)*sgn(x) + 4095*(sqrt(c)*x - sqrt(c*x^2 + b))^8*B*b^6*c
^(13/2)*sgn(x) - 2730*(sqrt(c)*x - sqrt(c*x^2 + b))^8*A*b^5*c^(15/2)*sgn(x)
 - 1365*(sqrt(c)*x - sqrt(c*x^2 + b))^6*B*b^7*c^(13/2)*sgn(x) + 910*(sqrt(c
)*x - sqrt(c*x^2 + b))^6*A*b^6*c^(15/2)*sgn(x) + 315*(sqrt(c)*x - sqrt(c*x^
2 + b))^4*B*b^8*c^(13/2)*sgn(x) - 210*(sqrt(c)*x - sqrt(c*x^2 + b))^4*A*b^7
*c^(15/2)*sgn(x) - 45*(sqrt(c)*x - sqrt(c*x^2 + b))^2*B*b^9*c^(13/2)*sgn(x)
 + 30*(sqrt(c)*x - sqrt(c*x^2 + b))^2*A*b^8*c^(15/2)*sgn(x) + 3*B*b^10*c^(1
3/2)*sgn(x) - 2*A*b^9*c^(15/2)*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b
)^15
```

3.119 $\int x^4 \left(A+Bx^2\right) \left(bx^2+cx^4\right)^{3/2} \, dx$

Optimal. Leaf size=168

$$\frac{16b^3\left(bx^2+cx^4\right)^{5/2}(8bB-13Ac)}{15015c^5x^5}-\frac{8b^2\left(bx^2+cx^4\right)^{5/2}(8bB-13Ac)}{3003c^4x^3}+\frac{2b\left(bx^2+cx^4\right)^{5/2}(8bB-13Ac)}{429c^3x}-\frac{x\left(bx^2+cx^4\right)^{5/2}}{143c}$$

```
[Out] (16*b^3*(8*b*B - 13*A*c)*(b*x^2 + c*x^4)^(5/2))/(15015*c^5*x^5) - (8*b^2*(8
*b*B - 13*A*c)*(b*x^2 + c*x^4)^(5/2))/(3003*c^4*x^3) + (2*b*(8*b*B - 13*A*c
)*(b*x^2 + c*x^4)^(5/2))/(429*c^3*x) - ((8*b*B - 13*A*c)*x*(b*x^2 + c*x^4)^
(5/2))/(143*c^2) + (B*x^3*(b*x^2 + c*x^4)^(5/2))/(13*c)
```

Rubi [A] time = 0.297179, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2039, 2016, 2002, 2014}

$$\frac{16b^3\left(bx^2+cx^4\right)^{5/2}(8bB-13Ac)}{15015c^5x^5}-\frac{8b^2\left(bx^2+cx^4\right)^{5/2}(8bB-13Ac)}{3003c^4x^3}+\frac{2b\left(bx^2+cx^4\right)^{5/2}(8bB-13Ac)}{429c^3x}-\frac{x\left(bx^2+cx^4\right)^{5/2}}{143c}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (16*b^3*(8*b*B - 13*A*c)*(b*x^2 + c*x^4)^(5/2))/(15015*c^5*x^5) - (8*b^2*(8
*b*B - 13*A*c)*(b*x^2 + c*x^4)^(5/2))/(3003*c^4*x^3) + (2*b*(8*b*B - 13*A*c
)*(b*x^2 + c*x^4)^(5/2))/(429*c^3*x) - ((8*b*B - 13*A*c)*x*(b*x^2 + c*x^4)^
(5/2))/(143*c^2) + (B*x^3*(b*x^2 + c*x^4)^(5/2))/(13*c)
```

Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int x^4\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}dx=\frac{Bx^3\left(bx^2+cx^4\right)^{5/2}}{13c}-\frac{(8bB-13Ac)\int x^4\left(bx^2+cx^4\right)^{3/2}dx}{13c}$$
$$=-\frac{(8bB-13Ac)x\left(bx^2+cx^4\right)^{5/2}}{143c^2}+\frac{Bx^3\left(bx^2+cx^4\right)^{5/2}}{13c}+\frac{(6b(8bB-13Ac))\int x^2\left(bx^2\right.}{143c^2}$$
$$=\frac{2b(8bB-13Ac)\left(bx^2+cx^4\right)^{5/2}}{429c^3x}-\frac{(8bB-13Ac)x\left(bx^2+cx^4\right)^{5/2}}{143c^2}+\frac{Bx^3\left(bx^2+cx^4\right)^{5/}}{13c}$$
$$=-\frac{8b^2(8bB-13Ac)\left(bx^2+cx^4\right)^{5/2}}{3003c^4x^3}+\frac{2b(8bB-13Ac)\left(bx^2+cx^4\right)^{5/2}}{429c^3x}-\frac{(8bB-13Ac}{14}$$
$$=\frac{16b^3(8bB-13Ac)\left(bx^2+cx^4\right)^{5/2}}{15015c^5x^5}-\frac{8b^2(8bB-13Ac)\left(bx^2+cx^4\right)^{5/2}}{3003c^4x^3}+\frac{2b(8bB-13}{4}$$

Mathematica [A] time = 0.0826835, size = 113, normalized size = 0.67

$$\frac{x\left(b+cx^2\right)^3\left(40b^2c^2x^2\left(13A+14Bx^2\right)-16b^3c\left(13A+20Bx^2\right)-70bc^3x^4\left(13A+12Bx^2\right)+105c^4x^6\left(13A+11Bx^2\right)+12\right.}{15015c^5\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(b + c*x^2)^3*(128*b^4*B + 105*c^4*x^6*(13*A + 11*B*x^2) - 70*b*c^3*x^4*
(13*A + 12*B*x^2) + 40*b^2*c^2*x^2*(13*A + 14*B*x^2) - 16*b^3*c*(13*A + 20*
B*x^2)))/(15015*c^5*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.005, size = 115, normalized size = 0.7

$$-\frac{\left(cx^2+b\right)\left(-1155\,Bx^8c^4-1365\,Ac^4x^6+840\,Bbc^3x^6+910\,Abc^3x^4-560\,Bb^2c^2x^4-520\,Ab^2c^2x^2+320\,Bb^3cx^2+208\,Ab^3\right.}{15015\,c^5x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/15015*(c*x^2+b)*(-1155*B*c^4*x^8-1365*A*c^4*x^6+840*B*b*c^3*x^6+910*A*b*
c^3*x^4-560*B*b^2*c^2*x^4-520*A*b^2*c^2*x^2+320*B*b^3*c*x^2+208*A*b^3*c-128
*B*b^4)*(c*x^4+b*x^2)^(3/2)/c^5/x^3
```

Maxima [A] time = 1.24136, size = 203, normalized size = 1.21

$$\frac{\left(105\,c^5x^{10}+140\,bc^4x^8+5\,b^2c^3x^6-6\,b^3c^2x^4+8\,b^4cx^2-16\,b^5\right)\sqrt{cx^2+b}A}{1155\,c^4}+\frac{\left(1155\,c^6x^{12}+1470\,bc^5x^{10}+35\,b^2c^4x^8-40\,b\right.}{1\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/1155*(105*c^5*x^10 + 140*b*c^4*x^8 + 5*b^2*c^3*x^6 - 6*b^3*c^2*x^4 + 8*b^
4*c*x^2 - 16*b^5)*sqrt(c*x^2 + b)*A/c^4 + 1/15015*(1155*c^6*x^12 + 1470*b*c
^5*x^10 + 35*b^2*c^4*x^8 - 40*b^3*c^3*x^6 + 48*b^4*c^2*x^4 - 64*b^5*c*x^2 +
 128*b^6)*sqrt(c*x^2 + b)*B/c^5
```

Fricas [A] time = 1.08841, size = 350, normalized size = 2.08

$$\frac{\left(1155\,Bc^6x^{12}+105\left(14\,Bbc^5+13\,Ac^6\right)x^{10}+35\left(Bb^2c^4+52\,Abc^5\right)x^8+128\,Bb^6-208\,Ab^5c-5\left(8\,Bb^3c^3-13\,Ab^2c^4\right)x^6+\right.}{15015\,c^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15015*(1155*B*c^6*x^12 + 105*(14*B*b*c^5 + 13*A*c^6)*x^10 + 35*(B*b^2*c^4
 + 52*A*b*c^5)*x^8 + 128*B*b^6 - 208*A*b^5*c - 5*(8*B*b^3*c^3 - 13*A*b^2*c^
4)*x^6 + 6*(8*B*b^4*c^2 - 13*A*b^3*c^3)*x^4 - 8*(8*B*b^5*c - 13*A*b^4*c^2)*
x^2)*sqrt(c*x^4 + b*x^2)/(c^5*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}\left(A+Bx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**4*(x**2*(b + c*x**2))**(3/2)*(A + B*x**2), x)
```

Giac [B] time = 1.24968, size = 440, normalized size = 2.62

$$\frac{143\left(35\left(cx^2+b\right)^{\frac{9}{2}}-135\left(cx^2+b\right)^{\frac{7}{2}}b+189\left(cx^2+b\right)^{\frac{5}{2}}b^2-105\left(cx^2+b\right)^{\frac{3}{2}}b^3\right)Ab\,\mathrm{sgn}(x)}{c^3}+\frac{13\left(315\left(cx^2+b\right)^{\frac{11}{2}}-1540\left(cx^2+b\right)^{\frac{9}{2}}b+2970\left(cx^2+b\right)^{\frac{7}{2}}b^2-2772\left(cx^2+b\right)^{\frac{5}{2}}b^3\right.}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/45045*(143*(35*(c*x^2 + b)^(9/2) - 135*(c*x^2 + b)^(7/2)*b + 189*(c*x^2 +
 b)^(5/2)*b^2 - 105*(c*x^2 + b)^(3/2)*b^3)*A*b*sgn(x)/c^3 + 13*(315*(c*x^2
+ b)^(11/2) - 1540*(c*x^2 + b)^(9/2)*b + 2970*(c*x^2 + b)^(7/2)*b^2 - 2772*
(c*x^2 + b)^(5/2)*b^3 + 1155*(c*x^2 + b)^(3/2)*b^4)*B*b*sgn(x)/c^4 + 13*(31
5*(c*x^2 + b)^(11/2) - 1540*(c*x^2 + b)^(9/2)*b + 2970*(c*x^2 + b)^(7/2)*b^
2 - 2772*(c*x^2 + b)^(5/2)*b^3 + 1155*(c*x^2 + b)^(3/2)*b^4)*A*sgn(x)/c^3 +
 5*(693*(c*x^2 + b)^(13/2) - 4095*(c*x^2 + b)^(11/2)*b + 10010*(c*x^2 + b)^
(9/2)*b^2 - 12870*(c*x^2 + b)^(7/2)*b^3 + 9009*(c*x^2 + b)^(5/2)*b^4 - 3003
*(c*x^2 + b)^(3/2)*b^5)*B*sgn(x)/c^4)/c - 16/15015*(8*B*b^(13/2) - 13*A*b^(
```

```
11/2)*c)*sgn(x)/c^5
```

3.120 $\int x^2 \left(A+Bx^2\right) \left(bx^2+cx^4\right)^{3/2} \, dx$

Optimal. Leaf size=131

$$-\frac{8b^2\left(bx^2+cx^4\right)^{5/2}(6bB-11Ac)}{3465c^4x^5}-\frac{\left(bx^2+cx^4\right)^{5/2}(6bB-11Ac)}{99c^2x}+\frac{4b\left(bx^2+cx^4\right)^{5/2}(6bB-11Ac)}{693c^3x^3}+\frac{Bx\left(bx^2+cx^4\right)^{5/2}}{11c}$$

```
[Out] (-8*b^2*(6*b*B - 11*A*c)*(b*x^2 + c*x^4)^(5/2))/(3465*c^4*x^5) + (4*b*(6*b*
B - 11*A*c)*(b*x^2 + c*x^4)^(5/2))/(693*c^3*x^3) - ((6*b*B - 11*A*c)*(b*x^2
 + c*x^4)^(5/2))/(99*c^2*x) + (B*x*(b*x^2 + c*x^4)^(5/2))/(11*c)
```

Rubi [A] time = 0.241394, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2039, 2016, 2002, 2014}

$$-\frac{8b^2\left(bx^2+cx^4\right)^{5/2}(6bB-11Ac)}{3465c^4x^5}-\frac{\left(bx^2+cx^4\right)^{5/2}(6bB-11Ac)}{99c^2x}+\frac{4b\left(bx^2+cx^4\right)^{5/2}(6bB-11Ac)}{693c^3x^3}+\frac{Bx\left(bx^2+cx^4\right)^{5/2}}{11c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```


```
[Out] (-8*b^2*(6*b*B - 11*A*c)*(b*x^2 + c*x^4)^(5/2))/(3465*c^4*x^5) + (4*b*(6*b*
B - 11*A*c)*(b*x^2 + c*x^4)^(5/2))/(693*c^3*x^3) - ((6*b*B - 11*A*c)*(b*x^2
 + c*x^4)^(5/2))/(99*c^2*x) + (B*x*(b*x^2 + c*x^4)^(5/2))/(11*c)
```


Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
```

```
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int x^2\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}dx=\frac{Bx\left(bx^2+cx^4\right)^{5/2}}{11c}-\frac{(6bB-11Ac)\int x^2\left(bx^2+cx^4\right)^{3/2}dx}{11c}$$

$$=-\frac{(6bB-11Ac)\left(bx^2+cx^4\right)^{5/2}}{99c^2x}+\frac{Bx\left(bx^2+cx^4\right)^{5/2}}{11c}+\frac{(4b(6bB-11Ac))\int\left(bx^2+cx^4\right)}{99c^2}$$

$$=\frac{4b(6bB-11Ac)\left(bx^2+cx^4\right)^{5/2}}{693c^3x^3}-\frac{(6bB-11Ac)\left(bx^2+cx^4\right)^{5/2}}{99c^2x}+\frac{Bx\left(bx^2+cx^4\right)^{5/2}}{11c}-$$

$$=-\frac{8b^2(6bB-11Ac)\left(bx^2+cx^4\right)^{5/2}}{3465c^4x^5}+\frac{4b(6bB-11Ac)\left(bx^2+cx^4\right)^{5/2}}{693c^3x^3}-\frac{(6bB-11Ac)\left(\right.}{99c^2}$$

Mathematica [A] time = 0.067365, size = 92, normalized size = 0.7

$$\frac{x\left(b+cx^2\right)^3\left(8b^2c\left(11A+15Bx^2\right)-10bc^2x^2\left(22A+21Bx^2\right)+35c^3x^4\left(11A+9Bx^2\right)-48b^3B\right)}{3465c^4\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(b + c*x^2)^3*(-48*b^3*B + 35*c^3*x^4*(11*A + 9*B*x^2) + 8*b^2*c*(11*A +
 15*B*x^2) - 10*b*c^2*x^2*(22*A + 21*B*x^2)))/(3465*c^4*Sqrt[x^2*(b + c*x^2
)])
```

Maple [A] time = 0.006, size = 91, normalized size = 0.7

$$\frac{(cx^2+b)(315\,Bc^3x^6+385\,Ax^4c^3-210\,Bx^4bc^2-220\,Ax^2bc^2+120\,Bx^2b^2c+88\,Ab^2c-48\,Bb^3)}{3465\,c^4x^3}(cx^4+bx^2)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/3465*(c*x^2+b)*(315*B*c^3*x^6+385*A*c^3*x^4-210*B*b*c^2*x^4-220*A*b*c^2*x
^2+120*B*b^2*c*x^2+88*A*b^2*c-48*B*b^3)*(c*x^4+b*x^2)^(3/2)/c^4/x^3
```

Maxima [A] time = 1.28856, size = 173, normalized size = 1.32

$$\frac{(35\,c^4x^8+50\,bc^3x^6+3\,b^2c^2x^4-4\,b^3cx^2+8\,b^4)\sqrt{cx^2+b}A}{315\,c^3}+\frac{(105\,c^5x^{10}+140\,bc^4x^8+5\,b^2c^3x^6-6\,b^3c^2x^4+8\,b^4cx^2-1}{1155\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/315*(35*c^4*x^8 + 50*b*c^3*x^6 + 3*b^2*c^2*x^4 - 4*b^3*c*x^2 + 8*b^4)*sqr
t(c*x^2 + b)*A/c^3 + 1/1155*(105*c^5*x^10 + 140*b*c^4*x^8 + 5*b^2*c^3*x^6 -
 6*b^3*c^2*x^4 + 8*b^4*c*x^2 - 16*b^5)*sqrt(c*x^2 + b)*B/c^4
```

Fricas [A] time = 1.20251, size = 294, normalized size = 2.24

$$\frac{(315\,Bc^5x^{10}+35\,(12\,Bbc^4+11\,Ac^5)x^8+5\,(3\,Bb^2c^3+110\,Abc^4)x^6-48\,Bb^5+88\,Ab^4c-3\,(6\,Bb^3c^2-11\,Ab^2c^3)x^4+4\,(}{3465\,c^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3465*(315*B*c^5*x^10 + 35*(12*B*b*c^4 + 11*A*c^5)*x^8 + 5*(3*B*b^2*c^3 +
110*A*b*c^4)*x^6 - 48*B*b^5 + 88*A*b^4*c - 3*(6*B*b^3*c^2 - 11*A*b^2*c^3)*x
^4 + 4*(6*B*b^4*c - 11*A*b^3*c^2)*x^2)*sqrt(c*x^4 + b*x^2)/(c^4*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \left(x^2 \left(b + cx^2\right)\right)^{\frac{3}{2}} \left(A + Bx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**2*(x**2*(b + c*x**2))**(3/2)*(A + B*x**2), x)
```

Giac [B] time = 1.22388, size = 363, normalized size = 2.77

$$\frac{\frac{33\left(15\left(cx^2+b\right)^{\frac{7}{2}}-42\left(cx^2+b\right)^{\frac{5}{2}}b+35\left(cx^2+b\right)^{\frac{3}{2}}b^2\right)Ab\operatorname{sgn}(x)}{c^2}+\frac{11\left(35\left(cx^2+b\right)^{\frac{9}{2}}-135\left(cx^2+b\right)^{\frac{7}{2}}b+189\left(cx^2+b\right)^{\frac{5}{2}}b^2-105\left(cx^2+b\right)^{\frac{3}{2}}b^3\right)Bb\operatorname{sgn}(x)}{c^3}+\frac{11\left(35\left(cx^2+b\right)\right.}{}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3465*(33*(15*(c*x^2 + b)^(7/2) - 42*(c*x^2 + b)^(5/2)*b + 35*(c*x^2 + b)^
(3/2)*b^2)*A*b*sgn(x)/c^2 + 11*(35*(c*x^2 + b)^(9/2) - 135*(c*x^2 + b)^(7/2
)*b + 189*(c*x^2 + b)^(5/2)*b^2 - 105*(c*x^2 + b)^(3/2)*b^3)*B*b*sgn(x)/c^3
 + 11*(35*(c*x^2 + b)^(9/2) - 135*(c*x^2 + b)^(7/2)*b + 189*(c*x^2 + b)^(5/
2)*b^2 - 105*(c*x^2 + b)^(3/2)*b^3)*A*sgn(x)/c^2 + (315*(c*x^2 + b)^(11/2)
- 1540*(c*x^2 + b)^(9/2)*b + 2970*(c*x^2 + b)^(7/2)*b^2 - 2772*(c*x^2 + b)^
(5/2)*b^3 + 1155*(c*x^2 + b)^(3/2)*b^4)*B*sgn(x)/c^3)/c + 8/3465*(6*B*b^(11
/2) - 11*A*b^(9/2)*c)*sgn(x)/c^4
```

3.121 $\int \left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=96

$$-\frac{\left(bx^2+cx^4\right)^{5/2}(4bB-9Ac)}{63c^2x^3}+\frac{2b\left(bx^2+cx^4\right)^{5/2}(4bB-9Ac)}{315c^3x^5}+\frac{B\left(bx^2+cx^4\right)^{5/2}}{9cx}$$

```
[Out] (2*b*(4*b*B - 9*A*c)*(b*x^2 + c*x^4)^(5/2))/(315*c^3*x^5) - ((4*b*B - 9*A*c
)*(b*x^2 + c*x^4)^(5/2))/(63*c^2*x^3) + (B*(b*x^2 + c*x^4)^(5/2))/(9*c*x)
```

Rubi [A] time = 0.069621, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {1145, 2002, 2014}

$$-\frac{\left(bx^2+cx^4\right)^{5/2}(4bB-9Ac)}{63c^2x^3}+\frac{2b\left(bx^2+cx^4\right)^{5/2}(4bB-9Ac)}{315c^3x^5}+\frac{B\left(bx^2+cx^4\right)^{5/2}}{9cx}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*b*(4*b*B - 9*A*c)*(b*x^2 + c*x^4)^(5/2))/(315*c^3*x^5) - ((4*b*B - 9*A*c
)*(b*x^2 + c*x^4)^(5/2))/(63*c^2*x^3) + (B*(b*x^2 + c*x^4)^(5/2))/(9*c*x)
```

Rule 1145

```
Int[((d_) + (e_.)*(x_)^2)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
Simp[(e*(b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 3)*x), x] - Dist[(b*e*(2*p + 1)
- c*d*(4*p + 3))/(c*(4*p + 3)), Int[(b*x^2 + c*x^4)^p, x], x] /; FreeQ[{b,
c, d, e, p}, x] &&  !IntegerQ[p] && NeQ[4*p + 3, 0] && NeQ[b*e*(2*p + 1) -
c*d*(4*p + 3), 0]
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx &= \frac{B\left(bx^2+cx^4\right)^{5/2}}{9cx}-\frac{(4bB-9Ac)\int \left(bx^2+cx^4\right)^{3/2}\,dx}{9c}\\
&= -\frac{(4bB-9Ac)\left(bx^2+cx^4\right)^{5/2}}{63c^2x^3}+\frac{B\left(bx^2+cx^4\right)^{5/2}}{9cx}+\frac{(2b(4bB-9Ac))\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^2}\,dx}{63c^2}\\
&= \frac{2b(4bB-9Ac)\left(bx^2+cx^4\right)^{5/2}}{315c^3x^5}-\frac{(4bB-9Ac)\left(bx^2+cx^4\right)^{5/2}}{63c^2x^3}+\frac{B\left(bx^2+cx^4\right)^{5/2}}{9cx}
\end{aligned}$$

Mathematica [A] time = 0.047036, size = 71, normalized size = 0.74

$$\frac{x\left(b+cx^2\right)^3\left(-2bc\left(9A+10Bx^2\right)+5c^2x^2\left(9A+7Bx^2\right)+8b^2B\right)}{315c^3\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(b + c*x^2)^3*(8*b^2*B + 5*c^2*x^2*(9*A + 7*B*x^2) - 2*b*c*(9*A + 10*B*x
^2)))/(315*c^3*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.004, size = 67, normalized size = 0.7

$$-\frac{\left(cx^2+b\right)\left(-35\,Bc^2x^4-45\,Ax^2c^2+20\,Bx^2bc+18\,Abc-8\,Bb^2\right)}{315\,c^3x^3}\left(cx^4+bx^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/315*(c*x^2+b)*(-35*B*c^2*x^4-45*A*c^2*x^2+20*B*b*c*x^2+18*A*b*c-8*B*b^2)
*(c*x^4+b*x^2)^(3/2)/c^3/x^3
```

Maxima [A] time = 1.27695, size = 142, normalized size = 1.48

$$\frac{\left(5\,c^3x^6+8\,bc^2x^4+b^2cx^2-2\,b^3\right)\sqrt{cx^2+b}A}{35\,c^2}+\frac{\left(35\,c^4x^8+50\,bc^3x^6+3\,b^2c^2x^4-4\,b^3cx^2+8\,b^4\right)\sqrt{cx^2+b}B}{315\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/35*(5*c^3*x^6 + 8*b*c^2*x^4 + b^2*c*x^2 - 2*b^3)*sqrt(c*x^2 + b)*A/c^2 +
1/315*(35*c^4*x^8 + 50*b*c^3*x^6 + 3*b^2*c^2*x^4 - 4*b^3*c*x^2 + 8*b^4)*sqr
t(c*x^2 + b)*B/c^3
```

Fricas [A] time = 1.08831, size = 228, normalized size = 2.38

$$\frac{\left(35\,Bc^4x^8+5\left(10\,Bbc^3+9\,Ac^4\right)x^6+8\,Bb^4-18\,Ab^3c+3\left(Bb^2c^2+24\,Abc^3\right)x^4-\left(4\,Bb^3c-9\,Ab^2c^2\right)x^2\right)\sqrt{cx^4+bx^2}}{315\,c^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/315*(35*B*c^4*x^8 + 5*(10*B*b*c^3 + 9*A*c^4)*x^6 + 8*B*b^4 - 18*A*b^3*c +
 3*(B*b^2*c^2 + 24*A*b*c^3)*x^4 - (4*B*b^3*c - 9*A*b^2*c^2)*x^2)*sqrt(c*x^4
 + b*x^2)/(c^3*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}\left(A+Bx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2), x)
```

Giac [B] time = 1.13837, size = 288, normalized size = 3.

$$\frac{\frac{21\left(3\left(cx^2+b\right)^{\frac{5}{2}}-5\left(cx^2+b\right)^{\frac{3}{2}}b\right)Ab\mathrm{sgn}(x)}{c}+\frac{3\left(15\left(cx^2+b\right)^{\frac{7}{2}}-42\left(cx^2+b\right)^{\frac{5}{2}}b+35\left(cx^2+b\right)^{\frac{3}{2}}b^2\right)Bb\mathrm{sgn}(x)}{c^2}+\frac{3\left(15\left(cx^2+b\right)^{\frac{7}{2}}-42\left(cx^2+b\right)^{\frac{5}{2}}b+35\left(cx^2+b\right)^{\frac{3}{2}}b^2\right)A\mathrm{sgn}(}{c}}{315\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/315*(21*(3*(c*x^2 + b)^(5/2) - 5*(c*x^2 + b)^(3/2)*b)*A*b*sgn(x)/c + 3*(1
5*(c*x^2 + b)^(7/2) - 42*(c*x^2 + b)^(5/2)*b + 35*(c*x^2 + b)^(3/2)*b^2)*B*
b*sgn(x)/c^2 + 3*(15*(c*x^2 + b)^(7/2) - 42*(c*x^2 + b)^(5/2)*b + 35*(c*x^2
 + b)^(3/2)*b^2)*A*sgn(x)/c + (35*(c*x^2 + b)^(9/2) - 135*(c*x^2 + b)^(7/2)
*b + 189*(c*x^2 + b)^(5/2)*b^2 - 105*(c*x^2 + b)^(3/2)*b^3)*B*sgn(x)/c^2)/c
 - 2/315*(4*B*b^(9/2) - 9*A*b^(7/2)*c)*sgn(x)/c^3
```

3.122 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=61

$$\frac{B\left(bx^2+cx^4\right)^{5/2}}{7cx^3}-\frac{\left(bx^2+cx^4\right)^{5/2}(2bB-7Ac)}{35c^2x^5}$$

```
[Out] -((2*b*B - 7*A*c)*(b*x^2 + c*x^4)^(5/2))/(35*c^2*x^5) + (B*(b*x^2 + c*x^4)^
(5/2))/(7*c*x^3)
```

Rubi [A] time = 0.159236, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2039, 2014}

$$\frac{B\left(bx^2+cx^4\right)^{5/2}}{7cx^3}-\frac{\left(bx^2+cx^4\right)^{5/2}(2bB-7Ac)}{35c^2x^5}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^2,x]
```

```
[Out] -((2*b*B - 7*A*c)*(b*x^2 + c*x^4)^(5/2))/(35*c^2*x^5) + (B*(b*x^2 + c*x^4)^
(5/2))/(7*c*x^3)
```

Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^2}\,dx = \frac{B(bx^2+cx^4)^{5/2}}{7cx^3} - \frac{(2bB-7Ac)\int \frac{(bx^2+cx^4)^{3/2}}{x^2}\,dx}{7c}$$
$$= -\frac{(2bB-7Ac)(bx^2+cx^4)^{5/2}}{35c^2x^5} + \frac{B(bx^2+cx^4)^{5/2}}{7cx^3}$$

Mathematica [A] time = 0.0312384, size = 48, normalized size = 0.79

$$\frac{x(b+cx^2)^3(7Ac-2bB+5Bcx^2)}{35c^2\sqrt{x^2(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^2,x]
```

```
[Out] (x*(b + c*x^2)^3*(-2*b*B + 7*A*c + 5*B*c*x^2))/(35*c^2*Sqrt[x^2*(b + c*x^2)
])
```

Maple [A] time = 0.003, size = 45, normalized size = 0.7

$$\frac{(cx^2+b)(5\,Bcx^2+7\,Ac-2\,Bb)}{35\,c^2x^3}(cx^4+bx^2)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^2,x)
```

```
[Out] 1/35*(c*x^2+b)*(5*B*c*x^2+7*A*c-2*B*b)*(c*x^4+b*x^2)^(3/2)/c^2/x^3
```

Maxima [A] time = 1.24526, size = 108, normalized size = 1.77

$$\frac{(c^2x^4+2\,bcx^2+b^2)\sqrt{cx^2+b}A}{5\,c} + \frac{(5\,c^3x^6+8\,bc^2x^4+b^2cx^2-2\,b^3)\sqrt{cx^2+b}B}{35\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/5*(c^2*x^4 + 2*b*c*x^2 + b^2)*sqrt(c*x^2 + b)*A/c + 1/35*(5*c^3*x^6 + 8*b
*c^2*x^4 + b^2*c*x^2 - 2*b^3)*sqrt(c*x^2 + b)*B/c^2
```

Fricas [A] time = 1.10781, size = 171, normalized size = 2.8

$$\frac{\left(5\,Bc^3x^6+\left(8\,Bbc^2+7\,Ac^3\right)x^4-2\,Bb^3+7\,Ab^2c+\left(Bb^2c+14\,Abc^2\right)x^2\right)\sqrt{cx^4+bx^2}}{35\,c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/35*(5*B*c^3*x^6 + (8*B*b*c^2 + 7*A*c^3)*x^4 - 2*B*b^3 + 7*A*b^2*c + (B*b^
2*c + 14*A*b*c^2)*x^2)*sqrt(c*x^4 + b*x^2)/(c^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}\left(A+Bx^2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**2,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**2, x)
```

Giac [B] time = 1.12936, size = 203, normalized size = 3.33

$$\frac{35\left(cx^2+b\right)^{\frac{3}{2}}Ab\,\mathrm{sgn}\left(x\right)+7\left(3\left(cx^2+b\right)^{\frac{5}{2}}-5\left(cx^2+b\right)^{\frac{3}{2}}b\right)A\,\mathrm{sgn}\left(x\right)+\frac{7\left(3\left(cx^2+b\right)^{\frac{5}{2}}-5\left(cx^2+b\right)^{\frac{3}{2}}b\right)Bb\,\mathrm{sgn}(x)}{c}+\frac{\left(15\left(cx^2+b\right)^{\frac{7}{2}}-42\left(cx^2\cdots\right.\right.}{\cdots}}{105\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/105*(35*(c*x^2 + b)^(3/2)*A*b*sgn(x) + 7*(3*(c*x^2 + b)^(5/2) - 5*(c*x^2
+ b)^(3/2)*b)*A*sgn(x) + 7*(3*(c*x^2 + b)^(5/2) - 5*(c*x^2 + b)^(3/2)*b)*B*
b*sgn(x)/c + (15*(c*x^2 + b)^(7/2) - 42*(c*x^2 + b)^(5/2)*b + 35*(c*x^2 + b
)^(3/2)*b^2)*B*sgn(x)/c)/c + 1/35*(2*B*b^(7/2) - 7*A*b^(5/2)*c)*sgn(x)/c^2
```

3.123 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=102

$$-Ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)+\frac{A\left(bx^2+cx^4\right)^{3/2}}{3x^3}+\frac{Ab\sqrt{bx^2+cx^4}}{x}+\frac{B\left(bx^2+cx^4\right)^{5/2}}{5cx^5}$$

```
[Out] (A*b*Sqrt[b*x^2 + c*x^4])/x + (A*(b*x^2 + c*x^4)^(3/2))/(3*x^3) + (B*(b*x^2
 + c*x^4)^(5/2))/(5*c*x^5) - A*b^(3/2)*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x
^4]]
```

Rubi [A] time = 0.205408, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2039, 2021, 2008, 206}

$$-Ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)+\frac{A\left(bx^2+cx^4\right)^{3/2}}{3x^3}+\frac{Ab\sqrt{bx^2+cx^4}}{x}+\frac{B\left(bx^2+cx^4\right)^{5/2}}{5cx^5}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^4,x]
```

```
[Out] (A*b*Sqrt[b*x^2 + c*x^4])/x + (A*(b*x^2 + c*x^4)^(3/2))/(3*x^3) + (B*(b*x^2
 + c*x^4)^(5/2))/(5*c*x^5) - A*b^(3/2)*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x
^4]]
```

Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
```

```
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}}{x^4}\,dx &= \frac{B\left(bx^2+cx^4\right)^{5/2}}{5cx^5}+A\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^4}\,dx\\
&= \frac{A\left(bx^2+cx^4\right)^{3/2}}{3x^3}+\frac{B\left(bx^2+cx^4\right)^{5/2}}{5cx^5}+(Ab)\int \frac{\sqrt{bx^2+cx^4}}{x^2}\,dx\\
&= \frac{Ab\sqrt{bx^2+cx^4}}{x}+\frac{A\left(bx^2+cx^4\right)^{3/2}}{3x^3}+\frac{B\left(bx^2+cx^4\right)^{5/2}}{5cx^5}+\left(Ab^2\right)\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx\\
&= \frac{Ab\sqrt{bx^2+cx^4}}{x}+\frac{A\left(bx^2+cx^4\right)^{3/2}}{3x^3}+\frac{B\left(bx^2+cx^4\right)^{5/2}}{5cx^5}-\left(Ab^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,
\end{aligned}
$$

$$
= \frac{Ab\sqrt{bx^2+cx^4}}{x}+\frac{A\left(bx^2+cx^4\right)^{3/2}}{3x^3}+\frac{B\left(bx^2+cx^4\right)^{5/2}}{5cx^5}-Ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)
$$

Mathematica [A] time = 0.0879823, size = 109, normalized size = 1.07

$$
\frac{\left(x^2\left(b+cx^2\right)\right)^{3/2}\left(-15Ab^{3/2}c\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)+5Ac\left(b+cx^2\right)^{3/2}+15Abc\sqrt{b+cx^2}+3B\left(b+cx^2\right)^{5/2}\right)}{15cx^3\left(b+cx^2\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^4,x]
```

```
[Out] ((x^2*(b + c*x^2))^(3/2)*(15*A*b*c*Sqrt[b + c*x^2] + 5*A*c*(b + c*x^2)^(3/2
) + 3*B*(b + c*x^2)^(5/2) - 15*A*b^(3/2)*c*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]]
))/(15*c*x^3*(b + c*x^2)^(3/2))
```

Maple [A] time = 0.009, size = 99, normalized size = 1.

$$-\frac{1}{15\,cx^3}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(-3\,B\left(cx^2+b\right)^{5/2}+15\,Ab^{3/2}\ln\left(2\,\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)c-5\,A\left(cx^2+b\right)^{3/2}c-15\,A\sqrt{cx^2+b}bc\right)\left(cx^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^4,x)
```

```
[Out] -1/15*(c*x^4+b*x^2)^(3/2)*(-3*B*(c*x^2+b)^(5/2)+15*A*b^(3/2)*ln(2*(b^(1/2)*
(c*x^2+b)^(1/2)+b)/x)*c-5*A*(c*x^2+b)^(3/2)*c-15*A*(c*x^2+b)^(1/2)*b*c)/x^3
/(c*x^2+b)^(3/2)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(Bx^2+A\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^4, x)
```

Fricas [A] time = 1.16754, size = 467, normalized size = 4.58

$$\left[\frac{15\,Ab^{\frac{3}{2}}cx\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)+2\left(3\,Bc^2x^4+3\,Bb^2+20\,Abc+\left(6\,Bbc+5\,Ac^2\right)x^2\right)\sqrt{cx^4+bx^2}}{30\,cx},\;\frac{15\,A\sqrt{-b}bcx\,\mathrm{arc}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/30*(15*A*b^(3/2)*c*x*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b)
)/x^3) + 2*(3*B*c^2*x^4 + 3*B*b^2 + 20*A*b*c + (6*B*b*c + 5*A*c^2)*x^2)*sqr
t(c*x^4 + b*x^2))/(c*x), 1/15*(15*A*sqrt(-b)*b*c*x*arctan(sqrt(c*x^4 + b*x^
2)*sqrt(-b)/(c*x^3 + b*x)) + (3*B*c^2*x^4 + 3*B*b^2 + 20*A*b*c + (6*B*b*c +
 5*A*c^2)*x^2)*sqrt(c*x^4 + b*x^2))/(c*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}\left(A+Bx^2\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**4,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**4, x)
```

Giac [A] time = 1.18216, size = 189, normalized size = 1.85

$$\frac{Ab^2\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)\operatorname{sgn}(x)}{\sqrt{-b}}-\frac{\left(15\,Ab^2c\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+3\,B\sqrt{-b}b^{\frac{5}{2}}+20\,A\sqrt{-b}b^{\frac{3}{2}}c\right)\operatorname{sgn}(x)}{15\,\sqrt{-b}c}+\frac{3\left(cx^2+b\right)^{\frac{5}{2}}Bc^4\operatorname{sgn}(x)+\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] A*b^2*arctan(sqrt(c*x^2 + b)/sqrt(-b))*sgn(x)/sqrt(-b) - 1/15*(15*A*b^2*c*a
rctan(sqrt(b)/sqrt(-b)) + 3*B*sqrt(-b)*b^(5/2) + 20*A*sqrt(-b)*b^(3/2)*c)*s
gn(x)/(sqrt(-b)*c) + 1/15*(3*(c*x^2 + b)^(5/2)*B*c^4*sgn(x) + 5*(c*x^2 + b)
^(3/2)*A*c^5*sgn(x) + 15*sqrt(c*x^2 + b)*A*b*c^5*sgn(x))/c^5
```

3.124 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^6}\,dx$

Optimal. Leaf size=133

$$\frac{(bx^2+cx^4)^{3/2}(3Ac+2bB)}{6bx^3}+\frac{\sqrt{bx^2+cx^4}(3Ac+2bB)}{2x}-\frac{1}{2}\sqrt{b}(3Ac+2bB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)-\frac{A(bx^2+cx^4)^{5/2}}{2bx^7}$$

```
[Out] ((2*b*B + 3*A*c)*Sqrt[b*x^2 + c*x^4])/(2*x) + ((2*b*B + 3*A*c)*(b*x^2 + c*x
^4)^(3/2))/(6*b*x^3) - (A*(b*x^2 + c*x^4)^(5/2))/(2*b*x^7) - (Sqrt[b]*(2*b*
B + 3*A*c)*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/2
```

Rubi [A] time = 0.219648, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2038, 2021, 2008, 206}

$$\frac{(bx^2+cx^4)^{3/2}(3Ac+2bB)}{6bx^3}+\frac{\sqrt{bx^2+cx^4}(3Ac+2bB)}{2x}-\frac{1}{2}\sqrt{b}(3Ac+2bB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)-\frac{A(bx^2+cx^4)^{5/2}}{2bx^7}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^6,x]
```


```
[Out] ((2*b*B + 3*A*c)*Sqrt[b*x^2 + c*x^4])/(2*x) + ((2*b*B + 3*A*c)*(b*x^2 + c*x
^4)^(3/2))/(6*b*x^3) - (A*(b*x^2 + c*x^4)^(5/2))/(2*b*x^7) - (Sqrt[b]*(2*b*
B + 3*A*c)*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/2
```


Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^6}\,dx = -\frac{A(bx^2+cx^4)^{5/2}}{2bx^7} - \frac{(-2bB-3Ac)\int \frac{(bx^2+cx^4)^{3/2}}{x^4}\,dx}{2b}$$

$$= \frac{(2bB+3Ac)(bx^2+cx^4)^{3/2}}{6bx^3} - \frac{A(bx^2+cx^4)^{5/2}}{2bx^7} - \frac{1}{2}(-2bB-3Ac)\int \frac{\sqrt{bx^2+cx^4}}{x^2}\,dx$$

$$= \frac{(2bB+3Ac)\sqrt{bx^2+cx^4}}{2x} + \frac{(2bB+3Ac)(bx^2+cx^4)^{3/2}}{6bx^3} - \frac{A(bx^2+cx^4)^{5/2}}{2bx^7} + \frac{1}{2}(b(2bB+$$

$$= \frac{(2bB+3Ac)\sqrt{bx^2+cx^4}}{2x} + \frac{(2bB+3Ac)(bx^2+cx^4)^{3/2}}{6bx^3} - \frac{A(bx^2+cx^4)^{5/2}}{2bx^7} - \frac{1}{2}(b(2bB+$$

$$= \frac{(2bB+3Ac)\sqrt{bx^2+cx^4}}{2x} + \frac{(2bB+3Ac)(bx^2+cx^4)^{3/2}}{6bx^3} - \frac{A(bx^2+cx^4)^{5/2}}{2bx^7} - \frac{1}{2}\sqrt{b}(2bB$$

Mathematica [A] time = 0.0628259, size = 109, normalized size = 0.82

$$\frac{\sqrt{x^2(b+cx^2)}\left(\sqrt{b+cx^2}\left(-3Ab+6Acx^2+8bBx^2+2Bcx^4\right)-3\sqrt{b}x^2(3Ac+2bB)\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)\right)}{6x^3\sqrt{b+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^6,x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(Sqrt[b + c*x^2]*(-3*A*b + 8*b*B*x^2 + 6*A*c*x^2 + 2
*B*c*x^4) - 3*Sqrt[b]*(2*b*B + 3*A*c)*x^2*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]])
)/(6*x^3*Sqrt[b + c*x^2])
```

Maple [A] time = 0.009, size = 172, normalized size = 1.3

$$-\frac{1}{6\,bx^5}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(9\,Ab^{3/2}\ln\left(2\,\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)x^2c-3\,A\left(cx^2+b\right)^{3/2}x^2c+6\,Bb^{5/2}\ln\left(2\,\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)x^2-2\,B\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^6,x)
```

```
[Out] -1/6*(c*x^4+b*x^2)^(3/2)*(9*A*b^(3/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*x
^2*c-3*A*(c*x^2+b)^(3/2)*x^2*c+6*B*b^(5/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)
/x)*x^2-2*B*(c*x^2+b)^(3/2)*x^2*b+3*A*(c*x^2+b)^(5/2)-9*A*(c*x^2+b)^(1/2)*x
^2*b*c-6*B*(c*x^2+b)^(1/2)*x^2*b^2)/x^5/(c*x^2+b)^(3/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(Bx^2+A\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^6, x)
```

Fricas [A] time = 1.11061, size = 447, normalized size = 3.36

$$\left[\frac{3\,(2\,Bb+3\,Ac)\sqrt{b}x^3\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)+2\left(2\,Bcx^4+2\,(4\,Bb+3\,Ac)x^2-3\,Ab\right)\sqrt{cx^4+bx^2}}{12\,x^3},\frac{3\,(2\,Bb+3\,Ac)\sqrt{-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(2*B*b + 3*A*c)*sqrt(b)*x^3*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b
*x^2)*sqrt(b))/x^3) + 2*(2*B*c*x^4 + 2*(4*B*b + 3*A*c)*x^2 - 3*A*b)*sqrt(c*
x^4 + b*x^2))/x^3, 1/6*(3*(2*B*b + 3*A*c)*sqrt(-b)*x^3*arctan(sqrt(c*x^4 +
b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + (2*B*c*x^4 + 2*(4*B*b + 3*A*c)*x^2 - 3*A*b
)*sqrt(c*x^4 + b*x^2))/x^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}\left(A+Bx^2\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**6,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**6, x)
```

Giac [A] time = 1.35209, size = 155, normalized size = 1.17

$$\frac{2\left(cx^2+b\right)^{\frac{3}{2}}Bc\mathrm{sgn}\left(x\right)+6\sqrt{cx^2+b}Bbc\mathrm{sgn}\left(x\right)+6\sqrt{cx^2+b}Ac^2\mathrm{sgn}\left(x\right)-\frac{3\sqrt{cx^2+b}Abc\mathrm{sgn}(x)}{x^2}+\frac{3\left(2\,Bb^2c\mathrm{sgn}(x)+3\,Abc^2\mathrm{sgn}(x)\right)\mathrm{arcta}}{\sqrt{-b}}}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/6*(2*(c*x^2 + b)^(3/2)*B*c*sgn(x) + 6*sqrt(c*x^2 + b)*B*b*c*sgn(x) + 6*sq
rt(c*x^2 + b)*A*c^2*sgn(x) - 3*sqrt(c*x^2 + b)*A*b*c*sgn(x)/x^2 + 3*(2*B*b^
2*c*sgn(x) + 3*A*b*c^2*sgn(x))*arctan(sqrt(c*x^2 + b)/sqrt(-b))/sqrt(-b))/c
```

3.125 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^8}\,dx$

Optimal. Leaf size=135

$$-\frac{(bx^2+cx^4)^{3/2}(Ac+4bB)}{8bx^5}+\frac{3c\sqrt{bx^2+cx^4}(Ac+4bB)}{8bx}-\frac{3c(Ac+4bB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8\sqrt{b}}-\frac{A(bx^2+cx^4)^{5/2}}{4bx^9}$$

```
[Out] (3*c*(4*b*B + A*c)*Sqrt[b*x^2 + c*x^4])/(8*b*x) - ((4*b*B + A*c)*(b*x^2 + c
*x^4)^(3/2))/(8*b*x^5) - (A*(b*x^2 + c*x^4)^(5/2))/(4*b*x^9) - (3*c*(4*b*B
+ A*c)*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(8*Sqrt[b])
```

Rubi [A] time = 0.21611, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2038, 2020, 2021, 2008, 206}

$$-\frac{(bx^2+cx^4)^{3/2}(Ac+4bB)}{8bx^5}+\frac{3c\sqrt{bx^2+cx^4}(Ac+4bB)}{8bx}-\frac{3c(Ac+4bB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8\sqrt{b}}-\frac{A(bx^2+cx^4)^{5/2}}{4bx^9}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^8,x]
```

```
[Out] (3*c*(4*b*B + A*c)*Sqrt[b*x^2 + c*x^4])/(8*b*x) - ((4*b*B + A*c)*(b*x^2 + c
*x^4)^(3/2))/(8*b*x^5) - (A*(b*x^2 + c*x^4)^(5/2))/(4*b*x^9) - (3*c*(4*b*B
+ A*c)*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(8*Sqrt[b])
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}}{x^8}\,dx = -\frac{A\left(bx^2+cx^4\right)^{5/2}}{4bx^9} - \frac{(-4bB-Ac)\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^6}\,dx}{4b}$$

$$= -\frac{(4bB+Ac)\left(bx^2+cx^4\right)^{3/2}}{8bx^5} - \frac{A\left(bx^2+cx^4\right)^{5/2}}{4bx^9} + \frac{(3c(4bB+Ac))\int \frac{\sqrt{bx^2+cx^4}}{x^2}\,dx}{8b}$$

$$= \frac{3c(4bB+Ac)\sqrt{bx^2+cx^4}}{8bx} - \frac{(4bB+Ac)\left(bx^2+cx^4\right)^{3/2}}{8bx^5} - \frac{A\left(bx^2+cx^4\right)^{5/2}}{4bx^9} + \frac{1}{8}(3c(4bB$$

$$= \frac{3c(4bB+Ac)\sqrt{bx^2+cx^4}}{8bx} - \frac{(4bB+Ac)\left(bx^2+cx^4\right)^{3/2}}{8bx^5} - \frac{A\left(bx^2+cx^4\right)^{5/2}}{4bx^9} - \frac{1}{8}(3c(4bB$$

$$= \frac{3c(4bB+Ac)\sqrt{bx^2+cx^4}}{8bx} - \frac{(4bB+Ac)\left(bx^2+cx^4\right)^{3/2}}{8bx^5} - \frac{A\left(bx^2+cx^4\right)^{5/2}}{4bx^9} - \frac{3c(4bB+}{}$$

Mathematica [C] time = 0.0350848, size = 63, normalized size = 0.47

$$\frac{\left(x^2\left(b+cx^2\right)\right)^{5/2}\left(cx^4(Ac+4bB)\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{cx^2}{b}+1\right)-5Ab^2\right)}{20b^3x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^8,x]
```

```
[Out] ((x^2*(b + c*x^2))^(5/2)*(-5*A*b^2 + c*(4*b*B + A*c)*x^4*Hypergeometric2F1[
2, 5/2, 7/2, 1 + (c*x^2)/b]))/(20*b^3*x^9)
```

Maple [A] time = 0.01, size = 213, normalized size = 1.6

$$-\frac{1}{8\,b^2x^7}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(3\,Ab^{3/2}\ln\left(2\,\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)x^4c^2-A\left(cx^2+b\right)^{\frac{3}{2}}x^4c^2+12\,Bb^{5/2}\ln\left(2\,\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)x^4c-4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^8,x)
```

```
[Out] -1/8*(c*x^4+b*x^2)^(3/2)*(3*A*b^(3/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*x
^4*c^2-A*(c*x^2+b)^(3/2)*x^4*c^2+12*B*b^(5/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)
+b)/x)*x^4*c-4*B*(c*x^2+b)^(3/2)*x^4*b*c+A*(c*x^2+b)^(5/2)*x^2*c-3*A*(c*x^2
+b)^(1/2)*x^4*b*c^2+4*B*(c*x^2+b)^(5/2)*x^2*b-12*B*(c*x^2+b)^(1/2)*x^4*b^2*
c+2*A*(c*x^2+b)^(5/2)*b)/x^7/(c*x^2+b)^(3/2)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(Bx^2+A\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^8,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^8, x)
```

Fricas [A] time = 1.14898, size = 479, normalized size = 3.55

$$\left[\frac{3\left(4\,Bbc + Ac^2\right)\sqrt{b}x^5 \log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right) + 2\left(8\,Bbcx^4 - 2\,Ab^2 - \left(4\,Bb^2 + 5\,Abc\right)x^2\right)\sqrt{cx^4 + bx^2}}{16\,bx^5}, \frac{3\left(4\,Bbc + Ac^2\right)}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^8,x, algorithm="fricas")
```

```
[Out] [1/16*(3*(4*B*b*c + A*c^2)*sqrt(b)*x^5*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 +
 b*x^2)*sqrt(b))/x^3) + 2*(8*B*b*c*x^4 - 2*A*b^2 - (4*B*b^2 + 5*A*b*c)*x^2)
*sqrt(c*x^4 + b*x^2))/(b*x^5), 1/8*(3*(4*B*b*c + A*c^2)*sqrt(-b)*x^5*arctan
(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + (8*B*b*c*x^4 - 2*A*b^2 - (4*
B*b^2 + 5*A*b*c)*x^2)*sqrt(c*x^4 + b*x^2))/(b*x^5)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(b + cx^2\right)\right)^{\frac{3}{2}}\left(A + Bx^2\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**8,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**8, x)
```

Giac [A] time = 1.30842, size = 196, normalized size = 1.45

$$\frac{8\sqrt{cx^2+b}Bc^2\mathrm{sgn}\left(x\right) + \frac{3\left(4\,Bbc^2\mathrm{sgn}(x)+Ac^3\mathrm{sgn}(x)\right)\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}} - \frac{4\left(cx^2+b\right)^{\frac{3}{2}}Bbc^2\mathrm{sgn}(x)-4\sqrt{cx^2+b}Bb^2c^2\mathrm{sgn}(x)+5\left(cx^2+b\right)^{\frac{3}{2}}Ac^3\mathrm{sgn}(x)-3\sqrt{cx^2}}{c^2x^4}}{8\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^8,x, algorithm="giac")
```

```
[Out] 1/8*(8*sqrt(c*x^2 + b)*B*c^2*sgn(x) + 3*(4*B*b*c^2*sgn(x) + A*c^3*sgn(x))*a
rctan(sqrt(c*x^2 + b)/sqrt(-b))/sqrt(-b) - (4*(c*x^2 + b)^(3/2)*B*b*c^2*sgn
(x) - 4*sqrt(c*x^2 + b)*B*b^2*c^2*sgn(x) + 5*(c*x^2 + b)^(3/2)*A*c^3*sgn(x)
 - 3*sqrt(c*x^2 + b)*A*b*c^3*sgn(x))/(c^2*x^4))/c
```

3.126 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{10}}\,dx$

Optimal. Leaf size=140

$$-\frac{c^2(6bB-Ac)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{16b^{3/2}}-\frac{(bx^2+cx^4)^{3/2}(6bB-Ac)}{24bx^7}-\frac{c\sqrt{bx^2+cx^4}(6bB-Ac)}{16bx^3}-\frac{A(bx^2+cx^4)^{5/2}}{6bx^{11}}$$

```
[Out] -(c*(6*b*B - A*c)*Sqrt[b*x^2 + c*x^4])/(16*b*x^3) - ((6*b*B - A*c)*(b*x^2 +
 c*x^4)^(3/2))/(24*b*x^7) - (A*(b*x^2 + c*x^4)^(5/2))/(6*b*x^11) - (c^2*(6*
b*B - A*c)*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(16*b^(3/2))
```

Rubi [A] time = 0.223342, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2038, 2020, 2008, 206}

$$-\frac{c^2(6bB-Ac)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{16b^{3/2}}-\frac{(bx^2+cx^4)^{3/2}(6bB-Ac)}{24bx^7}-\frac{c\sqrt{bx^2+cx^4}(6bB-Ac)}{16bx^3}-\frac{A(bx^2+cx^4)^{5/2}}{6bx^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^10,x]
```

```
[Out] -(c*(6*b*B - A*c)*Sqrt[b*x^2 + c*x^4])/(16*b*x^3) - ((6*b*B - A*c)*(b*x^2 +
 c*x^4)^(3/2))/(24*b*x^7) - (A*(b*x^2 + c*x^4)^(5/2))/(6*b*x^11) - (c^2*(6*
b*B - A*c)*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(16*b^(3/2))
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}}{x^{10}}\,dx = -\frac{A\left(bx^2+cx^4\right)^{5/2}}{6bx^{11}} - \frac{(-6bB+Ac)\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^8}\,dx}{6b}$$

$$= -\frac{(6bB-Ac)\left(bx^2+cx^4\right)^{3/2}}{24bx^7} - \frac{A\left(bx^2+cx^4\right)^{5/2}}{6bx^{11}} + \frac{(c(6bB-Ac))\int \frac{\sqrt{bx^2+cx^4}}{x^4}\,dx}{8b}$$

$$= -\frac{c(6bB-Ac)\sqrt{bx^2+cx^4}}{16bx^3} - \frac{(6bB-Ac)\left(bx^2+cx^4\right)^{3/2}}{24bx^7} - \frac{A\left(bx^2+cx^4\right)^{5/2}}{6bx^{11}} + \frac{\left(c^2(6bB\cdots\right.}{\cdots}$$

$$= -\frac{c(6bB-Ac)\sqrt{bx^2+cx^4}}{16bx^3} - \frac{(6bB-Ac)\left(bx^2+cx^4\right)^{3/2}}{24bx^7} - \frac{A\left(bx^2+cx^4\right)^{5/2}}{6bx^{11}} - \frac{\left(c^2(6bB\cdots\right.}{\cdots}$$

$$= -\frac{c(6bB-Ac)\sqrt{bx^2+cx^4}}{16bx^3} - \frac{(6bB-Ac)\left(bx^2+cx^4\right)^{3/2}}{24bx^7} - \frac{A\left(bx^2+cx^4\right)^{5/2}}{6bx^{11}} - \frac{c^2(6bB-\cdots}{\cdots}$$

Mathematica [A] time = 0.127024, size = 121, normalized size = 0.86

$$-\frac{\left(b+cx^2\right)\left(A\left(8b^2+14bcx^2+3c^2x^4\right)+6bBx^2\left(2b+5cx^2\right)\right)+3c^2x^6\sqrt{\frac{cx^2}{b}+1}(6bB-Ac)\tanh^{-1}\left(\sqrt{\frac{cx^2}{b}+1}\right)}{48bx^5\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^10,x]
```

```
[Out] -((b + c*x^2)*(6*b*B*x^2*(2*b + 5*c*x^2) + A*(8*b^2 + 14*b*c*x^2 + 3*c^2*x^
4)) + 3*c^2*(6*b*B - A*c)*x^6*Sqrt[1 + (c*x^2)/b]*ArcTanh[Sqrt[1 + (c*x^2)/
b]])/(48*b*x^5*Sqrt[x^2*(b + c*x^2)])
```

Maple [B] time = 0.013, size = 259, normalized size = 1.9

$$\frac{1}{48\,x^9b^3}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(3\,Ab^{3/2}\ln\left(2\,\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)x^6c^3-A\left(cx^2+b\right)^{\frac{3}{2}}x^6c^3-18\,Bb^{5/2}\ln\left(2\,\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)x^6c^2+6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^10,x)
```

```
[Out] 1/48*(c*x^4+b*x^2)^(3/2)*(3*A*b^(3/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*x
^6*c^3-A*(c*x^2+b)^(3/2)*x^6*c^3-18*B*b^(5/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)
+b)/x)*x^6*c^2+6*B*(c*x^2+b)^(3/2)*x^6*b*c^2+A*(c*x^2+b)^(5/2)*x^4*c^2-3*A*
(c*x^2+b)^(1/2)*x^6*b*c^3-6*B*(c*x^2+b)^(5/2)*x^4*b*c+18*B*(c*x^2+b)^(1/2)*
x^6*b^2*c^2+2*A*(c*x^2+b)^(5/2)*x^2*b*c-12*B*(c*x^2+b)^(5/2)*x^2*b^2-8*A*(c
*x^2+b)^(5/2)*b^2)/x^9/(c*x^2+b)^(3/2)/b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(Bx^2+A\right)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^10,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^10, x)
```

Fricas [A] time = 1.16548, size = 549, normalized size = 3.92

$$\left[-\frac{3\left(6\,Bbc^{2}-Ac^{3}\right)\sqrt{b}x^{7}\log\left(-\frac{cx^{3}+2\,bx+2\,\sqrt{cx^{4}+bx^{2}}\sqrt{b}}{x^{3}}\right)+2\left(3\left(10\,Bb^{2}c+Abc^{2}\right)x^{4}+8\,Ab^{3}+2\left(6\,Bb^{3}+7\,Ab^{2}c\right)x^{2}\right)\sqrt{cx^{4}+}}{96\,b^{2}x^{7}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^10,x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(6*B*b*c^2 - A*c^3)*sqrt(b)*x^7*log(-(c*x^3 + 2*b*x + 2*sqrt(c*x^
4 + b*x^2)*sqrt(b))/x^3) + 2*(3*(10*B*b^2*c + A*b*c^2)*x^4 + 8*A*b^3 + 2*(6
*B*b^3 + 7*A*b^2*c)*x^2)*sqrt(c*x^4 + b*x^2))/(b^2*x^7), 1/48*(3*(6*B*b*c^2
 - A*c^3)*sqrt(-b)*x^7*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) -
 (3*(10*B*b^2*c + A*b*c^2)*x^4 + 8*A*b^3 + 2*(6*B*b^3 + 7*A*b^2*c)*x^2)*sqr
t(c*x^4 + b*x^2))/(b^2*x^7)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^{2}\left(b+cx^{2}\right)\right)^{\frac{3}{2}}\left(A+Bx^{2}\right)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**10,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**10, x)
```

Giac [A] time = 1.32323, size = 236, normalized size = 1.69

$$\frac{\frac{3\left(6\,Bbc^{3}\mathrm{sgn}(x)-Ac^{4}\mathrm{sgn}(x)\right)\arctan\left(\frac{\sqrt{cx^{2}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b}-\frac{30\left(cx^{2}+b\right)^{\frac{5}{2}}Bbc^{3}\mathrm{sgn}(x)-48\left(cx^{2}+b\right)^{\frac{3}{2}}Bb^{2}c^{3}\mathrm{sgn}(x)+18\,\sqrt{cx^{2}+b}Bb^{3}c^{3}\mathrm{sgn}(x)+3\left(cx^{2}+b\right)^{\frac{5}{2}}Ac^{4}\mathrm{sgn}(x)+8\left(cx^{2}+\right.}{bc^{3}x^{6}}}{48\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^10,x, algorithm="giac")
```

```
[Out] 1/48*(3*(6*B*b*c^3*sgn(x) - A*c^4*sgn(x))*arctan(sqrt(c*x^2 + b)/sqrt(-b))/
(sqrt(-b)*b) - (30*(c*x^2 + b)^(5/2)*B*b*c^3*sgn(x) - 48*(c*x^2 + b)^(3/2)*
B*b^2*c^3*sgn(x) + 18*sqrt(c*x^2 + b)*B*b^3*c^3*sgn(x) + 3*(c*x^2 + b)^(5/2
)*A*c^4*sgn(x) + 8*(c*x^2 + b)^(3/2)*A*b*c^4*sgn(x) - 3*sqrt(c*x^2 + b)*A*b
^2*c^4*sgn(x))/(b*c^3*x^6))/c
```

3.127 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{12}}\,dx$

Optimal. Leaf size=177

$$-\frac{c^2\sqrt{bx^2+cx^4}(8bB-3Ac)}{128b^2x^3}+\frac{c^3(8bB-3Ac)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{128b^{5/2}}-\frac{c\sqrt{bx^2+cx^4}(8bB-3Ac)}{64bx^5}-\frac{\left(bx^2+cx^4\right)^{3/2}(8bB-3A}{48bx^9}$$

```
[Out] -(c*(8*b*B - 3*A*c)*Sqrt[b*x^2 + c*x^4])/(64*b*x^5) - (c^2*(8*b*B - 3*A*c)*
Sqrt[b*x^2 + c*x^4])/(128*b^2*x^3) - ((8*b*B - 3*A*c)*(b*x^2 + c*x^4)^(3/2)
)/(48*b*x^9) - (A*(b*x^2 + c*x^4)^(5/2))/(8*b*x^13) + (c^3*(8*b*B - 3*A*c)*
ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(128*b^(5/2))
```

Rubi [A] time = 0.281209, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2038, 2020, 2025, 2008, 206}

$$-\frac{c^2\sqrt{bx^2+cx^4}(8bB-3Ac)}{128b^2x^3}+\frac{c^3(8bB-3Ac)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{128b^{5/2}}-\frac{c\sqrt{bx^2+cx^4}(8bB-3Ac)}{64bx^5}-\frac{\left(bx^2+cx^4\right)^{3/2}(8bB-3A}{48bx^9}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^12,x]
```

```
[Out] -(c*(8*b*B - 3*A*c)*Sqrt[b*x^2 + c*x^4])/(64*b*x^5) - (c^2*(8*b*B - 3*A*c)*
Sqrt[b*x^2 + c*x^4])/(128*b^2*x^3) - ((8*b*B - 3*A*c)*(b*x^2 + c*x^4)^(3/2)
)/(48*b*x^9) - (A*(b*x^2 + c*x^4)^(5/2))/(8*b*x^13) + (c^3*(8*b*B - 3*A*c)*
ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(128*b^(5/2))
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{12}}dx = -\frac{A(bx^2+cx^4)^{5/2}}{8bx^{13}} - \frac{(-8bB+3Ac)\int \frac{(bx^2+cx^4)^{3/2}}{x^{10}}dx}{8b}$$
$$= -\frac{(8bB-3Ac)(bx^2+cx^4)^{3/2}}{48bx^9} - \frac{A(bx^2+cx^4)^{5/2}}{8bx^{13}} + \frac{(c(8bB-3Ac))\int \frac{\sqrt{bx^2+cx^4}}{x^6}dx}{16b}$$
$$= -\frac{c(8bB-3Ac)\sqrt{bx^2+cx^4}}{64bx^5} - \frac{(8bB-3Ac)(bx^2+cx^4)^{3/2}}{48bx^9} - \frac{A(bx^2+cx^4)^{5/2}}{8bx^{13}} + \frac{(c^2(8b}{}$$
$$= -\frac{c(8bB-3Ac)\sqrt{bx^2+cx^4}}{64bx^5} - \frac{c^2(8bB-3Ac)\sqrt{bx^2+cx^4}}{128b^2x^3} - \frac{(8bB-3Ac)(bx^2+cx^4)^{3/2}}{48bx^9}$$
$$= -\frac{c(8bB-3Ac)\sqrt{bx^2+cx^4}}{64bx^5} - \frac{c^2(8bB-3Ac)\sqrt{bx^2+cx^4}}{128b^2x^3} - \frac{(8bB-3Ac)(bx^2+cx^4)^{3/2}}{48bx^9}$$
$$= -\frac{c(8bB-3Ac)\sqrt{bx^2+cx^4}}{64bx^5} - \frac{c^2(8bB-3Ac)\sqrt{bx^2+cx^4}}{128b^2x^3} - \frac{(8bB-3Ac)(bx^2+cx^4)^{3/2}}{48bx^9}$$

Mathematica [C] time = 0.0347221, size = 66, normalized size = 0.37

$$\frac{(x^2(b+cx^2))^{5/2}\left(c^3x^8(8bB-3Ac)\,{}_2F_1\left(\frac{5}{2},4;\frac{7}{2};\frac{cx^2}{b}+1\right)-5Ab^4\right)}{40b^5x^{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^12,x]
```

```
[Out] ((x^2*(b + c*x^2))^(5/2)*(-5*A*b^4 + c^3*(8*b*B - 3*A*c)*x^8*Hypergeometric
2F1[5/2, 4, 7/2, 1 + (c*x^2)/b]))/(40*b^5*x^13)
```

Maple [A] time = 0.017, size = 302, normalized size = 1.7

$$-\frac{1}{384\,x^{11}b^4}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(9\,Ab^{3/2}\ln\left(2\,\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)x^8c^4-3\,A\left(cx^2+b\right)^{3/2}x^8c^4-24\,Bb^{5/2}\ln\left(2\,\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^12,x)
```

```
[Out] -1/384*(c*x^4+b*x^2)^(3/2)*(9*A*b^(3/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)
*x^8*c^4-3*A*(c*x^2+b)^(3/2)*x^8*c^4-24*B*b^(5/2)*ln(2*(b^(1/2)*(c*x^2+b)^(
1/2)+b)/x)*x^8*c^3+8*B*(c*x^2+b)^(3/2)*x^8*b*c^3+3*A*(c*x^2+b)^(5/2)*x^6*c^
3-9*A*(c*x^2+b)^(1/2)*x^8*b*c^4-8*B*(c*x^2+b)^(5/2)*x^6*b*c^2+24*B*(c*x^2+b
)^(1/2)*x^8*b^2*c^3+6*A*(c*x^2+b)^(5/2)*x^4*b*c^2-16*B*(c*x^2+b)^(5/2)*x^4*
b^2*c-24*A*(c*x^2+b)^(5/2)*x^2*b^2*c+64*B*(c*x^2+b)^(5/2)*x^2*b^3+48*A*(c*x
^2+b)^(5/2)*b^3)/x^11/(c*x^2+b)^(3/2)/b^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2\right)^{\frac{3}{2}}\left(Bx^2 + A\right)}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^12,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^12, x)
```

Fricas [A] time = 1.23321, size = 664, normalized size = 3.75

$$\left[-\frac{3\left(8\,Bbc^3 - 3\,Ac^4\right)\sqrt{b}x^9\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right) + 2\left(3\left(8\,Bb^2c^2 - 3\,Abc^3\right)x^6 + 48\,Ab^4 + 2\left(56\,Bb^3c + 3\,Ab^2c^2\right)x^4 + \right.}{768\,b^3x^9}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^12,x, algorithm="fricas")
```

```
[Out] [-1/768*(3*(8*B*b*c^3 - 3*A*c^4)*sqrt(b)*x^9*log(-(c*x^3 + 2*b*x - 2*sqrt(c
*x^4 + b*x^2)*sqrt(b))/x^3) + 2*(3*(8*B*b^2*c^2 - 3*A*b*c^3)*x^6 + 48*A*b^4
 + 2*(56*B*b^3*c + 3*A*b^2*c^2)*x^4 + 8*(8*B*b^4 + 9*A*b^3*c)*x^2)*sqrt(c*x
^4 + b*x^2))/(b^3*x^9), -1/384*(3*(8*B*b*c^3 - 3*A*c^4)*sqrt(-b)*x^9*arctan
(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + (3*(8*B*b^2*c^2 - 3*A*b*c^3)
*x^6 + 48*A*b^4 + 2*(56*B*b^3*c + 3*A*b^2*c^2)*x^4 + 8*(8*B*b^4 + 9*A*b^3*c
)*x^2)*sqrt(c*x^4 + b*x^2))/(b^3*x^9)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2 \left(b + cx^2\right)\right)^{\frac{3}{2}} \left(A + Bx^2\right)}{x^{12}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**12,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**12, x)
```

Giac [A] time = 1.29051, size = 289, normalized size = 1.63

$$-\frac{\frac{3\left(8\,Bbc^4\text{sgn}(x)-3\,Ac^5\text{sgn}(x)\right)\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^2} + \frac{24\left(cx^2+b\right)^{\frac{7}{2}}Bbc^4\text{sgn}(x)+40\left(cx^2+b\right)^{\frac{5}{2}}Bb^2c^4\text{sgn}(x)-88\left(cx^2+b\right)^{\frac{3}{2}}Bb^3c^4\text{sgn}(x)+24\sqrt{cx^2+b}Bb^4c^4\text{sgn}(x)-9}{b^2c^4x^8}}{384\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^12,x, algorithm="giac")
```

```
[Out] -1/384*(3*(8*B*b*c^4*sgn(x) - 3*A*c^5*sgn(x))*arctan(sqrt(c*x^2 + b)/sqrt(-
b))/(sqrt(-b)*b^2) + (24*(c*x^2 + b)^(7/2)*B*b*c^4*sgn(x) + 40*(c*x^2 + b)^
(5/2)*B*b^2*c^4*sgn(x) - 88*(c*x^2 + b)^(3/2)*B*b^3*c^4*sgn(x) + 24*sqrt(c*
x^2 + b)*B*b^4*c^4*sgn(x) - 9*(c*x^2 + b)^(7/2)*A*c^5*sgn(x) + 33*(c*x^2 +
b)^(5/2)*A*b*c^5*sgn(x) + 33*(c*x^2 + b)^(3/2)*A*b^2*c^5*sgn(x) - 9*sqrt(c*
x^2 + b)*A*b^3*c^5*sgn(x))/(b^2*c^4*x^8))/c
```

3.128 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{14}} dx$

Optimal. Leaf size=214

$$\frac{3c^3\sqrt{bx^2+cx^4}(2bB-Ac)}{256b^3x^3}-\frac{c^2\sqrt{bx^2+cx^4}(2bB-Ac)}{128b^2x^5}-\frac{3c^4(2bB-Ac)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{256b^{7/2}}-\frac{c\sqrt{bx^2+cx^4}(2bB-Ac)}{32bx^7}-\ldots$$

```
[Out] -(c*(2*b*B - A*c)*Sqrt[b*x^2 + c*x^4])/(32*b*x^7) - (c^2*(2*b*B - A*c)*Sqrt
[b*x^2 + c*x^4])/(128*b^2*x^5) + (3*c^3*(2*b*B - A*c)*Sqrt[b*x^2 + c*x^4])/
(256*b^3*x^3) - ((2*b*B - A*c)*(b*x^2 + c*x^4)^(3/2))/(16*b*x^11) - (A*(b*x
^2 + c*x^4)^(5/2))/(10*b*x^15) - (3*c^4*(2*b*B - A*c)*ArcTanh[(Sqrt[b]*x)/S
qrt[b*x^2 + c*x^4]])/(256*b^(7/2))
```

Rubi [A] time = 0.339416, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2038, 2020, 2025, 2008, 206}

$$\frac{3c^3\sqrt{bx^2+cx^4}(2bB-Ac)}{256b^3x^3}-\frac{c^2\sqrt{bx^2+cx^4}(2bB-Ac)}{128b^2x^5}-\frac{3c^4(2bB-Ac)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{256b^{7/2}}-\frac{c\sqrt{bx^2+cx^4}(2bB-Ac)}{32bx^7}-\ldots$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^14,x]
```


```
[Out] -(c*(2*b*B - A*c)*Sqrt[b*x^2 + c*x^4])/(32*b*x^7) - (c^2*(2*b*B - A*c)*Sqrt
[b*x^2 + c*x^4])/(128*b^2*x^5) + (3*c^3*(2*b*B - A*c)*Sqrt[b*x^2 + c*x^4])/
(256*b^3*x^3) - ((2*b*B - A*c)*(b*x^2 + c*x^4)^(3/2))/(16*b*x^11) - (A*(b*x
^2 + c*x^4)^(5/2))/(10*b*x^15) - (3*c^4*(2*b*B - A*c)*ArcTanh[(Sqrt[b]*x)/S
qrt[b*x^2 + c*x^4]])/(256*b^(7/2))
```


Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
```

```
0]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{14}}\,dx = -\frac{A(bx^2+cx^4)^{5/2}}{10bx^{15}} - \frac{(-10bB+5Ac)\int \frac{(bx^2+cx^4)^{3/2}}{x^{12}}\,dx}{10b}$$

$$= -\frac{(2bB-Ac)(bx^2+cx^4)^{3/2}}{16bx^{11}} - \frac{A(bx^2+cx^4)^{5/2}}{10bx^{15}} + \frac{(3c(2bB-Ac))\int \frac{\sqrt{bx^2+cx^4}}{x^8}\,dx}{16b}$$

$$= -\frac{c(2bB-Ac)\sqrt{bx^2+cx^4}}{32bx^7} - \frac{(2bB-Ac)(bx^2+cx^4)^{3/2}}{16bx^{11}} - \frac{A(bx^2+cx^4)^{5/2}}{10bx^{15}} + \frac{(c^2(2bB-}{}$$

$$= -\frac{c(2bB-Ac)\sqrt{bx^2+cx^4}}{32bx^7} - \frac{c^2(2bB-Ac)\sqrt{bx^2+cx^4}}{128b^2x^5} - \frac{(2bB-Ac)(bx^2+cx^4)^{3/2}}{16bx^{11}} - \frac{A}{}$$

$$= -\frac{c(2bB-Ac)\sqrt{bx^2+cx^4}}{32bx^7} - \frac{c^2(2bB-Ac)\sqrt{bx^2+cx^4}}{128b^2x^5} + \frac{3c^3(2bB-Ac)\sqrt{bx^2+cx^4}}{256b^3x^3} - \frac{(2}{}$$

$$= -\frac{c(2bB-Ac)\sqrt{bx^2+cx^4}}{32bx^7} - \frac{c^2(2bB-Ac)\sqrt{bx^2+cx^4}}{128b^2x^5} + \frac{3c^3(2bB-Ac)\sqrt{bx^2+cx^4}}{256b^3x^3} - \frac{(2}{}$$

$$= -\frac{c(2bB-Ac)\sqrt{bx^2+cx^4}}{32bx^7} - \frac{c^2(2bB-Ac)\sqrt{bx^2+cx^4}}{128b^2x^5} + \frac{3c^3(2bB-Ac)\sqrt{bx^2+cx^4}}{256b^3x^3} - \frac{(2}{}$$

Mathematica [C] time = 0.0329536, size = 65, normalized size = 0.3

$$-\frac{\left(x^2(b+cx^2)\right)^{5/2}\left(Ab^5+c^4x^{10}(2bB-Ac)\,{}_2F_1\left(\frac{5}{2},5;\frac{7}{2};\frac{cx^2}{b}+1\right)\right)}{10b^6x^{15}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^14,x]
```

```
[Out] -((x^2*(b + c*x^2))^(5/2)*(A*b^5 + c^4*(2*b*B - A*c)*x^10*Hypergeometric2F1
[5/2, 5, 7/2, 1 + (c*x^2)/b]))/(10*b^6*x^15)
```

Maple [A] time = 0.033, size = 344, normalized size = 1.6

$$\frac{1}{1280\,x^{13}b^5}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(-5\,A\left(cx^2+b\right)^{3/2}x^{10}c^5+15\,Ab^{3/2}\ln\left(2\,\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)x^{10}c^5+10\,B\left(cx^2+b\right)^{3/2}x^{10}bc^4-30\,B\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^14,x)
```

```
[Out] 1/1280*(c*x^4+b*x^2)^(3/2)*(-5*A*(c*x^2+b)^(3/2)*x^10*c^5+15*A*b^(3/2)*ln(2
*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*x^10*c^5+10*B*(c*x^2+b)^(3/2)*x^10*b*c^4-30
*B*b^(5/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*x^10*c^4+5*A*(c*x^2+b)^(5/2)
*x^8*c^4-15*A*(c*x^2+b)^(1/2)*x^10*b*c^5-10*B*(c*x^2+b)^(5/2)*x^8*b*c^3+30*
B*(c*x^2+b)^(1/2)*x^10*b^2*c^4+10*A*(c*x^2+b)^(5/2)*x^6*b*c^3-20*B*(c*x^2+b
)^(5/2)*x^6*b^2*c^2-40*A*(c*x^2+b)^(5/2)*x^4*b^2*c^2+80*B*(c*x^2+b)^(5/2)*x
^4*b^3*c+80*A*(c*x^2+b)^(5/2)*x^2*b^3*c-160*B*(c*x^2+b)^(5/2)*x^2*b^4-128*A
*(c*x^2+b)^(5/2)*b^4)/x^13/(c*x^2+b)^(3/2)/b^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(Bx^2+A\right)}{x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^14,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^14, x)
```

Fricas [A] time = 1.4328, size = 765, normalized size = 3.57

$$\left[-\frac{15\left(2\,Bbc^4-Ac^5\right)\sqrt{b}x^{11}\log\left(-\frac{cx^3+2\,bx+2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)-2\left(15\left(2\,Bb^2c^3-Abc^4\right)x^8-10\left(2\,Bb^3c^2-Ab^2c^3\right)x^6-128\,Ab^5-\right.}{2560\,b^4x^{11}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^14,x, algorithm="fricas")
```

```
[Out] [-1/2560*(15*(2*B*b*c^4 - A*c^5)*sqrt(b)*x^11*log(-(c*x^3 + 2*b*x + 2*sqrt(
c*x^4 + b*x^2)*sqrt(b))/x^3) - 2*(15*(2*B*b^2*c^3 - A*b*c^4)*x^8 - 10*(2*B*
b^3*c^2 - A*b^2*c^3)*x^6 - 128*A*b^5 - 8*(30*B*b^4*c + A*b^3*c^2)*x^4 - 16*
(10*B*b^5 + 11*A*b^4*c)*x^2)*sqrt(c*x^4 + b*x^2))/(b^4*x^11), 1/1280*(15*(2
*B*b*c^4 - A*c^5)*sqrt(-b)*x^11*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3
+ b*x)) + (15*(2*B*b^2*c^3 - A*b*c^4)*x^8 - 10*(2*B*b^3*c^2 - A*b^2*c^3)*x^
```

```
6 - 128*A*b^5 - 8*(30*B*b^4*c + A*b^3*c^2)*x^4 - 16*(10*B*b^5 + 11*A*b^4*c)
*x^2)*sqrt(c*x^4 + b*x^2))/(b^4*x^11)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}\left(A+Bx^2\right)}{x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**14,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**14, x)
```

Giac [A] time = 1.29525, size = 316, normalized size = 1.48

$$\frac{\frac{15\left(2Bbc^5\mathrm{sgn}(x)-Ac^6\mathrm{sgn}(x)\right)\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b^3}+\frac{30\left(cx^2+b\right)^{\frac{9}{2}}Bbc^5\mathrm{sgn}(x)-140\left(cx^2+b\right)^{\frac{7}{2}}Bb^2c^5\mathrm{sgn}(x)+140\left(cx^2+b\right)^{\frac{3}{2}}Bb^4c^5\mathrm{sgn}(x)-30\sqrt{cx^2+b}\,Bb^5c^5\mathrm{sgn}(x)-15}{}}{1280\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^14,x, algorithm="giac")
```

```
[Out] 1/1280*(15*(2*B*b*c^5*sgn(x) - A*c^6*sgn(x))*arctan(sqrt(c*x^2 + b)/sqrt(-b
))/(sqrt(-b)*b^3) + (30*(c*x^2 + b)^(9/2)*B*b*c^5*sgn(x) - 140*(c*x^2 + b)^
(7/2)*B*b^2*c^5*sgn(x) + 140*(c*x^2 + b)^(3/2)*B*b^4*c^5*sgn(x) - 30*sqrt(c
*x^2 + b)*B*b^5*c^5*sgn(x) - 15*(c*x^2 + b)^(9/2)*A*c^6*sgn(x) + 70*(c*x^2
+ b)^(7/2)*A*b*c^6*sgn(x) - 128*(c*x^2 + b)^(5/2)*A*b^2*c^6*sgn(x) - 70*(c*
x^2 + b)^(3/2)*A*b^3*c^6*sgn(x) + 15*sqrt(c*x^2 + b)*A*b^4*c^6*sgn(x))/(b^3
*c^5*x^10))/c
```

3.129 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{16}}\,dx$

Optimal. Leaf size=251

$$-\frac{c^4\sqrt{bx^2+cx^4}(12bB-7Ac)}{1024b^4x^3}+\frac{c^3\sqrt{bx^2+cx^4}(12bB-7Ac)}{1536b^3x^5}-\frac{c^2\sqrt{bx^2+cx^4}(12bB-7Ac)}{1920b^2x^7}+\frac{c^5(12bB-7Ac)\tanh^{-1}\left(\frac{\sqrt{b}}{}\right.}{1024b^{9/2}}$$

```
[Out] -(c*(12*b*B - 7*A*c)*Sqrt[b*x^2 + c*x^4])/(320*b*x^9) - (c^2*(12*b*B - 7*A*
c)*Sqrt[b*x^2 + c*x^4])/(1920*b^2*x^7) + (c^3*(12*b*B - 7*A*c)*Sqrt[b*x^2 +
 c*x^4])/(1536*b^3*x^5) - (c^4*(12*b*B - 7*A*c)*Sqrt[b*x^2 + c*x^4])/(1024*
b^4*x^3) - ((12*b*B - 7*A*c)*(b*x^2 + c*x^4)^(3/2))/(120*b*x^13) - (A*(b*x^
2 + c*x^4)^(5/2))/(12*b*x^17) + (c^5*(12*b*B - 7*A*c)*ArcTanh[(Sqrt[b]*x)/S
qrt[b*x^2 + c*x^4]])/(1024*b^(9/2))
```

Rubi [A] time = 0.389782, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2038, 2020, 2025, 2008, 206}

$$-\frac{c^4\sqrt{bx^2+cx^4}(12bB-7Ac)}{1024b^4x^3}+\frac{c^3\sqrt{bx^2+cx^4}(12bB-7Ac)}{1536b^3x^5}-\frac{c^2\sqrt{bx^2+cx^4}(12bB-7Ac)}{1920b^2x^7}+\frac{c^5(12bB-7Ac)\tanh^{-1}\left(\frac{\sqrt{b}}{}\right.}{1024b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^16,x]
```


```
[Out] -(c*(12*b*B - 7*A*c)*Sqrt[b*x^2 + c*x^4])/(320*b*x^9) - (c^2*(12*b*B - 7*A*
c)*Sqrt[b*x^2 + c*x^4])/(1920*b^2*x^7) + (c^3*(12*b*B - 7*A*c)*Sqrt[b*x^2 +
 c*x^4])/(1536*b^3*x^5) - (c^4*(12*b*B - 7*A*c)*Sqrt[b*x^2 + c*x^4])/(1024*
b^4*x^3) - ((12*b*B - 7*A*c)*(b*x^2 + c*x^4)^(3/2))/(120*b*x^13) - (A*(b*x^
2 + c*x^4)^(5/2))/(12*b*x^17) + (c^5*(12*b*B - 7*A*c)*ArcTanh[(Sqrt[b]*x)/S
qrt[b*x^2 + c*x^4]])/(1024*b^(9/2))
```


Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
```

```
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{16}}\,dx = -\frac{A(bx^2+cx^4)^{5/2}}{12bx^{17}} - \frac{(-12bB+7Ac)\int \frac{(bx^2+cx^4)^{3/2}}{x^{14}}dx}{12b}$$

$$= -\frac{(12bB-7Ac)(bx^2+cx^4)^{3/2}}{120bx^{13}} - \frac{A(bx^2+cx^4)^{5/2}}{12bx^{17}} + \frac{(c(12bB-7Ac))\int \frac{\sqrt{bx^2+cx^4}}{x^{10}}dx}{40b}$$

$$= -\frac{c(12bB-7Ac)\sqrt{bx^2+cx^4}}{320bx^9} - \frac{(12bB-7Ac)(bx^2+cx^4)^{3/2}}{120bx^{13}} - \frac{A(bx^2+cx^4)^{5/2}}{12bx^{17}} + \frac{(c^2(}{}$$

$$= -\frac{c(12bB-7Ac)\sqrt{bx^2+cx^4}}{320bx^9} - \frac{c^2(12bB-7Ac)\sqrt{bx^2+cx^4}}{1920b^2x^7} - \frac{(12bB-7Ac)(bx^2+cx^4}{120bx^{13}}$$

$$= -\frac{c(12bB-7Ac)\sqrt{bx^2+cx^4}}{320bx^9} - \frac{c^2(12bB-7Ac)\sqrt{bx^2+cx^4}}{1920b^2x^7} + \frac{c^3(12bB-7Ac)\sqrt{bx^2+c}}{1536b^3x^5}$$

$$= -\frac{c(12bB-7Ac)\sqrt{bx^2+cx^4}}{320bx^9} - \frac{c^2(12bB-7Ac)\sqrt{bx^2+cx^4}}{1920b^2x^7} + \frac{c^3(12bB-7Ac)\sqrt{bx^2+c}}{1536b^3x^5}$$

$$= -\frac{c(12bB-7Ac)\sqrt{bx^2+cx^4}}{320bx^9} - \frac{c^2(12bB-7Ac)\sqrt{bx^2+cx^4}}{1920b^2x^7} + \frac{c^3(12bB-7Ac)\sqrt{bx^2+c}}{1536b^3x^5}$$

$$= -\frac{c(12bB-7Ac)\sqrt{bx^2+cx^4}}{320bx^9} - \frac{c^2(12bB-7Ac)\sqrt{bx^2+cx^4}}{1920b^2x^7} + \frac{c^3(12bB-7Ac)\sqrt{bx^2+c}}{1536b^3x^5}$$

Mathematica [C] time = 0.0366375, size = 66, normalized size = 0.26

$$\frac{(x^2(b+cx^2))^{5/2}\left(c^5x^{12}(12bB-7Ac)\,{}_2F_1\left(\frac{5}{2},6;\frac{7}{2};\frac{cx^2}{b}+1\right)-5Ab^6\right)}{60b^7x^{17}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^16,x]
```

```
[Out] ((x^2*(b + c*x^2))^(5/2)*(-5*A*b^6 + c^5*(12*b*B - 7*A*c)*x^12*Hypergeometr
ic2F1[5/2, 6, 7/2, 1 + (c*x^2)/b]))/(60*b^7*x^17)
```

Maple [A] time = 0.062, size = 386, normalized size = 1.5

$$-\frac{1}{15360\,x^{15}b^6}(cx^4+bx^2)^{\frac{3}{2}}\left(105\,Ab^{3/2}\ln\left(2\,\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)x^{12}c^6-35\,A(cx^2+b)^{3/2}x^{12}c^6-180\,Bb^{5/2}\ln\left(2\,\frac{\sqrt{b}\sqrt{cx^2}}{x}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^16,x)
```

```
[Out] -1/15360*(c*x^4+b*x^2)^(3/2)*(105*A*b^(3/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b
)/x)*x^12*c^6-35*A*(c*x^2+b)^(3/2)*x^12*c^6-180*B*b^(5/2)*ln(2*(b^(1/2)*(c*
x^2+b)^(1/2)+b)/x)*x^12*c^5+60*B*(c*x^2+b)^(3/2)*x^12*b*c^5+35*A*(c*x^2+b)^
(5/2)*x^10*c^5-105*A*(c*x^2+b)^(1/2)*x^12*b*c^6-60*B*(c*x^2+b)^(5/2)*x^10*b
*c^4+180*B*(c*x^2+b)^(1/2)*x^12*b^2*c^5+70*A*(c*x^2+b)^(5/2)*x^8*b*c^4-120*
B*(c*x^2+b)^(5/2)*x^8*b^2*c^3-280*A*(c*x^2+b)^(5/2)*x^6*b^2*c^3+480*B*(c*x^
2+b)^(5/2)*x^6*b^3*c^2+560*A*(c*x^2+b)^(5/2)*x^4*b^3*c^2-960*B*(c*x^2+b)^(5
/2)*x^4*b^4*c-896*A*(c*x^2+b)^(5/2)*x^2*b^4*c+1536*B*(c*x^2+b)^(5/2)*x^2*b^
5+1280*A*(c*x^2+b)^(5/2)*b^5)/x^15/(c*x^2+b)^(3/2)/b^6
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2\right)^{\frac{3}{2}}\left(Bx^2 + A\right)}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^16,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^16, x)
```

Fricas [A] time = 1.71437, size = 905, normalized size = 3.61

$$\left[-\frac{15\left(12\,Bbc^5 - 7\,Ac^6\right)\sqrt{b}x^{13}\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right) + 2\left(15\left(12\,Bb^2c^4 - 7\,Abc^5\right)x^{10} - 10\left(12\,Bb^3c^3 - 7\,Ab^2c^4\right)x^8 + 12\right.}{30720\,b^5x^{13}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^16,x, algorithm="fricas")
```

```
[Out] [-1/30720*(15*(12*B*b*c^5 - 7*A*c^6)*sqrt(b)*x^13*log(-(c*x^3 + 2*b*x - 2*s
qrt(c*x^4 + b*x^2)*sqrt(b))/x^3) + 2*(15*(12*B*b^2*c^4 - 7*A*b*c^5)*x^10 -
10*(12*B*b^3*c^3 - 7*A*b^2*c^4)*x^8 + 1280*A*b^6 + 8*(12*B*b^4*c^2 - 7*A*b^
```

```
3*c^3)*x^6 + 48*(44*B*b^5*c + A*b^4*c^2)*x^4 + 128*(12*B*b^6 + 13*A*b^5*c)*
x^2)*sqrt(c*x^4 + b*x^2))/(b^5*x^13), -1/15360*(15*(12*B*b*c^5 - 7*A*c^6)*s
qrt(-b)*x^13*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + (15*(12*B
*b^2*c^4 - 7*A*b*c^5)*x^10 - 10*(12*B*b^3*c^3 - 7*A*b^2*c^4)*x^8 + 1280*A*b
^6 + 8*(12*B*b^4*c^2 - 7*A*b^3*c^3)*x^6 + 48*(44*B*b^5*c + A*b^4*c^2)*x^4 +
 128*(12*B*b^6 + 13*A*b^5*c)*x^2)*sqrt(c*x^4 + b*x^2))/(b^5*x^13)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}\left(A+Bx^2\right)}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**16,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**16, x)
```

Giac [A] time = 1.29499, size = 397, normalized size = 1.58

$$-\frac{15\left(12\,Bbc^6\mathrm{sgn}(x)-7\,Ac^7\mathrm{sgn}(x)\right)\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^4}+\frac{180\left(cx^2+b\right)^{\frac{11}{2}}Bbc^6\mathrm{sgn}(x)-1020\left(cx^2+b\right)^{\frac{9}{2}}Bb^2c^6\mathrm{sgn}(x)+2376\left(cx^2+b\right)^{\frac{7}{2}}Bb^3c^6\mathrm{sgn}(x)-696\left(cx^2+b\right)^{\frac{5}{2}}Bb\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^16,x, algorithm="giac")
```

```
[Out] -1/15360*(15*(12*B*b*c^6*sgn(x) - 7*A*c^7*sgn(x))*arctan(sqrt(c*x^2 + b)/sq
rt(-b))/(sqrt(-b)*b^4) + (180*(c*x^2 + b)^(11/2)*B*b*c^6*sgn(x) - 1020*(c*x
^2 + b)^(9/2)*B*b^2*c^6*sgn(x) + 2376*(c*x^2 + b)^(7/2)*B*b^3*c^6*sgn(x) -
696*(c*x^2 + b)^(5/2)*B*b^4*c^6*sgn(x) - 1020*(c*x^2 + b)^(3/2)*B*b^5*c^6*s
gn(x) + 180*sqrt(c*x^2 + b)*B*b^6*c^6*sgn(x) - 105*(c*x^2 + b)^(11/2)*A*c^7
*sgn(x) + 595*(c*x^2 + b)^(9/2)*A*b*c^7*sgn(x) - 1386*(c*x^2 + b)^(7/2)*A*b
^2*c^7*sgn(x) + 1686*(c*x^2 + b)^(5/2)*A*b^3*c^7*sgn(x) + 595*(c*x^2 + b)^(
3/2)*A*b^4*c^7*sgn(x) - 105*sqrt(c*x^2 + b)*A*b^5*c^7*sgn(x))/(b^4*c^6*x^12
))/c
```

3.130 $\int \frac{x^7(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=176

$$-\frac{5b^2\sqrt{bx^2+cx^4}(7bB-8Ac)}{128c^4}+\frac{5b^3(7bB-8Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{9/2}}-\frac{x^4\sqrt{bx^2+cx^4}(7bB-8Ac)}{48c^2}+\frac{5bx^2\sqrt{bx^2+cx^4}(7bB-}{192c^3}$$

```
[Out] (-5*b^2*(7*b*B - 8*A*c)*Sqrt[b*x^2 + c*x^4])/(128*c^4) + (5*b*(7*b*B - 8*A*
c)*x^2*Sqrt[b*x^2 + c*x^4])/(192*c^3) - ((7*b*B - 8*A*c)*x^4*Sqrt[b*x^2 + c
*x^4])/(48*c^2) + (B*x^6*Sqrt[b*x^2 + c*x^4])/(8*c) + (5*b^3*(7*b*B - 8*A*c
)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(128*c^(9/2))
```

Rubi [A] time = 0.328448, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2034, 794, 670, 640, 620, 206}

$$-\frac{5b^2\sqrt{bx^2+cx^4}(7bB-8Ac)}{128c^4}+\frac{5b^3(7bB-8Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{9/2}}-\frac{x^4\sqrt{bx^2+cx^4}(7bB-8Ac)}{48c^2}+\frac{5bx^2\sqrt{bx^2+cx^4}(7bB-}{192c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```


```
[Out] (-5*b^2*(7*b*B - 8*A*c)*Sqrt[b*x^2 + c*x^4])/(128*c^4) + (5*b*(7*b*B - 8*A*
c)*x^2*Sqrt[b*x^2 + c*x^4])/(192*c^3) - ((7*b*B - 8*A*c)*x^4*Sqrt[b*x^2 + c
*x^4])/(48*c^2) + (B*x^6*Sqrt[b*x^2 + c*x^4])/(8*c) + (5*b^3*(7*b*B - 8*A*c
)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(128*c^(9/2))
```


Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] && !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^7 \left(A+Bx^2\right)}{\sqrt{bx^2+cx^4}}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{x^3(A+Bx)}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)$$

$$= \frac{Bx^6\sqrt{bx^2+cx^4}}{8c} + \frac{\left(3(-bB+Ac)+\frac{1}{2}(-bB+2Ac)\right)\mathrm{Subst}\left(\int \frac{x^3}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{8c}$$

$$= -\frac{(7bB-8Ac)x^4\sqrt{bx^2+cx^4}}{48c^2} + \frac{Bx^6\sqrt{bx^2+cx^4}}{8c} + \frac{(5b(7bB-8Ac))\,\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{96c^2}$$

$$= \frac{5b(7bB-8Ac)x^2\sqrt{bx^2+cx^4}}{192c^3} - \frac{(7bB-8Ac)x^4\sqrt{bx^2+cx^4}}{48c^2} + \frac{Bx^6\sqrt{bx^2+cx^4}}{8c} - \frac{\left(5b^2(7bB-8Ac)\right)\mathrm{S}}{}$$

$$= -\frac{5b^2(7bB-8Ac)\sqrt{bx^2+cx^4}}{128c^4} + \frac{5b(7bB-8Ac)x^2\sqrt{bx^2+cx^4}}{192c^3} - \frac{(7bB-8Ac)x^4\sqrt{bx^2+cx^4}}{48c^2} + \frac{Bx^6\sqrt{}}{}$$

$$= -\frac{5b^2(7bB-8Ac)\sqrt{bx^2+cx^4}}{128c^4} + \frac{5b(7bB-8Ac)x^2\sqrt{bx^2+cx^4}}{192c^3} - \frac{(7bB-8Ac)x^4\sqrt{bx^2+cx^4}}{48c^2} + \frac{Bx^6\sqrt{}}{}$$

$$= -\frac{5b^2(7bB-8Ac)\sqrt{bx^2+cx^4}}{128c^4} + \frac{5b(7bB-8Ac)x^2\sqrt{bx^2+cx^4}}{192c^3} - \frac{(7bB-8Ac)x^4\sqrt{bx^2+cx^4}}{48c^2} + \frac{Bx^6\sqrt{}}{}$$

Mathematica [A] time = 0.167758, size = 145, normalized size = 0.82

$$\frac{x\left(15b^3\sqrt{b+cx^2}(7bB-8Ac)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b+cx^2}}\right)-\sqrt{c}x\left(b+cx^2\right)\left(-10b^2c\left(12A+7Bx^2\right)+8bc^2x^2\left(10A+7Bx^2\right)-16c^3x^4\left(4\right.\right.\right.}{384c^{9/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*(-(Sqrt[c]*x*(b + c*x^2)*(105*b^3*B - 16*c^3*x^4*(4*A + 3*B*x^2) + 8*b*c
^2*x^2*(10*A + 7*B*x^2) - 10*b^2*c*(12*A + 7*B*x^2))) + 15*b^3*(7*b*B - 8*A
*c)*Sqrt[b + c*x^2]*ArcTanh[(Sqrt[c]*x)/Sqrt[b + c*x^2]]))/(384*c^(9/2)*Sqr
t[x^2*(b + c*x^2)])
```

Maple [A] time = 0.013, size = 211, normalized size = 1.2

$$\frac{x}{384}\sqrt{cx^2+b}\left(48\,Bc^{9/2}\sqrt{cx^2+b}x^7+64\,Ac^{9/2}\sqrt{cx^2+b}x^5-56\,Bc^{7/2}\sqrt{cx^2+b}x^5b-80\,Ac^{7/2}\sqrt{cx^2+b}x^3b+70\,Bc^{5/2}\sqrt{cx^2+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/384*x*(c*x^2+b)^(1/2)*(48*B*c^(9/2)*(c*x^2+b)^(1/2)*x^7+64*A*c^(9/2)*(c*x
^2+b)^(1/2)*x^5-56*B*c^(7/2)*(c*x^2+b)^(1/2)*x^5*b-80*A*c^(7/2)*(c*x^2+b)^(
1/2)*x^3*b+70*B*c^(5/2)*(c*x^2+b)^(1/2)*x^3*b^2+120*A*c^(5/2)*(c*x^2+b)^(1/
2)*x*b^2-105*B*c^(3/2)*(c*x^2+b)^(1/2)*x*b^3-120*A*ln(x*c^(1/2)+(c*x^2+b)^(
1/2))*b^3*c^2+105*B*ln(x*c^(1/2)+(c*x^2+b)^(1/2))*b^4*c)/(c*x^4+b*x^2)^(1/2
)/c^(11/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.21276, size = 626, normalized size = 3.56

$$\left[-\frac{15\left(7\,Bb^4-8\,Ab^3c\right)\sqrt{c}\log\left(-2\,cx^2-b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\left(48\,Bc^4x^6-105\,Bb^3c+120\,Ab^2c^2-8\left(7\,Bbc^3-8\,Ac^4\right)\right.}{768\,c^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/768*(15*(7*B*b^4 - 8*A*b^3*c)*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 +
 b*x^2)*sqrt(c)) - 2*(48*B*c^4*x^6 - 105*B*b^3*c + 120*A*b^2*c^2 - 8*(7*B*b
*c^3 - 8*A*c^4)*x^4 + 10*(7*B*b^2*c^2 - 8*A*b*c^3)*x^2)*sqrt(c*x^4 + b*x^2)
)/c^5, -1/384*(15*(7*B*b^4 - 8*A*b^3*c)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)
*sqrt(-c)/(c*x^2 + b)) - (48*B*c^4*x^6 - 105*B*b^3*c + 120*A*b^2*c^2 - 8*(7
*B*b*c^3 - 8*A*c^4)*x^4 + 10*(7*B*b^2*c^2 - 8*A*b*c^3)*x^2)*sqrt(c*x^4 + b*
x^2))/c^5]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7 \left(A + Bx^2\right)}{\sqrt{x^2 \left(b + cx^2\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**7*(A + B*x**2)/sqrt(x**2*(b + c*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2 + A\right)x^7}{\sqrt{cx^4 + bx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^7/sqrt(c*x^4 + b*x^2), x)
```

3.131 $\int \frac{x^5(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=139

$$-\frac{b^2(5bB-6Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{7/2}}-\frac{x^2\sqrt{bx^2+cx^4}(5bB-6Ac)}{24c^2}+\frac{b\sqrt{bx^2+cx^4}(5bB-6Ac)}{16c^3}+\frac{Bx^4\sqrt{bx^2+cx^4}}{6c}$$

```
[Out] (b*(5*b*B - 6*A*c)*Sqrt[b*x^2 + c*x^4])/(16*c^3) - ((5*b*B - 6*A*c)*x^2*Sqr
t[b*x^2 + c*x^4])/(24*c^2) + (B*x^4*Sqrt[b*x^2 + c*x^4])/(6*c) - (b^2*(5*b*
B - 6*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(16*c^(7/2))
```

Rubi [A] time = 0.274916, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2034, 794, 670, 640, 620, 206}

$$-\frac{b^2(5bB-6Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{7/2}}-\frac{x^2\sqrt{bx^2+cx^4}(5bB-6Ac)}{24c^2}+\frac{b\sqrt{bx^2+cx^4}(5bB-6Ac)}{16c^3}+\frac{Bx^4\sqrt{bx^2+cx^4}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```


```
[Out] (b*(5*b*B - 6*A*c)*Sqrt[b*x^2 + c*x^4])/(16*c^3) - ((5*b*B - 6*A*c)*x^2*Sqr
t[b*x^2 + c*x^4])/(24*c^2) + (B*x^4*Sqrt[b*x^2 + c*x^4])/(6*c) - (b^2*(5*b*
B - 6*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(16*c^(7/2))
```


Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
```

```
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^5\left(A+Bx^2\right)}{\sqrt{bx^2+cx^4}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2(A+Bx)}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)$$
$$= \frac{Bx^4\sqrt{bx^2+cx^4}}{6c} + \frac{\left(2(-bB+Ac)+\frac{1}{2}(-bB+2Ac)\right)\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{6c}$$
$$= -\frac{(5bB-6Ac)x^2\sqrt{bx^2+cx^4}}{24c^2} + \frac{Bx^4\sqrt{bx^2+cx^4}}{6c} + \frac{(b(5bB-6Ac))\operatorname{Subst}\left(\int \frac{x}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{16c^2}$$
$$= \frac{b(5bB-6Ac)\sqrt{bx^2+cx^4}}{16c^3} - \frac{(5bB-6Ac)x^2\sqrt{bx^2+cx^4}}{24c^2} + \frac{Bx^4\sqrt{bx^2+cx^4}}{6c} - \frac{\left(b^2(5bB-6Ac)\right)\operatorname{Sub}}{3}$$
$$= \frac{b(5bB-6Ac)\sqrt{bx^2+cx^4}}{16c^3} - \frac{(5bB-6Ac)x^2\sqrt{bx^2+cx^4}}{24c^2} + \frac{Bx^4\sqrt{bx^2+cx^4}}{6c} - \frac{\left(b^2(5bB-6Ac)\right)\operatorname{Sub}}{}$$
$$= \frac{b(5bB-6Ac)\sqrt{bx^2+cx^4}}{16c^3} - \frac{(5bB-6Ac)x^2\sqrt{bx^2+cx^4}}{24c^2} + \frac{Bx^4\sqrt{bx^2+cx^4}}{6c} - \frac{b^2(5bB-6Ac)\tanh^{-}}{16c^{7/2}}$$

Mathematica [A] time = 0.114082, size = 123, normalized size = 0.88

$$\frac{x\left(\sqrt{c}x\left(b+cx^2\right)\left(-2bc\left(9A+5Bx^2\right)+4c^2x^2\left(3A+2Bx^2\right)+15b^2B\right)-3b^2\sqrt{b+cx^2}(5bB-6Ac)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b+cx^2}}\right)\right)}{48c^{7/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*(Sqrt[c]*x*(b + c*x^2)*(15*b^2*B + 4*c^2*x^2*(3*A + 2*B*x^2) - 2*b*c*(9*
A + 5*B*x^2)) - 3*b^2*(5*b*B - 6*A*c)*Sqrt[b + c*x^2]*ArcTanh[(Sqrt[c]*x)/S
qrt[b + c*x^2]]))/(48*c^(7/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.008, size = 169, normalized size = 1.2

$$\frac{x}{48}\sqrt{cx^2+b}\left(8\,Bc^{7/2}\sqrt{cx^2+bx}^5+12\,Ac^{7/2}\sqrt{cx^2+bx}^3-10\,Bc^{5/2}\sqrt{cx^2+bx}^3b-18\,Ac^{5/2}\sqrt{cx^2+bx}b+15\,Bc^{3/2}\sqrt{cx^2+bx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/48*x*(c*x^2+b)^(1/2)*(8*B*c^(7/2)*(c*x^2+b)^(1/2)*x^5+12*A*c^(7/2)*(c*x^2
+b)^(1/2)*x^3-10*B*c^(5/2)*(c*x^2+b)^(1/2)*x^3*b-18*A*c^(5/2)*(c*x^2+b)^(1/
2)*x*b+15*B*c^(3/2)*(c*x^2+b)^(1/2)*x*b^2+18*A*ln(x*c^(1/2)+(c*x^2+b)^(1/2)
)*b^2*c^2-15*B*ln(x*c^(1/2)+(c*x^2+b)^(1/2))*b^3*c)/(c*x^4+b*x^2)^(1/2)/c^(
9/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.11245, size = 512, normalized size = 3.68

$$\left[-\frac{3\left(5\,Bb^3-6\,Ab^2c\right)\sqrt{c}\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\left(8\,Bc^3x^4+15\,Bb^2c-18\,Abc^2-2\left(5\,Bbc^2-6\,Ac^3\right)x^2\right)\sqrt{cx}}{96\,c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(5*B*b^3 - 6*A*b^2*c)*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b
*x^2)*sqrt(c)) - 2*(8*B*c^3*x^4 + 15*B*b^2*c - 18*A*b*c^2 - 2*(5*B*b*c^2 -
6*A*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^4, 1/48*(3*(5*B*b^3 - 6*A*b^2*c)*sqrt(
-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) + (8*B*c^3*x^4 + 15*B*
b^2*c - 18*A*b*c^2 - 2*(5*B*b*c^2 - 6*A*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5\left(A+Bx^2\right)}{\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**5*(A + B*x**2)/sqrt(x**2*(b + c*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)x^5}{\sqrt{cx^4 + bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^5/sqrt(c*x^4 + b*x^2), x)
```

3.132 $\int \frac{x^3(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=83

$$\frac{b(3bB-4Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{5/2}}-\frac{\sqrt{bx^2+cx^4}\left(-4Ac+3bB-2Bcx^2\right)}{8c^2}$$

```
[Out] -((3*b*B - 4*A*c - 2*B*c*x^2)*Sqrt[b*x^2 + c*x^4])/(8*c^2) + (b*(3*b*B - 4*
A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(8*c^(5/2))
```

Rubi [A] time = 0.171246, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2034, 779, 620, 206}

$$\frac{b(3bB-4Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{5/2}}-\frac{\sqrt{bx^2+cx^4}\left(-4Ac+3bB-2Bcx^2\right)}{8c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] -((3*b*B - 4*A*c - 2*B*c*x^2)*Sqrt[b*x^2 + c*x^4])/(8*c^2) + (b*(3*b*B - 4*
A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(8*c^(5/2))
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
```

```
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\left(A+Bx^2\right)}{\sqrt{bx^2+cx^4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x(A+Bx)}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)\\
&= -\frac{\left(3bB-4Ac-2Bcx^2\right)\sqrt{bx^2+cx^4}}{8c^2}+\frac{(b(3bB-4Ac))\operatorname{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{16c^2}\\
&= -\frac{\left(3bB-4Ac-2Bcx^2\right)\sqrt{bx^2+cx^4}}{8c^2}+\frac{(b(3bB-4Ac))\operatorname{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^2}\\
&= -\frac{\left(3bB-4Ac-2Bcx^2\right)\sqrt{bx^2+cx^4}}{8c^2}+\frac{b(3bB-4Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.0901898, size = 97, normalized size = 1.17

$$\frac{x\left(\sqrt{c}x\left(b+cx^2\right)\left(4Ac-3bB+2Bcx^2\right)+b\sqrt{b+cx^2}(3bB-4Ac)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b+cx^2}}\right)\right)}{8c^{5/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*(Sqrt[c]*x*(b + c*x^2)*(-3*b*B + 4*A*c + 2*B*c*x^2) + b*(3*b*B - 4*A*c)*
Sqrt[b + c*x^2]*ArcTanh[(Sqrt[c]*x)/Sqrt[b + c*x^2]]))/(8*c^(5/2)*Sqrt[x^2*
(b + c*x^2)])
```

Maple [A] time = 0.007, size = 127, normalized size = 1.5

$$\frac{x}{8}\sqrt{cx^2+b}\left(2\,Bc^{5/2}\sqrt{cx^2+b}x^3+4\,Ac^{5/2}\sqrt{cx^2+b}x-3\,Bc^{3/2}\sqrt{cx^2+b}xb-4\,A\ln\left(x\sqrt{c}+\sqrt{cx^2+b}\right)bc^2+3\,B\ln\left(x\sqrt{c}+\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/8*x*(c*x^2+b)^(1/2)*(2*B*c^(5/2)*(c*x^2+b)^(1/2)*x^3+4*A*c^(5/2)*(c*x^2+b
)^(1/2)*x-3*B*c^(3/2)*(c*x^2+b)^(1/2)*x*b-4*A*ln(x*c^(1/2)+(c*x^2+b)^(1/2))
*b*c^2+3*B*ln(x*c^(1/2)+(c*x^2+b)^(1/2))*b^2*c)/(c*x^4+b*x^2)^(1/2)/c^(7/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.0668, size = 404, normalized size = 4.87

$$\left[-\frac{\left(3\,Bb^2-4\,Abc\right)\sqrt{c}\log\left(-2\,cx^2-b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\left(2\,Bc^2x^2-3\,Bbc+4\,Ac^2\right)\sqrt{cx^4+bx^2}}{16\,c^3},-\frac{\left(3\,Bb^2-4\,Abc\right)\sqrt{-c}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((3*B*b^2 - 4*A*b*c)*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 + b*x^2
)*sqrt(c)) - 2*(2*B*c^2*x^2 - 3*B*b*c + 4*A*c^2)*sqrt(c*x^4 + b*x^2))/c^3,
-1/8*((3*B*b^2 - 4*A*b*c)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x
```

```
^2 + b)) - (2*B*c^2*x^2 - 3*B*b*c + 4*A*c^2)*sqrt(c*x^4 + b*x^2))/c^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 (A + Bx^2)}{\sqrt{x^2 (b + cx^2)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**3*(A + B*x**2)/sqrt(x**2*(b + c*x**2)), x)
```

Giac [A] time = 1.23462, size = 123, normalized size = 1.48

$$\frac{1}{8}\sqrt{cx^4 + bx^2}\left(\frac{2Bx^2}{c} - \frac{3Bb - 4Ac}{c^2}\right) - \frac{(3Bb^2 - 4Abc)\log\left(\left|-2\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2}\right)\sqrt{c} - b\right|\right)}{16c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(c*x^4 + b*x^2)*(2*B*x^2/c - (3*B*b - 4*A*c)/c^2) - 1/16*(3*B*b^2 -
 4*A*b*c)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))*sqrt(c) - b))/c^(5
/2)
```

3.133 $\int \frac{x(A+Bx^2)}{\sqrt{bx^2+cx^4}} dx$

Optimal. Leaf size=66

$$\frac{B\sqrt{bx^2+cx^4}}{2c} - \frac{(bB-2Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{3/2}}$$

```
[Out] (B*Sqrt[b*x^2 + c*x^4])/(2*c) - ((b*B - 2*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b
*x^2 + c*x^4]])/(2*c^(3/2))
```

Rubi [A] time = 0.123152, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2034, 640, 620, 206}

$$\frac{B\sqrt{bx^2+cx^4}}{2c} - \frac{(bB-2Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (B*Sqrt[b*x^2 + c*x^4])/(2*c) - ((b*B - 2*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b
*x^2 + c*x^4]])/(2*c^(3/2))
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(A+Bx^2\right)}{\sqrt{bx^2+cx^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{A+Bx}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)\\
&= \frac{B\sqrt{bx^2+cx^4}}{2c}+\frac{(-bB+2Ac)\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{4c}\\
&= \frac{B\sqrt{bx^2+cx^4}}{2c}+\frac{(-bB+2Ac)\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{2c}\\
&= \frac{B\sqrt{bx^2+cx^4}}{2c}-\frac{(bB-2Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0434992, size = 81, normalized size = 1.23

$$\frac{x\left(B\sqrt{c}x\left(b+cx^2\right)-\sqrt{b+cx^2}(bB-2Ac)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b+cx^2}}\right)\right)}{2c^{3/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*(B*Sqrt[c]*x*(b + c*x^2) - (b*B - 2*A*c)*Sqrt[b + c*x^2]*ArcTanh[(Sqrt[c
]*x)/Sqrt[b + c*x^2]]))/(2*c^(3/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.006, size = 88, normalized size = 1.3

$$\frac{x}{2}\sqrt{cx^{2}+b}\left(Bc^{\frac{3}{2}}\sqrt{cx^{2}+b}\,x+2\,A\ln\left(x\sqrt{c}+\sqrt{cx^{2}+b}\right)c^{2}-B\ln\left(x\sqrt{c}+\sqrt{cx^{2}+b}\right)bc\right)\frac{1}{\sqrt{cx^{4}+bx^{2}}}c^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/2*x*(c*x^2+b)^(1/2)*(B*c^(3/2)*(c*x^2+b)^(1/2)*x+2*A*ln(x*c^(1/2)+(c*x^2+
b)^(1/2))*c^2-B*ln(x*c^(1/2)+(c*x^2+b)^(1/2))*b*c)/(c*x^4+b*x^2)^(1/2)/c^(5
/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.1539, size = 302, normalized size = 4.58

$$\left[\frac{2\sqrt{cx^{4}+bx^{2}}Bc-(Bb-2\,Ac)\sqrt{c}\log\left(-2\,cx^{2}-b-2\sqrt{cx^{4}+bx^{2}}\sqrt{c}\right)}{4\,c^{2}},\frac{\sqrt{cx^{4}+bx^{2}}Bc+(Bb-2\,Ac)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}}\sqrt{-c}}{cx^{2}+b}\right)}{2\,c^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(c*x^4 + b*x^2)*B*c - (B*b - 2*A*c)*sqrt(c)*log(-2*c*x^2 - b -
2*sqrt(c*x^4 + b*x^2)*sqrt(c)))/c^2, 1/2*(sqrt(c*x^4 + b*x^2)*B*c + (B*b -
2*A*c)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)))/c^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(A+Bx^2\right)}{\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x*(A + B*x**2)/sqrt(x**2*(b + c*x**2)), x)
```

Giac [A] time = 1.23461, size = 90, normalized size = 1.36

$$\frac{\sqrt{cx^4+bx^2}B}{2\,c}+\frac{(Bb-2\,Ac)\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2}\right)\sqrt{c}-b\right|\right)}{4\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^4 + b*x^2)*B/c + 1/4*(B*b - 2*A*c)*log(abs(-2*(sqrt(c)*x^2 - s
qrt(c*x^4 + b*x^2))*sqrt(c) - b))/c^(3/2)
```

3.134 $\int \frac{A+Bx^2}{x\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=57

$$\frac{B\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}-\frac{A\sqrt{bx^2+cx^4}}{bx^2}$$

```
[Out] -((A*Sqrt[b*x^2 + c*x^4])/(b*x^2)) + (B*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 +
c*x^4]])/Sqrt[c]
```

Rubi [A] time = 0.150843, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2034, 792, 620, 206}

$$\frac{B\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}-\frac{A\sqrt{bx^2+cx^4}}{bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -((A*Sqrt[b*x^2 + c*x^4])/(b*x^2)) + (B*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 +
c*x^4]])/Sqrt[c]
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
```

```
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^2}{x\sqrt{bx^2+cx^4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{x\sqrt{bx+cx^2}}\,dx,x,x^2\right)\\
&= -\frac{A\sqrt{bx^2+cx^4}}{bx^2}+\frac{1}{2}B\operatorname{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)\\
&= -\frac{A\sqrt{bx^2+cx^4}}{bx^2}+B\operatorname{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x^2}{\sqrt{bx^2+cx^4}}\right)\\
&= -\frac{A\sqrt{bx^2+cx^4}}{bx^2}+\frac{B\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}
\end{aligned}
$$

Mathematica [A] time = 0.0292608, size = 74, normalized size = 1.3

$$
\frac{bBx\sqrt{b+cx^2}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b+cx^2}}\right)-A\sqrt{c}\left(b+cx^2\right)}{b\sqrt{c}\sqrt{x^2\left(b+cx^2\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-(A*Sqrt[c]*(b + c*x^2)) + b*B*x*Sqrt[b + c*x^2]*ArcTanh[(Sqrt[c]*x)/Sqrt[
b + c*x^2]])/(b*Sqrt[c]*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.009, size = 67, normalized size = 1.2

$$-\frac{1}{b}\sqrt{cx^2+b}\left(-B\ln\left(x\sqrt{c}+\sqrt{cx^2+b}\right)bx+A\sqrt{cx^2+b}\sqrt{c}\right)\frac{1}{\sqrt{cx^4+bx^2}}\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -(c*x^2+b)^(1/2)*(-B*ln(x*c^(1/2)+(c*x^2+b)^(1/2))*b*x+A*(c*x^2+b)^(1/2)*c^
(1/2))/(c*x^4+b*x^2)^(1/2)/c^(1/2)/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.03812, size = 298, normalized size = 5.23

$$\left[\frac{Bb\sqrt{c}x^2\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\,\sqrt{cx^4+bx^2}Ac}{2\,bcx^2},-\frac{Bb\sqrt{-c}x^2\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right)+\sqrt{cx^4+bx^2}Ac}{bcx^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(B*b*sqrt(c)*x^2*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 2
*sqrt(c*x^4 + b*x^2)*A*c)/(b*c*x^2), -(B*b*sqrt(-c)*x^2*arctan(sqrt(c*x^4 +
 b*x^2)*sqrt(-c)/(c*x^2 + b)) + sqrt(c*x^4 + b*x^2)*A*c)/(b*c*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + Bx^2}{x\sqrt{x^2\left(b + cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x*sqrt(x**2*(b + c*x**2))), x)
```

Giac [A] time = 1.182, size = 54, normalized size = 0.95

$$-\frac{B\arctan\left(\frac{\sqrt{c+\frac{b}{x^2}}}{\sqrt{-c}}\right)}{\sqrt{-c}} - \frac{A\sqrt{c + \frac{b}{x^2}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -B*arctan(sqrt(c + b/x^2)/sqrt(-c))/sqrt(-c) - A*sqrt(c + b/x^2)/b
```

3.135 $\int \frac{A+Bx^2}{x^3\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=61

$$-\frac{\sqrt{bx^2+cx^4}(3bB-2Ac)}{3b^2x^2}-\frac{A\sqrt{bx^2+cx^4}}{3bx^4}$$

```
[Out] -(A*Sqrt[b*x^2 + c*x^4])/(3*b*x^4) - ((3*b*B - 2*A*c)*Sqrt[b*x^2 + c*x^4])/
(3*b^2*x^2)
```

Rubi [A] time = 0.1684, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2034, 792, 650}

$$-\frac{\sqrt{bx^2+cx^4}(3bB-2Ac)}{3b^2x^2}-\frac{A\sqrt{bx^2+cx^4}}{3bx^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^3*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -(A*Sqrt[b*x^2 + c*x^4])/(3*b*x^4) - ((3*b*B - 2*A*c)*Sqrt[b*x^2 + c*x^4])/
(3*b^2*x^2)
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
```

```
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{x^3\sqrt{bx^2+cx^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{A+Bx}{x^2\sqrt{bx+cx^2}}\,dx,x,x^2\right)\\ &= -\frac{A\sqrt{bx^2+cx^4}}{3bx^4}+\frac{\left(-2(-bB+Ac)+\frac{1}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{1}{x\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{3b}\\ &= -\frac{A\sqrt{bx^2+cx^4}}{3bx^4}-\frac{(3bB-2Ac)\sqrt{bx^2+cx^4}}{3b^2x^2}\end{aligned}$$

Mathematica [A] time = 0.0238092, size = 43, normalized size = 0.7

$$-\frac{\sqrt{x^2\left(b+cx^2\right)}\left(A\left(b-2cx^2\right)+3bBx^2\right)}{3b^2x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^3*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -(Sqrt[x^2*(b + c*x^2)]*(3*b*B*x^2 + A*(b - 2*c*x^2)))/(3*b^2*x^4)
```

Maple [A] time = 0.005, size = 47, normalized size = 0.8

$$-\frac{\left(cx^2+b\right)\left(-2\,Ax^2c+3\,Bx^2b+Ab\right)}{3\,b^2x^2}\frac{1}{\sqrt{cx^4+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^3/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/3*(c*x^2+b)*(-2*A*c*x^2+3*B*b*x^2+A*b)/x^2/b^2/(c*x^4+b*x^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.07458, size = 86, normalized size = 1.41

$$-\frac{\sqrt{cx^4+bx^2}\left((3\,Bb-2\,Ac)x^2+Ab\right)}{3\,b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(c*x^4 + b*x^2)*((3*B*b - 2*A*c)*x^2 + A*b)/(b^2*x^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A+Bx^2}{x^3\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**3/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**3*sqrt(x**2*(b + c*x**2))), x)
```

Giac [A] time = 1.16651, size = 58, normalized size = 0.95

$$-\frac{3\,Bb\sqrt{c+\frac{b}{x^2}}+A\left(c+\frac{b}{x^2}\right)^{\frac{3}{2}}-3\,A\sqrt{c+\frac{b}{x^2}}c}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*(3*B*b*sqrt(c + b/x^2) + A*(c + b/x^2)^(3/2) - 3*A*sqrt(c + b/x^2)*c)/
b^2
```

3.136 $\int \frac{A+Bx^2}{x^5\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=96

$$\frac{2c\sqrt{bx^2+cx^4}(5bB-4Ac)}{15b^3x^2}-\frac{\sqrt{bx^2+cx^4}(5bB-4Ac)}{15b^2x^4}-\frac{A\sqrt{bx^2+cx^4}}{5bx^6}$$

```
[Out] -(A*Sqrt[b*x^2 + c*x^4])/(5*b*x^6) - ((5*b*B - 4*A*c)*Sqrt[b*x^2 + c*x^4])/
(15*b^2*x^4) + (2*c*(5*b*B - 4*A*c)*Sqrt[b*x^2 + c*x^4])/(15*b^3*x^2)
```

Rubi [A] time = 0.209251, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2034, 792, 658, 650}

$$\frac{2c\sqrt{bx^2+cx^4}(5bB-4Ac)}{15b^3x^2}-\frac{\sqrt{bx^2+cx^4}(5bB-4Ac)}{15b^2x^4}-\frac{A\sqrt{bx^2+cx^4}}{5bx^6}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^5*Sqrt[b*x^2 + c*x^4]),x]
```


```
[Out] -(A*Sqrt[b*x^2 + c*x^4])/(5*b*x^6) - ((5*b*B - 4*A*c)*Sqrt[b*x^2 + c*x^4])/
(15*b^2*x^4) + (2*c*(5*b*B - 4*A*c)*Sqrt[b*x^2 + c*x^4])/(15*b^3*x^2)
```


Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
```

```
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^2}{x^5\sqrt{bx^2+cx^4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{x^3\sqrt{bx+cx^2}}\,dx,x,x^2\right)\\
&= -\frac{A\sqrt{bx^2+cx^4}}{5bx^6}+\frac{\left(-3(-bB+Ac)+\frac{1}{2}(-bB+2Ac)\right)\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{5b}\\
&= -\frac{A\sqrt{bx^2+cx^4}}{5bx^6}-\frac{(5bB-4Ac)\sqrt{bx^2+cx^4}}{15b^2x^4}-\frac{(c(5bB-4Ac))\operatorname{Subst}\left(\int \frac{1}{x\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{15b^2}\\
&= -\frac{A\sqrt{bx^2+cx^4}}{5bx^6}-\frac{(5bB-4Ac)\sqrt{bx^2+cx^4}}{15b^2x^4}+\frac{2c(5bB-4Ac)\sqrt{bx^2+cx^4}}{15b^3x^2}
\end{aligned}$$

Mathematica [A] time = 0.0258534, size = 64, normalized size = 0.67

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(A\left(-3b^2+4bcx^2-8c^2x^4\right)-5bBx^2\left(b-2cx^2\right)\right)}{15b^3x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^5*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(-5*b*B*x^2*(b - 2*c*x^2) + A*(-3*b^2 + 4*b*c*x^2 -
8*c^2*x^4)))/(15*b^3*x^6)
```

Maple [A] time = 0.005, size = 70, normalized size = 0.7

$$-\frac{\left(cx^{2}+b\right)\left(8\,Ac^{2}x^{4}-10\,Bx^{4}bc-4\,Abcx^{2}+5\,Bx^{2}b^{2}+3\,Ab^{2}\right)}{15\,b^{3}x^{4}}\frac{1}{\sqrt{cx^{4}+bx^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^5/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/15*(c*x^2+b)*(8*A*c^2*x^4-10*B*b*c*x^4-4*A*b*c*x^2+5*B*b^2*x^2+3*A*b^2)/
x^4/b^3/(c*x^4+b*x^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.18404, size = 135, normalized size = 1.41

$$\frac{\left(2\left(5\,Bbc-4\,Ac^{2}\right)x^{4}-3\,Ab^{2}-\left(5\,Bb^{2}-4\,Abc\right)x^{2}\right)\sqrt{cx^{4}+bx^{2}}}{15\,b^{3}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(2*(5*B*b*c - 4*A*c^2)*x^4 - 3*A*b^2 - (5*B*b^2 - 4*A*b*c)*x^2)*sqrt(c
*x^4 + b*x^2)/(b^3*x^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A+Bx^2}{x^5\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**5/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**5*sqrt(x**2*(b + c*x**2))), x)
```

Giac [A] time = 1.15571, size = 99, normalized size = 1.03

$$-\frac{5\,Bb\left(c+\frac{b}{x^2}\right)^{\frac{3}{2}}+3\,A\left(c+\frac{b}{x^2}\right)^{\frac{5}{2}}-15\,Bb\sqrt{c+\frac{b}{x^2}}c-10\,A\left(c+\frac{b}{x^2}\right)^{\frac{3}{2}}c+15\,A\sqrt{c+\frac{b}{x^2}}c^2}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/15*(5*B*b*(c + b/x^2)^(3/2) + 3*A*(c + b/x^2)^(5/2) - 15*B*b*sqrt(c + b/
x^2)*c - 10*A*(c + b/x^2)^(3/2)*c + 15*A*sqrt(c + b/x^2)*c^2)/b^3
```

3.137 $\int \frac{A+Bx^2}{x^7\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=133

$$-\frac{8c^2\sqrt{bx^2+cx^4}(7bB-6Ac)}{105b^4x^2}+\frac{4c\sqrt{bx^2+cx^4}(7bB-6Ac)}{105b^3x^4}-\frac{\sqrt{bx^2+cx^4}(7bB-6Ac)}{35b^2x^6}-\frac{A\sqrt{bx^2+cx^4}}{7bx^8}$$

```
[Out] -(A*Sqrt[b*x^2 + c*x^4])/(7*b*x^8) - ((7*b*B - 6*A*c)*Sqrt[b*x^2 + c*x^4])/
(35*b^2*x^6) + (4*c*(7*b*B - 6*A*c)*Sqrt[b*x^2 + c*x^4])/(105*b^3*x^4) - (8
*c^2*(7*b*B - 6*A*c)*Sqrt[b*x^2 + c*x^4])/(105*b^4*x^2)
```

Rubi [A] time = 0.253481, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2034, 792, 658, 650}

$$-\frac{8c^2\sqrt{bx^2+cx^4}(7bB-6Ac)}{105b^4x^2}+\frac{4c\sqrt{bx^2+cx^4}(7bB-6Ac)}{105b^3x^4}-\frac{\sqrt{bx^2+cx^4}(7bB-6Ac)}{35b^2x^6}-\frac{A\sqrt{bx^2+cx^4}}{7bx^8}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^7*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -(A*Sqrt[b*x^2 + c*x^4])/(7*b*x^8) - ((7*b*B - 6*A*c)*Sqrt[b*x^2 + c*x^4])/
(35*b^2*x^6) + (4*c*(7*b*B - 6*A*c)*Sqrt[b*x^2 + c*x^4])/(105*b^3*x^4) - (8
*c^2*(7*b*B - 6*A*c)*Sqrt[b*x^2 + c*x^4])/(105*b^4*x^2)
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
```

```
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^2}{x^7\sqrt{bx^2+cx^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{A+Bx}{x^4\sqrt{bx+cx^2}}\,dx,x,x^2\right)\\
&= -\frac{A\sqrt{bx^2+cx^4}}{7bx^8} + \frac{\left(-4(-bB+Ac)+\frac{1}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{1}{x^3\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{7b}\\
&= -\frac{A\sqrt{bx^2+cx^4}}{7bx^8} - \frac{(7bB-6Ac)\sqrt{bx^2+cx^4}}{35b^2x^6} - \frac{(2c(7bB-6Ac))\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{35b^2}\\
&= -\frac{A\sqrt{bx^2+cx^4}}{7bx^8} - \frac{(7bB-6Ac)\sqrt{bx^2+cx^4}}{35b^2x^6} + \frac{4c(7bB-6Ac)\sqrt{bx^2+cx^4}}{105b^3x^4} + \frac{\left(4c^2(7bB-6Ac)\right)\text{Sub}}{10}\\
&= -\frac{A\sqrt{bx^2+cx^4}}{7bx^8} - \frac{(7bB-6Ac)\sqrt{bx^2+cx^4}}{35b^2x^6} + \frac{4c(7bB-6Ac)\sqrt{bx^2+cx^4}}{105b^3x^4} - \frac{8c^2(7bB-6Ac)\sqrt{bx^2}}{105b^4x^2}
\end{aligned}$$

Mathematica [A] time = 0.0307912, size = 89, normalized size = 0.67

$$-\frac{\sqrt{x^2\left(b+cx^2\right)}\left(3A\left(-6b^2cx^2+5b^3+8bc^2x^4-16c^3x^6\right)+7bBx^2\left(3b^2-4bcx^2+8c^2x^4\right)\right)}{105b^4x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^7*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -(Sqrt[x^2*(b + c*x^2)]*(7*b*B*x^2*(3*b^2 - 4*b*c*x^2 + 8*c^2*x^4) + 3*A*(5
*b^3 - 6*b^2*c*x^2 + 8*b*c^2*x^4 - 16*c^3*x^6)))/(105*b^4*x^8)
```

Maple [A] time = 0.006, size = 94, normalized size = 0.7

$$-\frac{(cx^2+b)\left(-48\,Ac^3x^6+56\,Bx^6bc^2+24\,Abc^2x^4-28\,Bx^4b^2c-18\,Ab^2cx^2+21\,Bx^2b^3+15\,Ab^3\right)}{105\,x^6b^4}\frac{1}{\sqrt{cx^4+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^7/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/105*(c*x^2+b)*(-48*A*c^3*x^6+56*B*b*c^2*x^6+24*A*b*c^2*x^4-28*B*b^2*c*x^
4-18*A*b^2*c*x^2+21*B*b^3*x^2+15*A*b^3)/x^6/b^4/(c*x^4+b*x^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.15395, size = 190, normalized size = 1.43

$$-\frac{\left(8\left(7\,Bbc^2-6\,Ac^3\right)x^6-4\left(7\,Bb^2c-6\,Abc^2\right)x^4+15\,Ab^3+3\left(7\,Bb^3-6\,Ab^2c\right)x^2\right)\sqrt{cx^4+bx^2}}{105\,b^4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/105*(8*(7*B*b*c^2 - 6*A*c^3)*x^6 - 4*(7*B*b^2*c - 6*A*b*c^2)*x^4 + 15*A*
b^3 + 3*(7*B*b^3 - 6*A*b^2*c)*x^2)*sqrt(c*x^4 + b*x^2)/(b^4*x^8)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + Bx^2}{x^7\sqrt{x^2\left(b + cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**7/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**7*sqrt(x**2*(b + c*x**2))), x)
```

Giac [A] time = 1.16977, size = 140, normalized size = 1.05

$$-\frac{21\,Bb\left(c + \frac{b}{x^2}\right)^{\frac{5}{2}} + 15\,A\left(c + \frac{b}{x^2}\right)^{\frac{7}{2}} - 70\,Bb\left(c + \frac{b}{x^2}\right)^{\frac{3}{2}}c - 63\,A\left(c + \frac{b}{x^2}\right)^{\frac{5}{2}}c + 105\,Bb\sqrt{c + \frac{b}{x^2}}c^2 + 105\,A\left(c + \frac{b}{x^2}\right)^{\frac{3}{2}}c^2 - 105\,A}{105\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/105*(21*B*b*(c + b/x^2)^(5/2) + 15*A*(c + b/x^2)^(7/2) - 70*B*b*(c + b/x
^2)^(3/2)*c - 63*A*(c + b/x^2)^(5/2)*c + 105*B*b*sqrt(c + b/x^2)*c^2 + 105*
A*(c + b/x^2)^(3/2)*c^2 - 105*A*sqrt(c + b/x^2)*c^3)/b^4
```

3.138 $\int \frac{A+Bx^2}{x^9\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=170

$$\frac{16c^3\sqrt{bx^2+cx^4}(9bB-8Ac)}{315b^5x^2}-\frac{8c^2\sqrt{bx^2+cx^4}(9bB-8Ac)}{315b^4x^4}+\frac{2c\sqrt{bx^2+cx^4}(9bB-8Ac)}{105b^3x^6}-\frac{\sqrt{bx^2+cx^4}(9bB-8Ac)}{63b^2x^8}-A\sqrt{}$$

```
[Out] -(A*Sqrt[b*x^2 + c*x^4])/(9*b*x^10) - ((9*b*B - 8*A*c)*Sqrt[b*x^2 + c*x^4])
/(63*b^2*x^8) + (2*c*(9*b*B - 8*A*c)*Sqrt[b*x^2 + c*x^4])/(105*b^3*x^6) - (
8*c^2*(9*b*B - 8*A*c)*Sqrt[b*x^2 + c*x^4])/(315*b^4*x^4) + (16*c^3*(9*b*B -
 8*A*c)*Sqrt[b*x^2 + c*x^4])/(315*b^5*x^2)
```

Rubi [A] time = 0.30266, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2034, 792, 658, 650}

$$\frac{16c^3\sqrt{bx^2+cx^4}(9bB-8Ac)}{315b^5x^2}-\frac{8c^2\sqrt{bx^2+cx^4}(9bB-8Ac)}{315b^4x^4}+\frac{2c\sqrt{bx^2+cx^4}(9bB-8Ac)}{105b^3x^6}-\frac{\sqrt{bx^2+cx^4}(9bB-8Ac)}{63b^2x^8}-A\sqrt{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^9*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -(A*Sqrt[b*x^2 + c*x^4])/(9*b*x^10) - ((9*b*B - 8*A*c)*Sqrt[b*x^2 + c*x^4])
/(63*b^2*x^8) + (2*c*(9*b*B - 8*A*c)*Sqrt[b*x^2 + c*x^4])/(105*b^3*x^6) - (
8*c^2*(9*b*B - 8*A*c)*Sqrt[b*x^2 + c*x^4])/(315*b^4*x^4) + (16*c^3*(9*b*B -
 8*A*c)*Sqrt[b*x^2 + c*x^4])/(315*b^5*x^2)
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
```

```
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^9\sqrt{bx^2+cx^4}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{x^5\sqrt{bx+cx^2}}\,dx, x, x^2\right)$$

$$= -\frac{A\sqrt{bx^2+cx^4}}{9bx^{10}} + \frac{\left(-5(-bB+Ac)+\frac{1}{2}(-bB+2Ac)\right)\operatorname{Subst}\left(\int \frac{1}{x^4\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{9b}$$

$$= -\frac{A\sqrt{bx^2+cx^4}}{9bx^{10}} - \frac{(9bB-8Ac)\sqrt{bx^2+cx^4}}{63b^2x^8} - \frac{(c(9bB-8Ac))\operatorname{Subst}\left(\int \frac{1}{x^3\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{21b^2}$$

$$= -\frac{A\sqrt{bx^2+cx^4}}{9bx^{10}} - \frac{(9bB-8Ac)\sqrt{bx^2+cx^4}}{63b^2x^8} + \frac{2c(9bB-8Ac)\sqrt{bx^2+cx^4}}{105b^3x^6} + \frac{\left(4c^2(9bB-8Ac)\right)\operatorname{Sub}}{1}$$

$$= -\frac{A\sqrt{bx^2+cx^4}}{9bx^{10}} - \frac{(9bB-8Ac)\sqrt{bx^2+cx^4}}{63b^2x^8} + \frac{2c(9bB-8Ac)\sqrt{bx^2+cx^4}}{105b^3x^6} - \frac{8c^2(9bB-8Ac)\sqrt{bx^2}}{315b^4x^4}$$

$$= -\frac{A\sqrt{bx^2+cx^4}}{9bx^{10}} - \frac{(9bB-8Ac)\sqrt{bx^2+cx^4}}{63b^2x^8} + \frac{2c(9bB-8Ac)\sqrt{bx^2+cx^4}}{105b^3x^6} - \frac{8c^2(9bB-8Ac)\sqrt{bx^2}}{315b^4x^4}$$

Mathematica [A] time = 0.0576179, size = 94, normalized size = 0.55

$$\frac{x^2\left(\frac{cx^2}{b}+1\right)\left(-6b^2cx^2+5b^3+8bc^2x^4-16c^3x^6\right)(8Ac-9bB)-35Ab^3\left(b+cx^2\right)}{315b^4x^8\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^9*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-35*A*b^3*(b + c*x^2) + (-9*b*B + 8*A*c)*x^2*(1 + (c*x^2)/b)*(5*b^3 - 6*b^
2*c*x^2 + 8*b*c^2*x^4 - 16*c^3*x^6))/(315*b^4*x^8*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.006, size = 118, normalized size = 0.7

$$-\frac{\left(cx^2+b\right)\left(128\,Ac^4x^8-144\,Bbc^3x^8-64\,Abc^3x^6+72\,Bb^2c^2x^6+48\,Ab^2c^2x^4-54\,Bb^3cx^4-40\,Ab^3cx^2+45\,Bb^4x^2+35\,A\right.}{315\,x^8b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^9/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/315*(c*x^2+b)*(128*A*c^4*x^8-144*B*b*c^3*x^8-64*A*b*c^3*x^6+72*B*b^2*c^2
*x^6+48*A*b^2*c^2*x^4-54*B*b^3*c*x^4-40*A*b^3*c*x^2+45*B*b^4*x^2+35*A*b^4)/
x^8/b^5/(c*x^4+b*x^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^9/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.26573, size = 240, normalized size = 1.41

$$\frac{\left(16\left(9\,Bbc^3-8\,Ac^4\right)x^8-8\left(9\,Bb^2c^2-8\,Abc^3\right)x^6-35\,Ab^4+6\left(9\,Bb^3c-8\,Ab^2c^2\right)x^4-5\left(9\,Bb^4-8\,Ab^3c\right)x^2\right)\sqrt{cx^4+bx^2}}{315\,b^5x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^9/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/315*(16*(9*B*b*c^3 - 8*A*c^4)*x^8 - 8*(9*B*b^2*c^2 - 8*A*b*c^3)*x^6 - 35*
A*b^4 + 6*(9*B*b^3*c - 8*A*b^2*c^2)*x^4 - 5*(9*B*b^4 - 8*A*b^3*c)*x^2)*sqrt
(c*x^4 + b*x^2)/(b^5*x^10)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A+Bx^2}{x^9\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**9/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**9*sqrt(x**2*(b + c*x**2))), x)
```

Giac [A] time = 1.18785, size = 182, normalized size = 1.07

$$-\frac{45\,Bb\left(c+\frac{b}{x^2}\right)^{\frac{7}{2}}+35\,A\left(c+\frac{b}{x^2}\right)^{\frac{9}{2}}-189\,Bb\left(c+\frac{b}{x^2}\right)^{\frac{5}{2}}c-180\,A\left(c+\frac{b}{x^2}\right)^{\frac{7}{2}}c+315\,Bb\left(c+\frac{b}{x^2}\right)^{\frac{3}{2}}c^2+378\,A\left(c+\frac{b}{x^2}\right)^{\frac{5}{2}}c^2-31}{315\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^9/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/315*(45*B*b*(c + b/x^2)^(7/2) + 35*A*(c + b/x^2)^(9/2) - 189*B*b*(c + b/
x^2)^(5/2)*c - 180*A*(c + b/x^2)^(7/2)*c + 315*B*b*(c + b/x^2)^(3/2)*c^2 +
378*A*(c + b/x^2)^(5/2)*c^2 - 315*B*b*sqrt(c + b/x^2)*c^3 - 420*A*(c + b/x^
2)^(3/2)*c^3 + 315*A*sqrt(c + b/x^2)*c^4)/b^5
```

3.139 $\int \frac{x^6(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=131

$$-\frac{8b^2\sqrt{bx^2+cx^4}(6bB-7Ac)}{105c^4x}-\frac{x^3\sqrt{bx^2+cx^4}(6bB-7Ac)}{35c^2}+\frac{4bx\sqrt{bx^2+cx^4}(6bB-7Ac)}{105c^3}+\frac{Bx^5\sqrt{bx^2+cx^4}}{7c}$$

```
[Out] (-8*b^2*(6*b*B - 7*A*c)*Sqrt[b*x^2 + c*x^4])/(105*c^4*x) + (4*b*(6*b*B - 7*
A*c)*x*Sqrt[b*x^2 + c*x^4])/(105*c^3) - ((6*b*B - 7*A*c)*x^3*Sqrt[b*x^2 + c
*x^4])/(35*c^2) + (B*x^5*Sqrt[b*x^2 + c*x^4])/(7*c)
```

Rubi [A] time = 0.239866, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2039, 2016, 1588}

$$-\frac{8b^2\sqrt{bx^2+cx^4}(6bB-7Ac)}{105c^4x}-\frac{x^3\sqrt{bx^2+cx^4}(6bB-7Ac)}{35c^2}+\frac{4bx\sqrt{bx^2+cx^4}(6bB-7Ac)}{105c^3}+\frac{Bx^5\sqrt{bx^2+cx^4}}{7c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```


```
[Out] (-8*b^2*(6*b*B - 7*A*c)*Sqrt[b*x^2 + c*x^4])/(105*c^4*x) + (4*b*(6*b*B - 7*
A*c)*x*Sqrt[b*x^2 + c*x^4])/(105*c^3) - ((6*b*B - 7*A*c)*x^3*Sqrt[b*x^2 + c
*x^4])/(35*c^2) + (B*x^5*Sqrt[b*x^2 + c*x^4])/(7*c)
```


Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
```

```
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^6\left(A+Bx^2\right)}{\sqrt{bx^2+cx^4}}\,dx = \frac{Bx^5\sqrt{bx^2+cx^4}}{7c}-\frac{(6bB-7Ac)\int \frac{x^6}{\sqrt{bx^2+cx^4}}\,dx}{7c}$$

$$= -\frac{(6bB-7Ac)x^3\sqrt{bx^2+cx^4}}{35c^2}+\frac{Bx^5\sqrt{bx^2+cx^4}}{7c}+\frac{(4b(6bB-7Ac))\int \frac{x^4}{\sqrt{bx^2+cx^4}}\,dx}{35c^2}$$

$$= \frac{4b(6bB-7Ac)x\sqrt{bx^2+cx^4}}{105c^3}-\frac{(6bB-7Ac)x^3\sqrt{bx^2+cx^4}}{35c^2}+\frac{Bx^5\sqrt{bx^2+cx^4}}{7c}-\frac{\left(8b^2(6bB-7Ac)\right)}{105c^3}$$

$$= -\frac{8b^2(6bB-7Ac)\sqrt{bx^2+cx^4}}{105c^4x}+\frac{4b(6bB-7Ac)x\sqrt{bx^2+cx^4}}{105c^3}-\frac{(6bB-7Ac)x^3\sqrt{bx^2+cx^4}}{35c^2}+\frac{Bx^5}{}$$

Mathematica [A] time = 0.0599342, size = 85, normalized size = 0.65

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(8b^2c\left(7A+3Bx^2\right)-2bc^2x^2\left(14A+9Bx^2\right)+3c^3x^4\left(7A+5Bx^2\right)-48b^3B\right)}{105c^4x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(-48*b^3*B + 8*b^2*c*(7*A + 3*B*x^2) + 3*c^3*x^4*(7*
A + 5*B*x^2) - 2*b*c^2*x^2*(14*A + 9*B*x^2)))/(105*c^4*x)
```

Maple [A] time = 0.005, size = 89, normalized size = 0.7

$$\frac{\left(cx^{2}+b\right)\left(15\,Bc^{3}x^{6}+21\,Ax^{4}c^{3}-18\,Bx^{4}bc^{2}-28\,Ax^{2}bc^{2}+24\,Bx^{2}b^{2}c+56\,Ab^{2}c-48\,Bb^{3}\right)x}{105\,c^{4}}\frac{1}{\sqrt{cx^{4}+bx^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/105*(c*x^2+b)*(15*B*c^3*x^6+21*A*c^3*x^4-18*B*b*c^2*x^4-28*A*b*c^2*x^2+24
*B*b^2*c*x^2+56*A*b^2*c-48*B*b^3)*x/c^4/(c*x^4+b*x^2)^(1/2)
```

Maxima [A] time = 1.18615, size = 143, normalized size = 1.09

$$\frac{\left(3\,c^{3}x^{6}-bc^{2}x^{4}+4\,b^{2}cx^{2}+8\,b^{3}\right)A}{15\,\sqrt{cx^{2}+b}c^{3}}+\frac{\left(5\,c^{4}x^{8}-bc^{3}x^{6}+2\,b^{2}c^{2}x^{4}-8\,b^{3}cx^{2}-16\,b^{4}\right)B}{35\,\sqrt{cx^{2}+b}c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*c^3*x^6 - b*c^2*x^4 + 4*b^2*c*x^2 + 8*b^3)*A/(sqrt(c*x^2 + b)*c^3)
+ 1/35*(5*c^4*x^8 - b*c^3*x^6 + 2*b^2*c^2*x^4 - 8*b^3*c*x^2 - 16*b^4)*B/(sq
rt(c*x^2 + b)*c^4)
```

Fricas [A] time = 1.10809, size = 184, normalized size = 1.4

$$\frac{\left(15\,Bc^{3}x^{6}-3\left(6\,Bbc^{2}-7\,Ac^{3}\right)x^{4}-48\,Bb^{3}+56\,Ab^{2}c+4\left(6\,Bb^{2}c-7\,Abc^{2}\right)x^{2}\right)\sqrt{cx^{4}+bx^{2}}}{105\,c^{4}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*B*c^3*x^6 - 3*(6*B*b*c^2 - 7*A*c^3)*x^4 - 48*B*b^3 + 56*A*b^2*c +
 4*(6*B*b^2*c - 7*A*b*c^2)*x^2)*sqrt(c*x^4 + b*x^2)/(c^4*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6 \left(A + Bx^2\right)}{\sqrt{x^2 \left(b + cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**6*(A + B*x**2)/sqrt(x**2*(b + c*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2 + A\right)x^6}{\sqrt{cx^4 + bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^6/sqrt(c*x^4 + b*x^2), x)
```

3.140 $\int \frac{x^4(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=94

$$-\frac{x\sqrt{bx^2+cx^4}(4bB-5Ac)}{15c^2}+\frac{2b\sqrt{bx^2+cx^4}(4bB-5Ac)}{15c^3x}+\frac{Bx^3\sqrt{bx^2+cx^4}}{5c}$$

```
[Out] (2*b*(4*b*B - 5*A*c)*Sqrt[b*x^2 + c*x^4])/(15*c^3*x) - ((4*b*B - 5*A*c)*x*S
qrt[b*x^2 + c*x^4])/(15*c^2) + (B*x^3*Sqrt[b*x^2 + c*x^4])/(5*c)
```

Rubi [A] time = 0.193394, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2039, 2016, 1588}

$$-\frac{x\sqrt{bx^2+cx^4}(4bB-5Ac)}{15c^2}+\frac{2b\sqrt{bx^2+cx^4}(4bB-5Ac)}{15c^3x}+\frac{Bx^3\sqrt{bx^2+cx^4}}{5c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*b*(4*b*B - 5*A*c)*Sqrt[b*x^2 + c*x^4])/(15*c^3*x) - ((4*b*B - 5*A*c)*x*S
qrt[b*x^2 + c*x^4])/(15*c^2) + (B*x^3*Sqrt[b*x^2 + c*x^4])/(5*c)
```

Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
```

```
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4\left(A+Bx^2\right)}{\sqrt{bx^2+cx^4}}\,dx &= \frac{Bx^3\sqrt{bx^2+cx^4}}{5c}-\frac{(4bB-5Ac)\int \frac{x^4}{\sqrt{bx^2+cx^4}}\,dx}{5c}\\
&= -\frac{(4bB-5Ac)x\sqrt{bx^2+cx^4}}{15c^2}+\frac{Bx^3\sqrt{bx^2+cx^4}}{5c}+\frac{(2b(4bB-5Ac))\int \frac{x^2}{\sqrt{bx^2+cx^4}}\,dx}{15c^2}\\
&= \frac{2b(4bB-5Ac)\sqrt{bx^2+cx^4}}{15c^3x}-\frac{(4bB-5Ac)x\sqrt{bx^2+cx^4}}{15c^2}+\frac{Bx^3\sqrt{bx^2+cx^4}}{5c}
\end{aligned}
$$

Mathematica [A] time = 0.042267, size = 63, normalized size = 0.67

$$
\frac{\sqrt{x^2\left(b+cx^2\right)}\left(-2bc\left(5A+2Bx^2\right)+c^2x^2\left(5A+3Bx^2\right)+8b^2B\right)}{15c^3x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(8*b^2*B - 2*b*c*(5*A + 2*B*x^2) + c^2*x^2*(5*A + 3*
B*x^2)))/(15*c^3*x)
```

Maple [A] time = 0.005, size = 65, normalized size = 0.7

$$
-\frac{\left(cx^2+b\right)\left(-3\,Bc^2x^4-5\,Ax^2c^2+4\,Bx^2bc+10\,Abc-8\,Bb^2\right)x}{15\,c^3}\frac{1}{\sqrt{cx^4+bx^2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/15*(c*x^2+b)*(-3*B*c^2*x^4-5*A*c^2*x^2+4*B*b*c*x^2+10*A*b*c-8*B*b^2)*x/c
^3/(c*x^4+b*x^2)^(1/2)
```

Maxima [A] time = 1.31643, size = 112, normalized size = 1.19

$$\frac{\left(c^2x^4 - bcx^2 - 2\,b^2\right)A}{3\,\sqrt{cx^2 + b}c^2} + \frac{\left(3\,c^3x^6 - bc^2x^4 + 4\,b^2cx^2 + 8\,b^3\right)B}{15\,\sqrt{cx^2 + b}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(c^2*x^4 - b*c*x^2 - 2*b^2)*A/(sqrt(c*x^2 + b)*c^2) + 1/15*(3*c^3*x^6 -
 b*c^2*x^4 + 4*b^2*c*x^2 + 8*b^3)*B/(sqrt(c*x^2 + b)*c^3)
```

Fricas [A] time = 1.01806, size = 128, normalized size = 1.36

$$\frac{\left(3\,Bc^2x^4 + 8\,Bb^2 - 10\,Abc - \left(4\,Bbc - 5\,Ac^2\right)x^2\right)\sqrt{cx^4 + bx^2}}{15\,c^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*B*c^2*x^4 + 8*B*b^2 - 10*A*b*c - (4*B*b*c - 5*A*c^2)*x^2)*sqrt(c*x^
4 + b*x^2)/(c^3*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4\left(A + Bx^2\right)}{\sqrt{x^2\left(b + cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**4*(A + B*x**2)/sqrt(x**2*(b + c*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)x^4}{\sqrt{cx^4 + bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^4/sqrt(c*x^4 + b*x^2), x)
```

3.141 $\int \frac{x^2(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=59

$$\frac{Bx\sqrt{bx^2+cx^4}}{3c}-\frac{\sqrt{bx^2+cx^4}(2bB-3Ac)}{3c^2x}$$

```
[Out] -((2*b*B - 3*A*c)*Sqrt[b*x^2 + c*x^4])/(3*c^2*x) + (B*x*Sqrt[b*x^2 + c*x^4]
)/(3*c)
```

Rubi [A] time = 0.135175, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2039, 1588}

$$\frac{Bx\sqrt{bx^2+cx^4}}{3c}-\frac{\sqrt{bx^2+cx^4}(2bB-3Ac)}{3c^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] -((2*b*B - 3*A*c)*Sqrt[b*x^2 + c*x^4])/(3*c^2*x) + (B*x*Sqrt[b*x^2 + c*x^4]
)/(3*c)
```

Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
```

```
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^2 \left(A+Bx^2\right)}{\sqrt{bx^2+cx^4}}\,dx = \frac{Bx\sqrt{bx^2+cx^4}}{3c} - \frac{(2bB-3Ac)\int \frac{x^2}{\sqrt{bx^2+cx^4}}\,dx}{3c}$$
$$= -\frac{(2bB-3Ac)\sqrt{bx^2+cx^4}}{3c^2x} + \frac{Bx\sqrt{bx^2+cx^4}}{3c}$$

Mathematica [A] time = 0.0276201, size = 40, normalized size = 0.68

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(3Ac-2bB+Bcx^2\right)}{3c^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(-2*b*B + 3*A*c + B*c*x^2))/(3*c^2*x)
```

Maple [A] time = 0.004, size = 42, normalized size = 0.7

$$\frac{\left(cx^2+b\right)\left(Bcx^2+3\,Ac-2\,Bb\right)x}{3\,c^2}\frac{1}{\sqrt{cx^4+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/3*(c*x^2+b)*(B*c*x^2+3*A*c-2*B*b)*x/c^2/(c*x^4+b*x^2)^(1/2)
```

Maxima [A] time = 1.15904, size = 68, normalized size = 1.15

$$\frac{\sqrt{cx^2+b}A}{c} + \frac{\left(c^2x^4-bcx^2-2\,b^2\right)B}{3\,\sqrt{cx^2+b}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(c*x^2 + b)*A/c + 1/3*(c^2*x^4 - b*c*x^2 - 2*b^2)*B/(sqrt(c*x^2 + b)*c^
2)
```

Fricas [A] time = 1.00099, size = 80, normalized size = 1.36

$$\frac{\sqrt{cx^4 + bx^2}\left(Bcx^2 - 2\,Bb + 3\,Ac\right)}{3\,c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(c*x^4 + b*x^2)*(B*c*x^2 - 2*B*b + 3*A*c)/(c^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\left(A + Bx^2\right)}{\sqrt{x^2\left(b + cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**2*(A + B*x**2)/sqrt(x**2*(b + c*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2 + A\right)x^2}{\sqrt{cx^4 + bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")

[Out] integrate((B*x^2 + A)*x^2/sqrt(c*x^4 + b*x^2), x)
```

3.142 $\int \frac{A+Bx^2}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=55

$$\frac{B\sqrt{bx^2+cx^4}}{cx}-\frac{A\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{b}}$$

```
[Out] (B*Sqrt[b*x^2 + c*x^4])/(c*x) - (A*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]]
)/Sqrt[b]
```

Rubi [A] time = 0.0197414, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {1145, 2008, 206}

$$\frac{B\sqrt{bx^2+cx^4}}{cx}-\frac{A\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (B*Sqrt[b*x^2 + c*x^4])/(c*x) - (A*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]]
)/Sqrt[b]
```

Rule 1145

```
Int[((d_) + (e_.)*(x_)^2)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
Simp[(e*(b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 3)*x), x] - Dist[(b*e*(2*p + 1)
- c*d*(4*p + 3))/(c*(4*p + 3)), Int[(b*x^2 + c*x^4)^p, x], x] /; FreeQ[{b,
c, d, e, p}, x] &&  !IntegerQ[p] && NeQ[4*p + 3, 0] && NeQ[b*e*(2*p + 1) -
c*d*(4*p + 3), 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{\sqrt{bx^2+cx^4}}\,dx &= \frac{B\sqrt{bx^2+cx^4}}{cx}+A\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx\\ &= \frac{B\sqrt{bx^2+cx^4}}{cx}-A\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2+cx^4}}\right)\\ &= \frac{B\sqrt{bx^2+cx^4}}{cx}-\frac{A\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.0319559, size = 73, normalized size = 1.33

$$\frac{x\left(\sqrt{b}B\left(b+cx^2\right)-Ac\sqrt{b+cx^2}\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)\right)}{\sqrt{b}c\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*(Sqrt[b]*B*(b + c*x^2) - A*c*Sqrt[b + c*x^2]*ArcTanh[Sqrt[b + c*x^2]/Sqr
t[b]]))/(Sqrt[b]*c*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.008, size = 72, normalized size = 1.3

$$-\frac{x}{c}\sqrt{cx^2+b}\left(A\ln\left(2\,\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)c-B\sqrt{cx^2+b}\sqrt{b}\right)\frac{1}{\sqrt{cx^4+bx^2}}\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -x*(c*x^2+b)^(1/2)*(A*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*c-B*(c*x^2+b)^(1/
2)*b^(1/2))/(c*x^4+b*x^2)^(1/2)/c/b^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\sqrt{cx^4 + bx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/sqrt(c*x^4 + b*x^2), x)
```

Fricas [A] time = 1.06763, size = 300, normalized size = 5.45

$$\left[\frac{A\sqrt{b}cx\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)+2\,\sqrt{cx^4+bx^2}Bb}{2\,bcx}, \frac{A\sqrt{-b}cx\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-b}}{cx^3+bx}\right)+\sqrt{cx^4+bx^2}Bb}{bcx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(A*sqrt(b)*c*x*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/x^
3) + 2*sqrt(c*x^4 + b*x^2)*B*b)/(b*c*x), (A*sqrt(-b)*c*x*arctan(sqrt(c*x^4
+ b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + sqrt(c*x^4 + b*x^2)*B*b)/(b*c*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + Bx^2}{\sqrt{x^2\left(b + cx^2\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/sqrt(x**2*(b + c*x**2)), x)
```

Giac [A] time = 1.19273, size = 81, normalized size = 1.47

$$\frac{A\log\left(\left(\sqrt{c+\frac{b}{x^2}}-\frac{\sqrt{b}}{x}\right)^2\right)}{2\sqrt{b}}-\frac{2B\sqrt{b}}{\left(\sqrt{c+\frac{b}{x^2}}-\frac{\sqrt{b}}{x}\right)^2-c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*A*log((sqrt(c + b/x^2) - sqrt(b)/x)^2)/sqrt(b) - 2*B*sqrt(b)/((sqrt(c +
 b/x^2) - sqrt(b)/x)^2 - c)
```

3.143 $\int \frac{A+Bx^2}{x^2\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=68

$$-\frac{(2bB-Ac)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}-\frac{A\sqrt{bx^2+cx^4}}{2bx^3}$$

```
[Out] -(A*Sqrt[b*x^2 + c*x^4])/(2*b*x^3) - ((2*b*B - A*c)*ArcTanh[(Sqrt[b]*x)/Sqr
t[b*x^2 + c*x^4]])/(2*b^(3/2))
```

Rubi [A] time = 0.117278, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2038, 2008, 206}

$$-\frac{(2bB-Ac)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}-\frac{A\sqrt{bx^2+cx^4}}{2bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^2*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -(A*Sqrt[b*x^2 + c*x^4])/(2*b*x^3) - ((2*b*B - A*c)*ArcTanh[(Sqrt[b]*x)/Sqr
t[b*x^2 + c*x^4]])/(2*b^(3/2))
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
```

```
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^2}{x^2\sqrt{bx^2+cx^4}}\,dx &= -\frac{A\sqrt{bx^2+cx^4}}{2bx^3}-\frac{(-2bB+Ac)\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{2b}\\
&= -\frac{A\sqrt{bx^2+cx^4}}{2bx^3}+\frac{(-2bB+Ac)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2+cx^4}}\right)}{2b}\\
&= -\frac{A\sqrt{bx^2+cx^4}}{2bx^3}-\frac{(2bB-Ac)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0524825, size = 87, normalized size = 1.28

$$\frac{x\sqrt{b+cx^2}\left(-\frac{2\left(bB-\frac{Ac}{2}\right)\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)}{b^{3/2}}-\frac{A\sqrt{b+cx^2}}{bx^2}\right)}{2\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^2*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (x*Sqrt[b + c*x^2]*(-((A*Sqrt[b + c*x^2])/(b*x^2)) - (2*(b*B - (A*c)/2)*Arc
Tanh[Sqrt[b + c*x^2]/Sqrt[b]])/b^(3/2)))/(2*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.009, size = 105, normalized size = 1.5

$$-\frac{1}{2x}\sqrt{cx^2+b}\left(2B\ln\left(2\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)x^2b^2-A\ln\left(2\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)x^2bc+Ab^{\frac{3}{2}}\sqrt{cx^2+b}\right)\frac{1}{\sqrt{cx^4+bx^2}}b^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^2/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/2/x*(c*x^2+b)^(1/2)*(2*B*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*x^2*b^2-A*l
n(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*x^2*b*c+A*b^(3/2)*(c*x^2+b)^(1/2))/(c*x^
4+b*x^2)^(1/2)/b^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\sqrt{cx^4 + bx^2}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^2), x)
```

Fricas [A] time = 1.06317, size = 344, normalized size = 5.06

$$\left[-\frac{(2\,Bb - Ac)\sqrt{b}x^3\log\left(-\frac{cx^3+2\,bx+2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right) + 2\,\sqrt{cx^4 + bx^2}Ab}{4\,b^2x^3},\ \frac{(2\,Bb - Ac)\sqrt{-b}x^3\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-b}}{cx^3+bx}\right) - \sqrt{cx^4 + bx^2}}{2\,b^2x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((2*B*b - A*c)*sqrt(b)*x^3*log(-(c*x^3 + 2*b*x + 2*sqrt(c*x^4 + b*x^2
)*sqrt(b))/x^3) + 2*sqrt(c*x^4 + b*x^2)*A*b)/(b^2*x^3), 1/2*((2*B*b - A*c)*
sqrt(-b)*x^3*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) - sqrt(c*x^
4 + b*x^2)*A*b)/(b^2*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + Bx^2}{x^2 \sqrt{x^2 \left(b + cx^2\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**2/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**2*sqrt(x**2*(b + c*x**2))), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.144 $\int \frac{A+Bx^2}{x^4\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=103

$$-\frac{\sqrt{bx^2+cx^4}(4bB-3Ac)}{8b^2x^3}+\frac{c(4bB-3Ac)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8b^{5/2}}-\frac{A\sqrt{bx^2+cx^4}}{4bx^5}$$

```
[Out] -(A*Sqrt[b*x^2 + c*x^4])/(4*b*x^5) - ((4*b*B - 3*A*c)*Sqrt[b*x^2 + c*x^4])/
(8*b^2*x^3) + (c*(4*b*B - 3*A*c)*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/
(8*b^(5/2))
```

Rubi [A] time = 0.165402, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2038, 2025, 2008, 206}

$$-\frac{\sqrt{bx^2+cx^4}(4bB-3Ac)}{8b^2x^3}+\frac{c(4bB-3Ac)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8b^{5/2}}-\frac{A\sqrt{bx^2+cx^4}}{4bx^5}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^4*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -(A*Sqrt[b*x^2 + c*x^4])/(4*b*x^5) - ((4*b*B - 3*A*c)*Sqrt[b*x^2 + c*x^4])/
(8*b^2*x^3) + (c*(4*b*B - 3*A*c)*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/
(8*b^(5/2))
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^2}{x^4\sqrt{bx^2+cx^4}}\,dx &= -\frac{A\sqrt{bx^2+cx^4}}{4bx^5}-\frac{(-4bB+3Ac)\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx}{4b}\\
&= -\frac{A\sqrt{bx^2+cx^4}}{4bx^5}-\frac{(4bB-3Ac)\sqrt{bx^2+cx^4}}{8b^2x^3}-\frac{(c(4bB-3Ac))\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{8b^2}\\
&= -\frac{A\sqrt{bx^2+cx^4}}{4bx^5}-\frac{(4bB-3Ac)\sqrt{bx^2+cx^4}}{8b^2x^3}+\frac{(c(4bB-3Ac))\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2+cx^4}}\right)}{8b^2}\\
&= -\frac{A\sqrt{bx^2+cx^4}}{4bx^5}-\frac{(4bB-3Ac)\sqrt{bx^2+cx^4}}{8b^2x^3}+\frac{c(4bB-3Ac)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8b^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.149729, size = 104, normalized size = 1.01

$$-\frac{\sqrt{x^2\left(b+cx^2\right)}\left(b\sqrt{\frac{cx^2}{b}+1}\left(2Ab-3Acx^2+4bBx^2\right)+cx^4(3Ac-4bB)\tanh^{-1}\left(\sqrt{\frac{cx^2}{b}+1}\right)\right)}{8b^3x^5\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^4*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -(Sqrt[x^2*(b + c*x^2)]*(b*(2*A*b + 4*b*B*x^2 - 3*A*c*x^2)*Sqrt[1 + (c*x^2)
/b] + c*(-4*b*B + 3*A*c)*x^4*ArcTanh[Sqrt[1 + (c*x^2)/b]]))/(8*b^3*x^5*Sqrt
[1 + (c*x^2)/b])
```

Maple [A] time = 0.01, size = 146, normalized size = 1.4

$$-\frac{1}{8\,x^3}\sqrt{cx^2+b}\left(3\,A\ln\left(2\,\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)x^4bc^2+4\,Bb^{5/2}\sqrt{cx^2+b}x^2-4\,B\ln\left(2\,\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)x^4b^2c-3\,Ab^{3/2}\sqrt{cx^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^4/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/8*(c*x^2+b)^(1/2)*(3*A*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*x^4*b*c^2+4*B
*b^(5/2)*(c*x^2+b)^(1/2)*x^2-4*B*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*x^4*b^
2*c-3*A*b^(3/2)*(c*x^2+b)^(1/2)*x^2*c+2*A*b^(5/2)*(c*x^2+b)^(1/2))/x^3/(c*x
^4+b*x^2)^(1/2)/b^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{Bx^2+A}{\sqrt{cx^4+bx^2}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^4), x)
```

Fricas [A] time = 1.42975, size = 450, normalized size = 4.37

$$\left[-\frac{\left(4\,Bbc-3\,Ac^2\right)\sqrt{b}x^5\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)+2\,\sqrt{cx^4+bx^2}\left(2\,Ab^2+\left(4\,Bb^2-3\,Abc\right)x^2\right)}{16\,b^3x^5},-\frac{\left(4\,Bbc-3\,Ac^2\right)\sqrt{-b}x^5}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((4*B*b*c - 3*A*c^2)*sqrt(b)*x^5*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4
+ b*x^2)*sqrt(b))/x^3) + 2*sqrt(c*x^4 + b*x^2)*(2*A*b^2 + (4*B*b^2 - 3*A*b*
c)*x^2))/(b^3*x^5), -1/8*((4*B*b*c - 3*A*c^2)*sqrt(-b)*x^5*arctan(sqrt(c*x^
4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + sqrt(c*x^4 + b*x^2)*(2*A*b^2 + (4*B*b^
2 - 3*A*b*c)*x^2))/(b^3*x^5)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + Bx^2}{x^4\sqrt{x^2\left(b + cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**4/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**4*sqrt(x**2*(b + c*x**2))), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.145 $\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=184

$$-\frac{5b^2(7bB-6Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{9/2}}+\frac{x^4\sqrt{bx^2+cx^4}(7bB-6Ac)}{6bc^2}-\frac{5x^2\sqrt{bx^2+cx^4}(7bB-6Ac)}{24c^3}+\frac{5b\sqrt{bx^2+cx^4}(7bB-6}{16c^4}$$

```
[Out] -(((b*B - A*c)*x^8)/(b*c*Sqrt[b*x^2 + c*x^4])) + (5*b*(7*b*B - 6*A*c)*Sqrt[
b*x^2 + c*x^4])/(16*c^4) - (5*(7*b*B - 6*A*c)*x^2*Sqrt[b*x^2 + c*x^4])/(24*
c^3) + ((7*b*B - 6*A*c)*x^4*Sqrt[b*x^2 + c*x^4])/(6*b*c^2) - (5*b^2*(7*b*B
- 6*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(16*c^(9/2))
```

Rubi [A] time = 0.33328, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2034, 788, 670, 640, 620, 206}

$$-\frac{5b^2(7bB-6Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{9/2}}+\frac{x^4\sqrt{bx^2+cx^4}(7bB-6Ac)}{6bc^2}-\frac{5x^2\sqrt{bx^2+cx^4}(7bB-6Ac)}{24c^3}+\frac{5b\sqrt{bx^2+cx^4}(7bB-6}{16c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^9*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(((b*B - A*c)*x^8)/(b*c*Sqrt[b*x^2 + c*x^4])) + (5*b*(7*b*B - 6*A*c)*Sqrt[
b*x^2 + c*x^4])/(16*c^4) - (5*(7*b*B - 6*A*c)*x^2*Sqrt[b*x^2 + c*x^4])/(24*
c^3) + ((7*b*B - 6*A*c)*x^4*Sqrt[b*x^2 + c*x^4])/(6*b*c^2) - (5*b^2*(7*b*B
- 6*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(16*c^(9/2))
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rule 788

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((g*(c*d - b*e) + c*e*f)*(d + e*x)^m*(a
 + b*x + c*x^2)^(p + 1))/(c*(p + 1)*(2*c*d - b*e)), x] - Dist[(e*(m*(g*(c*d
 - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g)))/(c*(p + 1)*(2*c*d - b*e)), Int
[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d,
e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ
[p, -1] && GtQ[m, 0]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^9\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^4(A+Bx)}{\left(bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)$$

$$= -\frac{(bB-Ac)x^8}{bc\sqrt{bx^2+cx^4}} + \frac{1}{2}\left(-\frac{6A}{b}+\frac{7B}{c}\right)\operatorname{Subst}\left(\int \frac{x^3}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)$$

$$= -\frac{(bB-Ac)x^8}{bc\sqrt{bx^2+cx^4}} + \frac{(7bB-6Ac)x^4\sqrt{bx^2+cx^4}}{6bc^2} - \frac{(5(7bB-6Ac))\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{12c^2}$$

$$= -\frac{(bB-Ac)x^8}{bc\sqrt{bx^2+cx^4}} - \frac{5(7bB-6Ac)x^2\sqrt{bx^2+cx^4}}{24c^3} + \frac{(7bB-6Ac)x^4\sqrt{bx^2+cx^4}}{6bc^2} + \frac{(5b(7bB-6Ac))\,\text{Su}}{}$$

$$= -\frac{(bB-Ac)x^8}{bc\sqrt{bx^2+cx^4}} + \frac{5b(7bB-6Ac)\sqrt{bx^2+cx^4}}{16c^4} - \frac{5(7bB-6Ac)x^2\sqrt{bx^2+cx^4}}{24c^3} + \frac{(7bB-6Ac)x^4\sqrt{bx^2}}{6bc^2}$$

$$= -\frac{(bB-Ac)x^8}{bc\sqrt{bx^2+cx^4}} + \frac{5b(7bB-6Ac)\sqrt{bx^2+cx^4}}{16c^4} - \frac{5(7bB-6Ac)x^2\sqrt{bx^2+cx^4}}{24c^3} + \frac{(7bB-6Ac)x^4\sqrt{bx^2}}{6bc^2}$$

$$= -\frac{(bB-Ac)x^8}{bc\sqrt{bx^2+cx^4}} + \frac{5b(7bB-6Ac)\sqrt{bx^2+cx^4}}{16c^4} - \frac{5(7bB-6Ac)x^2\sqrt{bx^2+cx^4}}{24c^3} + \frac{(7bB-6Ac)x^4\sqrt{bx^2}}{6bc^2}$$

Mathematica [A] time = 0.179336, size = 136, normalized size = 0.74

$$\frac{x\left(\sqrt{c}x\left(b^2\left(35Bcx^2-90Ac\right)-2bc^2x^2\left(15A+7Bx^2\right)+4c^3x^4\left(3A+2Bx^2\right)+105b^3B\right)-15b^{5/2}\sqrt{\frac{cx^2}{b}+1}(7bB-6Ac)\sinh^{-}\right.}{48c^{9/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^9*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(Sqrt[c]*x*(105*b^3*B + 4*c^3*x^4*(3*A + 2*B*x^2) - 2*b*c^2*x^2*(15*A +
7*B*x^2) + b^2*(-90*A*c + 35*B*c*x^2)) - 15*b^(5/2)*(7*b*B - 6*A*c)*Sqrt[1
+ (c*x^2)/b]*ArcSinh[(Sqrt[c]*x)/Sqrt[b]]))/(48*c^(9/2)*Sqrt[x^2*(b + c*x^2
)])
```

Maple [A] time = 0.015, size = 166, normalized size = 0.9

$$\frac{\left(cx^2+b\right)x^3}{48}\left(8\,Bc^{9/2}x^7+12\,Ac^{9/2}x^5-14\,Bc^{7/2}x^5b-30\,Ac^{7/2}x^3b+35\,Bc^{5/2}x^3b^2-90\,Ac^{5/2}xb^2+105\,Bc^{3/2}xb^3+90\,A\ln\left(x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/48*x^3*(c*x^2+b)*(8*B*c^(9/2)*x^7+12*A*c^(9/2)*x^5-14*B*c^(7/2)*x^5*b-30*
A*c^(7/2)*x^3*b+35*B*c^(5/2)*x^3*b^2-90*A*c^(5/2)*x*b^2+105*B*c^(3/2)*x*b^3
+90*A*ln(x*c^(1/2)+(c*x^2+b)^(1/2))*(c*x^2+b)^(1/2)*b^2*c^2-105*B*ln(x*c^(1
/2)+(c*x^2+b)^(1/2))*(c*x^2+b)^(1/2)*b^3*c)/(c*x^4+b*x^2)^(3/2)/c^(11/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.53562, size = 738, normalized size = 4.01

$$\left[-\frac{15\left(7\,Bb^4-6\,Ab^3c+\left(7\,Bb^3c-6\,Ab^2c^2\right)x^2\right)\sqrt{c}\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\left(8\,Bc^4x^6+105\,Bb^3c-90\,Ab^2c^2\right.}{96\left(c^6x^2+bc^5\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(15*(7*B*b^4 - 6*A*b^3*c + (7*B*b^3*c - 6*A*b^2*c^2)*x^2)*sqrt(c)*lo
g(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 2*(8*B*c^4*x^6 + 105*B*b^
3*c - 90*A*b^2*c^2 - 2*(7*B*b*c^3 - 6*A*c^4)*x^4 + 5*(7*B*b^2*c^2 - 6*A*b*c
^3)*x^2)*sqrt(c*x^4 + b*x^2))/(c^6*x^2 + b*c^5), 1/48*(15*(7*B*b^4 - 6*A*b^
3*c + (7*B*b^3*c - 6*A*b^2*c^2)*x^2)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sq
rt(-c)/(c*x^2 + b)) + (8*B*c^4*x^6 + 105*B*b^3*c - 90*A*b^2*c^2 - 2*(7*B*b*
c^3 - 6*A*c^4)*x^4 + 5*(7*B*b^2*c^2 - 6*A*b*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/
(c^6*x^2 + b*c^5)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^9 \left(A + Bx^2\right)}{\left(x^2 \left(b + cx^2\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**9*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2 + A\right)x^9}{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^9/(c*x^4 + b*x^2)^(3/2), x)
```

3.146 $\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=147

$$\frac{x^2\sqrt{bx^2+cx^4}(5bB-4Ac)}{4bc^2}-\frac{3\sqrt{bx^2+cx^4}(5bB-4Ac)}{8c^3}+\frac{3b(5bB-4Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{7/2}}-\frac{x^6(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

```
[Out] -(((b*B - A*c)*x^6)/(b*c*Sqrt[b*x^2 + c*x^4])) - (3*(5*b*B - 4*A*c)*Sqrt[b*
x^2 + c*x^4])/(8*c^3) + ((5*b*B - 4*A*c)*x^2*Sqrt[b*x^2 + c*x^4])/(4*b*c^2)
 + (3*b*(5*b*B - 4*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(8*c^(7
/2))
```

Rubi [A] time = 0.280451, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2034, 788, 670, 640, 620, 206}

$$\frac{x^2\sqrt{bx^2+cx^4}(5bB-4Ac)}{4bc^2}-\frac{3\sqrt{bx^2+cx^4}(5bB-4Ac)}{8c^3}+\frac{3b(5bB-4Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{7/2}}-\frac{x^6(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(((b*B - A*c)*x^6)/(b*c*Sqrt[b*x^2 + c*x^4])) - (3*(5*b*B - 4*A*c)*Sqrt[b*
x^2 + c*x^4])/(8*c^3) + ((5*b*B - 4*A*c)*x^2*Sqrt[b*x^2 + c*x^4])/(4*b*c^2)
 + (3*b*(5*b*B - 4*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(8*c^(7
/2))
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rule 788

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((g*(c*d - b*e) + c*e*f)*(d + e*x)^m*(a
 + b*x + c*x^2)^(p + 1))/(c*(p + 1)*(2*c*d - b*e)), x] - Dist[(e*(m*(g*(c*d
 - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g)))/(c*(p + 1)*(2*c*d - b*e)), Int
[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d,
e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ
[p, -1] && GtQ[m, 0]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^7 \left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^3(A+Bx)}{\left(bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)$$
$$= -\frac{(bB-Ac)x^6}{bc\sqrt{bx^2+cx^4}} + \frac{1}{2}\left(-\frac{4A}{b}+\frac{5B}{c}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)$$
$$= -\frac{(bB-Ac)x^6}{bc\sqrt{bx^2+cx^4}} + \frac{(5bB-4Ac)x^2\sqrt{bx^2+cx^4}}{4bc^2} - \frac{(3(5bB-4Ac))\,\text{Subst}\left(\int \frac{x}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{8c^2}$$
$$= -\frac{(bB-Ac)x^6}{bc\sqrt{bx^2+cx^4}} - \frac{3(5bB-4Ac)\sqrt{bx^2+cx^4}}{8c^3} + \frac{(5bB-4Ac)x^2\sqrt{bx^2+cx^4}}{4bc^2} + \frac{(3b(5bB-4Ac))\,\text{Su}}{}$$
$$= -\frac{(bB-Ac)x^6}{bc\sqrt{bx^2+cx^4}} - \frac{3(5bB-4Ac)\sqrt{bx^2+cx^4}}{8c^3} + \frac{(5bB-4Ac)x^2\sqrt{bx^2+cx^4}}{4bc^2} + \frac{(3b(5bB-4Ac))\,\text{Su}}{}$$
$$= -\frac{(bB-Ac)x^6}{bc\sqrt{bx^2+cx^4}} - \frac{3(5bB-4Ac)\sqrt{bx^2+cx^4}}{8c^3} + \frac{(5bB-4Ac)x^2\sqrt{bx^2+cx^4}}{4bc^2} + \frac{3b(5bB-4Ac)\tanh}{8c^{7/2}}$$

Mathematica [A] time = 0.136921, size = 113, normalized size = 0.77

$$\frac{x\left(\sqrt{c}x\left(bc\left(12A-5Bx^2\right)+2c^2x^2\left(2A+Bx^2\right)-15b^2B\right)+3b^{3/2}\sqrt{\frac{cx^2}{b}+1}(5bB-4Ac)\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)\right)}{8c^{7/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(Sqrt[c]*x*(-15*b^2*B + b*c*(12*A - 5*B*x^2) + 2*c^2*x^2*(2*A + B*x^2))
+ 3*b^(3/2)*(5*b*B - 4*A*c)*Sqrt[1 + (c*x^2)/b]*ArcSinh[(Sqrt[c]*x)/Sqrt[b]
]))/(8*c^(7/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.009, size = 140, normalized size = 1.

$$\frac{\left(cx^2+b\right)x^3}{8}\left(2\,Bc^{7/2}x^5+4\,Ac^{7/2}x^3-5\,Bc^{5/2}x^3b+12\,Ac^{5/2}xb-15\,Bc^{3/2}xb^2-12\,A\ln\left(x\sqrt{c}+\sqrt{cx^2+b}\right)\sqrt{cx^2+b}bc^2+1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/8*x^3*(c*x^2+b)*(2*B*c^(7/2)*x^5+4*A*c^(7/2)*x^3-5*B*c^(5/2)*x^3*b+12*A*c
^(5/2)*x*b-15*B*c^(3/2)*x*b^2-12*A*ln(x*c^(1/2)+(c*x^2+b)^(1/2))*(c*x^2+b)^
(1/2)*b*c^2+15*B*ln(x*c^(1/2)+(c*x^2+b)^(1/2))*(c*x^2+b)^(1/2)*b^2*c)/(c*x^
4+b*x^2)^(3/2)/c^(9/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.48536, size = 625, normalized size = 4.25

$$\left[-\frac{3\left(5\,Bb^3-4\,Ab^2c+\left(5\,Bb^2c-4\,Abc^2\right)x^2\right)\sqrt{c}\log\left(-2\,cx^2-b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\left(2\,Bc^3x^4-15\,Bb^2c+12\,Abc^2-\left(5\right.\right.}{16\left(c^5x^2+bc^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(3*(5*B*b^3 - 4*A*b^2*c + (5*B*b^2*c - 4*A*b*c^2)*x^2)*sqrt(c)*log(-
2*c*x^2 - b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 2*(2*B*c^3*x^4 - 15*B*b^2*c
+ 12*A*b*c^2 - (5*B*b*c^2 - 4*A*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/(c^5*x^2 + b
*c^4), -1/8*(3*(5*B*b^3 - 4*A*b^2*c + (5*B*b^2*c - 4*A*b*c^2)*x^2)*sqrt(-c)
*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) - (2*B*c^3*x^4 - 15*B*b^2
*c + 12*A*b*c^2 - (5*B*b*c^2 - 4*A*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/(c^5*x^2
+ b*c^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7 \left(A + Bx^2\right)}{\left(x^2 \left(b + cx^2\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**7*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2 + A\right)x^7}{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^7/(c*x^4 + b*x^2)^(3/2), x)
```

3.147 $\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^{3/2}} \, dx$

Optimal. Leaf size=112

$$\frac{\sqrt{bx^2+cx^4}(3bB-2Ac)}{2bc^2}-\frac{(3bB-2Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{5/2}}-\frac{x^4(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

```
[Out] -(((b*B - A*c)*x^4)/(b*c*Sqrt[b*x^2 + c*x^4])) + ((3*b*B - 2*A*c)*Sqrt[b*x^
2 + c*x^4])/(2*b*c^2) - ((3*b*B - 2*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 +
 c*x^4]])/(2*c^(5/2))
```

Rubi [A] time = 0.241677, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2034, 788, 640, 620, 206}

$$\frac{\sqrt{bx^2+cx^4}(3bB-2Ac)}{2bc^2}-\frac{(3bB-2Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{5/2}}-\frac{x^4(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(((b*B - A*c)*x^4)/(b*c*Sqrt[b*x^2 + c*x^4])) + ((3*b*B - 2*A*c)*Sqrt[b*x^
2 + c*x^4])/(2*b*c^2) - ((3*b*B - 2*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 +
 c*x^4]])/(2*c^(5/2))
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rule 788

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((g*(c*d - b*e) + c*e*f)*(d + e*x)^m*(a
```

```
 + b*x + c*x^2)^(p + 1))/(c*(p + 1)*(2*c*d - b*e)), x] - Dist[(e*(m*(g*(c*d
 - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g)))/(c*(p + 1)*(2*c*d - b*e)), Int
[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d,
e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ
[p, -1] && GtQ[m, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^{3/2}}\,dx &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{x^2(A+Bx)}{\left(bx+cx^2\right)^{3/2}}\,dx,x,x^2\right)\\
&= -\frac{(bB-Ac)x^4}{bc\sqrt{bx^2+cx^4}}+\frac{1}{2}\left(-\frac{2A}{b}+\frac{3B}{c}\right)\mathrm{Subst}\left(\int \frac{x}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)\\
&= -\frac{(bB-Ac)x^4}{bc\sqrt{bx^2+cx^4}}+\frac{(3bB-2Ac)\sqrt{bx^2+cx^4}}{2bc^2}-\frac{(3bB-2Ac)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{4c^2}\\
&= -\frac{(bB-Ac)x^4}{bc\sqrt{bx^2+cx^4}}+\frac{(3bB-2Ac)\sqrt{bx^2+cx^4}}{2bc^2}-\frac{(3bB-2Ac)\,\mathrm{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^2}\\
&= -\frac{(bB-Ac)x^4}{bc\sqrt{bx^2+cx^4}}+\frac{(3bB-2Ac)\sqrt{bx^2+cx^4}}{2bc^2}-\frac{(3bB-2Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.112973, size = 91, normalized size = 0.81

$$\frac{x\left(\sqrt{c}x\left(-2Ac+3bB+Bcx^2\right)-\sqrt{b}\sqrt{\frac{cx^2}{b}+1}(3bB-2Ac)\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)\right)}{2c^{5/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(Sqrt[c]*x*(3*b*B - 2*A*c + B*c*x^2) - Sqrt[b]*(3*b*B - 2*A*c)*Sqrt[1 +
(c*x^2)/b]*ArcSinh[(Sqrt[c]*x)/Sqrt[b]]))/(2*c^(5/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.008, size = 115, normalized size = 1.

$$-\frac{\left(cx^2+b\right)x^3}{2}\left(-Bc^{\frac{5}{2}}x^3+2\,Ac^{5/2}x-3\,Bc^{3/2}xb-2\,A\ln\left(x\sqrt{c}+\sqrt{cx^2+b}\right)\sqrt{cx^2+b}c^2+3\,B\ln\left(x\sqrt{c}+\sqrt{cx^2+b}\right)\sqrt{cx^2+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/2*x^3*(c*x^2+b)*(-B*c^(5/2)*x^3+2*A*c^(5/2)*x-3*B*c^(3/2)*x*b-2*A*ln(x*c
^(1/2)+(c*x^2+b)^(1/2))*(c*x^2+b)^(1/2)*c^2+3*B*ln(x*c^(1/2)+(c*x^2+b)^(1/2
))*(c*x^2+b)^(1/2)*b*c)/(c*x^4+b*x^2)^(3/2)/c^(7/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.42984, size = 504, normalized size = 4.5

$$\left[-\frac{\left(3\,Bb^2-2\,Abc+\left(3\,Bbc-2\,Ac^2\right)x^2\right)\sqrt{c}\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\left(Bc^2x^2+3\,Bbc-2\,Ac^2\right)\sqrt{cx^4+bx^2}}{4\left(c^4x^2+bc^3\right)},-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((3*B*b^2 - 2*A*b*c + (3*B*b*c - 2*A*c^2)*x^2)*sqrt(c)*log(-2*c*x^2 -
 b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 2*(B*c^2*x^2 + 3*B*b*c - 2*A*c^2)*sqr
t(c*x^4 + b*x^2))/(c^4*x^2 + b*c^3), 1/2*((3*B*b^2 - 2*A*b*c + (3*B*b*c - 2
*A*c^2)*x^2)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) + (B
*c^2*x^2 + 3*B*b*c - 2*A*c^2)*sqrt(c*x^4 + b*x^2))/(c^4*x^2 + b*c^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5\left(A+Bx^2\right)}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**5*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2+A\right)x^5}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^5/(c*x^4 + b*x^2)^(3/2), x)
```

3.148 $\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=67

$$\frac{B\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{c^{3/2}}-\frac{x^2(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

```
[Out] -(((b*B - A*c)*x^2)/(b*c*Sqrt[b*x^2 + c*x^4])) + (B*ArcTanh[(Sqrt[c]*x^2)/S
qrt[b*x^2 + c*x^4]])/c^(3/2)
```

Rubi [A] time = 0.176754, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2034, 777, 620, 206}

$$\frac{B\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{c^{3/2}}-\frac{x^2(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(((b*B - A*c)*x^2)/(b*c*Sqrt[b*x^2 + c*x^4])) + (B*ArcTanh[(Sqrt[c]*x^2)/S
qrt[b*x^2 + c*x^4]])/c^(3/2)
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rule 777

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x)*(a + b*x + c*x^2)^(p +
1))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(
```

```
2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^{3/2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x(A+Bx)}{\left(bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)\\
&= -\frac{(bB-Ac)x^2}{bc\sqrt{bx^2+cx^4}}+\frac{B\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{2c}\\
&= -\frac{(bB-Ac)x^2}{bc\sqrt{bx^2+cx^4}}+\frac{B\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{c}\\
&= -\frac{(bB-Ac)x^2}{bc\sqrt{bx^2+cx^4}}+\frac{B\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{c^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0697558, size = 75, normalized size = 1.12

$$\frac{x\left(\sqrt{c}x(Ac-bB)+b^{3/2}B\sqrt{\frac{cx^2}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)\right)}{bc^{3/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(Sqrt[c]*(-(b*B) + A*c)*x + b^(3/2)*B*Sqrt[1 + (c*x^2)/b]*ArcSinh[(Sqrt[
c]*x)/Sqrt[b]]))/(b*c^(3/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.005, size = 75, normalized size = 1.1

$$\frac{(cx^2+b)\,x^3}{b}\left(Ac^{\frac{5}{2}}x - Bc^{\frac{3}{2}}xb + B\ln\left(x\sqrt{c}+\sqrt{cx^2+b}\right)\sqrt{cx^2+b}bc\right)\left(cx^4+bx^2\right)^{-\frac{3}{2}}c^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] x^3*(c*x^2+b)*(A*c^(5/2)*x-B*c^(3/2)*x*b+B*ln(x*c^(1/2)+(c*x^2+b)^(1/2))*(c
*x^2+b)^(1/2)*b*c)/(c*x^4+b*x^2)^(3/2)/c^(5/2)/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.35979, size = 396, normalized size = 5.91

$$\left[\frac{(Bbcx^2+Bb^2)\sqrt{c}\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\,\sqrt{cx^4+bx^2}(Bbc-Ac^2)}{2\,(bc^3x^2+b^2c^2)}, -\frac{(Bbcx^2+Bb^2)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2}}{cx^2+b}\right.}{bc^3x^2+b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((B*b*c*x^2 + B*b^2)*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*
sqrt(c)) - 2*sqrt(c*x^4 + b*x^2)*(B*b*c - A*c^2))/(b*c^3*x^2 + b^2*c^2), -(
```

```
(B*b*c*x^2 + B*b^2)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b
)) + sqrt(c*x^4 + b*x^2)*(B*b*c - A*c^2))/(b*c^3*x^2 + b^2*c^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \left(A + Bx^2\right)}{\left(x^2 \left(b + cx^2\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**3*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.149 $\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=37

$$-\frac{Ab-x^2(bB-2Ac)}{b^2\sqrt{bx^2+cx^4}}$$

```
[Out] -((A*b - (b*B - 2*A*c)*x^2)/(b^2*Sqrt[b*x^2 + c*x^4]))
```

Rubi [A] time = 0.121296, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2034, 636}

$$-\frac{Ab-x^2(bB-2Ac)}{b^2\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]

[Out] -((A*b - (b*B - 2*A*c)*x^2)/(b^2*Sqrt[b*x^2 + c*x^4]))
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{A+Bx}{\left(bx+cx^2\right)^{3/2}}\,dx,x,x^2\right)$$
$$= -\frac{Ab-(bB-2Ac)x^2}{b^2\sqrt{bx^2+cx^4}}$$

Mathematica [A] time = 0.0187079, size = 37, normalized size = 1.

$$\frac{bBx^2-A\left(b+2cx^2\right)}{b^2\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (b*B*x^2 - A*(b + 2*c*x^2))/(b^2*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.004, size = 47, normalized size = 1.3

$$-\frac{\left(cx^2+b\right)x^2\left(2\,Ax^2c-Bx^2b+Ab\right)}{b^2}\left(cx^4+bx^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -(c*x^2+b)*x^2*(2*A*c*x^2-B*b*x^2+A*b)/b^2/(c*x^4+b*x^2)^(3/2)
```

Maxima [A] time = 1.1736, size = 88, normalized size = 2.38

$$-A\left(\frac{2\,cx^2}{\sqrt{cx^4+bx^2}b^2}+\frac{1}{\sqrt{cx^4+bx^2}b}\right)+\frac{Bx^2}{\sqrt{cx^4+bx^2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -A*(2*c*x^2/(sqrt(c*x^4 + b*x^2)*b^2) + 1/(sqrt(c*x^4 + b*x^2)*b)) + B*x^2/
(sqrt(c*x^4 + b*x^2)*b)
```

Fricas [A] time = 1.32803, size = 93, normalized size = 2.51

$$\frac{\sqrt{cx^4 + bx^2}\left((Bb - 2\,Ac)x^2 - Ab\right)}{b^2cx^4 + b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^4 + b*x^2)*((B*b - 2*A*c)*x^2 - A*b)/(b^2*c*x^4 + b^3*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(A + Bx^2\right)}{\left(x^2\left(b + cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

Giac [A] time = 1.18676, size = 49, normalized size = 1.32

$$\frac{\frac{(Bb - 2\,Ac)x^2}{b^2} - \frac{A}{b}}{\sqrt{cx^4 + bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] ((B*b - 2*A*c)*x^2/b^2 - A/b)/sqrt(c*x^4 + b*x^2)
```

3.150 $\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{\left(b+2cx^2\right)(3bB-4Ac)}{3b^3\sqrt{bx^2+cx^4}}-\frac{A}{3bx^2\sqrt{bx^2+cx^4}}$$

```
[Out] -A/(3*b*x^2*Sqrt[b*x^2 + c*x^4]) - ((3*b*B - 4*A*c)*(b + 2*c*x^2))/(3*b^3*S
qrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.163637, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2034, 792, 613}

$$-\frac{\left(b+2cx^2\right)(3bB-4Ac)}{3b^3\sqrt{bx^2+cx^4}}-\frac{A}{3bx^2\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] -A/(3*b*x^2*Sqrt[b*x^2 + c*x^4]) - ((3*b*B - 4*A*c)*(b + 2*c*x^2))/(3*b^3*S
qrt[b*x^2 + c*x^4])
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
```

```
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)^{3/2}}\,dx &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{A+Bx}{x\left(bx+cx^2\right)^{3/2}}\,dx,x,x^2\right)\\ &= -\frac{A}{3bx^2\sqrt{bx^2+cx^4}}+\frac{\left(bB-Ac+\frac{1}{2}(bB-2Ac)\right)\mathrm{Subst}\left(\int \frac{1}{\left(bx+cx^2\right)^{3/2}}\,dx,x,x^2\right)}{3b}\\ &= -\frac{A}{3bx^2\sqrt{bx^2+cx^4}}-\frac{(3bB-4Ac)\left(b+2cx^2\right)}{3b^3\sqrt{bx^2+cx^4}}\end{aligned}$$

Mathematica [A] time = 0.0224842, size = 64, normalized size = 0.97

$$\frac{A\left(-b^2+4bcx^2+8c^2x^4\right)-3bBx^2\left(b+2cx^2\right)}{3b^3x^2\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-3*b*B*x^2*(b + 2*c*x^2) + A*(-b^2 + 4*b*c*x^2 + 8*c^2*x^4))/(3*b^3*x^2*Sq
rt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.005, size = 66, normalized size = 1.

$$-\frac{\left(cx^2+b\right)\left(-8\,Ac^2x^4+6\,Bx^4bc-4\,Abcx^2+3\,Bx^2b^2+Ab^2\right)}{3\,b^3}\left(cx^4+bx^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/3*(c*x^2+b)*(-8*A*c^2*x^4+6*B*b*c*x^4-4*A*b*c*x^2+3*B*b^2*x^2+A*b^2)/b^3
/(c*x^4+b*x^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.25112, size = 149, normalized size = 2.26

$$-\frac{\left(2\left(3\,Bbc-4\,Ac^2\right)x^4+Ab^2+\left(3\,Bb^2-4\,Abc\right)x^2\right)\sqrt{cx^4+bx^2}}{3\left(b^3cx^6+b^4x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*(3*B*b*c - 4*A*c^2)*x^4 + A*b^2 + (3*B*b^2 - 4*A*b*c)*x^2)*sqrt(c*x
^4 + b*x^2)/(b^3*c*x^6 + b^4*x^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A+Bx^2}{x\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral((A + B*x**2)/(x*(x**2*(b + c*x**2))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\left(cx^4 + bx^2\right)^{\frac{3}{2}} x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/((c*x^4 + b*x^2)^(3/2)*x), x)
```

3.151 $\int \frac{A+Bx^2}{x^3\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=101

$$\frac{4c\left(b+2cx^2\right)(5bB-6Ac)}{15b^4\sqrt{bx^2+cx^4}}-\frac{5bB-6Ac}{15b^2x^2\sqrt{bx^2+cx^4}}-\frac{A}{5bx^4\sqrt{bx^2+cx^4}}$$

```
[Out] -A/(5*b*x^4*Sqrt[b*x^2 + c*x^4]) - (5*b*B - 6*A*c)/(15*b^2*x^2*Sqrt[b*x^2 +
 c*x^4]) + (4*c*(5*b*B - 6*A*c)*(b + 2*c*x^2))/(15*b^4*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.220201, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2034, 792, 658, 613}

$$\frac{4c\left(b+2cx^2\right)(5bB-6Ac)}{15b^4\sqrt{bx^2+cx^4}}-\frac{5bB-6Ac}{15b^2x^2\sqrt{bx^2+cx^4}}-\frac{A}{5bx^4\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^3*(b*x^2 + c*x^4)^(3/2)),x]
```


```
[Out] -A/(5*b*x^4*Sqrt[b*x^2 + c*x^4]) - (5*b*B - 6*A*c)/(15*b^2*x^2*Sqrt[b*x^2 +
 c*x^4]) + (4*c*(5*b*B - 6*A*c)*(b + 2*c*x^2))/(15*b^4*Sqrt[b*x^2 + c*x^4])
```


Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
```

```
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^2}{x^3\left(bx^2+cx^4\right)^{3/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{x^2\left(bx+cx^2\right)^{3/2}}\,dx,x,x^2\right)\\
&= -\frac{A}{5bx^4\sqrt{bx^2+cx^4}}+\frac{\left(\frac{1}{2}(bB-2Ac)-2(-bB+Ac)\right)\operatorname{Subst}\left(\int \frac{1}{x\left(bx+cx^2\right)^{3/2}}\,dx,x,x^2\right)}{5b}\\
&= -\frac{A}{5bx^4\sqrt{bx^2+cx^4}}-\frac{5bB-6Ac}{15b^2x^2\sqrt{bx^2+cx^4}}-\frac{(2c(5bB-6Ac))\operatorname{Subst}\left(\int \frac{1}{\left(bx+cx^2\right)^{3/2}}\,dx,x,x^2\right)}{15b^2}\\
&= -\frac{A}{5bx^4\sqrt{bx^2+cx^4}}-\frac{5bB-6Ac}{15b^2x^2\sqrt{bx^2+cx^4}}+\frac{4c(5bB-6Ac)\left(b+2cx^2\right)}{15b^4\sqrt{bx^2+cx^4}}
\end{aligned}$$

Mathematica [A] time = 0.0268884, size = 85, normalized size = 0.84

$$\frac{-3A\left(-2b^2cx^2+b^3+8bc^2x^4+16c^3x^6\right)-5bBx^2\left(b^2-4bcx^2-8c^2x^4\right)}{15b^4x^4\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^3*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-5*b*B*x^2*(b^2 - 4*b*c*x^2 - 8*c^2*x^4) - 3*A*(b^3 - 2*b^2*c*x^2 + 8*b*c^
2*x^4 + 16*c^3*x^6))/(15*b^4*x^4*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.006, size = 94, normalized size = 0.9

$$-\frac{(cx^2+b)\left(48\,Ac^3x^6-40\,Bx^6bc^2+24\,Abc^2x^4-20\,Bx^4b^2c-6\,Ab^2cx^2+5\,Bx^2b^3+3\,Ab^3\right)}{15\,x^2b^4}\left(cx^4+bx^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^3/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/15*(c*x^2+b)*(48*A*c^3*x^6-40*B*b*c^2*x^6+24*A*b*c^2*x^4-20*B*b^2*c*x^4-
6*A*b^2*c*x^2+5*B*b^3*x^2+3*A*b^3)/x^2/b^4/(c*x^4+b*x^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.3184, size = 200, normalized size = 1.98

$$\frac{\left(8\left(5\,Bbc^2-6\,Ac^3\right)x^6+4\left(5\,Bb^2c-6\,Abc^2\right)x^4-3\,Ab^3-\left(5\,Bb^3-6\,Ab^2c\right)x^2\right)\sqrt{cx^4+bx^2}}{15\left(b^4cx^8+b^5x^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(8*(5*B*b*c^2 - 6*A*c^3)*x^6 + 4*(5*B*b^2*c - 6*A*b*c^2)*x^4 - 3*A*b^3
 - (5*B*b^3 - 6*A*b^2*c)*x^2)*sqrt(c*x^4 + b*x^2)/(b^4*c*x^8 + b^5*x^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + Bx^2}{x^3 \left(x^2 \left(b + cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**3/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**3*(x**2*(b + c*x**2))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\left(cx^4 + bx^2\right)^{\frac{3}{2}} x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/((c*x^4 + b*x^2)^(3/2)*x^3), x)
```

3.152 $\int \frac{A+Bx^2}{x^5\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=138

$$-\frac{8c^2\left(b+2cx^2\right)(7bB-8Ac)}{35b^5\sqrt{bx^2+cx^4}}+\frac{2c(7bB-8Ac)}{35b^3x^2\sqrt{bx^2+cx^4}}-\frac{7bB-8Ac}{35b^2x^4\sqrt{bx^2+cx^4}}-\frac{A}{7bx^6\sqrt{bx^2+cx^4}}$$

```
[Out] -A/(7*b*x^6*Sqrt[b*x^2 + c*x^4]) - (7*b*B - 8*A*c)/(35*b^2*x^4*Sqrt[b*x^2 +
 c*x^4]) + (2*c*(7*b*B - 8*A*c))/(35*b^3*x^2*Sqrt[b*x^2 + c*x^4]) - (8*c^2*
(7*b*B - 8*A*c)*(b + 2*c*x^2))/(35*b^5*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.266861, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2034, 792, 658, 613}

$$-\frac{8c^2\left(b+2cx^2\right)(7bB-8Ac)}{35b^5\sqrt{bx^2+cx^4}}+\frac{2c(7bB-8Ac)}{35b^3x^2\sqrt{bx^2+cx^4}}-\frac{7bB-8Ac}{35b^2x^4\sqrt{bx^2+cx^4}}-\frac{A}{7bx^6\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^5*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] -A/(7*b*x^6*Sqrt[b*x^2 + c*x^4]) - (7*b*B - 8*A*c)/(35*b^2*x^4*Sqrt[b*x^2 +
 c*x^4]) + (2*c*(7*b*B - 8*A*c))/(35*b^3*x^2*Sqrt[b*x^2 + c*x^4]) - (8*c^2*
(7*b*B - 8*A*c)*(b + 2*c*x^2))/(35*b^5*Sqrt[b*x^2 + c*x^4])
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
```

```
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^2}{x^5\left(bx^2+cx^4\right)^{3/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{x^3\left(bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)\\
&= -\frac{A}{7bx^6\sqrt{bx^2+cx^4}}+\frac{\left(\frac{1}{2}(bB-2Ac)-3(-bB+Ac)\right)\operatorname{Subst}\left(\int \frac{1}{x^2\left(bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)}{7b}\\
&= -\frac{A}{7bx^6\sqrt{bx^2+cx^4}}-\frac{7bB-8Ac}{35b^2x^4\sqrt{bx^2+cx^4}}-\frac{(3c(7bB-8Ac))\operatorname{Subst}\left(\int \frac{1}{x\left(bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)}{35b^2}\\
&= -\frac{A}{7bx^6\sqrt{bx^2+cx^4}}-\frac{7bB-8Ac}{35b^2x^4\sqrt{bx^2+cx^4}}+\frac{2c(7bB-8Ac)}{35b^3x^2\sqrt{bx^2+cx^4}}+\frac{\left(4c^2(7bB-8Ac)\right)\operatorname{Subst}\left(\int \cdots\right.}{35b^3}\\
&= -\frac{A}{7bx^6\sqrt{bx^2+cx^4}}-\frac{7bB-8Ac}{35b^2x^4\sqrt{bx^2+cx^4}}+\frac{2c(7bB-8Ac)}{35b^3x^2\sqrt{bx^2+cx^4}}-\frac{8c^2(7bB-8Ac)\left(b+2cx^2\right)}{35b^5\sqrt{bx^2+cx^4}}
\end{aligned}
$$

Mathematica [A] time = 0.0340571, size = 75, normalized size = 0.54

$$\frac{x^2\left(-2b^2cx^2+b^3+8bc^2x^4+16c^3x^6\right)(8Ac-7bB)-5Ab^4}{35b^5x^6\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^5*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-5*A*b^4 + (-7*b*B + 8*A*c)*x^2*(b^3 - 2*b^2*c*x^2 + 8*b*c^2*x^4 + 16*c^3*
x^6))/(35*b^5*x^6*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.006, size = 118, normalized size = 0.9

$$-\frac{\left(cx^2+b\right)\left(-128\,Ac^4x^8+112\,Bbc^3x^8-64\,Abc^3x^6+56\,Bb^2c^2x^6+16\,Ab^2c^2x^4-14\,Bb^3cx^4-8\,Ab^3cx^2+7\,Bb^4x^2+5\,Ab^4\right.}{35\,x^4b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^5/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/35*(c*x^2+b)*(-128*A*c^4*x^8+112*B*b*c^3*x^8-64*A*b*c^3*x^6+56*B*b^2*c^2
*x^6+16*A*b^2*c^2*x^4-14*B*b^3*c*x^4-8*A*b^3*c*x^2+7*B*b^4*x^2+5*A*b^4)/x^4
/b^5/(c*x^4+b*x^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.5006, size = 252, normalized size = 1.83

$$-\frac{\left(16\left(7\,Bbc^{3}-8\,Ac^{4}\right)x^{8}+8\left(7\,Bb^{2}c^{2}-8\,Abc^{3}\right)x^{6}+5\,Ab^{4}-2\left(7\,Bb^{3}c-8\,Ab^{2}c^{2}\right)x^{4}+\left(7\,Bb^{4}-8\,Ab^{3}c\right)x^{2}\right)\sqrt{cx^{4}+bx^{2}}}{35\left(b^{5}cx^{10}+b^{6}x^{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/35*(16*(7*B*b*c^3 - 8*A*c^4)*x^8 + 8*(7*B*b^2*c^2 - 8*A*b*c^3)*x^6 + 5*A
*b^4 - 2*(7*B*b^3*c - 8*A*b^2*c^2)*x^4 + (7*B*b^4 - 8*A*b^3*c)*x^2)*sqrt(c*
x^4 + b*x^2)/(b^5*c*x^10 + b^6*x^8)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A+Bx^{2}}{x^{5}\left(x^{2}\left(b+cx^{2}\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**5/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**5*(x**2*(b + c*x**2))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^{2}+A}{\left(cx^{4}+bx^{2}\right)^{\frac{3}{2}}x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/((c*x^4 + b*x^2)^(3/2)*x^5), x)
```

3.153 $\int \frac{x^8\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=139

$$\frac{x^3\sqrt{bx^2+cx^4}(6bB-5Ac)}{5bc^2}-\frac{4x\sqrt{bx^2+cx^4}(6bB-5Ac)}{15c^3}+\frac{8b\sqrt{bx^2+cx^4}(6bB-5Ac)}{15c^4x}-\frac{x^7(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

```
[Out] -(((b*B - A*c)*x^7)/(b*c*Sqrt[b*x^2 + c*x^4])) + (8*b*(6*b*B - 5*A*c)*Sqrt[
b*x^2 + c*x^4])/(15*c^4*x) - (4*(6*b*B - 5*A*c)*x*Sqrt[b*x^2 + c*x^4])/(15*
c^3) + ((6*b*B - 5*A*c)*x^3*Sqrt[b*x^2 + c*x^4])/(5*b*c^2)
```

Rubi [A] time = 0.248238, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2037, 2016, 1588}

$$\frac{x^3\sqrt{bx^2+cx^4}(6bB-5Ac)}{5bc^2}-\frac{4x\sqrt{bx^2+cx^4}(6bB-5Ac)}{15c^3}+\frac{8b\sqrt{bx^2+cx^4}(6bB-5Ac)}{15c^4x}-\frac{x^7(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(((b*B - A*c)*x^7)/(b*c*Sqrt[b*x^2 + c*x^4])) + (8*b*(6*b*B - 5*A*c)*Sqrt[
b*x^2 + c*x^4])/(15*c^4*x) - (4*(6*b*B - 5*A*c)*x*Sqrt[b*x^2 + c*x^4])/(15*
c^3) + ((6*b*B - 5*A*c)*x^3*Sqrt[b*x^2 + c*x^4])/(5*b*c^2)
```

Rule 2037

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> -Simp[(e^(j - 1)*(b*c - a*d)*(e*x)^(m - j +
 1)*(a*x^j + b*x^(j + n))^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(e^j*(a*d*(m
+ j*p + 1) - b*c*(m + n + p*(j + n) + 1)))/(a*b*n*(p + 1)), Int[(e*x)^(m -
j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m, n},
 x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &
& GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
```

```
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^{3/2}}\,dx &= -\frac{(bB-Ac)x^7}{bc\sqrt{bx^2+cx^4}}+\frac{(6bB-5Ac)\int \frac{x^6}{\sqrt{bx^2+cx^4}}\,dx}{bc}\\
&= -\frac{(bB-Ac)x^7}{bc\sqrt{bx^2+cx^4}}+\frac{(6bB-5Ac)x^3\sqrt{bx^2+cx^4}}{5bc^2}-\frac{(4(6bB-5Ac))\int \frac{x^4}{\sqrt{bx^2+cx^4}}\,dx}{5c^2}\\
&= -\frac{(bB-Ac)x^7}{bc\sqrt{bx^2+cx^4}}-\frac{4(6bB-5Ac)x\sqrt{bx^2+cx^4}}{15c^3}+\frac{(6bB-5Ac)x^3\sqrt{bx^2+cx^4}}{5bc^2}+\frac{(8b(6bB-5Ac))\int}{15c^3}\\
&= -\frac{(bB-Ac)x^7}{bc\sqrt{bx^2+cx^4}}+\frac{8b(6bB-5Ac)\sqrt{bx^2+cx^4}}{15c^4x}-\frac{4(6bB-5Ac)x\sqrt{bx^2+cx^4}}{15c^3}+\frac{(6bB-5Ac)x^3\sqrt{bx}}{5bc^2}
\end{aligned}$$

Mathematica [A] time = 0.0508943, size = 82, normalized size = 0.59

$$\frac{x\left(-8b^2c\left(5A-3Bx^2\right)-2bc^2x^2\left(10A+3Bx^2\right)+c^3x^4\left(5A+3Bx^2\right)+48b^3B\right)}{15c^4\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(48*b^3*B - 8*b^2*c*(5*A - 3*B*x^2) + c^3*x^4*(5*A + 3*B*x^2) - 2*b*c^2*
x^2*(10*A + 3*B*x^2)))/(15*c^4*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.005, size = 91, normalized size = 0.7

$$-\frac{\left(cx^{2}+b\right)\left(-3\,Bc^{3}x^{6}-5\,Ax^{4}c^{3}+6\,Bx^{4}bc^{2}+20\,Ax^{2}bc^{2}-24\,Bx^{2}b^{2}c+40\,Ab^{2}c-48\,Bb^{3}\right)x^{3}}{15\,c^{4}}\left(cx^{4}+bx^{2}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/15*(c*x^2+b)*(-3*B*c^3*x^6-5*A*c^3*x^4+6*B*b*c^2*x^4+20*A*b*c^2*x^2-24*B
*b^2*c*x^2+40*A*b^2*c-48*B*b^3)*x^3/c^4/(c*x^4+b*x^2)^(3/2)
```

Maxima [A] time = 1.19335, size = 111, normalized size = 0.8

$$\frac{\left(c^{2}x^{4}-4\,bcx^{2}-8\,b^{2}\right)A}{3\,\sqrt{cx^{2}+b}c^{3}}+\frac{\left(c^{3}x^{6}-2\,bc^{2}x^{4}+8\,b^{2}cx^{2}+16\,b^{3}\right)B}{5\,\sqrt{cx^{2}+b}c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*(c^2*x^4 - 4*b*c*x^2 - 8*b^2)*A/(sqrt(c*x^2 + b)*c^3) + 1/5*(c^3*x^6 -
2*b*c^2*x^4 + 8*b^2*c*x^2 + 16*b^3)*B/(sqrt(c*x^2 + b)*c^4)
```

Fricas [A] time = 1.36381, size = 194, normalized size = 1.4

$$\frac{\left(3\,Bc^{3}x^{6}-\left(6\,Bbc^{2}-5\,Ac^{3}\right)x^{4}+48\,Bb^{3}-40\,Ab^{2}c+4\left(6\,Bb^{2}c-5\,Abc^{2}\right)x^{2}\right)\sqrt{cx^{4}+bx^{2}}}{15\left(c^{5}x^{3}+bc^{4}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*B*c^3*x^6 - (6*B*b*c^2 - 5*A*c^3)*x^4 + 48*B*b^3 - 40*A*b^2*c + 4*(
6*B*b^2*c - 5*A*b*c^2)*x^2)*sqrt(c*x^4 + b*x^2)/(c^5*x^3 + b*c^4*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^8 \left(A + Bx^2\right)}{\left(x^2 \left(b + cx^2\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**8*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2 + A\right)x^8}{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^8/(c*x^4 + b*x^2)^(3/2), x)
```

3.154 $\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=104

$$\frac{x\sqrt{bx^2+cx^4}(4bB-3Ac)}{3bc^2}-\frac{2\sqrt{bx^2+cx^4}(4bB-3Ac)}{3c^3x}-\frac{x^5(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

```
[Out] -(((b*B - A*c)*x^5)/(b*c*Sqrt[b*x^2 + c*x^4])) - (2*(4*b*B - 3*A*c)*Sqrt[b*
x^2 + c*x^4])/(3*c^3*x) + ((4*b*B - 3*A*c)*x*Sqrt[b*x^2 + c*x^4])/(3*b*c^2)
```

Rubi [A] time = 0.199874, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2037, 2016, 1588}

$$\frac{x\sqrt{bx^2+cx^4}(4bB-3Ac)}{3bc^2}-\frac{2\sqrt{bx^2+cx^4}(4bB-3Ac)}{3c^3x}-\frac{x^5(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```



```
[Out] -(((b*B - A*c)*x^5)/(b*c*Sqrt[b*x^2 + c*x^4])) - (2*(4*b*B - 3*A*c)*Sqrt[b*
x^2 + c*x^4])/(3*c^3*x) + ((4*b*B - 3*A*c)*x*Sqrt[b*x^2 + c*x^4])/(3*b*c^2)
```


Rule 2037

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> -Simp[(e^(j - 1)*(b*c - a*d)*(e*x)^(m - j +
 1)*(a*x^j + b*x^(j + n))^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(e^j*(a*d*(m
+ j*p + 1) - b*c*(m + n + p*(j + n) + 1)))/(a*b*n*(p + 1)), Int[(e*x)^(m -
j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m, n},
 x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &
& GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
```

```
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^6\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^{3/2}}\,dx &= -\frac{(bB-Ac)x^5}{bc\sqrt{bx^2+cx^4}}+\frac{(4bB-3Ac)\int \frac{x^4}{\sqrt{bx^2+cx^4}}\,dx}{bc}\\ &= -\frac{(bB-Ac)x^5}{bc\sqrt{bx^2+cx^4}}+\frac{(4bB-3Ac)x\sqrt{bx^2+cx^4}}{3bc^2}-\frac{(2(4bB-3Ac))\int \frac{x^2}{\sqrt{bx^2+cx^4}}\,dx}{3c^2}\\ &= -\frac{(bB-Ac)x^5}{bc\sqrt{bx^2+cx^4}}-\frac{2(4bB-3Ac)\sqrt{bx^2+cx^4}}{3c^3x}+\frac{(4bB-3Ac)x\sqrt{bx^2+cx^4}}{3bc^2}\end{aligned}$$

Mathematica [A] time = 0.0381198, size = 60, normalized size = 0.58

$$\frac{x\left(b\left(6Ac-4Bcx^2\right)+c^2x^2\left(3A+Bx^2\right)-8b^2B\right)}{3c^3\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(-8*b^2*B + c^2*x^2*(3*A + B*x^2) + b*(6*A*c - 4*B*c*x^2)))/(3*c^3*Sqrt[
x^2*(b + c*x^2)])
```

Maple [A] time = 0.005, size = 66, normalized size = 0.6

$$\frac{\left(cx^2+b\right)\left(Bc^2x^4+3\,Ax^2c^2-4\,Bx^2bc+6\,Abc-8\,Bb^2\right)x^3}{3\,c^3}\left(cx^4+bx^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/3*(c*x^2+b)*(B*c^2*x^4+3*A*c^2*x^2-4*B*b*c*x^2+6*A*b*c-8*B*b^2)*x^3/c^3/(
c*x^4+b*x^2)^(3/2)
```

Maxima [A] time = 1.1939, size = 80, normalized size = 0.77

$$\frac{\left(cx^2+2\,b\right)A}{\sqrt{cx^2+b}c^2}+\frac{\left(c^2x^4-4\,bcx^2-8\,b^2\right)B}{3\,\sqrt{cx^2+b}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] (c*x^2 + 2*b)*A/(sqrt(c*x^2 + b)*c^2) + 1/3*(c^2*x^4 - 4*b*c*x^2 - 8*b^2)*B
/(sqrt(c*x^2 + b)*c^3)
```

Fricas [A] time = 1.48939, size = 139, normalized size = 1.34

$$\frac{\left(Bc^2x^4-8\,Bb^2+6\,Abc-\left(4\,Bbc-3\,Ac^2\right)x^2\right)\sqrt{cx^4+bx^2}}{3\,\left(c^4x^3+bc^3x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(B*c^2*x^4 - 8*B*b^2 + 6*A*b*c - (4*B*b*c - 3*A*c^2)*x^2)*sqrt(c*x^4 +
b*x^2)/(c^4*x^3 + b*c^3*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6\left(A+Bx^2\right)}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**6*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)x^6}{(cx^4 + bx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^6/(c*x^4 + b*x^2)^(3/2), x)
```

3.155 $\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=69

$$\frac{\sqrt{bx^2+cx^4}(2bB-Ac)}{bc^2x}-\frac{x^3(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

```
[Out] -(((b*B - A*c)*x^3)/(b*c*Sqrt[b*x^2 + c*x^4])) + ((2*b*B - A*c)*Sqrt[b*x^2
+ c*x^4])/(b*c^2*x)
```

Rubi [A] time = 0.149797, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2037, 1588}

$$\frac{\sqrt{bx^2+cx^4}(2bB-Ac)}{bc^2x}-\frac{x^3(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(((b*B - A*c)*x^3)/(b*c*Sqrt[b*x^2 + c*x^4])) + ((2*b*B - A*c)*Sqrt[b*x^2
+ c*x^4])/(b*c^2*x)
```

Rule 2037

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> -Simp[(e^(j - 1)*(b*c - a*d)*(e*x)^(m - j +
 1)*(a*x^j + b*x^(j + n))^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(e^j*(a*d*(m
+ j*p + 1) - b*c*(m + n + p*(j + n) + 1)))/(a*b*n*(p + 1)), Int[(e*x)^(m -
j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m, n},
 x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &
& GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
```

```
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^{3/2}}\,dx&=-\frac{(bB-Ac)x^3}{bc\sqrt{bx^2+cx^4}}+\frac{(2bB-Ac)\int \frac{x^2}{\sqrt{bx^2+cx^4}}\,dx}{bc}\\&=-\frac{(bB-Ac)x^3}{bc\sqrt{bx^2+cx^4}}+\frac{(2bB-Ac)\sqrt{bx^2+cx^4}}{bc^2x}\end{aligned}$$

Mathematica [A] time = 0.0229948, size = 35, normalized size = 0.51

$$\frac{x\left(-Ac+2bB+Bcx^2\right)}{c^2\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(2*b*B - A*c + B*c*x^2))/(c^2*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.003, size = 44, normalized size = 0.6

$$-\frac{\left(cx^2+b\right)\left(-Bcx^2+Ac-2\,Bb\right)x^3}{c^2}\left(cx^4+bx^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -(c*x^2+b)*(-B*c*x^2+A*c-2*B*b)*x^3/c^2/(c*x^4+b*x^2)^(3/2)
```

Maxima [A] time = 1.21333, size = 53, normalized size = 0.77

$$\frac{\left(cx^2+2\,b\right)B}{\sqrt{cx^2+b}c^2}-\frac{A}{\sqrt{cx^2+b}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] (c*x^2 + 2*b)*B/(sqrt(c*x^2 + b)*c^2) - A/(sqrt(c*x^2 + b)*c)
```

Fricas [A] time = 1.27892, size = 88, normalized size = 1.28

$$\frac{\sqrt{cx^4 + bx^2}\left(Bcx^2 + 2\,Bb - Ac\right)}{c^3x^3 + bc^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^4 + b*x^2)*(B*c*x^2 + 2*B*b - A*c)/(c^3*x^3 + b*c^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4\left(A + Bx^2\right)}{\left(x^2\left(b + cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**4*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

Giac [A] time = 1.33152, size = 81, normalized size = 1.17

$$-\frac{2\,B\sqrt{b}}{\left(\left(\sqrt{c + \frac{b}{x^2}} - \frac{\sqrt{b}}{x}\right)^2 - c\right)c} + \frac{Bb - Ac}{\sqrt{c + \frac{b}{x^2}}c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -2*B*sqrt(b)/(((sqrt(c + b/x^2) - sqrt(b)/x)^2 - c)*c) + (B*b - A*c)/(sqrt(
c + b/x^2)*c^2*x)
```

3.156 $\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=64

$$-\frac{A\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{b^{3/2}}-\frac{x(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

```
[Out] -(((b*B - A*c)*x)/(b*c*Sqrt[b*x^2 + c*x^4])) - (A*ArcTanh[(Sqrt[b]*x)/Sqrt[
b*x^2 + c*x^4]])/b^(3/2)
```

Rubi [A] time = 0.138556, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2037, 2008, 206}

$$-\frac{A\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{b^{3/2}}-\frac{x(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(((b*B - A*c)*x)/(b*c*Sqrt[b*x^2 + c*x^4])) - (A*ArcTanh[(Sqrt[b]*x)/Sqrt[
b*x^2 + c*x^4]])/b^(3/2)
```

Rule 2037

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> -Simp[(e^(j - 1)*(b*c - a*d)*(e*x)^(m - j +
 1)*(a*x^j + b*x^(j + n))^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(e^j*(a*d*(m
+ j*p + 1) - b*c*(m + n + p*(j + n) + 1)))/(a*b*n*(p + 1)), Int[(e*x)^(m -
j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m, n},
 x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &
& GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
```

```
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^2\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^{3/2}}\,dx=-\frac{(bB-Ac)x}{bc\sqrt{bx^2+cx^4}}+\frac{A\int\frac{1}{\sqrt{bx^2+cx^4}}\,dx}{b}$$

$$=-\frac{(bB-Ac)x}{bc\sqrt{bx^2+cx^4}}-\frac{A\,\mathrm{Subst}\left(\int\frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2+cx^4}}\right)}{b}$$

$$=-\frac{(bB-Ac)x}{bc\sqrt{bx^2+cx^4}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{b^{3/2}}$$

Mathematica [A] time = 0.0309807, size = 73, normalized size = 1.14

$$-\frac{x\left(\sqrt{b}(bB-Ac)+Ac\sqrt{b+cx^2}\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)\right)}{b^{3/2}c\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -((x*(Sqrt[b]*(b*B - A*c) + A*c*Sqrt[b + c*x^2]*ArcTanh[Sqrt[b + c*x^2]/Sqr
t[b]]))/(b^(3/2)*c*Sqrt[x^2*(b + c*x^2)]))
```

Maple [A] time = 0.009, size = 79, normalized size = 1.2

$$\frac{\left(cx^2+b\right)x^3}{c}\left(Ab^{\frac{3}{2}}c-A\ln\left(2\,\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)\sqrt{cx^2+b}bc-Bb^{\frac{5}{2}}\right)\left(cx^4+bx^2\right)^{-\frac{3}{2}}b^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] x^3*(c*x^2+b)*(A*b^(3/2)*c-A*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*(c*x^2+b)^
(1/2)*b*c-B*b^(5/2))/(c*x^4+b*x^2)^(3/2)/c/b^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)x^2}{(cx^4 + bx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^2/(c*x^4 + b*x^2)^(3/2), x)
```

Fricas [A] time = 1.44314, size = 419, normalized size = 6.55

$$\left[\frac{\left(Ac^2x^3 + Abcx\right)\sqrt{b}\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right) - 2\,\sqrt{cx^4 + bx^2}\left(Bb^2 - Abc\right)}{2\left(b^2c^2x^3 + b^3cx\right)}, \frac{\left(Ac^2x^3 + Abcx\right)\sqrt{-b}\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-b}}{cx^3+bx}\right) -}{b^2c^2x^3 + b^3cx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((A*c^2*x^3 + A*b*c*x)*sqrt(b)*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*
x^2)*sqrt(b))/x^3) - 2*sqrt(c*x^4 + b*x^2)*(B*b^2 - A*b*c))/(b^2*c^2*x^3 +
b^3*c*x), ((A*c^2*x^3 + A*b*c*x)*sqrt(-b)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-
b)/(c*x^3 + b*x)) - sqrt(c*x^4 + b*x^2)*(B*b^2 - A*b*c))/(b^2*c^2*x^3 + b^3
*c*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \left(A + Bx^2\right)}{\left(x^2 \left(b + cx^2\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**2*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.157 $\int \frac{A+Bx^2}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=142

$$\frac{\sqrt{bx^2+cx^4}(2bB-3Ac)}{2b^2cx^3}-\frac{(2bB-3Ac)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2b^{5/2}}-\frac{2bB-3Ac}{3bcx\sqrt{bx^2+cx^4}}-\frac{B}{3cx\sqrt{bx^2+cx^4}}$$

```
[Out] -B/(3*c*x*Sqrt[b*x^2 + c*x^4]) - (2*b*B - 3*A*c)/(3*b*c*x*Sqrt[b*x^2 + c*x^
4]) + ((2*b*B - 3*A*c)*Sqrt[b*x^2 + c*x^4])/(2*b^2*c*x^3) - ((2*b*B - 3*A*c
)*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(2*b^(5/2))
```

Rubi [A] time = 0.0887823, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1145, 2006, 2025, 2008, 206}

$$\frac{\sqrt{bx^2+cx^4}(2bB-3Ac)}{2b^2cx^3}-\frac{(2bB-3Ac)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2b^{5/2}}-\frac{2bB-3Ac}{3bcx\sqrt{bx^2+cx^4}}-\frac{B}{3cx\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -B/(3*c*x*Sqrt[b*x^2 + c*x^4]) - (2*b*B - 3*A*c)/(3*b*c*x*Sqrt[b*x^2 + c*x^
4]) + ((2*b*B - 3*A*c)*Sqrt[b*x^2 + c*x^4])/(2*b^2*c*x^3) - ((2*b*B - 3*A*c
)*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(2*b^(5/2))
```

Rule 1145

```
Int[((d_) + (e_.)*(x_)^2)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
Simp[(e*(b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 3)*x), x] - Dist[(b*e*(2*p + 1)
- c*d*(4*p + 3))/(c*(4*p + 3)), Int[(b*x^2 + c*x^4)^p, x], x] /; FreeQ[{b,
c, d, e, p}, x] &&  !IntegerQ[p] && NeQ[4*p + 3, 0] && NeQ[b*e*(2*p + 1) -
c*d*(4*p + 3), 0]
```

Rule 2006

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Dist[(n*p + n - j + 1)/
(a*(n - j)*(p + 1)), Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a, b
```

```
}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+Bx^2}{\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{B}{3cx\sqrt{bx^2+cx^4}}+\frac{(-2bB+3Ac)\int \frac{1}{\left(bx^2+cx^4\right)^{3/2}}\,dx}{3c}$$

$$= -\frac{B}{3cx\sqrt{bx^2+cx^4}}-\frac{2bB-3Ac}{3bcx\sqrt{bx^2+cx^4}}+\frac{(-2bB+3Ac)\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx}{bc}$$

$$= -\frac{B}{3cx\sqrt{bx^2+cx^4}}-\frac{2bB-3Ac}{3bcx\sqrt{bx^2+cx^4}}+\frac{(2bB-3Ac)\sqrt{bx^2+cx^4}}{2b^2cx^3}+\frac{(2bB-3Ac)\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{2b^2}$$

$$= -\frac{B}{3cx\sqrt{bx^2+cx^4}}-\frac{2bB-3Ac}{3bcx\sqrt{bx^2+cx^4}}+\frac{(2bB-3Ac)\sqrt{bx^2+cx^4}}{2b^2cx^3}-\frac{(2bB-3Ac)\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx\right.}{2b^2}$$

$$= -\frac{B}{3cx\sqrt{bx^2+cx^4}}-\frac{2bB-3Ac}{3bcx\sqrt{bx^2+cx^4}}+\frac{(2bB-3Ac)\sqrt{bx^2+cx^4}}{2b^2cx^3}-\frac{(2bB-3Ac)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2b^{5/2}}$$

Mathematica [C] time = 0.0240062, size = 61, normalized size = 0.43

$$\frac{x^2(2bB-3Ac)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{cx^2}{b}+1\right)-Ab}{2b^2x\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-(A*b) + (2*b*B - 3*A*c)*x^2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (c*x^2)/b
])/(2*b^2*x*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.01, size = 129, normalized size = 0.9

$$-\frac{\left(cx^2+b\right)x}{2}\left(3\,Ab^{3/2}x^2c-3\,A\ln\left(2\,\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)\sqrt{cx^2+b}x^2bc-2\,Bb^{5/2}x^2+2\,B\ln\left(2\,\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)\sqrt{cx^2+b}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/2*x*(c*x^2+b)*(3*A*b^(3/2)*x^2*c-3*A*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)
*(c*x^2+b)^(1/2)*x^2*b*c-2*B*b^(5/2)*x^2+2*B*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+
b)/x)*(c*x^2+b)^(1/2)*x^2*b^2+A*b^(5/2))/(c*x^4+b*x^2)^(3/2)/b^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{Bx^2+A}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(c*x^4 + b*x^2)^(3/2), x)
```

Fricas [A] time = 1.48199, size = 549, normalized size = 3.87

$$\left[-\frac{\left(\left(2\,Bbc-3\,Ac^2\right)x^5+\left(2\,Bb^2-3\,Abc\right)x^3\right)\sqrt{b}\log\left(-\frac{cx^3+2\,bx+2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)+2\,\sqrt{cx^4+bx^2}\left(Ab^2-\left(2\,Bb^2-3\,Abc\right)x^2\right)}{4\left(b^3cx^5+b^4x^3\right)},\ -\frac{\left(\left(\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(((2*B*b*c - 3*A*c^2)*x^5 + (2*B*b^2 - 3*A*b*c)*x^3)*sqrt(b)*log(-(c*
x^3 + 2*b*x + 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/x^3) + 2*sqrt(c*x^4 + b*x^2)*(
A*b^2 - (2*B*b^2 - 3*A*b*c)*x^2))/(b^3*c*x^5 + b^4*x^3), 1/2*(((2*B*b*c - 3
*A*c^2)*x^5 + (2*B*b^2 - 3*A*b*c)*x^3)*sqrt(-b)*arctan(sqrt(c*x^4 + b*x^2)*
sqrt(-b)/(c*x^3 + b*x)) - sqrt(c*x^4 + b*x^2)*(A*b^2 - (2*B*b^2 - 3*A*b*c)*
x^2))/(b^3*c*x^5 + b^4*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A+Bx^2}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.158 $\int \frac{A+Bx^2}{x^2\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=137

$$-\frac{3\sqrt{bx^2+cx^4}(4bB-5Ac)}{8b^3x^3}+\frac{4bB-5Ac}{4b^2x\sqrt{bx^2+cx^4}}+\frac{3c(4bB-5Ac)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8b^{7/2}}-\frac{A}{4bx^3\sqrt{bx^2+cx^4}}$$

```
[Out] -A/(4*b*x^3*Sqrt[b*x^2 + c*x^4]) + (4*b*B - 5*A*c)/(4*b^2*x*Sqrt[b*x^2 + c*
x^4]) - (3*(4*b*B - 5*A*c)*Sqrt[b*x^2 + c*x^4])/(8*b^3*x^3) + (3*c*(4*b*B -
 5*A*c)*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(8*b^(7/2))
```

Rubi [A] time = 0.191907, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2038, 2006, 2025, 2008, 206}

$$-\frac{3\sqrt{bx^2+cx^4}(4bB-5Ac)}{8b^3x^3}+\frac{4bB-5Ac}{4b^2x\sqrt{bx^2+cx^4}}+\frac{3c(4bB-5Ac)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8b^{7/2}}-\frac{A}{4bx^3\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^2*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] -A/(4*b*x^3*Sqrt[b*x^2 + c*x^4]) + (4*b*B - 5*A*c)/(4*b^2*x*Sqrt[b*x^2 + c*
x^4]) - (3*(4*b*B - 5*A*c)*Sqrt[b*x^2 + c*x^4])/(8*b^3*x^3) + (3*c*(4*b*B -
 5*A*c)*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(8*b^(7/2))
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rule 2006

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Dist[(n*p + n - j + 1)/
(a*(n - j)*(p + 1)), Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a, b
}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+Bx^2}{x^2\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{A}{4bx^3\sqrt{bx^2+cx^4}} - \frac{(-4bB+5Ac)\int \frac{1}{\left(bx^2+cx^4\right)^{3/2}}\,dx}{4b}$$

$$= -\frac{A}{4bx^3\sqrt{bx^2+cx^4}} + \frac{4bB-5Ac}{4b^2x\sqrt{bx^2+cx^4}} + \frac{(3(4bB-5Ac))\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx}{4b^2}$$

$$= -\frac{A}{4bx^3\sqrt{bx^2+cx^4}} + \frac{4bB-5Ac}{4b^2x\sqrt{bx^2+cx^4}} - \frac{3(4bB-5Ac)\sqrt{bx^2+cx^4}}{8b^3x^3} - \frac{(3c(4bB-5Ac))\int \frac{1}{\sqrt{bx^2+cx^4}}}{8b^3}$$

$$= -\frac{A}{4bx^3\sqrt{bx^2+cx^4}} + \frac{4bB-5Ac}{4b^2x\sqrt{bx^2+cx^4}} - \frac{3(4bB-5Ac)\sqrt{bx^2+cx^4}}{8b^3x^3} + \frac{(3c(4bB-5Ac))\,\mathrm{Subst}\left(\int\right.}{8b^3}$$

$$= -\frac{A}{4bx^3\sqrt{bx^2+cx^4}} + \frac{4bB-5Ac}{4b^2x\sqrt{bx^2+cx^4}} - \frac{3(4bB-5Ac)\sqrt{bx^2+cx^4}}{8b^3x^3} + \frac{3c(4bB-5Ac)\tanh^{-1}\left(\frac{}{\sqrt{b}}\right.}{8b^{7/2}}$$

Mathematica [C] time = 0.0266065, size = 64, normalized size = 0.47

$$\frac{cx^4(5Ac-4bB)\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{cx^2}{b}+1\right)-Ab^2}{4b^3x^3\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^2*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-(A*b^2) + c*(-4*b*B + 5*A*c)*x^4*Hypergeometric2F1[-1/2, 2, 1/2, 1 + (c*x
^2)/b])/(4*b^3*x^3*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.01, size = 157, normalized size = 1.2

$$-\frac{cx^2+b}{8\,x}\left(12\,Bb^{5/2}x^4c-15\,Ab^{3/2}x^4c^2+15\,A\ln\left(2\,\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)\sqrt{cx^2+b}x^4bc^2+4\,Bb^{7/2}x^2-12\,B\ln\left(2\,\frac{\sqrt{b}\sqrt{cx^2+b}-}{x}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^2/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/8*(c*x^2+b)*(12*B*b^(5/2)*x^4*c-15*A*b^(3/2)*x^4*c^2+15*A*ln(2*(b^(1/2)*
(c*x^2+b)^(1/2)+b)/x)*(c*x^2+b)^(1/2)*x^4*b*c^2+4*B*b^(7/2)*x^2-12*B*ln(2*(
b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*(c*x^2+b)^(1/2)*x^4*b^2*c-5*A*b^(5/2)*x^2*c+2
*A*b^(7/2))/x/(c*x^4+b*x^2)^(3/2)/b^(9/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\left(cx^4 + bx^2\right)^{\frac{3}{2}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/((c*x^4 + b*x^2)^(3/2)*x^2), x)
```

Fricas [A] time = 1.4119, size = 671, normalized size = 4.9

$$\left[-\frac{3\left(\left(4\,Bbc^2 - 5\,Ac^3\right)x^7 + \left(4\,Bb^2c - 5\,Abc^2\right)x^5\right)\sqrt{b}\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right) + 2\left(3\left(4\,Bb^2c - 5\,Abc^2\right)x^4 + 2\,Ab^3 + \left(4\right.\right.}{16\left(b^4cx^7 + b^5x^5\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(3*((4*B*b*c^2 - 5*A*c^3)*x^7 + (4*B*b^2*c - 5*A*b*c^2)*x^5)*sqrt(b)
*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/x^3) + 2*(3*(4*B*b^2*
c - 5*A*b*c^2)*x^4 + 2*A*b^3 + (4*B*b^3 - 5*A*b^2*c)*x^2)*sqrt(c*x^4 + b*x^
2))/(b^4*c*x^7 + b^5*x^5), -1/8*(3*((4*B*b*c^2 - 5*A*c^3)*x^7 + (4*B*b^2*c
- 5*A*b*c^2)*x^5)*sqrt(-b)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x
)) + (3*(4*B*b^2*c - 5*A*b*c^2)*x^4 + 2*A*b^3 + (4*B*b^3 - 5*A*b^2*c)*x^2)*
sqrt(c*x^4 + b*x^2))/(b^4*c*x^7 + b^5*x^5)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + Bx^2}{x^2 \left(x^2 \left(b + cx^2\right)\right)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**2/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**2*(x**2*(b + c*x**2))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\left(cx^4 + bx^2\right)^{\frac{3}{2}} x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/((c*x^4 + b*x^2)^(3/2)*x^2), x)
```

3.159 $\int x^{7/2} \left(A+Bx^2\right) \left(bx^2+cx^4\right) dx$

Optimal. Leaf size=39

$$\frac{2}{17}x^{17/2}(Ac+bB)+\frac{2}{13}Abx^{13/2}+\frac{2}{21}Bcx^{21/2}$$

```
[Out] (2*A*b*x^(13/2))/13 + (2*(b*B + A*c)*x^(17/2))/17 + (2*B*c*x^(21/2))/21
```

Rubi [A] time = 0.0226728, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1584, 448}

$$\frac{2}{17}x^{17/2}(Ac+bB)+\frac{2}{13}Abx^{13/2}+\frac{2}{21}Bcx^{21/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4),x]
```

```
[Out] (2*A*b*x^(13/2))/13 + (2*(b*B + A*c)*x^(17/2))/17 + (2*B*c*x^(21/2))/21
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^{7/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)\,dx &= \int x^{11/2}\left(A+Bx^2\right)\left(b+cx^2\right)\,dx \\
&= \int \left(Abx^{11/2}+(bB+Ac)x^{15/2}+Bcx^{19/2}\right)\,dx \\
&= \frac{2}{13}Abx^{13/2}+\frac{2}{17}(bB+Ac)x^{17/2}+\frac{2}{21}Bcx^{21/2}
\end{aligned}
$$

Mathematica [A] time = 0.0157103, size = 33, normalized size = 0.85

$$
\frac{2x^{13/2}\left(273x^2(Ac+bB)+357Ab+221Bcx^4\right)}{4641}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4),x]
```

```
[Out] (2*x^(13/2)*(357*A*b + 273*(b*B + A*c)*x^2 + 221*B*c*x^4))/4641
```

Maple [A] time = 0.004, size = 32, normalized size = 0.8

$$
\frac{442\,Bcx^4+546\,Ax^2c+546\,Bx^2b+714\,Ab}{4641}x^{\frac{13}{2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2),x)
```

```
[Out] 2/4641*x^(13/2)*(221*B*c*x^4+273*A*c*x^2+273*B*b*x^2+357*A*b)
```

Maxima [A] time = 1.15532, size = 36, normalized size = 0.92

$$
\frac{2}{21}\,Bcx^{\frac{21}{2}}+\frac{2}{17}\,(Bb+Ac)x^{\frac{17}{2}}+\frac{2}{13}\,Abx^{\frac{13}{2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 2/21*B*c*x^(21/2) + 2/17*(B*b + A*c)*x^(17/2) + 2/13*A*b*x^(13/2)
```

Fricas [A] time = 1.299, size = 90, normalized size = 2.31

$$\frac{2}{4641}\left(221\,Bcx^{10}+273\,(Bb+Ac)x^{8}+357\,Abx^{6}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 2/4641*(221*B*c*x^10 + 273*(B*b + A*c)*x^8 + 357*A*b*x^6)*sqrt(x)
```

Sympy [A] time = 27.7593, size = 46, normalized size = 1.18

$$\frac{2Abx^{\frac{13}{2}}}{13}+\frac{2Acx^{\frac{17}{2}}}{17}+\frac{2Bbx^{\frac{17}{2}}}{17}+\frac{2Bcx^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(B*x**2+A)*(c*x**4+b*x**2),x)
```

```
[Out] 2*A*b*x**(13/2)/13 + 2*A*c*x**(17/2)/17 + 2*B*b*x**(17/2)/17 + 2*B*c*x**(21
/2)/21
```

Giac [A] time = 1.15707, size = 39, normalized size = 1.

$$\frac{2}{21}\,Bcx^{\frac{21}{2}}+\frac{2}{17}\,Bbx^{\frac{17}{2}}+\frac{2}{17}\,Acx^{\frac{17}{2}}+\frac{2}{13}\,Abx^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 2/21*B*c*x^(21/2) + 2/17*B*b*x^(17/2) + 2/17*A*c*x^(17/2) + 2/13*A*b*x^(13/
2)
```

3.160 $\int x^{5/2} \left(A+Bx^2\right) \left(bx^2+cx^4\right) \, dx$

Optimal. Leaf size=39

$$\frac{2}{15}x^{15/2}(Ac+bB)+\frac{2}{11}Abx^{11/2}+\frac{2}{19}Bcx^{19/2}$$

```
[Out] (2*A*b*x^(11/2))/11 + (2*(b*B + A*c)*x^(15/2))/15 + (2*B*c*x^(19/2))/19
```

Rubi [A] time = 0.0225822, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1584, 448}

$$\frac{2}{15}x^{15/2}(Ac+bB)+\frac{2}{11}Abx^{11/2}+\frac{2}{19}Bcx^{19/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4),x]
```

```
[Out] (2*A*b*x^(11/2))/11 + (2*(b*B + A*c)*x^(15/2))/15 + (2*B*c*x^(19/2))/19
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)\,dx &= \int x^{9/2}\left(A+Bx^2\right)\left(b+cx^2\right)\,dx\\ &= \int\left(Abx^{9/2}+(bB+Ac)x^{13/2}+Bcx^{17/2}\right)dx\\ &= \frac{2}{11}Abx^{11/2}+\frac{2}{15}(bB+Ac)x^{15/2}+\frac{2}{19}Bcx^{19/2}\end{aligned}$$

Mathematica [A] time = 0.0153222, size = 33, normalized size = 0.85

$$\frac{2x^{11/2}\left(209x^2(Ac+bB)+285Ab+165Bcx^4\right)}{3135}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4),x]
```

```
[Out] (2*x^(11/2)*(285*A*b + 209*(b*B + A*c)*x^2 + 165*B*c*x^4))/3135
```

Maple [A] time = 0.003, size = 32, normalized size = 0.8

$$\frac{330\,Bcx^4+418\,Ax^2c+418\,Bx^2b+570\,Ab}{3135}x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2),x)
```

```
[Out] 2/3135*x^(11/2)*(165*B*c*x^4+209*A*c*x^2+209*B*b*x^2+285*A*b)
```

Maxima [A] time = 1.13169, size = 36, normalized size = 0.92

$$\frac{2}{19}\,Bcx^{\frac{19}{2}}+\frac{2}{15}\,(Bb+Ac)x^{\frac{15}{2}}+\frac{2}{11}\,Abx^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 2/19*B*c*x^(19/2) + 2/15*(B*b + A*c)*x^(15/2) + 2/11*A*b*x^(11/2)
```

Fricas [A] time = 1.3187, size = 89, normalized size = 2.28

$$\frac{2}{3135}\left(165\,Bcx^9 + 209\,(Bb + Ac)x^7 + 285\,Abx^5\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 2/3135*(165*B*c*x^9 + 209*(B*b + A*c)*x^7 + 285*A*b*x^5)*sqrt(x)
```

Sympy [A] time = 19.5792, size = 46, normalized size = 1.18

$$\frac{2Abx^{\frac{11}{2}}}{11} + \frac{2Acx^{\frac{15}{2}}}{15} + \frac{2Bbx^{\frac{15}{2}}}{15} + \frac{2Bcx^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(B*x**2+A)*(c*x**4+b*x**2),x)
```

```
[Out] 2*A*b*x**(11/2)/11 + 2*A*c*x**(15/2)/15 + 2*B*b*x**(15/2)/15 + 2*B*c*x**(19
/2)/19
```

Giac [A] time = 1.15957, size = 39, normalized size = 1.

$$\frac{2}{19}\,Bcx^{\frac{19}{2}} + \frac{2}{15}\,Bbx^{\frac{15}{2}} + \frac{2}{15}\,Acx^{\frac{15}{2}} + \frac{2}{11}\,Abx^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 2/19*B*c*x^(19/2) + 2/15*B*b*x^(15/2) + 2/15*A*c*x^(15/2) + 2/11*A*b*x^(11/
2)
```

3.161 $\int x^{3/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)\,dx$

Optimal. Leaf size=39

$$\frac{2}{13}x^{13/2}(Ac+bB)+\frac{2}{9}Abx^{9/2}+\frac{2}{17}Bcx^{17/2}$$

```
[Out] (2*A*b*x^(9/2))/9 + (2*(b*B + A*c)*x^(13/2))/13 + (2*B*c*x^(17/2))/17
```

Rubi [A] time = 0.0229939, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1584, 448}

$$\frac{2}{13}x^{13/2}(Ac+bB)+\frac{2}{9}Abx^{9/2}+\frac{2}{17}Bcx^{17/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4),x]
```

```
[Out] (2*A*b*x^(9/2))/9 + (2*(b*B + A*c)*x^(13/2))/13 + (2*B*c*x^(17/2))/17
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int x^{3/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)dx = \int x^{7/2}\left(A+Bx^2\right)\left(b+cx^2\right)dx$$
$$= \int \left(Abx^{7/2}+(bB+Ac)x^{11/2}+Bcx^{15/2}\right)dx$$
$$= \frac{2}{9}Abx^{9/2}+\frac{2}{13}(bB+Ac)x^{13/2}+\frac{2}{17}Bcx^{17/2}$$

Mathematica [A] time = 0.0155499, size = 33, normalized size = 0.85

$$\frac{2x^{9/2}\left(153x^2(Ac+bB)+221Ab+117Bcx^4\right)}{1989}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4),x]
```

```
[Out] (2*x^(9/2)*(221*A*b + 153*(b*B + A*c)*x^2 + 117*B*c*x^4))/1989
```

Maple [A] time = 0.003, size = 32, normalized size = 0.8

$$\frac{234\,Bcx^4+306\,Ax^2c+306\,Bx^2b+442\,Ab}{1989}x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2),x)
```

```
[Out] 2/1989*x^(9/2)*(117*B*c*x^4+153*A*c*x^2+153*B*b*x^2+221*A*b)
```

Maxima [A] time = 1.06539, size = 36, normalized size = 0.92

$$\frac{2}{17}\,Bcx^{\frac{17}{2}}+\frac{2}{13}\,(Bb+Ac)x^{\frac{13}{2}}+\frac{2}{9}\,Abx^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 2/17*B*c*x^(17/2) + 2/13*(B*b + A*c)*x^(13/2) + 2/9*A*b*x^(9/2)
```

Fricas [A] time = 1.55594, size = 89, normalized size = 2.28

$$\frac{2}{1989}\left(117\,Bcx^{8}+153\,(Bb+Ac)x^{6}+221\,Abx^{4}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 2/1989*(117*B*c*x^8 + 153*(B*b + A*c)*x^6 + 221*A*b*x^4)*sqrt(x)
```

Sympy [A] time = 10.2908, size = 46, normalized size = 1.18

$$\frac{2Abx^{\frac{9}{2}}}{9}+\frac{2Acx^{\frac{13}{2}}}{13}+\frac{2Bbx^{\frac{13}{2}}}{13}+\frac{2Bcx^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(B*x**2+A)*(c*x**4+b*x**2),x)
```

```
[Out] 2*A*b*x**(9/2)/9 + 2*A*c*x**(13/2)/13 + 2*B*b*x**(13/2)/13 + 2*B*c*x**(17/2
)/17
```

Giac [A] time = 1.13722, size = 39, normalized size = 1.

$$\frac{2}{17}\,Bcx^{\frac{17}{2}}+\frac{2}{13}\,Bbx^{\frac{13}{2}}+\frac{2}{13}\,Acx^{\frac{13}{2}}+\frac{2}{9}\,Abx^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 2/17*B*c*x^(17/2) + 2/13*B*b*x^(13/2) + 2/13*A*c*x^(13/2) + 2/9*A*b*x^(9/2)
```

3.162 $\int \sqrt{x}\left(A+Bx^2\right)\left(bx^2+cx^4\right)\,dx$

Optimal. Leaf size=39

$$\frac{2}{11}x^{11/2}(Ac+bB)+\frac{2}{7}Abx^{7/2}+\frac{2}{15}Bcx^{15/2}$$

```
[Out] (2*A*b*x^(7/2))/7 + (2*(b*B + A*c)*x^(11/2))/11 + (2*B*c*x^(15/2))/15
```

Rubi [A] time = 0.0213687, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1584, 448}

$$\frac{2}{11}x^{11/2}(Ac+bB)+\frac{2}{7}Abx^{7/2}+\frac{2}{15}Bcx^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(A + B*x^2)*(b*x^2 + c*x^4),x]
```

```
[Out] (2*A*b*x^(7/2))/7 + (2*(b*B + A*c)*x^(11/2))/11 + (2*B*c*x^(15/2))/15
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(A+Bx^2\right)\left(bx^2+cx^4\right)\,dx &= \int x^{5/2}\left(A+Bx^2\right)\left(b+cx^2\right)\,dx\\ &= \int \left(Abx^{5/2}+(bB+Ac)x^{9/2}+Bcx^{13/2}\right)\,dx\\ &= \frac{2}{7}Abx^{7/2}+\frac{2}{11}(bB+Ac)x^{11/2}+\frac{2}{15}Bcx^{15/2}\end{aligned}$$

Mathematica [A] time = 0.0138779, size = 33, normalized size = 0.85

$$\frac{2x^{7/2}\left(105x^2(Ac+bB)+165Ab+77Bcx^4\right)}{1155}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(A + B*x^2)*(b*x^2 + c*x^4),x]
```

```
[Out] (2*x^(7/2)*(165*A*b + 105*(b*B + A*c)*x^2 + 77*B*c*x^4))/1155
```

Maple [A] time = 0.003, size = 32, normalized size = 0.8

$$\frac{154\,Bcx^4+210\,Ax^2c+210\,Bx^2b+330\,Ab}{1155}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)*x^(1/2),x)
```

```
[Out] 2/1155*x^(7/2)*(77*B*c*x^4+105*A*c*x^2+105*B*b*x^2+165*A*b)
```

Maxima [A] time = 1.18663, size = 36, normalized size = 0.92

$$\frac{2}{15}\,Bcx^{\frac{15}{2}}+\frac{2}{11}\,(Bb+Ac)x^{\frac{11}{2}}+\frac{2}{7}\,Abx^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*B*c*x^(15/2) + 2/11*(B*b + A*c)*x^(11/2) + 2/7*A*b*x^(7/2)
```

Fricas [A] time = 1.63969, size = 88, normalized size = 2.26

$$\frac{2}{1155}\left(77\,Bcx^{7}+105\,(Bb+Ac)x^{5}+165\,Abx^{3}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*B*c*x^7 + 105*(B*b + A*c)*x^5 + 165*A*b*x^3)*sqrt(x)
```

Sympy [A] time = 3.65863, size = 37, normalized size = 0.95

$$\frac{2Abx^{\frac{7}{2}}}{7}+\frac{2Bcx^{\frac{15}{2}}}{15}+\frac{2x^{\frac{11}{2}}\left(Ac+Bb\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)*x**(1/2),x)
```

```
[Out] 2*A*b*x**(7/2)/7 + 2*B*c*x**(15/2)/15 + 2*x**(11/2)*(A*c + B*b)/11
```

Giac [A] time = 1.14034, size = 39, normalized size = 1.

$$\frac{2}{15}\,Bcx^{\frac{15}{2}}+\frac{2}{11}\,Bbx^{\frac{11}{2}}+\frac{2}{11}\,Acx^{\frac{11}{2}}+\frac{2}{7}\,Abx^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*B*c*x^(15/2) + 2/11*B*b*x^(11/2) + 2/11*A*c*x^(11/2) + 2/7*A*b*x^(7/2)
```

3.163 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{\sqrt{x}}\,dx$

Optimal. Leaf size=39

$$\frac{2}{9}x^{9/2}(Ac+bB)+\frac{2}{5}Abx^{5/2}+\frac{2}{13}Bcx^{13/2}$$

```
[Out] (2*A*b*x^(5/2))/5 + (2*(b*B + A*c)*x^(9/2))/9 + (2*B*c*x^(13/2))/13
```

Rubi [A] time = 0.0210477, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1584, 448}

$$\frac{2}{9}x^{9/2}(Ac+bB)+\frac{2}{5}Abx^{5/2}+\frac{2}{13}Bcx^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4))/Sqrt[x],x]
```

```
[Out] (2*A*b*x^(5/2))/5 + (2*(b*B + A*c)*x^(9/2))/9 + (2*B*c*x^(13/2))/13
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{\sqrt{x}}\,dx = \int x^{3/2}(A+Bx^2)(b+cx^2)\,dx$$
$$= \int \left(Abx^{3/2} + (bB+Ac)x^{7/2} + Bcx^{11/2}\right) dx$$
$$= \frac{2}{5}Abx^{5/2} + \frac{2}{9}(bB+Ac)x^{9/2} + \frac{2}{13}Bcx^{13/2}$$

Mathematica [A] time = 0.0143855, size = 33, normalized size = 0.85

$$\frac{2}{585}x^{5/2}\left(65x^2(Ac+bB)+117Ab+45Bcx^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/Sqrt[x],x]
```

```
[Out] (2*x^(5/2)*(117*A*b + 65*(b*B + A*c)*x^2 + 45*B*c*x^4))/585
```

Maple [A] time = 0.003, size = 32, normalized size = 0.8

$$\frac{90\,Bcx^4+130\,Ax^2c+130\,Bx^2b+234\,Ab}{585}x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)/x^(1/2),x)
```

```
[Out] 2/585*x^(5/2)*(45*B*c*x^4+65*A*c*x^2+65*B*b*x^2+117*A*b)
```

Maxima [A] time = 1.02385, size = 36, normalized size = 0.92

$$\frac{2}{13}\,Bcx^{\frac{13}{2}}+\frac{2}{9}\,(Bb+Ac)x^{\frac{9}{2}}+\frac{2}{5}\,Abx^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/13*B*c*x^(13/2) + 2/9*(B*b + A*c)*x^(9/2) + 2/5*A*b*x^(5/2)
```

Fricas [A] time = 1.52941, size = 85, normalized size = 2.18

$$\frac{2}{585}\left(45\,Bcx^{6}+65\,(Bb+Ac)x^{4}+117\,Abx^{2}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/585*(45*B*c*x^6 + 65*(B*b + A*c)*x^4 + 117*A*b*x^2)*sqrt(x)
```

Sympy [A] time = 3.31652, size = 46, normalized size = 1.18

$$\frac{2Abx^{\frac{5}{2}}}{5}+\frac{2Acx^{\frac{9}{2}}}{9}+\frac{2Bbx^{\frac{9}{2}}}{9}+\frac{2Bcx^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)/x**(1/2),x)
```

```
[Out] 2*A*b*x**(5/2)/5 + 2*A*c*x**(9/2)/9 + 2*B*b*x**(9/2)/9 + 2*B*c*x**(13/2)/13
```

Giac [A] time = 1.14759, size = 39, normalized size = 1.

$$\frac{2}{13}\,Bcx^{\frac{13}{2}}+\frac{2}{9}\,Bbx^{\frac{9}{2}}+\frac{2}{9}\,Acx^{\frac{9}{2}}+\frac{2}{5}\,Abx^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/13*B*c*x^(13/2) + 2/9*B*b*x^(9/2) + 2/9*A*c*x^(9/2) + 2/5*A*b*x^(5/2)
```

3.164 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{3/2}} \, dx$

Optimal. Leaf size=39

$$\frac{2}{7}x^{7/2}(Ac+bB)+\frac{2}{3}Abx^{3/2}+\frac{2}{11}Bcx^{11/2}$$

```
[Out] (2*A*b*x^(3/2))/3 + (2*(b*B + A*c)*x^(7/2))/7 + (2*B*c*x^(11/2))/11
```

Rubi [A] time = 0.0212394, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1584, 448}

$$\frac{2}{7}x^{7/2}(Ac+bB)+\frac{2}{3}Abx^{3/2}+\frac{2}{11}Bcx^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4))/x^(3/2),x]
```

```
[Out] (2*A*b*x^(3/2))/3 + (2*(b*B + A*c)*x^(7/2))/7 + (2*B*c*x^(11/2))/11
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{3/2}}\,dx = \int \sqrt{x}\,(A+Bx^2)(b+cx^2)\,dx$$
$$= \int \left(Ab\sqrt{x}+(bB+Ac)x^{5/2}+Bcx^{9/2}\right)dx$$
$$= \frac{2}{3}Abx^{3/2}+\frac{2}{7}(bB+Ac)x^{7/2}+\frac{2}{11}Bcx^{11/2}$$

Mathematica [A] time = 0.0139968, size = 33, normalized size = 0.85

$$\frac{2}{231}x^{3/2}\left(33x^2(Ac+bB)+77Ab+21Bcx^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/x^(3/2),x]
```

```
[Out] (2*x^(3/2)*(77*A*b + 33*(b*B + A*c)*x^2 + 21*B*c*x^4))/231
```

Maple [A] time = 0.004, size = 32, normalized size = 0.8

$$\frac{42\,Bcx^4+66\,Ax^2c+66\,Bx^2b+154\,Ab}{231}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)/x^(3/2),x)
```

```
[Out] 2/231*x^(3/2)*(21*B*c*x^4+33*A*c*x^2+33*B*b*x^2+77*A*b)
```

Maxima [A] time = 1.19645, size = 36, normalized size = 0.92

$$\frac{2}{11}\,Bcx^{\frac{11}{2}}+\frac{2}{7}\,(Bb+Ac)x^{\frac{7}{2}}+\frac{2}{3}\,Abx^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/11*B*c*x^(11/2) + 2/7*(B*b + A*c)*x^(7/2) + 2/3*A*b*x^(3/2)
```

Fricas [A] time = 1.65898, size = 81, normalized size = 2.08

$$\frac{2}{231}\left(21\,Bcx^5 + 33\,(Bb + Ac)x^3 + 77\,Abx\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/231*(21*B*c*x^5 + 33*(B*b + A*c)*x^3 + 77*A*b*x)*sqrt(x)
```

Sympy [A] time = 2.66208, size = 46, normalized size = 1.18

$$\frac{2Abx^{\frac{3}{2}}}{3} + \frac{2Acx^{\frac{7}{2}}}{7} + \frac{2Bbx^{\frac{7}{2}}}{7} + \frac{2Bcx^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)/x**(3/2),x)
```

```
[Out] 2*A*b*x**(3/2)/3 + 2*A*c*x**(7/2)/7 + 2*B*b*x**(7/2)/7 + 2*B*c*x**(11/2)/11
```

Giac [A] time = 1.14787, size = 39, normalized size = 1.

$$\frac{2}{11}\,Bcx^{\frac{11}{2}} + \frac{2}{7}\,Bbx^{\frac{7}{2}} + \frac{2}{7}\,Acx^{\frac{7}{2}} + \frac{2}{3}\,Abx^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/11*B*c*x^(11/2) + 2/7*B*b*x^(7/2) + 2/7*A*c*x^(7/2) + 2/3*A*b*x^(3/2)
```

3.165 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{5/2}} dx$

Optimal. Leaf size=37

$$\frac{2}{5}x^{5/2}(Ac+bB)+2Ab\sqrt{x}+\frac{2}{9}Bcx^{9/2}$$

```
[Out] 2*A*b*Sqrt[x] + (2*(b*B + A*c)*x^(5/2))/5 + (2*B*c*x^(9/2))/9
```

Rubi [A] time = 0.0212557, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1584, 448}

$$\frac{2}{5}x^{5/2}(Ac+bB)+2Ab\sqrt{x}+\frac{2}{9}Bcx^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4))/x^(5/2),x]
```

```
[Out] 2*A*b*Sqrt[x] + (2*(b*B + A*c)*x^(5/2))/5 + (2*B*c*x^(9/2))/9
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{5/2}}\,dx = \int \frac{(A+Bx^2)(b+cx^2)}{\sqrt{x}}\,dx$$
$$= \int \left(\frac{Ab}{\sqrt{x}} + (bB+Ac)x^{3/2} + Bcx^{7/2}\right)dx$$
$$= 2Ab\sqrt{x} + \frac{2}{5}(bB+Ac)x^{5/2} + \frac{2}{9}Bcx^{9/2}$$

Mathematica [A] time = 0.0141017, size = 33, normalized size = 0.89

$$\frac{2}{45}\sqrt{x}\left(9x^2(Ac+bB)+45Ab+5Bcx^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/x^(5/2),x]
```

```
[Out] (2*Sqrt[x]*(45*A*b + 9*(b*B + A*c)*x^2 + 5*B*c*x^4))/45
```

Maple [A] time = 0.003, size = 32, normalized size = 0.9

$$\frac{10\,Bcx^4+18\,Ax^2c+18\,Bx^2b+90\,Ab}{45}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)/x^(5/2),x)
```

```
[Out] 2/45*x^(1/2)*(5*B*c*x^4+9*A*c*x^2+9*B*b*x^2+45*A*b)
```

Maxima [A] time = 1.12695, size = 36, normalized size = 0.97

$$\frac{2}{9}\,Bcx^{\frac{9}{2}} + \frac{2}{5}\,(Bb+Ac)x^{\frac{5}{2}} + 2\,Ab\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/9*B*c*x^(9/2) + 2/5*(B*b + A*c)*x^(5/2) + 2*A*b*sqrt(x)
```

Fricas [A] time = 1.5373, size = 74, normalized size = 2.

$$\frac{2}{45}\left(5\,Bcx^4 + 9\,(Bb + Ac)x^2 + 45\,Ab\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/45*(5*B*c*x^4 + 9*(B*b + A*c)*x^2 + 45*A*b)*sqrt(x)
```

Sympy [A] time = 4.54852, size = 44, normalized size = 1.19

$$2Ab\sqrt{x} + \frac{2Acx^{\frac{5}{2}}}{5} + \frac{2Bbx^{\frac{5}{2}}}{5} + \frac{2Bcx^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)/x**(5/2),x)
```

```
[Out] 2*A*b*sqrt(x) + 2*A*c*x**(5/2)/5 + 2*B*b*x**(5/2)/5 + 2*B*c*x**(9/2)/9
```

Giac [A] time = 1.13098, size = 39, normalized size = 1.05

$$\frac{2}{9}\,Bcx^{\frac{9}{2}} + \frac{2}{5}\,Bbx^{\frac{5}{2}} + \frac{2}{5}\,Acx^{\frac{5}{2}} + 2\,Ab\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/9*B*c*x^(9/2) + 2/5*B*b*x^(5/2) + 2/5*A*c*x^(5/2) + 2*A*b*sqrt(x)
```

3.166 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{7/2}}\,dx$

Optimal. Leaf size=37

$$\frac{2}{3}x^{3/2}(Ac+bB)-\frac{2Ab}{\sqrt{x}}+\frac{2}{7}Bcx^{7/2}$$

```
[Out] (-2*A*b)/Sqrt[x] + (2*(b*B + A*c)*x^(3/2))/3 + (2*B*c*x^(7/2))/7
```

Rubi [A] time = 0.0205999, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1584, 448}

$$\frac{2}{3}x^{3/2}(Ac+bB)-\frac{2Ab}{\sqrt{x}}+\frac{2}{7}Bcx^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4))/x^(7/2),x]
```

```
[Out] (-2*A*b)/Sqrt[x] + (2*(b*B + A*c)*x^(3/2))/3 + (2*B*c*x^(7/2))/7
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{7/2}}\,dx &= \int \frac{(A+Bx^2)(b+cx^2)}{x^{3/2}}\,dx\\ &= \int \left(\frac{Ab}{x^{3/2}}+(bB+Ac)\sqrt{x}+Bcx^{5/2}\right)dx\\ &= -\frac{2Ab}{\sqrt{x}}+\frac{2}{3}(bB+Ac)x^{3/2}+\frac{2}{7}Bcx^{7/2}\end{aligned}$$

Mathematica [A] time = 0.0098416, size = 35, normalized size = 0.95

$$\frac{2\left(-21Ab+7Acx^2+7bBx^2+3Bcx^4\right)}{21\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/x^(7/2),x]

[Out] (2*(-21*A*b + 7*b*B*x^2 + 7*A*c*x^2 + 3*B*c*x^4))/(21*Sqrt[x])
```

Maple [A] time = 0.004, size = 32, normalized size = 0.9

$$-\frac{-6\,Bcx^4-14\,Ax^2c-14\,Bx^2b+42\,Ab}{21}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)/x^(7/2),x)

[Out] -2/21/x^(1/2)*(-3*B*c*x^4-7*A*c*x^2-7*B*b*x^2+21*A*b)
```

Maxima [A] time = 1.14998, size = 36, normalized size = 0.97

$$\frac{2}{7}\,Bcx^{\frac{7}{2}}+\frac{2}{3}\,(Bb+Ac)x^{\frac{3}{2}}-\frac{2\,Ab}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/7*B*c*x^(7/2) + 2/3*(B*b + A*c)*x^(3/2) - 2*A*b/sqrt(x)
```

Fricas [A] time = 1.61498, size = 74, normalized size = 2.

$$\frac{2\left(3\,Bcx^{4}+7\,(Bb+Ac)x^{2}-21\,Ab\right)}{21\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/21*(3*B*c*x^4 + 7*(B*b + A*c)*x^2 - 21*A*b)/sqrt(x)
```

Sympy [A] time = 7.5185, size = 44, normalized size = 1.19

$$-\frac{2Ab}{\sqrt{x}}+\frac{2Acx^{\frac{3}{2}}}{3}+\frac{2Bbx^{\frac{3}{2}}}{3}+\frac{2Bcx^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)/x**(7/2),x)
```

```
[Out] -2*A*b/sqrt(x) + 2*A*c*x**(3/2)/3 + 2*B*b*x**(3/2)/3 + 2*B*c*x**(7/2)/7
```

Giac [A] time = 1.17079, size = 39, normalized size = 1.05

$$\frac{2}{7}\,Bcx^{\frac{7}{2}}+\frac{2}{3}\,Bbx^{\frac{3}{2}}+\frac{2}{3}\,Acx^{\frac{3}{2}}-\frac{2\,Ab}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/7*B*c*x^(7/2) + 2/3*B*b*x^(3/2) + 2/3*A*c*x^(3/2) - 2*A*b/sqrt(x)
```

3.167 $\int x^{7/2} \left(A+Bx^2\right) \left(bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=63

$$\frac{2}{17}Ab^2x^{17/2}+\frac{2}{25}cx^{25/2}(Ac+2bB)+\frac{2}{21}bx^{21/2}(2Ac+bB)+\frac{2}{29}Bc^2x^{29/2}$$

```
[Out] (2*A*b^2*x^(17/2))/17 + (2*b*(b*B + 2*A*c)*x^(21/2))/21 + (2*c*(2*b*B + A*c
)*x^(25/2))/25 + (2*B*c^2*x^(29/2))/29
```

Rubi [A] time = 0.0412536, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1584, 448}

$$\frac{2}{17}Ab^2x^{17/2}+\frac{2}{25}cx^{25/2}(Ac+2bB)+\frac{2}{21}bx^{21/2}(2Ac+bB)+\frac{2}{29}Bc^2x^{29/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*A*b^2*x^(17/2))/17 + (2*b*(b*B + 2*A*c)*x^(21/2))/21 + (2*c*(2*b*B + A*c
)*x^(25/2))/25 + (2*B*c^2*x^(29/2))/29
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int x^{7/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^2\,dx=\int x^{15/2}\left(A+Bx^2\right)\left(b+cx^2\right)^2\,dx$$
$$=\int\left(Ab^2x^{15/2}+b(bB+2Ac)x^{19/2}+c(2bB+Ac)x^{23/2}+Bc^2x^{27/2}\right)dx$$
$$=\frac{2}{17}Ab^2x^{17/2}+\frac{2}{21}b(bB+2Ac)x^{21/2}+\frac{2}{25}c(2bB+Ac)x^{25/2}+\frac{2}{29}Bc^2x^{29/2}$$

Mathematica [A] time = 0.0320792, size = 63, normalized size = 1.

$$\frac{2}{17}Ab^2x^{17/2}+\frac{2}{25}cx^{25/2}(Ac+2bB)+\frac{2}{21}bx^{21/2}(2Ac+bB)+\frac{2}{29}Bc^2x^{29/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*A*b^2*x^(17/2))/17 + (2*b*(b*B + 2*A*c)*x^(21/2))/21 + (2*c*(2*b*B + A*c
)*x^(25/2))/25 + (2*B*c^2*x^(29/2))/29
```

Maple [A] time = 0.005, size = 56, normalized size = 0.9

$$\frac{17850\,Bc^2x^6+20706\,Ac^2x^4+41412\,Bx^4bc+49300\,Abcx^2+24650\,Bx^2b^2+30450\,Ab^2}{258825}x^{\frac{17}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x)
```

```
[Out] 2/258825*x^(17/2)*(8925*B*c^2*x^6+10353*A*c^2*x^4+20706*B*b*c*x^4+24650*A*b
*c*x^2+12325*B*b^2*x^2+15225*A*b^2)
```

Maxima [A] time = 1.09971, size = 69, normalized size = 1.1

$$\frac{2}{29}\,Bc^2x^{\frac{29}{2}}+\frac{2}{25}\left(2\,Bbc+Ac^2\right)x^{\frac{25}{2}}+\frac{2}{17}\,Ab^2x^{\frac{17}{2}}+\frac{2}{21}\left(Bb^2+2\,Abc\right)x^{\frac{21}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 2/29*B*c^2*x^(29/2) + 2/25*(2*B*b*c + A*c^2)*x^(25/2) + 2/17*A*b^2*x^(17/2)
 + 2/21*(B*b^2 + 2*A*b*c)*x^(21/2)
```

Fricas [A] time = 1.57489, size = 157, normalized size = 2.49

$$\frac{2}{258825}\left(8925\,Bc^2x^{14}+10353\left(2\,Bbc+Ac^2\right)x^{12}+15225\,Ab^2x^8+12325\left(Bb^2+2\,Abc\right)x^{10}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 2/258825*(8925*B*c^2*x^14 + 10353*(2*B*b*c + A*c^2)*x^12 + 15225*A*b^2*x^8
+ 12325*(B*b^2 + 2*A*b*c)*x^10)*sqrt(x)
```

Sympy [A] time = 96.2608, size = 80, normalized size = 1.27

$$\frac{2Ab^2x^{\frac{17}{2}}}{17}+\frac{4Abcx^{\frac{21}{2}}}{21}+\frac{2Ac^2x^{\frac{25}{2}}}{25}+\frac{2Bb^2x^{\frac{21}{2}}}{21}+\frac{4Bbcx^{\frac{25}{2}}}{25}+\frac{2Bc^2x^{\frac{29}{2}}}{29}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(B*x**2+A)*(c*x**4+b*x**2)**2,x)
```

```
[Out] 2*A*b**2*x**(17/2)/17 + 4*A*b*c*x**(21/2)/21 + 2*A*c**2*x**(25/2)/25 + 2*B*
b**2*x**(21/2)/21 + 4*B*b*c*x**(25/2)/25 + 2*B*c**2*x**(29/2)/29
```

Giac [A] time = 1.17831, size = 72, normalized size = 1.14

$$\frac{2}{29}\,Bc^2x^{\frac{29}{2}}+\frac{4}{25}\,Bbcx^{\frac{25}{2}}+\frac{2}{25}\,Ac^2x^{\frac{25}{2}}+\frac{2}{21}\,Bb^2x^{\frac{21}{2}}+\frac{4}{21}\,Abcx^{\frac{21}{2}}+\frac{2}{17}\,Ab^2x^{\frac{17}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 2/29*B*c^2*x^(29/2) + 4/25*B*b*c*x^(25/2) + 2/25*A*c^2*x^(25/2) + 2/21*B*b^
2*x^(21/2) + 4/21*A*b*c*x^(21/2) + 2/17*A*b^2*x^(17/2)
```

3.168 $\int x^{5/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^2\,dx$

Optimal. Leaf size=63

$$\frac{2}{15}Ab^2x^{15/2}+\frac{2}{23}cx^{23/2}(Ac+2bB)+\frac{2}{19}bx^{19/2}(2Ac+bB)+\frac{2}{27}Bc^2x^{27/2}$$

```
[Out] (2*A*b^2*x^(15/2))/15 + (2*b*(b*B + 2*A*c)*x^(19/2))/19 + (2*c*(2*b*B + A*c
)*x^(23/2))/23 + (2*B*c^2*x^(27/2))/27
```

Rubi [A] time = 0.0388137, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1584, 448}

$$\frac{2}{15}Ab^2x^{15/2}+\frac{2}{23}cx^{23/2}(Ac+2bB)+\frac{2}{19}bx^{19/2}(2Ac+bB)+\frac{2}{27}Bc^2x^{27/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*A*b^2*x^(15/2))/15 + (2*b*(b*B + 2*A*c)*x^(19/2))/19 + (2*c*(2*b*B + A*c
)*x^(23/2))/23 + (2*B*c^2*x^(27/2))/27
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int x^{5/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^2\,dx=\int x^{13/2}\left(A+Bx^2\right)\left(b+cx^2\right)^2\,dx$$
$$=\int\left(Ab^2x^{13/2}+b(bB+2Ac)x^{17/2}+c(2bB+Ac)x^{21/2}+Bc^2x^{25/2}\right)dx$$
$$=\frac{2}{15}Ab^2x^{15/2}+\frac{2}{19}b(bB+2Ac)x^{19/2}+\frac{2}{23}c(2bB+Ac)x^{23/2}+\frac{2}{27}Bc^2x^{27/2}$$

Mathematica [A] time = 0.0297814, size = 53, normalized size = 0.84

$$\frac{2x^{15/2}\left(3933Ab^2+2565cx^4(Ac+2bB)+3105bx^2(2Ac+bB)+2185Bc^2x^6\right)}{58995}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*x^(15/2)*(3933*A*b^2 + 3105*b*(b*B + 2*A*c)*x^2 + 2565*c*(2*b*B + A*c)*x
^4 + 2185*B*c^2*x^6))/58995
```

Maple [A] time = 0.005, size = 56, normalized size = 0.9

$$\frac{4370\,Bc^2x^6+5130\,Ac^2x^4+10260\,Bx^4bc+12420\,Abcx^2+6210\,Bx^2b^2+7866\,Ab^2}{58995}x^{\frac{15}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x)
```

```
[Out] 2/58995*x^(15/2)*(2185*B*c^2*x^6+2565*A*c^2*x^4+5130*B*b*c*x^4+6210*A*b*c*x
^2+3105*B*b^2*x^2+3933*A*b^2)
```

Maxima [A] time = 1.15168, size = 69, normalized size = 1.1

$$\frac{2}{27}\,Bc^2x^{\frac{27}{2}}+\frac{2}{23}\left(2\,Bbc+Ac^2\right)x^{\frac{23}{2}}+\frac{2}{15}\,Ab^2x^{\frac{15}{2}}+\frac{2}{19}\left(Bb^2+2\,Abc\right)x^{\frac{19}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 2/27*B*c^2*x^(27/2) + 2/23*(2*B*b*c + A*c^2)*x^(23/2) + 2/15*A*b^2*x^(15/2)
 + 2/19*(B*b^2 + 2*A*b*c)*x^(19/2)
```

Fricas [A] time = 1.70015, size = 150, normalized size = 2.38

$$\frac{2}{58995}\left(2185\,Bc^2x^{13}+2565\left(2\,Bbc+Ac^2\right)x^{11}+3933\,Ab^2x^7+3105\left(Bb^2+2\,Abc\right)x^9\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 2/58995*(2185*B*c^2*x^13 + 2565*(2*B*b*c + A*c^2)*x^11 + 3933*A*b^2*x^7 + 3
105*(B*b^2 + 2*A*b*c)*x^9)*sqrt(x)
```

Sympy [A] time = 53.7629, size = 80, normalized size = 1.27

$$\frac{2Ab^2x^{\frac{15}{2}}}{15}+\frac{4Abcx^{\frac{19}{2}}}{19}+\frac{2Ac^2x^{\frac{23}{2}}}{23}+\frac{2Bb^2x^{\frac{19}{2}}}{19}+\frac{4Bbcx^{\frac{23}{2}}}{23}+\frac{2Bc^2x^{\frac{27}{2}}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(B*x**2+A)*(c*x**4+b*x**2)**2,x)
```

```
[Out] 2*A*b**2*x**(15/2)/15 + 4*A*b*c*x**(19/2)/19 + 2*A*c**2*x**(23/2)/23 + 2*B*
b**2*x**(19/2)/19 + 4*B*b*c*x**(23/2)/23 + 2*B*c**2*x**(27/2)/27
```

Giac [A] time = 1.16581, size = 72, normalized size = 1.14

$$\frac{2}{27}\,Bc^2x^{\frac{27}{2}}+\frac{4}{23}\,Bbcx^{\frac{23}{2}}+\frac{2}{23}\,Ac^2x^{\frac{23}{2}}+\frac{2}{19}\,Bb^2x^{\frac{19}{2}}+\frac{4}{19}\,Abcx^{\frac{19}{2}}+\frac{2}{15}\,Ab^2x^{\frac{15}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 2/27*B*c^2*x^(27/2) + 4/23*B*b*c*x^(23/2) + 2/23*A*c^2*x^(23/2) + 2/19*B*b^
2*x^(19/2) + 4/19*A*b*c*x^(19/2) + 2/15*A*b^2*x^(15/2)
```

3.169 $\int x^{3/2} \left(A + Bx^2\right) \left(bx^2 + cx^4\right)^2 \, dx$

Optimal. Leaf size=63

$$\frac{2}{13}Ab^2x^{13/2} + \frac{2}{21}cx^{21/2}(Ac + 2bB) + \frac{2}{17}bx^{17/2}(2Ac + bB) + \frac{2}{25}Bc^2x^{25/2}$$

```
[Out] (2*A*b^2*x^(13/2))/13 + (2*b*(b*B + 2*A*c)*x^(17/2))/17 + (2*c*(2*b*B + A*c
)*x^(21/2))/21 + (2*B*c^2*x^(25/2))/25
```

Rubi [A] time = 0.0380087, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1584, 448}

$$\frac{2}{13}Ab^2x^{13/2} + \frac{2}{21}cx^{21/2}(Ac + 2bB) + \frac{2}{17}bx^{17/2}(2Ac + bB) + \frac{2}{25}Bc^2x^{25/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*A*b^2*x^(13/2))/13 + (2*b*(b*B + 2*A*c)*x^(17/2))/17 + (2*c*(2*b*B + A*c
)*x^(21/2))/21 + (2*B*c^2*x^(25/2))/25
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int x^{3/2} \left(A+Bx^2\right) \left(bx^2+cx^4\right)^2 \, dx = \int x^{11/2} \left(A+Bx^2\right) \left(b+cx^2\right)^2 \, dx$$
$$= \int \left(Ab^2x^{11/2} + b(bB+2Ac)x^{15/2} + c(2bB+Ac)x^{19/2} + Bc^2x^{23/2}\right) dx$$
$$= \frac{2}{13}Ab^2x^{13/2} + \frac{2}{17}b(bB+2Ac)x^{17/2} + \frac{2}{21}c(2bB+Ac)x^{21/2} + \frac{2}{25}Bc^2x^{25/2}$$

Mathematica [A] time = 0.0294782, size = 63, normalized size = 1.

$$\frac{2}{13}Ab^2x^{13/2} + \frac{2}{21}cx^{21/2}(Ac+2bB) + \frac{2}{17}bx^{17/2}(2Ac+bB) + \frac{2}{25}Bc^2x^{25/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*A*b^2*x^(13/2))/13 + (2*b*(b*B + 2*A*c)*x^(17/2))/17 + (2*c*(2*b*B + A*c
)*x^(21/2))/21 + (2*B*c^2*x^(25/2))/25
```

Maple [A] time = 0.005, size = 56, normalized size = 0.9

$$\frac{9282\,Bc^2x^6 + 11050\,Ac^2x^4 + 22100\,Bx^4bc + 27300\,Abcx^2 + 13650\,Bx^2b^2 + 17850\,Ab^2}{116025}x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x)
```

```
[Out] 2/116025*x^(13/2)*(4641*B*c^2*x^6+5525*A*c^2*x^4+11050*B*b*c*x^4+13650*A*b*
c*x^2+6825*B*b^2*x^2+8925*A*b^2)
```

Maxima [A] time = 1.13105, size = 69, normalized size = 1.1

$$\frac{2}{25}\,Bc^2x^{\frac{25}{2}} + \frac{2}{21}\left(2\,Bbc + Ac^2\right)x^{\frac{21}{2}} + \frac{2}{13}\,Ab^2x^{\frac{13}{2}} + \frac{2}{17}\left(Bb^2 + 2\,Abc\right)x^{\frac{17}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 2/25*B*c^2*x^(25/2) + 2/21*(2*B*b*c + A*c^2)*x^(21/2) + 2/13*A*b^2*x^(13/2)
 + 2/17*(B*b^2 + 2*A*b*c)*x^(17/2)
```

Fricas [A] time = 1.53459, size = 151, normalized size = 2.4

$$\frac{2}{116025}\left(4641\,Bc^2x^{12} + 5525\left(2\,Bbc + Ac^2\right)x^{10} + 8925\,Ab^2x^6 + 6825\left(Bb^2 + 2\,Abc\right)x^8\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 2/116025*(4641*B*c^2*x^12 + 5525*(2*B*b*c + A*c^2)*x^10 + 8925*A*b^2*x^6 +
6825*(B*b^2 + 2*A*b*c)*x^8)*sqrt(x)
```

Sympy [A] time = 27.9737, size = 80, normalized size = 1.27

$$\frac{2Ab^2x^{\frac{13}{2}}}{13} + \frac{4Abcx^{\frac{17}{2}}}{17} + \frac{2Ac^2x^{\frac{21}{2}}}{21} + \frac{2Bb^2x^{\frac{17}{2}}}{17} + \frac{4Bbcx^{\frac{21}{2}}}{21} + \frac{2Bc^2x^{\frac{25}{2}}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(B*x**2+A)*(c*x**4+b*x**2)**2,x)
```

```
[Out] 2*A*b**2*x**(13/2)/13 + 4*A*b*c*x**(17/2)/17 + 2*A*c**2*x**(21/2)/21 + 2*B*
b**2*x**(17/2)/17 + 4*B*b*c*x**(21/2)/21 + 2*B*c**2*x**(25/2)/25
```

Giac [A] time = 1.21286, size = 72, normalized size = 1.14

$$\frac{2}{25}\,Bc^2x^{\frac{25}{2}} + \frac{4}{21}\,Bbcx^{\frac{21}{2}} + \frac{2}{21}\,Ac^2x^{\frac{21}{2}} + \frac{2}{17}\,Bb^2x^{\frac{17}{2}} + \frac{4}{17}\,Abcx^{\frac{17}{2}} + \frac{2}{13}\,Ab^2x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 2/25*B*c^2*x^(25/2) + 4/21*B*b*c*x^(21/2) + 2/21*A*c^2*x^(21/2) + 2/17*B*b^
2*x^(17/2) + 4/17*A*b*c*x^(17/2) + 2/13*A*b^2*x^(13/2)
```

3.170 $\int \sqrt{x}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^2\,dx$

Optimal. Leaf size=63

$$\frac{2}{11}Ab^2x^{11/2}+\frac{2}{19}cx^{19/2}(Ac+2bB)+\frac{2}{15}bx^{15/2}(2Ac+bB)+\frac{2}{23}Bc^2x^{23/2}$$

```
[Out] (2*A*b^2*x^(11/2))/11 + (2*b*(b*B + 2*A*c)*x^(15/2))/15 + (2*c*(2*b*B + A*c
)*x^(19/2))/19 + (2*B*c^2*x^(23/2))/23
```

Rubi [A] time = 0.0381059, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1584, 448}

$$\frac{2}{11}Ab^2x^{11/2}+\frac{2}{19}cx^{19/2}(Ac+2bB)+\frac{2}{15}bx^{15/2}(2Ac+bB)+\frac{2}{23}Bc^2x^{23/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*A*b^2*x^(11/2))/11 + (2*b*(b*B + 2*A*c)*x^(15/2))/15 + (2*c*(2*b*B + A*c
)*x^(19/2))/19 + (2*B*c^2*x^(23/2))/23
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \sqrt{x}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^2\,dx=\int x^{9/2}\left(A+Bx^2\right)\left(b+cx^2\right)^2\,dx$$
$$=\int\left(Ab^2x^{9/2}+b(bB+2Ac)x^{13/2}+c(2bB+Ac)x^{17/2}+Bc^2x^{21/2}\right)dx$$
$$=\frac{2}{11}Ab^2x^{11/2}+\frac{2}{15}b(bB+2Ac)x^{15/2}+\frac{2}{19}c(2bB+Ac)x^{19/2}+\frac{2}{23}Bc^2x^{23/2}$$

Mathematica [A] time = 0.0288025, size = 53, normalized size = 0.84

$$\frac{2x^{11/2}\left(6555Ab^2+3795cx^4(Ac+2bB)+4807bx^2(2Ac+bB)+3135Bc^2x^6\right)}{72105}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*x^(11/2)*(6555*A*b^2 + 4807*b*(b*B + 2*A*c)*x^2 + 3795*c*(2*b*B + A*c)*x
^4 + 3135*B*c^2*x^6))/72105
```

Maple [A] time = 0.005, size = 56, normalized size = 0.9

$$\frac{6270\,Bc^2x^6+7590\,Ac^2x^4+15180\,Bx^4bc+19228\,Abcx^2+9614\,Bx^2b^2+13110\,Ab^2}{72105}x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2*x^(1/2),x)
```

```
[Out] 2/72105*x^(11/2)*(3135*B*c^2*x^6+3795*A*c^2*x^4+7590*B*b*c*x^4+9614*A*b*c*x
^2+4807*B*b^2*x^2+6555*A*b^2)
```

Maxima [A] time = 1.12733, size = 69, normalized size = 1.1

$$\frac{2}{23}\,Bc^2x^{\frac{23}{2}}+\frac{2}{19}\left(2\,Bbc+Ac^2\right)x^{\frac{19}{2}}+\frac{2}{11}\,Ab^2x^{\frac{11}{2}}+\frac{2}{15}\left(Bb^2+2\,Abc\right)x^{\frac{15}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/23*B*c^2*x^(23/2) + 2/19*(2*B*b*c + A*c^2)*x^(19/2) + 2/11*A*b^2*x^(11/2)
 + 2/15*(B*b^2 + 2*A*b*c)*x^(15/2)
```

Fricas [A] time = 1.5609, size = 149, normalized size = 2.37

$$\frac{2}{72105}\left(3135\,Bc^2x^{11} + 3795\left(2\,Bbc + Ac^2\right)x^9 + 6555\,Ab^2x^5 + 4807\left(Bb^2 + 2\,Abc\right)x^7\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/72105*(3135*B*c^2*x^11 + 3795*(2*B*b*c + A*c^2)*x^9 + 6555*A*b^2*x^5 + 48
07*(B*b^2 + 2*A*b*c)*x^7)*sqrt(x)
```

Sympy [A] time = 6.59938, size = 66, normalized size = 1.05

$$\frac{2Ab^2x^{\frac{11}{2}}}{11} + \frac{2Bc^2x^{\frac{23}{2}}}{23} + \frac{2x^{\frac{19}{2}}\left(Ac^2 + 2Bbc\right)}{19} + \frac{2x^{\frac{15}{2}}\left(2Abc + Bb^2\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2*x**(1/2),x)
```

```
[Out] 2*A*b**2*x**(11/2)/11 + 2*B*c**2*x**(23/2)/23 + 2*x**(19/2)*(A*c**2 + 2*B*b
*c)/19 + 2*x**(15/2)*(2*A*b*c + B*b**2)/15
```

Giac [A] time = 1.17093, size = 72, normalized size = 1.14

$$\frac{2}{23}\,Bc^2x^{\frac{23}{2}} + \frac{4}{19}\,Bbcx^{\frac{19}{2}} + \frac{2}{19}\,Ac^2x^{\frac{19}{2}} + \frac{2}{15}\,Bb^2x^{\frac{15}{2}} + \frac{4}{15}\,Abcx^{\frac{15}{2}} + \frac{2}{11}\,Ab^2x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/23*B*c^2*x^(23/2) + 4/19*B*b*c*x^(19/2) + 2/19*A*c^2*x^(19/2) + 2/15*B*b^
2*x^(15/2) + 4/15*A*b*c*x^(15/2) + 2/11*A*b^2*x^(11/2)
```

3.171 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{\sqrt{x}} \, dx$

Optimal. Leaf size=63

$$\frac{2}{9}Ab^2x^{9/2} + \frac{2}{17}cx^{17/2}(Ac + 2bB) + \frac{2}{13}bx^{13/2}(2Ac + bB) + \frac{2}{21}Bc^2x^{21/2}$$

```
[Out] (2*A*b^2*x^(9/2))/9 + (2*b*(b*B + 2*A*c)*x^(13/2))/13 + (2*c*(2*b*B + A*c)*
x^(17/2))/17 + (2*B*c^2*x^(21/2))/21
```

Rubi [A] time = 0.0392227, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1584, 448}

$$\frac{2}{9}Ab^2x^{9/2} + \frac{2}{17}cx^{17/2}(Ac + 2bB) + \frac{2}{13}bx^{13/2}(2Ac + bB) + \frac{2}{21}Bc^2x^{21/2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/Sqrt[x],x]
```

```
[Out] (2*A*b^2*x^(9/2))/9 + (2*b*(b*B + 2*A*c)*x^(13/2))/13 + (2*c*(2*b*B + A*c)*
x^(17/2))/17 + (2*B*c^2*x^(21/2))/21
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{\sqrt{x}}\,dx = \int x^{7/2}(A+Bx^2)(b+cx^2)^2\,dx$$
$$= \int \left(Ab^2x^{7/2} + b(bB+2Ac)x^{11/2} + c(2bB+Ac)x^{15/2} + Bc^2x^{19/2}\right)dx$$
$$= \frac{2}{9}Ab^2x^{9/2} + \frac{2}{13}b(bB+2Ac)x^{13/2} + \frac{2}{17}c(2bB+Ac)x^{17/2} + \frac{2}{21}Bc^2x^{21/2}$$

Mathematica [A] time = 0.0288147, size = 53, normalized size = 0.84

$$\frac{2x^{9/2}\left(1547Ab^2 + 819cx^4(Ac+2bB) + 1071bx^2(2Ac+bB) + 663Bc^2x^6\right)}{13923}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/Sqrt[x],x]
```

```
[Out] (2*x^(9/2)*(1547*A*b^2 + 1071*b*(b*B + 2*A*c)*x^2 + 819*c*(2*b*B + A*c)*x^4
 + 663*B*c^2*x^6))/13923
```

Maple [A] time = 0.006, size = 56, normalized size = 0.9

$$\frac{1326\,Bc^2x^6 + 1638\,Ac^2x^4 + 3276\,Bx^4bc + 4284\,Abcx^2 + 2142\,Bx^2b^2 + 3094\,Ab^2}{13923}x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^(1/2),x)
```

```
[Out] 2/13923*x^(9/2)*(663*B*c^2*x^6+819*A*c^2*x^4+1638*B*b*c*x^4+2142*A*b*c*x^2+
1071*B*b^2*x^2+1547*A*b^2)
```

Maxima [A] time = 1.17118, size = 69, normalized size = 1.1

$$\frac{2}{21}\,Bc^2x^{\frac{21}{2}} + \frac{2}{17}\left(2\,Bbc + Ac^2\right)x^{\frac{17}{2}} + \frac{2}{9}\,Ab^2x^{\frac{9}{2}} + \frac{2}{13}\left(Bb^2 + 2\,Abc\right)x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/21*B*c^2*x^(21/2) + 2/17*(2*B*b*c + A*c^2)*x^(17/2) + 2/9*A*b^2*x^(9/2) +
 2/13*(B*b^2 + 2*A*b*c)*x^(13/2)
```

Fricas [A] time = 1.54821, size = 146, normalized size = 2.32

$$\frac{2}{13923}\left(663\,Bc^2x^{10} + 819\left(2\,Bbc + Ac^2\right)x^8 + 1547\,Ab^2x^4 + 1071\left(Bb^2 + 2\,Abc\right)x^6\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/13923*(663*B*c^2*x^10 + 819*(2*B*b*c + A*c^2)*x^8 + 1547*A*b^2*x^4 + 1071
*(B*b^2 + 2*A*b*c)*x^6)*sqrt(x)
```

Sympy [A] time = 11.9179, size = 80, normalized size = 1.27

$$\frac{2Ab^2x^{\frac{9}{2}}}{9} + \frac{4Abcx^{\frac{13}{2}}}{13} + \frac{2Ac^2x^{\frac{17}{2}}}{17} + \frac{2Bb^2x^{\frac{13}{2}}}{13} + \frac{4Bbcx^{\frac{17}{2}}}{17} + \frac{2Bc^2x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**(1/2),x)
```

```
[Out] 2*A*b**2*x**(9/2)/9 + 4*A*b*c*x**(13/2)/13 + 2*A*c**2*x**(17/2)/17 + 2*B*b*
*2*x**(13/2)/13 + 4*B*b*c*x**(17/2)/17 + 2*B*c**2*x**(21/2)/21
```

Giac [A] time = 1.16155, size = 72, normalized size = 1.14

$$\frac{2}{21}\,Bc^2x^{\frac{21}{2}} + \frac{4}{17}\,Bbcx^{\frac{17}{2}} + \frac{2}{17}\,Ac^2x^{\frac{17}{2}} + \frac{2}{13}\,Bb^2x^{\frac{13}{2}} + \frac{4}{13}\,Abcx^{\frac{13}{2}} + \frac{2}{9}\,Ab^2x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/21*B*c^2*x^(21/2) + 4/17*B*b*c*x^(17/2) + 2/17*A*c^2*x^(17/2) + 2/13*B*b^
2*x^(13/2) + 4/13*A*b*c*x^(13/2) + 2/9*A*b^2*x^(9/2)
```

3.172 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{3/2}}\,dx$

Optimal. Leaf size=63

$$\frac{2}{7}Ab^2x^{7/2}+\frac{2}{15}cx^{15/2}(Ac+2bB)+\frac{2}{11}bx^{11/2}(2Ac+bB)+\frac{2}{19}Bc^2x^{19/2}$$

```
[Out] (2*A*b^2*x^(7/2))/7 + (2*b*(b*B + 2*A*c)*x^(11/2))/11 + (2*c*(2*b*B + A*c)*
x^(15/2))/15 + (2*B*c^2*x^(19/2))/19
```

Rubi [A] time = 0.0393163, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1584, 448}

$$\frac{2}{7}Ab^2x^{7/2}+\frac{2}{15}cx^{15/2}(Ac+2bB)+\frac{2}{11}bx^{11/2}(2Ac+bB)+\frac{2}{19}Bc^2x^{19/2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^(3/2),x]
```

```
[Out] (2*A*b^2*x^(7/2))/7 + (2*b*(b*B + 2*A*c)*x^(11/2))/11 + (2*c*(2*b*B + A*c)*
x^(15/2))/15 + (2*B*c^2*x^(19/2))/19
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{3/2}}\,dx = \int x^{5/2}(A+Bx^2)(b+cx^2)^2\,dx$$
$$= \int \left(Ab^2x^{5/2} + b(bB+2Ac)x^{9/2} + c(2bB+Ac)x^{13/2} + Bc^2x^{17/2}\right)dx$$
$$= \frac{2}{7}Ab^2x^{7/2} + \frac{2}{11}b(bB+2Ac)x^{11/2} + \frac{2}{15}c(2bB+Ac)x^{15/2} + \frac{2}{19}Bc^2x^{19/2}$$

Mathematica [A] time = 0.0278801, size = 63, normalized size = 1.

$$\frac{2}{7}Ab^2x^{7/2} + \frac{2}{15}cx^{15/2}(Ac+2bB) + \frac{2}{11}bx^{11/2}(2Ac+bB) + \frac{2}{19}Bc^2x^{19/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^(3/2),x]
```

```
[Out] (2*A*b^2*x^(7/2))/7 + (2*b*(b*B + 2*A*c)*x^(11/2))/11 + (2*c*(2*b*B + A*c)*
x^(15/2))/15 + (2*B*c^2*x^(19/2))/19
```

Maple [A] time = 0.005, size = 56, normalized size = 0.9

$$\frac{2310\,Bc^2x^6 + 2926\,Ac^2x^4 + 5852\,Bx^4bc + 7980\,Abcx^2 + 3990\,Bx^2b^2 + 6270\,Ab^2}{21945}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^(3/2),x)
```

```
[Out] 2/21945*x^(7/2)*(1155*B*c^2*x^6+1463*A*c^2*x^4+2926*B*b*c*x^4+3990*A*b*c*x^
2+1995*B*b^2*x^2+3135*A*b^2)
```

Maxima [A] time = 1.17092, size = 69, normalized size = 1.1

$$\frac{2}{19}\,Bc^2x^{\frac{19}{2}} + \frac{2}{15}\left(2\,Bbc + Ac^2\right)x^{\frac{15}{2}} + \frac{2}{7}\,Ab^2x^{\frac{7}{2}} + \frac{2}{11}\left(Bb^2 + 2\,Abc\right)x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/19*B*c^2*x^(19/2) + 2/15*(2*B*b*c + A*c^2)*x^(15/2) + 2/7*A*b^2*x^(7/2) +
 2/11*(B*b^2 + 2*A*b*c)*x^(11/2)
```

Fricas [A] time = 1.56349, size = 147, normalized size = 2.33

$$\frac{2}{21945}\left(1155\,Bc^2x^9 + 1463\left(2\,Bbc + Ac^2\right)x^7 + 3135\,Ab^2x^3 + 1995\left(Bb^2 + 2\,Abc\right)x^5\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/21945*(1155*B*c^2*x^9 + 1463*(2*B*b*c + A*c^2)*x^7 + 3135*A*b^2*x^3 + 199
5*(B*b^2 + 2*A*b*c)*x^5)*sqrt(x)
```

Sympy [A] time = 13.4522, size = 80, normalized size = 1.27

$$\frac{2Ab^2x^{\frac{7}{2}}}{7} + \frac{4Abcx^{\frac{11}{2}}}{11} + \frac{2Ac^2x^{\frac{15}{2}}}{15} + \frac{2Bb^2x^{\frac{11}{2}}}{11} + \frac{4Bbcx^{\frac{15}{2}}}{15} + \frac{2Bc^2x^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**(3/2),x)
```

```
[Out] 2*A*b**2*x**(7/2)/7 + 4*A*b*c*x**(11/2)/11 + 2*A*c**2*x**(15/2)/15 + 2*B*b*
*2*x**(11/2)/11 + 4*B*b*c*x**(15/2)/15 + 2*B*c**2*x**(19/2)/19
```

Giac [A] time = 1.12225, size = 72, normalized size = 1.14

$$\frac{2}{19}\,Bc^2x^{\frac{19}{2}} + \frac{4}{15}\,Bbcx^{\frac{15}{2}} + \frac{2}{15}\,Ac^2x^{\frac{15}{2}} + \frac{2}{11}\,Bb^2x^{\frac{11}{2}} + \frac{4}{11}\,Abcx^{\frac{11}{2}} + \frac{2}{7}\,Ab^2x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/19*B*c^2*x^(19/2) + 4/15*B*b*c*x^(15/2) + 2/15*A*c^2*x^(15/2) + 2/11*B*b^
2*x^(11/2) + 4/11*A*b*c*x^(11/2) + 2/7*A*b^2*x^(7/2)
```

3.173 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{5/2}} \, dx$

Optimal. Leaf size=63

$$\frac{2}{5}Ab^2x^{5/2} + \frac{2}{13}cx^{13/2}(Ac + 2bB) + \frac{2}{9}bx^{9/2}(2Ac + bB) + \frac{2}{17}Bc^2x^{17/2}$$

```
[Out] (2*A*b^2*x^(5/2))/5 + (2*b*(b*B + 2*A*c)*x^(9/2))/9 + (2*c*(2*b*B + A*c)*x^
(13/2))/13 + (2*B*c^2*x^(17/2))/17
```

Rubi [A] time = 0.038272, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1584, 448}

$$\frac{2}{5}Ab^2x^{5/2} + \frac{2}{13}cx^{13/2}(Ac + 2bB) + \frac{2}{9}bx^{9/2}(2Ac + bB) + \frac{2}{17}Bc^2x^{17/2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^(5/2),x]
```

```
[Out] (2*A*b^2*x^(5/2))/5 + (2*b*(b*B + 2*A*c)*x^(9/2))/9 + (2*c*(2*b*B + A*c)*x^
(13/2))/13 + (2*B*c^2*x^(17/2))/17
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{5/2}}\,dx = \int x^{3/2}(A+Bx^2)(b+cx^2)^2\,dx$$
$$= \int \left(Ab^2x^{3/2} + b(bB+2Ac)x^{7/2} + c(2bB+Ac)x^{11/2} + Bc^2x^{15/2}\right)dx$$
$$= \frac{2}{5}Ab^2x^{5/2} + \frac{2}{9}b(bB+2Ac)x^{9/2} + \frac{2}{13}c(2bB+Ac)x^{13/2} + \frac{2}{17}Bc^2x^{17/2}$$

Mathematica [A] time = 0.0298686, size = 53, normalized size = 0.84

$$\frac{2x^{5/2}\left(1989Ab^2 + 765cx^4(Ac+2bB) + 1105bx^2(2Ac+bB) + 585Bc^2x^6\right)}{9945}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^(5/2),x]
```

```
[Out] (2*x^(5/2)*(1989*A*b^2 + 1105*b*(b*B + 2*A*c)*x^2 + 765*c*(2*b*B + A*c)*x^4
 + 585*B*c^2*x^6))/9945
```

Maple [A] time = 0.005, size = 56, normalized size = 0.9

$$\frac{1170\,Bc^2x^6 + 1530\,Ac^2x^4 + 3060\,Bx^4bc + 4420\,Abcx^2 + 2210\,Bx^2b^2 + 3978\,Ab^2}{9945}x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^(5/2),x)
```

```
[Out] 2/9945*x^(5/2)*(585*B*c^2*x^6+765*A*c^2*x^4+1530*B*b*c*x^4+2210*A*b*c*x^2+1
105*B*b^2*x^2+1989*A*b^2)
```

Maxima [A] time = 1.1389, size = 69, normalized size = 1.1

$$\frac{2}{17}\,Bc^2x^{\frac{17}{2}} + \frac{2}{13}\left(2\,Bbc + Ac^2\right)x^{\frac{13}{2}} + \frac{2}{5}\,Ab^2x^{\frac{5}{2}} + \frac{2}{9}\left(Bb^2 + 2\,Abc\right)x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/17*B*c^2*x^(17/2) + 2/13*(2*B*b*c + A*c^2)*x^(13/2) + 2/5*A*b^2*x^(5/2) +
 2/9*(B*b^2 + 2*A*b*c)*x^(9/2)
```

Fricas [A] time = 1.59709, size = 143, normalized size = 2.27

$$\frac{2}{9945}\left(585\,Bc^2x^8 + 765\left(2\,Bbc + Ac^2\right)x^6 + 1989\,Ab^2x^2 + 1105\left(Bb^2 + 2\,Abc\right)x^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/9945*(585*B*c^2*x^8 + 765*(2*B*b*c + A*c^2)*x^6 + 1989*A*b^2*x^2 + 1105*(
B*b^2 + 2*A*b*c)*x^4)*sqrt(x)
```

Sympy [A] time = 16.4983, size = 80, normalized size = 1.27

$$\frac{2Ab^2x^{\frac{5}{2}}}{5} + \frac{4Abcx^{\frac{9}{2}}}{9} + \frac{2Ac^2x^{\frac{13}{2}}}{13} + \frac{2Bb^2x^{\frac{9}{2}}}{9} + \frac{4Bbcx^{\frac{13}{2}}}{13} + \frac{2Bc^2x^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**(5/2),x)
```

```
[Out] 2*A*b**2*x**(5/2)/5 + 4*A*b*c*x**(9/2)/9 + 2*A*c**2*x**(13/2)/13 + 2*B*b**2
*x**(9/2)/9 + 4*B*b*c*x**(13/2)/13 + 2*B*c**2*x**(17/2)/17
```

Giac [A] time = 1.13223, size = 72, normalized size = 1.14

$$\frac{2}{17}\,Bc^2x^{\frac{17}{2}} + \frac{4}{13}\,Bbcx^{\frac{13}{2}} + \frac{2}{13}\,Ac^2x^{\frac{13}{2}} + \frac{2}{9}\,Bb^2x^{\frac{9}{2}} + \frac{4}{9}\,Abcx^{\frac{9}{2}} + \frac{2}{5}\,Ab^2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/17*B*c^2*x^(17/2) + 4/13*B*b*c*x^(13/2) + 2/13*A*c^2*x^(13/2) + 2/9*B*b^2
*x^(9/2) + 4/9*A*b*c*x^(9/2) + 2/5*A*b^2*x^(5/2)
```

3.174 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{7/2}}\,dx$

Optimal. Leaf size=63

$$\frac{2}{3}Ab^2x^{3/2}+\frac{2}{11}cx^{11/2}(Ac+2bB)+\frac{2}{7}bx^{7/2}(2Ac+bB)+\frac{2}{15}Bc^2x^{15/2}$$

```
[Out] (2*A*b^2*x^(3/2))/3 + (2*b*(b*B + 2*A*c)*x^(7/2))/7 + (2*c*(2*b*B + A*c)*x^
(11/2))/11 + (2*B*c^2*x^(15/2))/15
```

Rubi [A] time = 0.0379469, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1584, 448}

$$\frac{2}{3}Ab^2x^{3/2}+\frac{2}{11}cx^{11/2}(Ac+2bB)+\frac{2}{7}bx^{7/2}(2Ac+bB)+\frac{2}{15}Bc^2x^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^(7/2),x]
```

```
[Out] (2*A*b^2*x^(3/2))/3 + (2*b*(b*B + 2*A*c)*x^(7/2))/7 + (2*c*(2*b*B + A*c)*x^
(11/2))/11 + (2*B*c^2*x^(15/2))/15
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{7/2}}\,dx = \int \sqrt{x}(A+Bx^2)(b+cx^2)^2\,dx$$
$$= \int \left(Ab^2\sqrt{x}+b(bB+2Ac)x^{5/2}+c(2bB+Ac)x^{9/2}+Bc^2x^{13/2}\right)dx$$
$$= \frac{2}{3}Ab^2x^{3/2}+\frac{2}{7}b(bB+2Ac)x^{7/2}+\frac{2}{11}c(2bB+Ac)x^{11/2}+\frac{2}{15}Bc^2x^{15/2}$$

Mathematica [A] time = 0.0297101, size = 53, normalized size = 0.84

$$\frac{2x^{3/2}\left(385Ab^2+105cx^4(Ac+2bB)+165bx^2(2Ac+bB)+77Bc^2x^6\right)}{1155}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^(7/2),x]
```

```
[Out] (2*x^(3/2)*(385*A*b^2 + 165*b*(b*B + 2*A*c)*x^2 + 105*c*(2*b*B + A*c)*x^4 +
 77*B*c^2*x^6))/1155
```

Maple [A] time = 0.006, size = 56, normalized size = 0.9

$$\frac{154\,Bc^2x^6+210\,Ac^2x^4+420\,Bx^4bc+660\,Abcx^2+330\,Bx^2b^2+770\,Ab^2}{1155}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^(7/2),x)
```

```
[Out] 2/1155*x^(3/2)*(77*B*c^2*x^6+105*A*c^2*x^4+210*B*b*c*x^4+330*A*b*c*x^2+165*
B*b^2*x^2+385*A*b^2)
```

Maxima [A] time = 1.16944, size = 69, normalized size = 1.1

$$\frac{2}{15}\,Bc^2x^{\frac{15}{2}}+\frac{2}{11}\left(2\,Bbc+Ac^2\right)x^{\frac{11}{2}}+\frac{2}{3}\,Ab^2x^{\frac{3}{2}}+\frac{2}{7}\left(Bb^2+2\,Abc\right)x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/15*B*c^2*x^(15/2) + 2/11*(2*B*b*c + A*c^2)*x^(11/2) + 2/3*A*b^2*x^(3/2) +
 2/7*(B*b^2 + 2*A*b*c)*x^(7/2)
```

Fricas [A] time = 1.46583, size = 136, normalized size = 2.16

$$\frac{2}{1155}\left(77\,Bc^2x^7+105\left(2\,Bbc+Ac^2\right)x^5+385\,Ab^2x+165\left(Bb^2+2\,Abc\right)x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*B*c^2*x^7 + 105*(2*B*b*c + A*c^2)*x^5 + 385*A*b^2*x + 165*(B*b^2
 + 2*A*b*c)*x^3)*sqrt(x)
```

Sympy [A] time = 21.5161, size = 80, normalized size = 1.27

$$\frac{2Ab^2x^{\frac{3}{2}}}{3}+\frac{4Abcx^{\frac{7}{2}}}{7}+\frac{2Ac^2x^{\frac{11}{2}}}{11}+\frac{2Bb^2x^{\frac{7}{2}}}{7}+\frac{4Bbcx^{\frac{11}{2}}}{11}+\frac{2Bc^2x^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**(7/2),x)
```

```
[Out] 2*A*b**2*x**(3/2)/3 + 4*A*b*c*x**(7/2)/7 + 2*A*c**2*x**(11/2)/11 + 2*B*b**2
*x**(7/2)/7 + 4*B*b*c*x**(11/2)/11 + 2*B*c**2*x**(15/2)/15
```

Giac [A] time = 1.19147, size = 72, normalized size = 1.14

$$\frac{2}{15}\,Bc^2x^{\frac{15}{2}}+\frac{4}{11}\,Bbcx^{\frac{11}{2}}+\frac{2}{11}\,Ac^2x^{\frac{11}{2}}+\frac{2}{7}\,Bb^2x^{\frac{7}{2}}+\frac{4}{7}\,Abcx^{\frac{7}{2}}+\frac{2}{3}\,Ab^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/15*B*c^2*x^(15/2) + 4/11*B*b*c*x^(11/2) + 2/11*A*c^2*x^(11/2) + 2/7*B*b^2
*x^(7/2) + 4/7*A*b*c*x^(7/2) + 2/3*A*b^2*x^(3/2)
```

3.175 $\int x^{7/2} \left(A+Bx^2\right) \left(bx^2+cx^4\right)^3 \, dx$

Optimal. Leaf size=85

$$\frac{2}{25}b^2x^{25/2}(3Ac+bB)+\frac{2}{21}Ab^3x^{21/2}+\frac{2}{33}c^2x^{33/2}(Ac+3bB)+\frac{6}{29}bcx^{29/2}(Ac+bB)+\frac{2}{37}Bc^3x^{37/2}$$

```
[Out] (2*A*b^3*x^(21/2))/21 + (2*b^2*(b*B + 3*A*c)*x^(25/2))/25 + (6*b*c*(b*B + A
*c)*x^(29/2))/29 + (2*c^2*(3*b*B + A*c)*x^(33/2))/33 + (2*B*c^3*x^(37/2))/3
7
```

Rubi [A] time = 0.0523875, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1584, 448}

$$\frac{2}{25}b^2x^{25/2}(3Ac+bB)+\frac{2}{21}Ab^3x^{21/2}+\frac{2}{33}c^2x^{33/2}(Ac+3bB)+\frac{6}{29}bcx^{29/2}(Ac+bB)+\frac{2}{37}Bc^3x^{37/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*A*b^3*x^(21/2))/21 + (2*b^2*(b*B + 3*A*c)*x^(25/2))/25 + (6*b*c*(b*B + A
*c)*x^(29/2))/29 + (2*c^2*(3*b*B + A*c)*x^(33/2))/33 + (2*B*c^3*x^(37/2))/3
7
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int x^{7/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^3\,dx=\int x^{19/2}\left(A+Bx^2\right)\left(b+cx^2\right)^3\,dx$$
$$=\int\left(Ab^3x^{19/2}+b^2(bB+3Ac)x^{23/2}+3bc(bB+Ac)x^{27/2}+c^2(3bB+Ac)x^{31/2}+Bc^3x^{35}\right.$$
$$=\frac{2}{21}Ab^3x^{21/2}+\frac{2}{25}b^2(bB+3Ac)x^{25/2}+\frac{6}{29}bc(bB+Ac)x^{29/2}+\frac{2}{33}c^2(3bB+Ac)x^{33/2}+$$

Mathematica [A] time = 0.0471675, size = 85, normalized size = 1.

$$\frac{2}{25}b^2x^{25/2}(3Ac+bB)+\frac{2}{21}Ab^3x^{21/2}+\frac{2}{33}c^2x^{33/2}(Ac+3bB)+\frac{6}{29}bcx^{29/2}(Ac+bB)+\frac{2}{37}Bc^3x^{37/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*A*b^3*x^(21/2))/21 + (2*b^2*(b*B + 3*A*c)*x^(25/2))/25 + (6*b*c*(b*B + A
*c)*x^(29/2))/29 + (2*c^2*(3*b*B + A*c)*x^(33/2))/33 + (2*B*c^3*x^(37/2))/3
7
```

Maple [A] time = 0.006, size = 80, normalized size = 0.9

$$\frac{334950\,Bc^3x^8+375550\,Ac^3x^6+1126650\,Bx^6bc^2+1282050\,Abc^2x^4+1282050\,Bx^4b^2c+1487178\,Ab^2cx^2+495726\,Bx^2b}{6196575}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x)
```

```
[Out] 2/6196575*x^(21/2)*(167475*B*c^3*x^8+187775*A*c^3*x^6+563325*B*b*c^2*x^6+64
1025*A*b*c^2*x^4+641025*B*b^2*c*x^4+743589*A*b^2*c*x^2+247863*B*b^3*x^2+295
075*A*b^3)
```

Maxima [A] time = 1.08118, size = 99, normalized size = 1.16

$$\frac{2}{37}\,Bc^3x^{\frac{37}{2}}+\frac{2}{33}\left(3\,Bbc^2+Ac^3\right)x^{\frac{33}{2}}+\frac{6}{29}\left(Bb^2c+Abc^2\right)x^{\frac{29}{2}}+\frac{2}{21}\,Ab^3x^{\frac{21}{2}}+\frac{2}{25}\left(Bb^3+3\,Ab^2c\right)x^{\frac{25}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 2/37*B*c^3*x^(37/2) + 2/33*(3*B*b*c^2 + A*c^3)*x^(33/2) + 6/29*(B*b^2*c + A
*b*c^2)*x^(29/2) + 2/21*A*b^3*x^(21/2) + 2/25*(B*b^3 + 3*A*b^2*c)*x^(25/2)
```

Fricas [A] time = 1.4954, size = 217, normalized size = 2.55

$$\frac{2}{6196575}\left(167475\,Bc^3x^{18}+187775\left(3\,Bbc^2+Ac^3\right)x^{16}+641025\left(Bb^2c+Abc^2\right)x^{14}+295075\,Ab^3x^{10}+247863\left(Bb^3+3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 2/6196575*(167475*B*c^3*x^18 + 187775*(3*B*b*c^2 + A*c^3)*x^16 + 641025*(B*
b^2*c + A*b*c^2)*x^14 + 295075*A*b^3*x^10 + 247863*(B*b^3 + 3*A*b^2*c)*x^12
)*sqrt(x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(B*x**2+A)*(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11191, size = 104, normalized size = 1.22

$$\frac{2}{37}\,Bc^3x^{\frac{37}{2}}+\frac{2}{11}\,Bbc^2x^{\frac{33}{2}}+\frac{2}{33}\,Ac^3x^{\frac{33}{2}}+\frac{6}{29}\,Bb^2cx^{\frac{29}{2}}+\frac{6}{29}\,Abc^2x^{\frac{29}{2}}+\frac{2}{25}\,Bb^3x^{\frac{25}{2}}+\frac{6}{25}\,Ab^2cx^{\frac{25}{2}}+\frac{2}{21}\,Ab^3x^{\frac{21}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 2/37*B*c^3*x^(37/2) + 2/11*B*b*c^2*x^(33/2) + 2/33*A*c^3*x^(33/2) + 6/29*B*
b^2*c*x^(29/2) + 6/29*A*b*c^2*x^(29/2) + 2/25*B*b^3*x^(25/2) + 6/25*A*b^2*c
*x^(25/2) + 2/21*A*b^3*x^(21/2)
```

3.176 $\int x^{5/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^3\,dx$

Optimal. Leaf size=85

$$\frac{2}{23}b^2x^{23/2}(3Ac+bB)+\frac{2}{19}Ab^3x^{19/2}+\frac{2}{31}c^2x^{31/2}(Ac+3bB)+\frac{2}{9}bcx^{27/2}(Ac+bB)+\frac{2}{35}Bc^3x^{35/2}$$

```
[Out] (2*A*b^3*x^(19/2))/19 + (2*b^2*(b*B + 3*A*c)*x^(23/2))/23 + (2*b*c*(b*B + A
*c)*x^(27/2))/9 + (2*c^2*(3*b*B + A*c)*x^(31/2))/31 + (2*B*c^3*x^(35/2))/35
```

Rubi [A] time = 0.0497119, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1584, 448}

$$\frac{2}{23}b^2x^{23/2}(3Ac+bB)+\frac{2}{19}Ab^3x^{19/2}+\frac{2}{31}c^2x^{31/2}(Ac+3bB)+\frac{2}{9}bcx^{27/2}(Ac+bB)+\frac{2}{35}Bc^3x^{35/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*A*b^3*x^(19/2))/19 + (2*b^2*(b*B + 3*A*c)*x^(23/2))/23 + (2*b*c*(b*B + A
*c)*x^(27/2))/9 + (2*c^2*(3*b*B + A*c)*x^(31/2))/31 + (2*B*c^3*x^(35/2))/35
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int x^{5/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^3\,dx=\int x^{17/2}\left(A+Bx^2\right)\left(b+cx^2\right)^3\,dx$$
$$=\int\left(Ab^3x^{17/2}+b^2(bB+3Ac)x^{21/2}+3bc(bB+Ac)x^{25/2}+c^2(3bB+Ac)x^{29/2}+Bc^3x^{33}\right.$$
$$=\frac{2}{19}Ab^3x^{19/2}+\frac{2}{23}b^2(bB+3Ac)x^{23/2}+\frac{2}{9}bc(bB+Ac)x^{27/2}+\frac{2}{31}c^2(3bB+Ac)x^{31/2}+\frac{2}{3}$$

Mathematica [A] time = 0.0431332, size = 85, normalized size = 1.

$$\frac{2}{23}b^2x^{23/2}(3Ac+bB)+\frac{2}{19}Ab^3x^{19/2}+\frac{2}{31}c^2x^{31/2}(Ac+3bB)+\frac{2}{9}bcx^{27/2}(Ac+bB)+\frac{2}{35}Bc^3x^{35/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*A*b^3*x^(19/2))/19 + (2*b^2*(b*B + 3*A*c)*x^(23/2))/23 + (2*b*c*(b*B + A
*c)*x^(27/2))/9 + (2*c^2*(3*b*B + A*c)*x^(31/2))/31 + (2*B*c^3*x^(35/2))/35
```

Maple [A] time = 0.007, size = 80, normalized size = 0.9

$$\frac{243846\,Bc^3x^8+275310\,Ac^3x^6+825930\,Bx^6bc^2+948290\,Abc^2x^4+948290\,Bx^4b^2c+1113210\,Ab^2cx^2+371070\,Bx^2b^3+}{4267305}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x)
```

```
[Out] 2/4267305*x^(19/2)*(121923*B*c^3*x^8+137655*A*c^3*x^6+412965*B*b*c^2*x^6+47
4145*A*b*c^2*x^4+474145*B*b^2*c*x^4+556605*A*b^2*c*x^2+185535*B*b^3*x^2+224
595*A*b^3)
```

Maxima [A] time = 1.1156, size = 99, normalized size = 1.16

$$\frac{2}{35}\,Bc^3x^{\frac{35}{2}}+\frac{2}{31}\left(3\,Bbc^2+Ac^3\right)x^{\frac{31}{2}}+\frac{2}{9}\left(Bb^2c+Abc^2\right)x^{\frac{27}{2}}+\frac{2}{19}\,Ab^3x^{\frac{19}{2}}+\frac{2}{23}\left(Bb^3+3\,Ab^2c\right)x^{\frac{23}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 2/35*B*c^3*x^(35/2) + 2/31*(3*B*b*c^2 + A*c^3)*x^(31/2) + 2/9*(B*b^2*c + A*
b*c^2)*x^(27/2) + 2/19*A*b^3*x^(19/2) + 2/23*(B*b^3 + 3*A*b^2*c)*x^(23/2)
```

Fricas [A] time = 1.6826, size = 216, normalized size = 2.54

$$\frac{2}{4267305}\left(121923\,Bc^3x^{17}+137655\left(3\,Bbc^2+Ac^3\right)x^{15}+474145\left(Bb^2c+Abc^2\right)x^{13}+224595\,Ab^3x^9+185535\left(Bb^3+3\,\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 2/4267305*(121923*B*c^3*x^17 + 137655*(3*B*b*c^2 + A*c^3)*x^15 + 474145*(B*
b^2*c + A*b*c^2)*x^13 + 224595*A*b^3*x^9 + 185535*(B*b^3 + 3*A*b^2*c)*x^11)
*sqrt(x)
```

Sympy [A] time = 165.782, size = 114, normalized size = 1.34

$$\frac{2Ab^3x^{\frac{19}{2}}}{19}+\frac{6Ab^2cx^{\frac{23}{2}}}{23}+\frac{2Abc^2x^{\frac{27}{2}}}{9}+\frac{2Ac^3x^{\frac{31}{2}}}{31}+\frac{2Bb^3x^{\frac{23}{2}}}{23}+\frac{2Bb^2cx^{\frac{27}{2}}}{9}+\frac{6Bbc^2x^{\frac{31}{2}}}{31}+\frac{2Bc^3x^{\frac{35}{2}}}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(B*x**2+A)*(c*x**4+b*x**2)**3,x)
```

```
[Out] 2*A*b**3*x**(19/2)/19 + 6*A*b**2*c*x**(23/2)/23 + 2*A*b*c**2*x**(27/2)/9 +
2*A*c**3*x**(31/2)/31 + 2*B*b**3*x**(23/2)/23 + 2*B*b**2*c*x**(27/2)/9 + 6*
B*b*c**2*x**(31/2)/31 + 2*B*c**3*x**(35/2)/35
```

Giac [A] time = 1.12029, size = 104, normalized size = 1.22

$$\frac{2}{35}\,Bc^3x^{\frac{35}{2}}+\frac{6}{31}\,Bbc^2x^{\frac{31}{2}}+\frac{2}{31}\,Ac^3x^{\frac{31}{2}}+\frac{2}{9}\,Bb^2cx^{\frac{27}{2}}+\frac{2}{9}\,Abc^2x^{\frac{27}{2}}+\frac{2}{23}\,Bb^3x^{\frac{23}{2}}+\frac{6}{23}\,Ab^2cx^{\frac{23}{2}}+\frac{2}{19}\,Ab^3x^{\frac{19}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 2/35*B*c^3*x^(35/2) + 6/31*B*b*c^2*x^(31/2) + 2/31*A*c^3*x^(31/2) + 2/9*B*b
^2*c*x^(27/2) + 2/9*A*b*c^2*x^(27/2) + 2/23*B*b^3*x^(23/2) + 6/23*A*b^2*c*x
^(23/2) + 2/19*A*b^3*x^(19/2)
```

3.177 $\int x^{3/2} \left(A+Bx^2\right) \left(bx^2+cx^4\right)^3 \, dx$

Optimal. Leaf size=85

$$\frac{2}{21}b^2x^{21/2}(3Ac+bB)+\frac{2}{17}Ab^3x^{17/2}+\frac{2}{29}c^2x^{29/2}(Ac+3bB)+\frac{6}{25}bcx^{25/2}(Ac+bB)+\frac{2}{33}Bc^3x^{33/2}$$

```
[Out] (2*A*b^3*x^(17/2))/17 + (2*b^2*(b*B + 3*A*c)*x^(21/2))/21 + (6*b*c*(b*B + A
*c)*x^(25/2))/25 + (2*c^2*(3*b*B + A*c)*x^(29/2))/29 + (2*B*c^3*x^(33/2))/3
3
```

Rubi [A] time = 0.0501958, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1584, 448}

$$\frac{2}{21}b^2x^{21/2}(3Ac+bB)+\frac{2}{17}Ab^3x^{17/2}+\frac{2}{29}c^2x^{29/2}(Ac+3bB)+\frac{6}{25}bcx^{25/2}(Ac+bB)+\frac{2}{33}Bc^3x^{33/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*A*b^3*x^(17/2))/17 + (2*b^2*(b*B + 3*A*c)*x^(21/2))/21 + (6*b*c*(b*B + A
*c)*x^(25/2))/25 + (2*c^2*(3*b*B + A*c)*x^(29/2))/29 + (2*B*c^3*x^(33/2))/3
3
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int x^{3/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^3\,dx=\int x^{15/2}\left(A+Bx^2\right)\left(b+cx^2\right)^3\,dx$$
$$=\int\left(Ab^3x^{15/2}+b^2(bB+3Ac)x^{19/2}+3bc(bB+Ac)x^{23/2}+c^2(3bB+Ac)x^{27/2}+Bc^3x^{31/2}\right.$$
$$=\frac{2}{17}Ab^3x^{17/2}+\frac{2}{21}b^2(bB+3Ac)x^{21/2}+\frac{6}{25}bc(bB+Ac)x^{25/2}+\frac{2}{29}c^2(3bB+Ac)x^{29/2}+$$

Mathematica [A] time = 0.0409699, size = 85, normalized size = 1.

$$\frac{2}{21}b^2x^{21/2}(3Ac+bB)+\frac{2}{17}Ab^3x^{17/2}+\frac{2}{29}c^2x^{29/2}(Ac+3bB)+\frac{6}{25}bcx^{25/2}(Ac+bB)+\frac{2}{33}Bc^3x^{33/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*A*b^3*x^(17/2))/17 + (2*b^2*(b*B + 3*A*c)*x^(21/2))/21 + (6*b*c*(b*B + A
*c)*x^(25/2))/25 + (2*c^2*(3*b*B + A*c)*x^(29/2))/29 + (2*B*c^3*x^(33/2))/3
3
```

Maple [A] time = 0.005, size = 80, normalized size = 0.9

$$\frac{172550\,Bc^3x^8+196350\,Ac^3x^6+589050\,Bx^6bc^2+683298\,Abc^2x^4+683298\,Bx^4b^2c+813450\,Ab^2cx^2+271150\,Bx^2b^3+}{2847075}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x)
```

```
[Out] 2/2847075*x^(17/2)*(86275*B*c^3*x^8+98175*A*c^3*x^6+294525*B*b*c^2*x^6+3416
49*A*b*c^2*x^4+341649*B*b^2*c*x^4+406725*A*b^2*c*x^2+135575*B*b^3*x^2+16747
5*A*b^3)
```

Maxima [A] time = 1.19563, size = 99, normalized size = 1.16

$$\frac{2}{33}\,Bc^3x^{\frac{33}{2}}+\frac{2}{29}\left(3\,Bbc^2+Ac^3\right)x^{\frac{29}{2}}+\frac{6}{25}\left(Bb^2c+Abc^2\right)x^{\frac{25}{2}}+\frac{2}{17}\,Ab^3x^{\frac{17}{2}}+\frac{2}{21}\left(Bb^3+3\,Ab^2c\right)x^{\frac{21}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 2/33*B*c^3*x^(33/2) + 2/29*(3*B*b*c^2 + A*c^3)*x^(29/2) + 6/25*(B*b^2*c + A
*b*c^2)*x^(25/2) + 2/17*A*b^3*x^(17/2) + 2/21*(B*b^3 + 3*A*b^2*c)*x^(21/2)
```

Fricas [A] time = 1.84023, size = 213, normalized size = 2.51

$$\frac{2}{2847075}\left(86275\,Bc^3x^{16}+98175\left(3\,Bbc^2+Ac^3\right)x^{14}+341649\left(Bb^2c+Abc^2\right)x^{12}+167475\,Ab^3x^8+135575\left(Bb^3+3\,Ab\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 2/2847075*(86275*B*c^3*x^16 + 98175*(3*B*b*c^2 + A*c^3)*x^14 + 341649*(B*b^
2*c + A*b*c^2)*x^12 + 167475*A*b^3*x^8 + 135575*(B*b^3 + 3*A*b^2*c)*x^10)*s
qrt(x)
```

Sympy [A] time = 113.16, size = 114, normalized size = 1.34

$$\frac{2Ab^3x^{\frac{17}{2}}}{17}+\frac{2Ab^2cx^{\frac{21}{2}}}{7}+\frac{6Abc^2x^{\frac{25}{2}}}{25}+\frac{2Ac^3x^{\frac{29}{2}}}{29}+\frac{2Bb^3x^{\frac{21}{2}}}{21}+\frac{6Bb^2cx^{\frac{25}{2}}}{25}+\frac{6Bbc^2x^{\frac{29}{2}}}{29}+\frac{2Bc^3x^{\frac{33}{2}}}{33}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(B*x**2+A)*(c*x**4+b*x**2)**3,x)
```

```
[Out] 2*A*b**3*x**(17/2)/17 + 2*A*b**2*c*x**(21/2)/7 + 6*A*b*c**2*x**(25/2)/25 +
2*A*c**3*x**(29/2)/29 + 2*B*b**3*x**(21/2)/21 + 6*B*b**2*c*x**(25/2)/25 + 6
*B*b*c**2*x**(29/2)/29 + 2*B*c**3*x**(33/2)/33
```

Giac [A] time = 1.14716, size = 104, normalized size = 1.22

$$\frac{2}{33}\,Bc^3x^{\frac{33}{2}}+\frac{6}{29}\,Bbc^2x^{\frac{29}{2}}+\frac{2}{29}\,Ac^3x^{\frac{29}{2}}+\frac{6}{25}\,Bb^2cx^{\frac{25}{2}}+\frac{6}{25}\,Abc^2x^{\frac{25}{2}}+\frac{2}{21}\,Bb^3x^{\frac{21}{2}}+\frac{2}{7}\,Ab^2cx^{\frac{21}{2}}+\frac{2}{17}\,Ab^3x^{\frac{17}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 2/33*B*c^3*x^(33/2) + 6/29*B*b*c^2*x^(29/2) + 2/29*A*c^3*x^(29/2) + 6/25*B*
b^2*c*x^(25/2) + 6/25*A*b*c^2*x^(25/2) + 2/21*B*b^3*x^(21/2) + 2/7*A*b^2*c*
x^(21/2) + 2/17*A*b^3*x^(17/2)
```

3.178 $\int \sqrt{x}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^3\,dx$

Optimal. Leaf size=85

$$\frac{2}{19}b^2x^{19/2}(3Ac+bB)+\frac{2}{15}Ab^3x^{15/2}+\frac{2}{27}c^2x^{27/2}(Ac+3bB)+\frac{6}{23}bcx^{23/2}(Ac+bB)+\frac{2}{31}Bc^3x^{31/2}$$

```
[Out] (2*A*b^3*x^(15/2))/15 + (2*b^2*(b*B + 3*A*c)*x^(19/2))/19 + (6*b*c*(b*B + A
*c)*x^(23/2))/23 + (2*c^2*(3*b*B + A*c)*x^(27/2))/27 + (2*B*c^3*x^(31/2))/3
1
```

Rubi [A] time = 0.0498296, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1584, 448}

$$\frac{2}{19}b^2x^{19/2}(3Ac+bB)+\frac{2}{15}Ab^3x^{15/2}+\frac{2}{27}c^2x^{27/2}(Ac+3bB)+\frac{6}{23}bcx^{23/2}(Ac+bB)+\frac{2}{31}Bc^3x^{31/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(A + B*x^2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*A*b^3*x^(15/2))/15 + (2*b^2*(b*B + 3*A*c)*x^(19/2))/19 + (6*b*c*(b*B + A
*c)*x^(23/2))/23 + (2*c^2*(3*b*B + A*c)*x^(27/2))/27 + (2*B*c^3*x^(31/2))/3
1
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \sqrt{x}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^3\,dx = \int x^{13/2}\left(A+Bx^2\right)\left(b+cx^2\right)^3\,dx$$
$$= \int \left(Ab^3x^{13/2}+b^2(bB+3Ac)x^{17/2}+3bc(bB+Ac)x^{21/2}+c^2(3bB+Ac)x^{25/2}+Bc^3x^{29/}\right.$$
$$= \frac{2}{15}Ab^3x^{15/2}+\frac{2}{19}b^2(bB+3Ac)x^{19/2}+\frac{6}{23}bc(bB+Ac)x^{23/2}+\frac{2}{27}c^2(3bB+Ac)x^{27/2}+$$

Mathematica [A] time = 0.0399064, size = 85, normalized size = 1.

$$\frac{2}{19}b^2x^{19/2}(3Ac+bB)+\frac{2}{15}Ab^3x^{15/2}+\frac{2}{27}c^2x^{27/2}(Ac+3bB)+\frac{6}{23}bcx^{23/2}(Ac+bB)+\frac{2}{31}Bc^3x^{31/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(A + B*x^2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*A*b^3*x^(15/2))/15 + (2*b^2*(b*B + 3*A*c)*x^(19/2))/19 + (6*b*c*(b*B + A
*c)*x^(23/2))/23 + (2*c^2*(3*b*B + A*c)*x^(27/2))/27 + (2*B*c^3*x^(31/2))/3
1
```

Maple [A] time = 0.005, size = 80, normalized size = 0.9

$$\frac{117990\,Bc^3x^8+135470\,Ac^3x^6+406410\,Bx^6bc^2+477090\,Abc^2x^4+477090\,Bx^4b^2c+577530\,Ab^2cx^2+192510\,Bx^2b^3+}{1828845}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3*x^(1/2),x)
```

```
[Out] 2/1828845*x^(15/2)*(58995*B*c^3*x^8+67735*A*c^3*x^6+203205*B*b*c^2*x^6+2385
45*A*b*c^2*x^4+238545*B*b^2*c*x^4+288765*A*b^2*c*x^2+96255*B*b^3*x^2+121923
*A*b^3)
```

Maxima [A] time = 1.12662, size = 99, normalized size = 1.16

$$\frac{2}{31}\,Bc^3x^{\frac{31}{2}}+\frac{2}{27}\left(3\,Bbc^2+Ac^3\right)x^{\frac{27}{2}}+\frac{6}{23}\left(Bb^2c+Abc^2\right)x^{\frac{23}{2}}+\frac{2}{15}\,Ab^3x^{\frac{15}{2}}+\frac{2}{19}\left(Bb^3+3\,Ab^2c\right)x^{\frac{19}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/31*B*c^3*x^(31/2) + 2/27*(3*B*b*c^2 + A*c^3)*x^(27/2) + 6/23*(B*b^2*c + A
*b*c^2)*x^(23/2) + 2/15*A*b^3*x^(15/2) + 2/19*(B*b^3 + 3*A*b^2*c)*x^(19/2)
```

Fricas [A] time = 1.82087, size = 211, normalized size = 2.48

$$\frac{2}{1828845}\left(58995\,Bc^3x^{15}+67735\left(3\,Bbc^2+Ac^3\right)x^{13}+238545\left(Bb^2c+Abc^2\right)x^{11}+121923\,Ab^3x^7+96255\left(Bb^3+3\,Ab^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/1828845*(58995*B*c^3*x^15 + 67735*(3*B*b*c^2 + A*c^3)*x^13 + 238545*(B*b^
2*c + A*b*c^2)*x^11 + 121923*A*b^3*x^7 + 96255*(B*b^3 + 3*A*b^2*c)*x^9)*sqr
t(x)
```

Sympy [A] time = 18.2926, size = 95, normalized size = 1.12

$$\frac{2Ab^3x^{\frac{15}{2}}}{15}+\frac{2Bc^3x^{\frac{31}{2}}}{31}+\frac{2x^{\frac{27}{2}}\left(Ac^3+3Bbc^2\right)}{27}+\frac{2x^{\frac{23}{2}}\left(3Abc^2+3Bb^2c\right)}{23}+\frac{2x^{\frac{19}{2}}\left(3Ab^2c+Bb^3\right)}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3*x**(1/2),x)
```

```
[Out] 2*A*b**3*x**(15/2)/15 + 2*B*c**3*x**(31/2)/31 + 2*x**(27/2)*(A*c**3 + 3*B*b
*c**2)/27 + 2*x**(23/2)*(3*A*b*c**2 + 3*B*b**2*c)/23 + 2*x**(19/2)*(3*A*b**
2*c + B*b**3)/19
```

Giac [A] time = 1.11954, size = 104, normalized size = 1.22

$$\frac{2}{31}\,Bc^3x^{\frac{31}{2}}+\frac{2}{9}\,Bbc^2x^{\frac{27}{2}}+\frac{2}{27}\,Ac^3x^{\frac{27}{2}}+\frac{6}{23}\,Bb^2cx^{\frac{23}{2}}+\frac{6}{23}\,Abc^2x^{\frac{23}{2}}+\frac{2}{19}\,Bb^3x^{\frac{19}{2}}+\frac{6}{19}\,Ab^2cx^{\frac{19}{2}}+\frac{2}{15}\,Ab^3x^{\frac{15}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/31*B*c^3*x^(31/2) + 2/9*B*b*c^2*x^(27/2) + 2/27*A*c^3*x^(27/2) + 6/23*B*b
^2*c*x^(23/2) + 6/23*A*b*c^2*x^(23/2) + 2/19*B*b^3*x^(19/2) + 6/19*A*b^2*c*
x^(19/2) + 2/15*A*b^3*x^(15/2)
```

3.179 $\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^3}{\sqrt{x}}\,dx$

Optimal. Leaf size=85

$$\frac{2}{17}b^2x^{17/2}(3Ac+bB)+\frac{2}{13}Ab^3x^{13/2}+\frac{2}{25}c^2x^{25/2}(Ac+3bB)+\frac{2}{7}bcx^{21/2}(Ac+bB)+\frac{2}{29}Bc^3x^{29/2}$$

```
[Out] (2*A*b^3*x^(13/2))/13 + (2*b^2*(b*B + 3*A*c)*x^(17/2))/17 + (2*b*c*(b*B + A
*c)*x^(21/2))/7 + (2*c^2*(3*b*B + A*c)*x^(25/2))/25 + (2*B*c^3*x^(29/2))/29
```

Rubi [A] time = 0.0515566, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1584, 448}

$$\frac{2}{17}b^2x^{17/2}(3Ac+bB)+\frac{2}{13}Ab^3x^{13/2}+\frac{2}{25}c^2x^{25/2}(Ac+3bB)+\frac{2}{7}bcx^{21/2}(Ac+bB)+\frac{2}{29}Bc^3x^{29/2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/Sqrt[x],x]
```

```
[Out] (2*A*b^3*x^(13/2))/13 + (2*b^2*(b*B + 3*A*c)*x^(17/2))/17 + (2*b*c*(b*B + A
*c)*x^(21/2))/7 + (2*c^2*(3*b*B + A*c)*x^(25/2))/25 + (2*B*c^3*x^(29/2))/29
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{\sqrt{x}}\,dx = \int x^{11/2}(A+Bx^2)(b+cx^2)^3\,dx$$
$$= \int \left(Ab^3x^{11/2} + b^2(bB+3Ac)x^{15/2} + 3bc(bB+Ac)x^{19/2} + c^2(3bB+Ac)x^{23/2} + Bc^3x^{27/2}\right) d$$
$$= \frac{2}{13}Ab^3x^{13/2} + \frac{2}{17}b^2(bB+3Ac)x^{17/2} + \frac{2}{7}bc(bB+Ac)x^{21/2} + \frac{2}{25}c^2(3bB+Ac)x^{25/2} + \frac{2}{29}Bc$$

Mathematica [A] time = 0.0381778, size = 85, normalized size = 1.

$$\frac{2}{17}b^2x^{17/2}(3Ac+bB) + \frac{2}{13}Ab^3x^{13/2} + \frac{2}{25}c^2x^{25/2}(Ac+3bB) + \frac{2}{7}bcx^{21/2}(Ac+bB) + \frac{2}{29}Bc^3x^{29/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/Sqrt[x],x]
```

```
[Out] (2*A*b^3*x^(13/2))/13 + (2*b^2*(b*B + 3*A*c)*x^(17/2))/17 + (2*b*c*(b*B + A
*c)*x^(21/2))/7 + (2*c^2*(3*b*B + A*c)*x^(25/2))/25 + (2*B*c^3*x^(29/2))/29
```

Maple [A] time = 0.005, size = 80, normalized size = 0.9

$$\frac{77350\,Bc^3x^8 + 89726\,Ac^3x^6 + 269178\,Bx^6bc^2 + 320450\,Abc^2x^4 + 320450\,Bx^4b^2c + 395850\,Ab^2cx^2 + 131950\,Bx^2b^3 + 17}{1121575}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^(1/2),x)
```

```
[Out] 2/1121575*x^(13/2)*(38675*B*c^3*x^8+44863*A*c^3*x^6+134589*B*b*c^2*x^6+1602
25*A*b*c^2*x^4+160225*B*b^2*c*x^4+197925*A*b^2*c*x^2+65975*B*b^3*x^2+86275*
A*b^3)
```

Maxima [A] time = 1.16145, size = 99, normalized size = 1.16

$$\frac{2}{29}\,Bc^3x^{\frac{29}{2}} + \frac{2}{25}\left(3\,Bbc^2 + Ac^3\right)x^{\frac{25}{2}} + \frac{2}{7}\left(Bb^2c + Abc^2\right)x^{\frac{21}{2}} + \frac{2}{13}\,Ab^3x^{\frac{13}{2}} + \frac{2}{17}\left(Bb^3 + 3\,Ab^2c\right)x^{\frac{17}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/29*B*c^3*x^(29/2) + 2/25*(3*B*b*c^2 + A*c^3)*x^(25/2) + 2/7*(B*b^2*c + A*
b*c^2)*x^(21/2) + 2/13*A*b^3*x^(13/2) + 2/17*(B*b^3 + 3*A*b^2*c)*x^(17/2)
```

Fricas [A] time = 1.80342, size = 209, normalized size = 2.46

$$\frac{2}{1121575}\left(38675\,Bc^3x^{14}+44863\left(3\,Bbc^2+Ac^3\right)x^{12}+160225\left(Bb^2c+Abc^2\right)x^{10}+86275\,Ab^3x^6+65975\left(Bb^3+3\,Ab^2c\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/1121575*(38675*B*c^3*x^14 + 44863*(3*B*b*c^2 + A*c^3)*x^12 + 160225*(B*b^
2*c + A*b*c^2)*x^10 + 86275*A*b^3*x^6 + 65975*(B*b^3 + 3*A*b^2*c)*x^8)*sqrt
(x)
```

Sympy [A] time = 56.0395, size = 114, normalized size = 1.34

$$\frac{2Ab^3x^{\frac{13}{2}}}{13}+\frac{6Ab^2cx^{\frac{17}{2}}}{17}+\frac{2Abc^2x^{\frac{21}{2}}}{7}+\frac{2Ac^3x^{\frac{25}{2}}}{25}+\frac{2Bb^3x^{\frac{17}{2}}}{17}+\frac{2Bb^2cx^{\frac{21}{2}}}{7}+\frac{6Bbc^2x^{\frac{25}{2}}}{25}+\frac{2Bc^3x^{\frac{29}{2}}}{29}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**(1/2),x)
```

```
[Out] 2*A*b**3*x**(13/2)/13 + 6*A*b**2*c*x**(17/2)/17 + 2*A*b*c**2*x**(21/2)/7 +
2*A*c**3*x**(25/2)/25 + 2*B*b**3*x**(17/2)/17 + 2*B*b**2*c*x**(21/2)/7 + 6*
B*b*c**2*x**(25/2)/25 + 2*B*c**3*x**(29/2)/29
```

Giac [A] time = 1.13529, size = 104, normalized size = 1.22

$$\frac{2}{29}\,Bc^3x^{\frac{29}{2}}+\frac{6}{25}\,Bbc^2x^{\frac{25}{2}}+\frac{2}{25}\,Ac^3x^{\frac{25}{2}}+\frac{2}{7}\,Bb^2cx^{\frac{21}{2}}+\frac{2}{7}\,Abc^2x^{\frac{21}{2}}+\frac{2}{17}\,Bb^3x^{\frac{17}{2}}+\frac{6}{17}\,Ab^2cx^{\frac{17}{2}}+\frac{2}{13}\,Ab^3x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/29*B*c^3*x^(29/2) + 6/25*B*b*c^2*x^(25/2) + 2/25*A*c^3*x^(25/2) + 2/7*B*b
^2*c*x^(21/2) + 2/7*A*b*c^2*x^(21/2) + 2/17*B*b^3*x^(17/2) + 6/17*A*b^2*c*x
^(17/2) + 2/13*A*b^3*x^(13/2)
```

3.180 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{3/2}}\,dx$

Optimal. Leaf size=85

$$\frac{2}{15}b^2x^{15/2}(3Ac+bB)+\frac{2}{11}Ab^3x^{11/2}+\frac{2}{23}c^2x^{23/2}(Ac+3bB)+\frac{6}{19}bcx^{19/2}(Ac+bB)+\frac{2}{27}Bc^3x^{27/2}$$

```
[Out] (2*A*b^3*x^(11/2))/11 + (2*b^2*(b*B + 3*A*c)*x^(15/2))/15 + (6*b*c*(b*B + A
*c)*x^(19/2))/19 + (2*c^2*(3*b*B + A*c)*x^(23/2))/23 + (2*B*c^3*x^(27/2))/2
7
```

Rubi [A] time = 0.0496143, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1584, 448}

$$\frac{2}{15}b^2x^{15/2}(3Ac+bB)+\frac{2}{11}Ab^3x^{11/2}+\frac{2}{23}c^2x^{23/2}(Ac+3bB)+\frac{6}{19}bcx^{19/2}(Ac+bB)+\frac{2}{27}Bc^3x^{27/2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^(3/2),x]
```

```
[Out] (2*A*b^3*x^(11/2))/11 + (2*b^2*(b*B + 3*A*c)*x^(15/2))/15 + (6*b*c*(b*B + A
*c)*x^(19/2))/19 + (2*c^2*(3*b*B + A*c)*x^(23/2))/23 + (2*B*c^3*x^(27/2))/2
7
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{3/2}}\,dx = \int x^{9/2}(A+Bx^2)(b+cx^2)^3\,dx$$
$$= \int \left(Ab^3x^{9/2} + b^2(bB+3Ac)x^{13/2} + 3bc(bB+Ac)x^{17/2} + c^2(3bB+Ac)x^{21/2} + Bc^3x^{25/2}\right) dx$$
$$= \frac{2}{11}Ab^3x^{11/2} + \frac{2}{15}b^2(bB+3Ac)x^{15/2} + \frac{6}{19}bc(bB+Ac)x^{19/2} + \frac{2}{23}c^2(3bB+Ac)x^{23/2} + \frac{2}{27}B$$

Mathematica [A] time = 0.0367638, size = 85, normalized size = 1.

$$\frac{2}{15}b^2x^{15/2}(3Ac+bB) + \frac{2}{11}Ab^3x^{11/2} + \frac{2}{23}c^2x^{23/2}(Ac+3bB) + \frac{6}{19}bcx^{19/2}(Ac+bB) + \frac{2}{27}Bc^3x^{27/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^(3/2),x]
```

```
[Out] (2*A*b^3*x^(11/2))/11 + (2*b^2*(b*B + 3*A*c)*x^(15/2))/15 + (6*b*c*(b*B + A
*c)*x^(19/2))/19 + (2*c^2*(3*b*B + A*c)*x^(23/2))/23 + (2*B*c^3*x^(27/2))/2
7
```

Maple [A] time = 0.005, size = 80, normalized size = 0.9

$$\frac{48070\,Bc^3x^8 + 56430\,Ac^3x^6 + 169290\,Bx^6bc^2 + 204930\,Abc^2x^4 + 204930\,Bx^4b^2c + 259578\,Ab^2cx^2 + 86526\,Bx^2b^3 + 117}{648945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^(3/2),x)
```

```
[Out] 2/648945*x^(11/2)*(24035*B*c^3*x^8+28215*A*c^3*x^6+84645*B*b*c^2*x^6+102465
*A*b*c^2*x^4+102465*B*b^2*c*x^4+129789*A*b^2*c*x^2+43263*B*b^3*x^2+58995*A*
b^3)
```

Maxima [A] time = 1.10736, size = 99, normalized size = 1.16

$$\frac{2}{27}\,Bc^3x^{\frac{27}{2}} + \frac{2}{23}\left(3\,Bbc^2 + Ac^3\right)x^{\frac{23}{2}} + \frac{6}{19}\left(Bb^2c + Abc^2\right)x^{\frac{19}{2}} + \frac{2}{11}\,Ab^3x^{\frac{11}{2}} + \frac{2}{15}\left(Bb^3 + 3\,Ab^2c\right)x^{\frac{15}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/27*B*c^3*x^(27/2) + 2/23*(3*B*b*c^2 + A*c^3)*x^(23/2) + 6/19*(B*b^2*c + A
*b*c^2)*x^(19/2) + 2/11*A*b^3*x^(11/2) + 2/15*(B*b^3 + 3*A*b^2*c)*x^(15/2)
```

Fricas [A] time = 1.85796, size = 207, normalized size = 2.44

$$\frac{2}{648945}\left(24035\,Bc^3x^{13} + 28215\left(3\,Bbc^2 + Ac^3\right)x^{11} + 102465\left(Bb^2c + Abc^2\right)x^9 + 58995\,Ab^3x^5 + 43263\left(Bb^3 + 3\,Ab^2c\right)x^{\cdot}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/648945*(24035*B*c^3*x^13 + 28215*(3*B*b*c^2 + A*c^3)*x^11 + 102465*(B*b^2
*c + A*b*c^2)*x^9 + 58995*A*b^3*x^5 + 43263*(B*b^3 + 3*A*b^2*c)*x^7)*sqrt(x
)
```

Sympy [A] time = 59.9208, size = 114, normalized size = 1.34

$$\frac{2Ab^3x^{\frac{11}{2}}}{11} + \frac{2Ab^2cx^{\frac{15}{2}}}{5} + \frac{6Abc^2x^{\frac{19}{2}}}{19} + \frac{2Ac^3x^{\frac{23}{2}}}{23} + \frac{2Bb^3x^{\frac{15}{2}}}{15} + \frac{6Bb^2cx^{\frac{19}{2}}}{19} + \frac{6Bbc^2x^{\frac{23}{2}}}{23} + \frac{2Bc^3x^{\frac{27}{2}}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**(3/2),x)
```

```
[Out] 2*A*b**3*x**(11/2)/11 + 2*A*b**2*c*x**(15/2)/5 + 6*A*b*c**2*x**(19/2)/19 +
2*A*c**3*x**(23/2)/23 + 2*B*b**3*x**(15/2)/15 + 6*B*b**2*c*x**(19/2)/19 + 6
*B*b*c**2*x**(23/2)/23 + 2*B*c**3*x**(27/2)/27
```

Giac [A] time = 1.13949, size = 104, normalized size = 1.22

$$\frac{2}{27}\,Bc^3x^{\frac{27}{2}} + \frac{6}{23}\,Bbc^2x^{\frac{23}{2}} + \frac{2}{23}\,Ac^3x^{\frac{23}{2}} + \frac{6}{19}\,Bb^2cx^{\frac{19}{2}} + \frac{6}{19}\,Abc^2x^{\frac{19}{2}} + \frac{2}{15}\,Bb^3x^{\frac{15}{2}} + \frac{2}{5}\,Ab^2cx^{\frac{15}{2}} + \frac{2}{11}\,Ab^3x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/27*B*c^3*x^(27/2) + 6/23*B*b*c^2*x^(23/2) + 2/23*A*c^3*x^(23/2) + 6/19*B*
b^2*c*x^(19/2) + 6/19*A*b*c^2*x^(19/2) + 2/15*B*b^3*x^(15/2) + 2/5*A*b^2*c*
x^(15/2) + 2/11*A*b^3*x^(11/2)
```

3.181 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{5/2}} \, dx$

Optimal. Leaf size=85

$$\frac{2}{13}b^2x^{13/2}(3Ac+bB)+\frac{2}{9}Ab^3x^{9/2}+\frac{2}{21}c^2x^{21/2}(Ac+3bB)+\frac{6}{17}bcx^{17/2}(Ac+bB)+\frac{2}{25}Bc^3x^{25/2}$$

```
[Out] (2*A*b^3*x^(9/2))/9 + (2*b^2*(b*B + 3*A*c)*x^(13/2))/13 + (6*b*c*(b*B + A*c
)*x^(17/2))/17 + (2*c^2*(3*b*B + A*c)*x^(21/2))/21 + (2*B*c^3*x^(25/2))/25
```

Rubi [A] time = 0.0517297, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1584, 448}

$$\frac{2}{13}b^2x^{13/2}(3Ac+bB)+\frac{2}{9}Ab^3x^{9/2}+\frac{2}{21}c^2x^{21/2}(Ac+3bB)+\frac{6}{17}bcx^{17/2}(Ac+bB)+\frac{2}{25}Bc^3x^{25/2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^(5/2),x]
```

```
[Out] (2*A*b^3*x^(9/2))/9 + (2*b^2*(b*B + 3*A*c)*x^(13/2))/13 + (6*b*c*(b*B + A*c
)*x^(17/2))/17 + (2*c^2*(3*b*B + A*c)*x^(21/2))/21 + (2*B*c^3*x^(25/2))/25
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{5/2}}\,dx = \int x^{7/2}(A+Bx^2)(b+cx^2)^3\,dx$$
$$= \int \left(Ab^3x^{7/2} + b^2(bB+3Ac)x^{11/2} + 3bc(bB+Ac)x^{15/2} + c^2(3bB+Ac)x^{19/2} + Bc^3x^{23/2}\right) dx$$
$$= \frac{2}{9}Ab^3x^{9/2} + \frac{2}{13}b^2(bB+3Ac)x^{13/2} + \frac{6}{17}bc(bB+Ac)x^{17/2} + \frac{2}{21}c^2(3bB+Ac)x^{21/2} + \frac{2}{25}Bc^3$$

Mathematica [A] time = 0.0377509, size = 85, normalized size = 1.

$$\frac{2}{13}b^2x^{13/2}(3Ac+bB) + \frac{2}{9}Ab^3x^{9/2} + \frac{2}{21}c^2x^{21/2}(Ac+3bB) + \frac{6}{17}bcx^{17/2}(Ac+bB) + \frac{2}{25}Bc^3x^{25/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^(5/2),x]
```

```
[Out] (2*A*b^3*x^(9/2))/9 + (2*b^2*(b*B + 3*A*c)*x^(13/2))/13 + (6*b*c*(b*B + A*c
)*x^(17/2))/17 + (2*c^2*(3*b*B + A*c)*x^(21/2))/21 + (2*B*c^3*x^(25/2))/25
```

Maple [A] time = 0.004, size = 80, normalized size = 0.9

$$\frac{27846\,Bc^3x^8 + 33150\,Ac^3x^6 + 99450\,Bx^6bc^2 + 122850\,Abc^2x^4 + 122850\,Bx^4b^2c + 160650\,Ab^2cx^2 + 53550\,Bx^2b^3 + 7735}{348075}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^(5/2),x)
```

```
[Out] 2/348075*x^(9/2)*(13923*B*c^3*x^8+16575*A*c^3*x^6+49725*B*b*c^2*x^6+61425*A
*b*c^2*x^4+61425*B*b^2*c*x^4+80325*A*b^2*c*x^2+26775*B*b^3*x^2+38675*A*b^3)
```

Maxima [A] time = 1.16663, size = 99, normalized size = 1.16

$$\frac{2}{25}\,Bc^3x^{\frac{25}{2}} + \frac{2}{21}\left(3\,Bbc^2 + Ac^3\right)x^{\frac{21}{2}} + \frac{6}{17}\left(Bb^2c + Abc^2\right)x^{\frac{17}{2}} + \frac{2}{9}\,Ab^3x^{\frac{9}{2}} + \frac{2}{13}\left(Bb^3 + 3\,Ab^2c\right)x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/25*B*c^3*x^(25/2) + 2/21*(3*B*b*c^2 + A*c^3)*x^(21/2) + 6/17*(B*b^2*c + A
*b*c^2)*x^(17/2) + 2/9*A*b^3*x^(9/2) + 2/13*(B*b^3 + 3*A*b^2*c)*x^(13/2)
```

Fricas [A] time = 1.87996, size = 205, normalized size = 2.41

$$\frac{2}{348075}\left(13923\,Bc^3x^{12}+16575\left(3\,Bbc^2+Ac^3\right)x^{10}+61425\left(Bb^2c+Abc^2\right)x^8+38675\,Ab^3x^4+26775\left(Bb^3+3\,Ab^2c\right)x^6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/348075*(13923*B*c^3*x^12 + 16575*(3*B*b*c^2 + A*c^3)*x^10 + 61425*(B*b^2*
c + A*b*c^2)*x^8 + 38675*A*b^3*x^4 + 26775*(B*b^3 + 3*A*b^2*c)*x^6)*sqrt(x)
```

Sympy [A] time = 81.9809, size = 114, normalized size = 1.34

$$\frac{2Ab^3x^{\frac{9}{2}}}{9}+\frac{6Ab^2cx^{\frac{13}{2}}}{13}+\frac{6Abc^2x^{\frac{17}{2}}}{17}+\frac{2Ac^3x^{\frac{21}{2}}}{21}+\frac{2Bb^3x^{\frac{13}{2}}}{13}+\frac{6Bb^2cx^{\frac{17}{2}}}{17}+\frac{2Bbc^2x^{\frac{21}{2}}}{7}+\frac{2Bc^3x^{\frac{25}{2}}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**(5/2),x)
```

```
[Out] 2*A*b**3*x**(9/2)/9 + 6*A*b**2*c*x**(13/2)/13 + 6*A*b*c**2*x**(17/2)/17 + 2
*A*c**3*x**(21/2)/21 + 2*B*b**3*x**(13/2)/13 + 6*B*b**2*c*x**(17/2)/17 + 2*
B*b*c**2*x**(21/2)/7 + 2*B*c**3*x**(25/2)/25
```

Giac [A] time = 1.1221, size = 104, normalized size = 1.22

$$\frac{2}{25}\,Bc^3x^{\frac{25}{2}}+\frac{2}{7}\,Bbc^2x^{\frac{21}{2}}+\frac{2}{21}\,Ac^3x^{\frac{21}{2}}+\frac{6}{17}\,Bb^2cx^{\frac{17}{2}}+\frac{6}{17}\,Abc^2x^{\frac{17}{2}}+\frac{2}{13}\,Bb^3x^{\frac{13}{2}}+\frac{6}{13}\,Ab^2cx^{\frac{13}{2}}+\frac{2}{9}\,Ab^3x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/25*B*c^3*x^(25/2) + 2/7*B*b*c^2*x^(21/2) + 2/21*A*c^3*x^(21/2) + 6/17*B*b
^2*c*x^(17/2) + 6/17*A*b*c^2*x^(17/2) + 2/13*B*b^3*x^(13/2) + 6/13*A*b^2*c*
x^(13/2) + 2/9*A*b^3*x^(9/2)
```

3.182 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{7/2}} dx$

Optimal. Leaf size=85

$$\frac{2}{11}b^2x^{11/2}(3Ac+bB)+\frac{2}{7}Ab^3x^{7/2}+\frac{2}{19}c^2x^{19/2}(Ac+3bB)+\frac{2}{5}bcx^{15/2}(Ac+bB)+\frac{2}{23}Bc^3x^{23/2}$$

```
[Out] (2*A*b^3*x^(7/2))/7 + (2*b^2*(b*B + 3*A*c)*x^(11/2))/11 + (2*b*c*(b*B + A*c
)*x^(15/2))/5 + (2*c^2*(3*b*B + A*c)*x^(19/2))/19 + (2*B*c^3*x^(23/2))/23
```

Rubi [A] time = 0.0511572, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1584, 448}

$$\frac{2}{11}b^2x^{11/2}(3Ac+bB)+\frac{2}{7}Ab^3x^{7/2}+\frac{2}{19}c^2x^{19/2}(Ac+3bB)+\frac{2}{5}bcx^{15/2}(Ac+bB)+\frac{2}{23}Bc^3x^{23/2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^(7/2),x]
```


```
[Out] (2*A*b^3*x^(7/2))/7 + (2*b^2*(b*B + 3*A*c)*x^(11/2))/11 + (2*b*c*(b*B + A*c
)*x^(15/2))/5 + (2*c^2*(3*b*B + A*c)*x^(19/2))/19 + (2*B*c^3*x^(23/2))/23
```


Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^3}{x^{7/2}}\,dx=\int x^{5/2}\left(A+Bx^2\right)\left(b+cx^2\right)^3\,dx$$
$$=\int \left(Ab^3x^{5/2}+b^2(bB+3Ac)x^{9/2}+3bc(bB+Ac)x^{13/2}+c^2(3bB+Ac)x^{17/2}+Bc^3x^{21/2}\right)dx$$
$$=\frac{2}{7}Ab^3x^{7/2}+\frac{2}{11}b^2(bB+3Ac)x^{11/2}+\frac{2}{5}bc(bB+Ac)x^{15/2}+\frac{2}{19}c^2(3bB+Ac)x^{19/2}+\frac{2}{23}Bc^3x$$

Mathematica [A] time = 0.0372151, size = 85, normalized size = 1.

$$\frac{2}{11}b^2x^{11/2}(3Ac+bB)+\frac{2}{7}Ab^3x^{7/2}+\frac{2}{19}c^2x^{19/2}(Ac+3bB)+\frac{2}{5}bcx^{15/2}(Ac+bB)+\frac{2}{23}Bc^3x^{23/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^(7/2),x]
```

```
[Out] (2*A*b^3*x^(7/2))/7 + (2*b^2*(b*B + 3*A*c)*x^(11/2))/11 + (2*b*c*(b*B + A*c
)*x^(15/2))/5 + (2*c^2*(3*b*B + A*c)*x^(19/2))/19 + (2*B*c^3*x^(23/2))/23
```

Maple [A] time = 0.007, size = 80, normalized size = 0.9

$$\frac{14630\,Bc^3x^8+17710\,Ac^3x^6+53130\,Bx^6bc^2+67298\,Abc^2x^4+67298\,Bx^4b^2c+91770\,Ab^2cx^2+30590\,Bx^2b^3+48070\,Ab}{168245}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^(7/2),x)
```

```
[Out] 2/168245*x^(7/2)*(7315*B*c^3*x^8+8855*A*c^3*x^6+26565*B*b*c^2*x^6+33649*A*b
*c^2*x^4+33649*B*b^2*c*x^4+45885*A*b^2*c*x^2+15295*B*b^3*x^2+24035*A*b^3)
```

Maxima [A] time = 1.09849, size = 99, normalized size = 1.16

$$\frac{2}{23}\,Bc^3x^{\frac{23}{2}}+\frac{2}{19}\left(3\,Bbc^2+Ac^3\right)x^{\frac{19}{2}}+\frac{2}{5}\left(Bb^2c+Abc^2\right)x^{\frac{15}{2}}+\frac{2}{7}\,Ab^3x^{\frac{7}{2}}+\frac{2}{11}\left(Bb^3+3\,Ab^2c\right)x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/23*B*c^3*x^(23/2) + 2/19*(3*B*b*c^2 + A*c^3)*x^(19/2) + 2/5*(B*b^2*c + A*
b*c^2)*x^(15/2) + 2/7*A*b^3*x^(7/2) + 2/11*(B*b^3 + 3*A*b^2*c)*x^(11/2)
```

Fricas [A] time = 1.86223, size = 201, normalized size = 2.36

$$\frac{2}{168245}\left(7315\,Bc^3x^{11}+8855\left(3\,Bbc^2+Ac^3\right)x^9+33649\left(Bb^2c+Abc^2\right)x^7+24035\,Ab^3x^3+15295\left(Bb^3+3\,Ab^2c\right)x^5\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/168245*(7315*B*c^3*x^11 + 8855*(3*B*b*c^2 + A*c^3)*x^9 + 33649*(B*b^2*c +
 A*b*c^2)*x^7 + 24035*A*b^3*x^3 + 15295*(B*b^3 + 3*A*b^2*c)*x^5)*sqrt(x)
```

Sympy [A] time = 112.692, size = 114, normalized size = 1.34

$$\frac{2Ab^3x^{\frac{7}{2}}}{7}+\frac{6Ab^2cx^{\frac{11}{2}}}{11}+\frac{2Abc^2x^{\frac{15}{2}}}{5}+\frac{2Ac^3x^{\frac{19}{2}}}{19}+\frac{2Bb^3x^{\frac{11}{2}}}{11}+\frac{2Bb^2cx^{\frac{15}{2}}}{5}+\frac{6Bbc^2x^{\frac{19}{2}}}{19}+\frac{2Bc^3x^{\frac{23}{2}}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**(7/2),x)
```

```
[Out] 2*A*b**3*x**(7/2)/7 + 6*A*b**2*c*x**(11/2)/11 + 2*A*b*c**2*x**(15/2)/5 + 2*
A*c**3*x**(19/2)/19 + 2*B*b**3*x**(11/2)/11 + 2*B*b**2*c*x**(15/2)/5 + 6*B*
b*c**2*x**(19/2)/19 + 2*B*c**3*x**(23/2)/23
```

Giac [A] time = 1.13731, size = 104, normalized size = 1.22

$$\frac{2}{23}\,Bc^3x^{\frac{23}{2}}+\frac{6}{19}\,Bbc^2x^{\frac{19}{2}}+\frac{2}{19}\,Ac^3x^{\frac{19}{2}}+\frac{2}{5}\,Bb^2cx^{\frac{15}{2}}+\frac{2}{5}\,Abc^2x^{\frac{15}{2}}+\frac{2}{11}\,Bb^3x^{\frac{11}{2}}+\frac{6}{11}\,Ab^2cx^{\frac{11}{2}}+\frac{2}{7}\,Ab^3x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/23*B*c^3*x^(23/2) + 6/19*B*b*c^2*x^(19/2) + 2/19*A*c^3*x^(19/2) + 2/5*B*b
^2*c*x^(15/2) + 2/5*A*b*c^2*x^(15/2) + 2/11*B*b^3*x^(11/2) + 6/11*A*b^2*c*x
^(11/2) + 2/7*A*b^3*x^(7/2)
```

3.183 $\int \frac{x^{13/2}(A+Bx^2)}{bx^2+cx^4}\,dx$

Optimal. Leaf size=278

$$-\frac{b^{7/4}(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{15/4}}+\frac{b^{7/4}(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{15/4}}+\frac{b^{7/4}(bB-Ac)\tan^{-1}}{\sqrt{2}c^{15/4}}$$

```
[Out] (2*b*(b*B - A*c)*x^(3/2))/(3*c^3) - (2*(b*B - A*c)*x^(7/2))/(7*c^2) + (2*B*
x^(11/2))/(11*c) + (b^(7/4)*(b*B - A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x]
)/b^(1/4)])/(Sqrt[2]*c^(15/4)) - (b^(7/4)*(b*B - A*c)*ArcTan[1 + (Sqrt[2]*c
^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^(15/4)) - (b^(7/4)*(b*B - A*c)*Log[Sqr
t[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(15/4)) +
 (b^(7/4)*(b*B - A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[
c]*x])/(2*Sqrt[2]*c^(15/4))
```

Rubi [A] time = 0.274033, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1584, 459, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{b^{7/4}(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{15/4}}+\frac{b^{7/4}(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{15/4}}+\frac{b^{7/4}(bB-Ac)\tan^{-1}}{\sqrt{2}c^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```


```
[Out] (2*b*(b*B - A*c)*x^(3/2))/(3*c^3) - (2*(b*B - A*c)*x^(7/2))/(7*c^2) + (2*B*
x^(11/2))/(11*c) + (b^(7/4)*(b*B - A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x]
)/b^(1/4)])/(Sqrt[2]*c^(15/4)) - (b^(7/4)*(b*B - A*c)*ArcTan[1 + (Sqrt[2]*c
^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^(15/4)) - (b^(7/4)*(b*B - A*c)*Log[Sqr
t[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(15/4)) +
 (b^(7/4)*(b*B - A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[
c]*x])/(2*Sqrt[2]*c^(15/4))
```


Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^{13/2}\left(A+Bx^2\right)}{bx^2+cx^4}\,dx = \int \frac{x^{9/2}\left(A+Bx^2\right)}{b+cx^2}\,dx$$
$$= \frac{2Bx^{11/2}}{11c} - \frac{\left(2\left(\frac{11bB}{2}-\frac{11Ac}{2}\right)\right)\int \frac{x^{9/2}}{b+cx^2}\,dx}{11c}$$
$$= -\frac{2(bB-Ac)x^{7/2}}{7c^2} + \frac{2Bx^{11/2}}{11c} + \frac{(b(bB-Ac))\int \frac{x^{5/2}}{b+cx^2}\,dx}{c^2}$$
$$= \frac{2b(bB-Ac)x^{3/2}}{3c^3} - \frac{2(bB-Ac)x^{7/2}}{7c^2} + \frac{2Bx^{11/2}}{11c} - \frac{\left(b^2(bB-Ac)\right)\int \frac{\sqrt{x}}{b+cx^2}\,dx}{c^3}$$
$$= \frac{2b(bB-Ac)x^{3/2}}{3c^3} - \frac{2(bB-Ac)x^{7/2}}{7c^2} + \frac{2Bx^{11/2}}{11c} - \frac{\left(2b^2(bB-Ac)\right)\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c^3}$$
$$= \frac{2b(bB-Ac)x^{3/2}}{3c^3} - \frac{2(bB-Ac)x^{7/2}}{7c^2} + \frac{2Bx^{11/2}}{11c} + \frac{\left(b^2(bB-Ac)\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c^{7/2}} - \frac{(b}{}$$
$$= \frac{2b(bB-Ac)x^{3/2}}{3c^3} - \frac{2(bB-Ac)x^{7/2}}{7c^2} + \frac{2Bx^{11/2}}{11c} - \frac{\left(b^2(bB-Ac)\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2c^4}$$
$$= \frac{2b(bB-Ac)x^{3/2}}{3c^3} - \frac{2(bB-Ac)x^{7/2}}{7c^2} + \frac{2Bx^{11/2}}{11c} - \frac{b^{7/4}(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}c^{15/4}} + \cdots$$
$$= \frac{2b(bB-Ac)x^{3/2}}{3c^3} - \frac{2(bB-Ac)x^{7/2}}{7c^2} + \frac{2Bx^{11/2}}{11c} + \frac{b^{7/4}(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{15/4}} - \frac{b^{7/4}(bB-A}{}$$

Mathematica [A] time = 0.230452, size = 133, normalized size = 0.48

$$\frac{2x^{3/2}\left(-11bc\left(7A+3Bx^2\right)+3c^2x^2\left(11A+7Bx^2\right)+77b^2B\right)}{231c^3} + \frac{b(-b)^{3/4}(bB-Ac)\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)}{c^{15/4}} + \frac{(-b)^{7/4}(bB-Ac)\tanh^{-1}}{c^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (2*x^(3/2)*(77*b^2*B - 11*b*c*(7*A + 3*B*x^2) + 3*c^2*x^2*(11*A + 7*B*x^2))
)/(231*c^3) + ((-b)^(3/4)*b*(b*B - A*c)*ArcTan[(c^(1/4)*Sqrt[x])/(-b)^(1/4)
])/c^(15/4) + ((-b)^(7/4)*(b*B - A*c)*ArcTanh[(c^(1/4)*Sqrt[x])/(-b)^(1/4)]
)/c^(15/4)
```

Maple [A] time = 0.033, size = 336, normalized size = 1.2

$$\frac{2B}{11c}x^{\frac{11}{2}}+\frac{2A}{7c}x^{\frac{7}{2}}-\frac{2Bb}{7c^2}x^{\frac{7}{2}}-\frac{2Ab}{3c^2}x^{\frac{3}{2}}+\frac{2Bb^2}{3c^3}x^{\frac{3}{2}}+\frac{b^2\sqrt{2}A}{2c^3}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}+\frac{b^2\sqrt{2}A}{2c^3}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2),x)
```

```
[Out] 2/11*B*x^(11/2)/c+2/7/c*x^(7/2)*A-2/7/c^2*x^(7/2)*B*b-2/3/c^2*x^(3/2)*A*b+2
/3/c^3*x^(3/2)*B*b^2+1/2*b^2/c^3/(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)
^(1/4)*x^(1/2)+1)+1/2*b^2/c^3/(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1
/4)*x^(1/2)-1)+1/4*b^2/c^3/(b/c)^(1/4)*2^(1/2)*A*ln((x-(b/c)^(1/4)*x^(1/2)*
2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))-1/2*b^3/c
^4/(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-1/2*b^3/c^4/
(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-1/4*b^3/c^4/(b/
c)^(1/4)*2^(1/2)*B*ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^
(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.26914, size = 1914, normalized size = 6.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] -1/462*(924*c^3*(-(B^4*b^11 - 4*A*B^3*b^10*c + 6*A^2*B^2*b^9*c^2 - 4*A^3*B*
b^8*c^3 + A^4*b^7*c^4)/c^15)^(1/4)*arctan((sqrt((B^6*b^16 - 6*A*B^5*b^15*c
+ 15*A^2*B^4*b^14*c^2 - 20*A^3*B^3*b^13*c^3 + 15*A^4*B^2*b^12*c^4 - 6*A^5*B
*b^11*c^5 + A^6*b^10*c^6)*x - (B^4*b^11*c^7 - 4*A*B^3*b^10*c^8 + 6*A^2*B^2*
b^9*c^9 - 4*A^3*B*b^8*c^10 + A^4*b^7*c^11)*sqrt(-(B^4*b^11 - 4*A*B^3*b^10*c
 + 6*A^2*B^2*b^9*c^2 - 4*A^3*B*b^8*c^3 + A^4*b^7*c^4)/c^15))*c^4*(-(B^4*b^1
1 - 4*A*B^3*b^10*c + 6*A^2*B^2*b^9*c^2 - 4*A^3*B*b^8*c^3 + A^4*b^7*c^4)/c^1
5)^(1/4) + (B^3*b^8*c^4 - 3*A*B^2*b^7*c^5 + 3*A^2*B*b^6*c^6 - A^3*b^5*c^7)*
sqrt(x)*(-(B^4*b^11 - 4*A*B^3*b^10*c + 6*A^2*B^2*b^9*c^2 - 4*A^3*B*b^8*c^3
+ A^4*b^7*c^4)/c^15)^(1/4))/(B^4*b^11 - 4*A*B^3*b^10*c + 6*A^2*B^2*b^9*c^2
- 4*A^3*B*b^8*c^3 + A^4*b^7*c^4)) - 231*c^3*(-(B^4*b^11 - 4*A*B^3*b^10*c +
6*A^2*B^2*b^9*c^2 - 4*A^3*B*b^8*c^3 + A^4*b^7*c^4)/c^15)^(1/4)*log(c^11*(-(
B^4*b^11 - 4*A*B^3*b^10*c + 6*A^2*B^2*b^9*c^2 - 4*A^3*B*b^8*c^3 + A^4*b^7*c
^4)/c^15)^(3/4) - (B^3*b^8 - 3*A*B^2*b^7*c + 3*A^2*B*b^6*c^2 - A^3*b^5*c^3)
*sqrt(x)) + 231*c^3*(-(B^4*b^11 - 4*A*B^3*b^10*c + 6*A^2*B^2*b^9*c^2 - 4*A^
3*B*b^8*c^3 + A^4*b^7*c^4)/c^15)^(1/4)*log(-c^11*(-(B^4*b^11 - 4*A*B^3*b^10
*c + 6*A^2*B^2*b^9*c^2 - 4*A^3*B*b^8*c^3 + A^4*b^7*c^4)/c^15)^(3/4) - (B^3*
b^8 - 3*A*B^2*b^7*c + 3*A^2*B*b^6*c^2 - A^3*b^5*c^3)*sqrt(x)) - 4*(21*B*c^2
*x^5 - 33*(B*b*c - A*c^2)*x^3 + 77*(B*b^2 - A*b*c)*x)*sqrt(x))/c^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(13/2)*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16441, size = 402, normalized size = 1.45

$$-\frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{3}{4}}Bb^{2}-\left(bc^{3}\right)^{\frac{3}{4}}Abc\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2c^{6}}-\frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{3}{4}}Bb^{2}-\left(bc^{3}\right)^{\frac{3}{4}}Abc\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2c^{6}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*((b*c^3)^(3/4)*B*b^2 - (b*c^3)^(3/4)*A*b*c)*arctan(1/2*sqrt(2)
*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/c^6 - 1/2*sqrt(2)*((b*c^3)^
(3/4)*B*b^2 - (b*c^3)^(3/4)*A*b*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4)
 - 2*sqrt(x))/(b/c)^(1/4))/c^6 + 1/4*sqrt(2)*((b*c^3)^(3/4)*B*b^2 - (b*c^3)
^(3/4)*A*b*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^6 - 1/4*sq
rt(2)*((b*c^3)^(3/4)*B*b^2 - (b*c^3)^(3/4)*A*b*c)*log(-sqrt(2)*sqrt(x)*(b/c
)^(1/4) + x + sqrt(b/c))/c^6 + 2/231*(21*B*c^10*x^(11/2) - 33*B*b*c^9*x^(7/
2) + 33*A*c^10*x^(7/2) + 77*B*b^2*c^8*x^(3/2) - 77*A*b*c^9*x^(3/2))/c^11
```

3.184 $\int \frac{x^{11/2}(A+Bx^2)}{bx^2+cx^4}\,dx$

Optimal. Leaf size=276

$$\frac{b^{5/4}(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{13/4}}-\frac{b^{5/4}(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{13/4}}+\frac{b^{5/4}(bB-Ac)\tan^{-1}\left(1\right.}{\sqrt{2}c^{13/4}}$$

```
[Out] (2*b*(b*B - A*c)*Sqrt[x])/c^3 - (2*(b*B - A*c)*x^(5/2))/(5*c^2) + (2*B*x^(9
/2))/(9*c) + (b^(5/4)*(b*B - A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1
/4)])/(Sqrt[2]*c^(13/4)) - (b^(5/4)*(b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)
*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^(13/4)) + (b^(5/4)*(b*B - A*c)*Log[Sqrt[b] -
 Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(13/4)) - (b^(5
/4)*(b*B - A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])
/(2*Sqrt[2]*c^(13/4))
```

Rubi [A] time = 0.247373, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1584, 459, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{b^{5/4}(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{13/4}}-\frac{b^{5/4}(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{13/4}}+\frac{b^{5/4}(bB-Ac)\tan^{-1}\left(1\right.}{\sqrt{2}c^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```


```
[Out] (2*b*(b*B - A*c)*Sqrt[x])/c^3 - (2*(b*B - A*c)*x^(5/2))/(5*c^2) + (2*B*x^(9
/2))/(9*c) + (b^(5/4)*(b*B - A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1
/4)])/(Sqrt[2]*c^(13/4)) - (b^(5/4)*(b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)
*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^(13/4)) + (b^(5/4)*(b*B - A*c)*Log[Sqrt[b] -
 Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(13/4)) - (b^(5
/4)*(b*B - A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])
/(2*Sqrt[2]*c^(13/4))
```


Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{11/2}\left(A+Bx^2\right)}{bx^2+cx^4}\,dx = \int \frac{x^{7/2}\left(A+Bx^2\right)}{b+cx^2}\,dx$$

$$= \frac{2Bx^{9/2}}{9c} - \frac{\left(2\left(\frac{9bB}{2}-\frac{9Ac}{2}\right)\right)\int \frac{x^{7/2}}{b+cx^2}\,dx}{9c}$$

$$= -\frac{2(bB-Ac)x^{5/2}}{5c^2} + \frac{2Bx^{9/2}}{9c} + \frac{(b(bB-Ac))\int \frac{x^{3/2}}{b+cx^2}\,dx}{c^2}$$

$$= \frac{2b(bB-Ac)\sqrt{x}}{c^3} - \frac{2(bB-Ac)x^{5/2}}{5c^2} + \frac{2Bx^{9/2}}{9c} - \frac{\left(b^2(bB-Ac)\right)\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{c^3}$$

$$= \frac{2b(bB-Ac)\sqrt{x}}{c^3} - \frac{2(bB-Ac)x^{5/2}}{5c^2} + \frac{2Bx^{9/2}}{9c} - \frac{\left(2b^2(bB-Ac)\right)\mathrm{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c^3}$$

$$= \frac{2b(bB-Ac)\sqrt{x}}{c^3} - \frac{2(bB-Ac)x^{5/2}}{5c^2} + \frac{2Bx^{9/2}}{9c} - \frac{\left(b^{3/2}(bB-Ac)\right)\mathrm{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c^3} - \cdots$$

$$= \frac{2b(bB-Ac)\sqrt{x}}{c^3} - \frac{2(bB-Ac)x^{5/2}}{5c^2} + \frac{2Bx^{9/2}}{9c} - \frac{\left(b^{3/2}(bB-Ac)\right)\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2c^{7/2}} \cdots$$

$$= \frac{2b(bB-Ac)\sqrt{x}}{c^3} - \frac{2(bB-Ac)x^{5/2}}{5c^2} + \frac{2Bx^{9/2}}{9c} + \frac{b^{5/4}(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}c^{13/4}} - \cdots$$

$$= \frac{2b(bB-Ac)\sqrt{x}}{c^3} - \frac{2(bB-Ac)x^{5/2}}{5c^2} + \frac{2Bx^{9/2}}{9c} + \frac{b^{5/4}(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{13/4}} - \frac{b^{5/4}(bB-A\cdots}{\cdots}$$

Mathematica [A] time = 0.275619, size = 227, normalized size = 0.82

$$\frac{\frac{45\sqrt{2}b^{5/4}(bB-Ac)\left(\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)-\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)\right)}{\sqrt[4]{c}} + \frac{90\sqrt{2}b^{5/4}(bB-Ac)\left(\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)-\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)\right)}{\sqrt[4]{c}} + 72cx^{5/2}\cdots}{180c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (360*b*(b*B - A*c)*Sqrt[x] + 72*c*(-(b*B) + A*c)*x^(5/2) + 40*B*c^2*x^(9/2)
 + (90*Sqrt[2]*b^(5/4)*(b*B - A*c)*(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^
(1/4)] - ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]))/c^(1/4) + (45*Sqrt
```

```
[2]*b^(5/4)*(b*B - A*c)*(Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sq
rt[c]*x] - Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]))/c^(
1/4))/(180*c^3)
```

Maple [A] time = 0.008, size = 330, normalized size = 1.2

$$\frac{2B}{9c}x^{\frac{9}{2}}+\frac{2A}{5c}x^{\frac{5}{2}}-\frac{2Bb}{5c^2}x^{\frac{5}{2}}-2\frac{Ab\sqrt{x}}{c^2}+2\frac{Bb^2\sqrt{x}}{c^3}+\frac{b\sqrt{2}A}{2c^2}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)+\frac{b\sqrt{2}A}{2c^2}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2),x)
```

```
[Out] 2/9*B*x^(9/2)/c+2/5/c*A*x^(5/2)-2/5/c^2*B*x^(5/2)*b-2/c^2*A*b*x^(1/2)+2/c^3
*B*b^2*x^(1/2)+1/2*b/c^2*(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x
^(1/2)+1)+1/2*b/c^2*(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2
)-1)+1/4*b/c^2*(b/c)^(1/4)*2^(1/2)*A*ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c
)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))-1/2*b^2/c^3*(b/c)^(1/
4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-1/2*b^2/c^3*(b/c)^(1/4)*
2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-1/4*b^2/c^3*(b/c)^(1/4)*2^(
1/2)*B*ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2
)*2^(1/2)+(b/c)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.2737, size = 1477, normalized size = 5.35

$$\frac{180\,c^3\left(-\frac{B^4b^9-4\,AB^3b^8c+6\,A^2B^2b^7c^2-4\,A^3Bb^6c^3+A^4b^5c^4}{c^{13}}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{c^6\sqrt{-\frac{B^4b^9-4\,AB^3b^8c+6\,A^2B^2b^7c^2-4\,A^3Bb^6c^3+A^4b^5c^4}{c^{13}}}+\left(B^2b^4-2\,ABb^3c+A^2b^2c^2\right)xc}}{B^4b^9}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/90*(180*c^3*(-(B^4*b^9 - 4*A*B^3*b^8*c + 6*A^2*B^2*b^7*c^2 - 4*A^3*B*b^6*
c^3 + A^4*b^5*c^4)/c^13)^(1/4)*arctan((sqrt(c^6*sqrt(-(B^4*b^9 - 4*A*B^3*b^
8*c + 6*A^2*B^2*b^7*c^2 - 4*A^3*B*b^6*c^3 + A^4*b^5*c^4)/c^13) + (B^2*b^4 -
 2*A*B*b^3*c + A^2*b^2*c^2)*x)*c^10*(-(B^4*b^9 - 4*A*B^3*b^8*c + 6*A^2*B^2*
b^7*c^2 - 4*A^3*B*b^6*c^3 + A^4*b^5*c^4)/c^13)^(3/4) + (B*b^2*c^10 - A*b*c^
11)*sqrt(x)*(-(B^4*b^9 - 4*A*B^3*b^8*c + 6*A^2*B^2*b^7*c^2 - 4*A^3*B*b^6*c^
3 + A^4*b^5*c^4)/c^13)^(3/4))/(B^4*b^9 - 4*A*B^3*b^8*c + 6*A^2*B^2*b^7*c^2
- 4*A^3*B*b^6*c^3 + A^4*b^5*c^4)) + 45*c^3*(-(B^4*b^9 - 4*A*B^3*b^8*c + 6*A
^2*B^2*b^7*c^2 - 4*A^3*B*b^6*c^3 + A^4*b^5*c^4)/c^13)^(1/4)*log(c^3*(-(B^4*
b^9 - 4*A*B^3*b^8*c + 6*A^2*B^2*b^7*c^2 - 4*A^3*B*b^6*c^3 + A^4*b^5*c^4)/c^
13)^(1/4) - (B*b^2 - A*b*c)*sqrt(x)) - 45*c^3*(-(B^4*b^9 - 4*A*B^3*b^8*c +
6*A^2*B^2*b^7*c^2 - 4*A^3*B*b^6*c^3 + A^4*b^5*c^4)/c^13)^(1/4)*log(-c^3*(-(
B^4*b^9 - 4*A*B^3*b^8*c + 6*A^2*B^2*b^7*c^2 - 4*A^3*B*b^6*c^3 + A^4*b^5*c^4
)/c^13)^(1/4) - (B*b^2 - A*b*c)*sqrt(x)) + 4*(5*B*c^2*x^4 + 45*B*b^2 - 45*A
*b*c - 9*(B*b*c - A*c^2)*x^2)*sqrt(x))/c^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(11/2)*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16315, size = 402, normalized size = 1.46

$$-\frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{1}{4}}Bb^2-\left(bc^3\right)^{\frac{1}{4}}Abc\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,c^4}-\frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{1}{4}}Bb^2-\left(bc^3\right)^{\frac{1}{4}}Abc\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,c^4}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*((b*c^3)^(1/4)*B*b^2 - (b*c^3)^(1/4)*A*b*c)*arctan(1/2*sqrt(2)
*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/c^4 - 1/2*sqrt(2)*((b*c^3)^
(1/4)*B*b^2 - (b*c^3)^(1/4)*A*b*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4)
 - 2*sqrt(x))/(b/c)^(1/4))/c^4 - 1/4*sqrt(2)*((b*c^3)^(1/4)*B*b^2 - (b*c^3)
^(1/4)*A*b*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^4 + 1/4*sq
rt(2)*((b*c^3)^(1/4)*B*b^2 - (b*c^3)^(1/4)*A*b*c)*log(-sqrt(2)*sqrt(x)*(b/c
)^(1/4) + x + sqrt(b/c))/c^4 + 2/45*(5*B*c^8*x^(9/2) - 9*B*b*c^7*x^(5/2) +
9*A*c^8*x^(5/2) + 45*B*b^2*c^6*sqrt(x) - 45*A*b*c^7*sqrt(x))/c^9
```

3.185 $\int \frac{x^{9/2}\left(A+Bx^2\right)}{bx^2+cx^4}\,dx$

Optimal. Leaf size=257

$$\frac{b^{3/4}(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{11/4}}-\frac{b^{3/4}(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{11/4}}-\frac{b^{3/4}(bB-Ac)\tan^{-1}\left(1\right.}{\sqrt{2}c^{11/4}}$$

```
[Out] (-2*(b*B - A*c)*x^(3/2))/(3*c^2) + (2*B*x^(7/2))/(7*c) - (b^(3/4)*(b*B - A*
c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^(11/4)) + (b^(
3/4)*(b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^
(11/4)) + (b^(3/4)*(b*B - A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x
] + Sqrt[c]*x])/(2*Sqrt[2]*c^(11/4)) - (b^(3/4)*(b*B - A*c)*Log[Sqrt[b] + S
qrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(11/4))
```

Rubi [A] time = 0.217638, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1584, 459, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{b^{3/4}(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{11/4}}-\frac{b^{3/4}(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{11/4}}-\frac{b^{3/4}(bB-Ac)\tan^{-1}\left(1\right.}{\sqrt{2}c^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```


```
[Out] (-2*(b*B - A*c)*x^(3/2))/(3*c^2) + (2*B*x^(7/2))/(7*c) - (b^(3/4)*(b*B - A*
c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^(11/4)) + (b^(
3/4)*(b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^
(11/4)) + (b^(3/4)*(b*B - A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x
] + Sqrt[c]*x])/(2*Sqrt[2]*c^(11/4)) - (b^(3/4)*(b*B - A*c)*Log[Sqrt[b] + S
qrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(11/4))
```


Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^{9/2}\left(A+Bx^2\right)}{bx^2+cx^4}\,dx = \int \frac{x^{5/2}\left(A+Bx^2\right)}{b+cx^2}\,dx$$

$$= \frac{2Bx^{7/2}}{7c} - \frac{\left(2\left(\frac{7bB}{2}-\frac{7Ac}{2}\right)\right)\int \frac{x^{5/2}}{b+cx^2}\,dx}{7c}$$

$$= -\frac{2(bB-Ac)x^{3/2}}{3c^2} + \frac{2Bx^{7/2}}{7c} + \frac{(b(bB-Ac))\int \frac{\sqrt{x}}{b+cx^2}\,dx}{c^2}$$

$$= -\frac{2(bB-Ac)x^{3/2}}{3c^2} + \frac{2Bx^{7/2}}{7c} + \frac{(2b(bB-Ac))\,\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c^2}$$

$$= -\frac{2(bB-Ac)x^{3/2}}{3c^2} + \frac{2Bx^{7/2}}{7c} - \frac{(b(bB-Ac))\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c^{5/2}} + \frac{(b(bB-Ac))\,\text{Subst}\left(\int\right.}{c^{5/2}}$$

$$= -\frac{2(bB-Ac)x^{3/2}}{3c^2} + \frac{2Bx^{7/2}}{7c} + \frac{(b(bB-Ac))\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2c^3} + \frac{(b(bB-Ac))\,\text{Subs}}{}$$

$$= -\frac{2(bB-Ac)x^{3/2}}{3c^2} + \frac{2Bx^{7/2}}{7c} + \frac{b^{3/4}(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}c^{11/4}} - \frac{b^{3/4}(bB-Ac)\log\left(\sqrt{\right.}}{2}$$

$$= -\frac{2(bB-Ac)x^{3/2}}{3c^2} + \frac{2Bx^{7/2}}{7c} - \frac{b^{3/4}(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{11/4}} + \frac{b^{3/4}(bB-Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{b}}\right.}{\sqrt{2}c^{11/4}}$$

Mathematica [A] time = 0.126868, size = 110, normalized size = 0.43

$$\frac{2c^{3/4}x^{3/2}\left(7Ac-7bB+3Bcx^2\right)-21(-b)^{3/4}(bB-Ac)\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)+21(-b)^{3/4}(bB-Ac)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)}{21c^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (2*c^(3/4)*x^(3/2)*(-7*b*B + 7*A*c + 3*B*c*x^2) - 21*(-b)^(3/4)*(b*B - A*c)
*ArcTan[(c^(1/4)*Sqrt[x])/(-b)^(1/4)] + 21*(-b)^(3/4)*(b*B - A*c)*ArcTanh[(
c^(1/4)*Sqrt[x])/(-b)^(1/4)])/(21*c^(11/4))
```

Maple [A] time = 0.007, size = 308, normalized size = 1.2

$$\frac{2B}{7c}x^{\frac{7}{2}}+\frac{2A}{3c}x^{\frac{3}{2}}-\frac{2Bb}{3c^2}x^{\frac{3}{2}}-\frac{b\sqrt{2}A}{2c^2}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}-\frac{b\sqrt{2}A}{2c^2}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}-1\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}-\frac{b\sqrt{2}A}{4c^2}\ln\left(\left(x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2),x)
```

```
[Out] 2/7*B*x^(7/2)/c+2/3/c*x^(3/2)*A-2/3/c^2*x^(3/2)*B*b-1/2*b/c^2/(b/c)^(1/4)*2
^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-1/2*b/c^2/(b/c)^(1/4)*2^(1/2
)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-1/4*b/c^2/(b/c)^(1/4)*2^(1/2)*A*l
n((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2
)+(b/c)^(1/2)))+1/2*b^2/c^3/(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4
)*x^(1/2)+1)+1/2*b^2/c^3/(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x
^(1/2)-1)+1/4*b^2/c^3/(b/c)^(1/4)*2^(1/2)*B*ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/
2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.40833, size = 1823, normalized size = 7.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/42*(84*c^2*(-(B^4*b^7 - 4*A*B^3*b^6*c + 6*A^2*B^2*b^5*c^2 - 4*A^3*B*b^4*c
^3 + A^4*b^3*c^4)/c^11)^(1/4)*arctan((sqrt((B^6*b^10 - 6*A*B^5*b^9*c + 15*A
```

```
^2*B^4*b^8*c^2 - 20*A^3*B^3*b^7*c^3 + 15*A^4*B^2*b^6*c^4 - 6*A^5*B*b^5*c^5
+ A^6*b^4*c^6)*x - (B^4*b^7*c^5 - 4*A*B^3*b^6*c^6 + 6*A^2*B^2*b^5*c^7 - 4*A
^3*B*b^4*c^8 + A^4*b^3*c^9)*sqrt(-(B^4*b^7 - 4*A*B^3*b^6*c + 6*A^2*B^2*b^5*
c^2 - 4*A^3*B*b^4*c^3 + A^4*b^3*c^4)/c^11))*c^3*(-(B^4*b^7 - 4*A*B^3*b^6*c
+ 6*A^2*B^2*b^5*c^2 - 4*A^3*B*b^4*c^3 + A^4*b^3*c^4)/c^11)^(1/4) + (B^3*b^5
*c^3 - 3*A*B^2*b^4*c^4 + 3*A^2*B*b^3*c^5 - A^3*b^2*c^6)*sqrt(x)*(-(B^4*b^7
- 4*A*B^3*b^6*c + 6*A^2*B^2*b^5*c^2 - 4*A^3*B*b^4*c^3 + A^4*b^3*c^4)/c^11)^
(1/4))/(B^4*b^7 - 4*A*B^3*b^6*c + 6*A^2*B^2*b^5*c^2 - 4*A^3*B*b^4*c^3 + A^4
*b^3*c^4)) - 21*c^2*(-(B^4*b^7 - 4*A*B^3*b^6*c + 6*A^2*B^2*b^5*c^2 - 4*A^3*
B*b^4*c^3 + A^4*b^3*c^4)/c^11)^(1/4)*log(c^8*(-(B^4*b^7 - 4*A*B^3*b^6*c + 6
*A^2*B^2*b^5*c^2 - 4*A^3*B*b^4*c^3 + A^4*b^3*c^4)/c^11)^(3/4) - (B^3*b^5 -
3*A*B^2*b^4*c + 3*A^2*B*b^3*c^2 - A^3*b^2*c^3)*sqrt(x)) + 21*c^2*(-(B^4*b^7
 - 4*A*B^3*b^6*c + 6*A^2*B^2*b^5*c^2 - 4*A^3*B*b^4*c^3 + A^4*b^3*c^4)/c^11)
^(1/4)*log(-c^8*(-(B^4*b^7 - 4*A*B^3*b^6*c + 6*A^2*B^2*b^5*c^2 - 4*A^3*B*b^
4*c^3 + A^4*b^3*c^4)/c^11)^(3/4) - (B^3*b^5 - 3*A*B^2*b^4*c + 3*A^2*B*b^3*c
^2 - A^3*b^2*c^3)*sqrt(x)) + 4*(3*B*c*x^3 - 7*(B*b - A*c)*x)*sqrt(x))/c^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15171, size = 356, normalized size = 1.39

$$\frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{3}{4}} Bb - \left(bc^3\right)^{\frac{3}{4}} Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}} + 2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2c^5} + \frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{3}{4}} Bb - \left(bc^3\right)^{\frac{3}{4}} Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}} - 2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2c^5} - \frac{\sqrt{2}\left(\left(b\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2)*(sqr
t(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/c^5 + 1/2*sqrt(2)*((b*c^3)^(3/4)
*B*b - (b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt
(x))/(b/c)^(1/4))/c^5 - 1/4*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)
*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^5 + 1/4*sqrt(2)*((b*c^3
)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqr
t(b/c))/c^5 + 2/21*(3*B*c^6*x^(7/2) - 7*B*b*c^5*x^(3/2) + 7*A*c^6*x^(3/2))/
c^7
```

3.186 $\int \frac{x^{7/2}\left(A+Bx^2\right)}{bx^2+cx^4}\,dx$

Optimal. Leaf size=255

$$-\frac{2\sqrt{x}(bB-Ac)}{c^2}-\frac{\sqrt[4]{b}(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{9/4}}+\frac{\sqrt[4]{b}(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{9/4}}-\frac{\sqrt[4]{b}(b}{}$$

```
[Out] (-2*(b*B - A*c)*Sqrt[x])/c^2 + (2*B*x^(5/2))/(5*c) - (b^(1/4)*(b*B - A*c)*A
rcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^(9/4)) + (b^(1/4)*
(b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^(9/4)
) - (b^(1/4)*(b*B - A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sq
rt[c]*x])/(2*Sqrt[2]*c^(9/4)) + (b^(1/4)*(b*B - A*c)*Log[Sqrt[b] + Sqrt[2]*
b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(9/4))
```

Rubi [A] time = 0.209264, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1584, 459, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{2\sqrt{x}(bB-Ac)}{c^2}-\frac{\sqrt[4]{b}(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{9/4}}+\frac{\sqrt[4]{b}(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{9/4}}-\frac{\sqrt[4]{b}(b}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (-2*(b*B - A*c)*Sqrt[x])/c^2 + (2*B*x^(5/2))/(5*c) - (b^(1/4)*(b*B - A*c)*A
rcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^(9/4)) + (b^(1/4)*
(b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^(9/4)
) - (b^(1/4)*(b*B - A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sq
rt[c]*x])/(2*Sqrt[2]*c^(9/4)) + (b^(1/4)*(b*B - A*c)*Log[Sqrt[b] + Sqrt[2]*
b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(9/4))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{7/2}\left(A+Bx^2\right)}{bx^2+cx^4}\,dx = \int \frac{x^{3/2}\left(A+Bx^2\right)}{b+cx^2}\,dx$$

$$= \frac{2Bx^{5/2}}{5c} - \frac{\left(2\left(\frac{5bB}{2}-\frac{5Ac}{2}\right)\right)\int \frac{x^{3/2}}{b+cx^2}\,dx}{5c}$$

$$= -\frac{2(bB-Ac)\sqrt{x}}{c^2} + \frac{2Bx^{5/2}}{5c} + \frac{(b(bB-Ac))\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{c^2}$$

$$= -\frac{2(bB-Ac)\sqrt{x}}{c^2} + \frac{2Bx^{5/2}}{5c} + \frac{(2b(bB-Ac))\,\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c^2}$$

$$= -\frac{2(bB-Ac)\sqrt{x}}{c^2} + \frac{2Bx^{5/2}}{5c} + \frac{\left(\sqrt{b}(bB-Ac)\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c^2} + \frac{\left(\sqrt{b}(bB-Ac)\right)\text{Sub}}{}$$

$$= -\frac{2(bB-Ac)\sqrt{x}}{c^2} + \frac{2Bx^{5/2}}{5c} + \frac{\left(\sqrt{b}(bB-Ac)\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2c^{5/2}} + \frac{\left(\sqrt{b}(bB-Ac)\right)}{}$$

$$= -\frac{2(bB-Ac)\sqrt{x}}{c^2} + \frac{2Bx^{5/2}}{5c} - \frac{\sqrt[4]{b}(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}c^{9/4}} + \frac{\sqrt[4]{b}(bB-Ac)\log\left(\right.}{2}$$

$$= -\frac{2(bB-Ac)\sqrt{x}}{c^2} + \frac{2Bx^{5/2}}{5c} - \frac{\sqrt[4]{b}(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{9/4}} + \frac{\sqrt[4]{b}(bB-Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{b}}\right.}{\sqrt{2}c^{9/4}}$$

Mathematica [A] time = 0.201955, size = 208, normalized size = 0.82

$$\frac{-40\sqrt{x}(bB-Ac) + \frac{5\sqrt{2}\sqrt[4]{b}(Ac-bB)\left(\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)-\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)\right)}{\sqrt[4]{c}} - \frac{10\sqrt{2}\sqrt[4]{b}(bB-Ac)\left(\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)-\tan^{-1}\left(\frac{\sqrt{2}}{}\right.\right.}{\sqrt[4]{c}}}{20c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (-40*(b*B - A*c)*Sqrt[x] + 8*B*c*x^(5/2) - (10*Sqrt[2]*b^(1/4)*(b*B - A*c)*
(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] - ArcTan[1 + (Sqrt[2]*c^(1/4
)*Sqrt[x])/b^(1/4)]))/c^(1/4) + (5*Sqrt[2]*b^(1/4)*(-(b*B) + A*c)*(Log[Sqrt
[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] - Log[Sqrt[b] + Sqrt[2]*
b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]))/c^(1/4))/(20*c^2)
```

Maple [A] time = 0.008, size = 299, normalized size = 1.2

$$\frac{2B}{5c}x^{\frac{5}{2}}+2\frac{A\sqrt{x}}{c}-2\frac{Bb\sqrt{x}}{c^2}-\frac{\sqrt{2}A}{2c}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)-\frac{\sqrt{2}A}{2c}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}-1\right)-\frac{\sqrt{2}A}{4c}\sqrt[4]{\frac{b}{c}}\ln\left(\left(x+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2),x)
```

```
[Out] 2/5*B*x^(5/2)/c+2/c*A*x^(1/2)-2/c^2*B*b*x^(1/2)-1/2/c*(b/c)^(1/4)*2^(1/2)*A
*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-1/2/c*(b/c)^(1/4)*2^(1/2)*A*arctan(2
^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-1/4/c*(b/c)^(1/4)*2^(1/2)*A*ln((x+(b/c)^(1/4)
*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+
1/2/c^2*(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)*b+1/2/c
^2*(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)*b+1/4/c^2*(b
/c)^(1/4)*2^(1/2)*B*ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)
^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.07589, size = 1365, normalized size = 5.35

$$-\frac{20\,c^2\left(-\frac{B^4b^5-4\,AB^3b^4c+6\,A^2B^2b^3c^2-4\,A^3Bb^2c^3+A^4bc^4}{c^9}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{c^4\sqrt{-\frac{B^4b^5-4\,AB^3b^4c+6\,A^2B^2b^3c^2-4\,A^3Bb^2c^3+A^4bc^4}{c^9}}+\left(B^2b^2-2\,ABbc+A^2c^2\right)xc^7\left(-\frac{B^4b}{}\right.}}{B^4b^5-4\,AB^3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] -1/10*(20*c^2*(-(B^4*b^5 - 4*A*B^3*b^4*c + 6*A^2*B^2*b^3*c^2 - 4*A^3*B*b^2*
c^3 + A^4*b*c^4)/c^9)^(1/4)*arctan((sqrt(c^4*sqrt(-(B^4*b^5 - 4*A*B^3*b^4*c
 + 6*A^2*B^2*b^3*c^2 - 4*A^3*B*b^2*c^3 + A^4*b*c^4)/c^9) + (B^2*b^2 - 2*A*B
*b*c + A^2*c^2)*x)*c^7*(-(B^4*b^5 - 4*A*B^3*b^4*c + 6*A^2*B^2*b^3*c^2 - 4*A
^3*B*b^2*c^3 + A^4*b*c^4)/c^9)^(3/4) + (B*b*c^7 - A*c^8)*sqrt(x)*(-(B^4*b^5
 - 4*A*B^3*b^4*c + 6*A^2*B^2*b^3*c^2 - 4*A^3*B*b^2*c^3 + A^4*b*c^4)/c^9)^(3
/4))/(B^4*b^5 - 4*A*B^3*b^4*c + 6*A^2*B^2*b^3*c^2 - 4*A^3*B*b^2*c^3 + A^4*b
*c^4)) + 5*c^2*(-(B^4*b^5 - 4*A*B^3*b^4*c + 6*A^2*B^2*b^3*c^2 - 4*A^3*B*b^2
*c^3 + A^4*b*c^4)/c^9)^(1/4)*log(c^2*(-(B^4*b^5 - 4*A*B^3*b^4*c + 6*A^2*B^2
*b^3*c^2 - 4*A^3*B*b^2*c^3 + A^4*b*c^4)/c^9)^(1/4) - (B*b - A*c)*sqrt(x)) -
 5*c^2*(-(B^4*b^5 - 4*A*B^3*b^4*c + 6*A^2*B^2*b^3*c^2 - 4*A^3*B*b^2*c^3 + A
^4*b*c^4)/c^9)^(1/4)*log(-c^2*(-(B^4*b^5 - 4*A*B^3*b^4*c + 6*A^2*B^2*b^3*c^
2 - 4*A^3*B*b^2*c^3 + A^4*b*c^4)/c^9)^(1/4) - (B*b - A*c)*sqrt(x)) - 4*(B*c
*x^2 - 5*B*b + 5*A*c)*sqrt(x))/c^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12993, size = 355, normalized size = 1.39

$$\frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{1}{4}}Bb-\left(bc^3\right)^{\frac{1}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2c^3}+\frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{1}{4}}Bb-\left(bc^3\right)^{\frac{1}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2c^3}+\frac{\sqrt{2}\left(\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt(2)*(sqr
t(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/c^3 + 1/2*sqrt(2)*((b*c^3)^(1/4)
*B*b - (b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt
(x))/(b/c)^(1/4))/c^3 + 1/4*sqrt(2)*((b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)
*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^3 - 1/4*sqrt(2)*((b*c^3
)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqr
t(b/c))/c^3 + 2/5*(B*c^4*x^(5/2) - 5*B*b*c^3*sqrt(x) + 5*A*c^4*sqrt(x))/c^5
```

3.187 $\int \frac{x^{5/2}(A+Bx^2)}{bx^2+cx^4}\,dx$

Optimal. Leaf size=237

$$-\frac{(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}c^{7/4}}+\frac{(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}c^{7/4}}+\frac{(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}c^{7/4}}$$

```
[Out] (2*B*x^(3/2))/(3*c) + ((b*B - A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(
1/4)])/(Sqrt[2]*b^(1/4)*c^(7/4)) - ((b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)
*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(1/4)*c^(7/4)) - ((b*B - A*c)*Log[Sqrt[b] -
Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(1/4)*c^(7/4)) +
 ((b*B - A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(
2*Sqrt[2]*b^(1/4)*c^(7/4))
```

Rubi [A] time = 0.188252, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {1584, 459, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}c^{7/4}}+\frac{(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}c^{7/4}}+\frac{(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}c^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```


```
[Out] (2*B*x^(3/2))/(3*c) + ((b*B - A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(
1/4)])/(Sqrt[2]*b^(1/4)*c^(7/4)) - ((b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)
*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(1/4)*c^(7/4)) - ((b*B - A*c)*Log[Sqrt[b] -
Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(1/4)*c^(7/4)) +
 ((b*B - A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(
2*Sqrt[2]*b^(1/4)*c^(7/4))
```


Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{5/2}\left(A+Bx^2\right)}{bx^2+cx^4}\,dx &= \int \frac{\sqrt{x}\left(A+Bx^2\right)}{b+cx^2}\,dx \\
&= \frac{2Bx^{3/2}}{3c} - \frac{\left(2\left(\frac{3bB}{2}-\frac{3Ac}{2}\right)\right)\int \frac{\sqrt{x}}{b+cx^2}\,dx}{3c} \\
&= \frac{2Bx^{3/2}}{3c} - \frac{\left(4\left(\frac{3bB}{2}-\frac{3Ac}{2}\right)\right)\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{3c} \\
&= \frac{2Bx^{3/2}}{3c} + \frac{(bB-Ac)\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{c^{3/2}} - \frac{(bB-Ac)\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{c^{3/2}} \\
&= \frac{2Bx^{3/2}}{3c} - \frac{(bB-Ac)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{2c^2} - \frac{(bB-Ac)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,\ldots\right)}{2c^2} \\
&= \frac{2Bx^{3/2}}{3c} - \frac{(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}c^{7/4}} + \frac{(bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}c^{7/4}} \\
&= \frac{2Bx^{3/2}}{3c} + \frac{(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}c^{7/4}} - \frac{(bB-Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}c^{7/4}} - \frac{(bB-Ac)\log\left(\sqrt{b}\ldots\right.}{2\sqrt{2}\ldots}
\end{aligned}$$

Mathematica [A] time = 0.0748007, size = 95, normalized size = 0.4

$$\frac{(3Ac-3bB)\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)+3(bB-Ac)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)+2\sqrt[4]{-b}Bc^{3/4}x^{3/2}}{3\sqrt[4]{-b}c^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (2*(-b)^(1/4)*B*c^(3/4)*x^(3/2) + (-3*b*B + 3*A*c)*ArcTan[(c^(1/4)*Sqrt[x])
/(-b)^(1/4)] + 3*(b*B - A*c)*ArcTanh[(c^(1/4)*Sqrt[x])/(-b)^(1/4)])/(3*(-b)
^(1/4)*c^(7/4))
```

Maple [A] time = 0.008, size = 280, normalized size = 1.2

$$\frac{2B}{3c}x^{\frac{3}{2}}+\frac{\sqrt{2}A}{2c}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}-1\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}+\frac{\sqrt{2}A}{4c}\ln\left(\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}+\frac{\sqrt{2}A}{2c}\mathrm{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2),x)
```

```
[Out] 2/3*B*x^(3/2)/c+1/2/c/(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1
/2)-1)+1/4/c/(b/c)^(1/4)*2^(1/2)*A*ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^
(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+1/2/c/(b/c)^(1/4)*2^(1/
2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-1/2/c^2/(b/c)^(1/4)*2^(1/2)*B*ar
ctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)*b-1/4/c^2/(b/c)^(1/4)*2^(1/2)*B*ln((x-(
b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c
)^(1/2)))*b-1/2/c^2/(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2
)+1)*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.335, size = 1698, normalized size = 7.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/6*(4*B*x^(3/2) - 12*c*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*
A^3*B*b*c^3 + A^4*c^4)/(b*c^7))^(1/4)*arctan((sqrt((B^6*b^6 - 6*A*B^5*b^5*c
 + 15*A^2*B^4*b^4*c^2 - 20*A^3*B^3*b^3*c^3 + 15*A^4*B^2*b^2*c^4 - 6*A^5*B*b
*c^5 + A^6*c^6)*x - (B^4*b^5*c^3 - 4*A*B^3*b^4*c^4 + 6*A^2*B^2*b^3*c^5 - 4*
A^3*B*b^2*c^6 + A^4*b*c^7)*sqrt(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c
^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b*c^7)))*c^2*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*
A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b*c^7))^(1/4) + (B^3*b^3*c^2 -
3*A*B^2*b^2*c^3 + 3*A^2*B*b*c^4 - A^3*c^5)*sqrt(x)*(-(B^4*b^4 - 4*A*B^3*b^3
*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b*c^7))^(1/4))/(B^4*b^4
- 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)) + 3*c*(-(B^
4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b*c^7
))^(1/4)*log(b*c^5*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B
*b*c^3 + A^4*c^4)/(b*c^7))^(3/4) - (B^3*b^3 - 3*A*B^2*b^2*c + 3*A^2*B*b*c^2
 - A^3*c^3)*sqrt(x)) - 3*c*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 -
 4*A^3*B*b*c^3 + A^4*c^4)/(b*c^7))^(1/4)*log(-b*c^5*(-(B^4*b^4 - 4*A*B^3*b^
3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b*c^7))^(3/4) - (B^3*b^
3 - 3*A*B^2*b^2*c + 3*A^2*B*b*c^2 - A^3*c^3)*sqrt(x)))/c
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19203, size = 339, normalized size = 1.43

$$\frac{2\,Bx^{\frac{3}{2}}}{3\,c} - \frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{3}{4}}Bb - \left(bc^{3}\right)^{\frac{3}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}} + 2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,bc^{4}} - \frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{3}{4}}Bb - \left(bc^{3}\right)^{\frac{3}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}} - 2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.}{2\,bc^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 2/3*B*x^(3/2)/c - 1/2*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*arcta
n(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b*c^4) - 1/2*
sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2
)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b*c^4) + 1/4*sqrt(2)*((b*c^3)^(3/4
)*B*b - (b*c^3)^(3/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))
/(b*c^4) - 1/4*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*log(-sqrt(2)
*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^4)
```

3.188 $\int \frac{x^{3/2}(A+Bx^2)}{bx^2+cx^4} dx$

Optimal. Leaf size=235

$$\frac{(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{3/4}c^{5/4}}-\frac{(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{3/4}c^{5/4}}+\frac{(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right.}{\sqrt{2}b^{3/4}c^{5/4}}$$

```
[Out] (2*B*Sqrt[x])/c + ((b*B - A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)
])/(Sqrt[2]*b^(3/4)*c^(5/4)) - ((b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqr
t[x])/b^(1/4)])/(Sqrt[2]*b^(3/4)*c^(5/4)) + ((b*B - A*c)*Log[Sqrt[b] - Sqrt
[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(3/4)*c^(5/4)) - ((b
*B - A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sq
rt[2]*b^(3/4)*c^(5/4))
```

Rubi [A] time = 0.183632, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {1584, 459, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{3/4}c^{5/4}}-\frac{(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{3/4}c^{5/4}}+\frac{(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right.}{\sqrt{2}b^{3/4}c^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (2*B*Sqrt[x])/c + ((b*B - A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)
])/(Sqrt[2]*b^(3/4)*c^(5/4)) - ((b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqr
t[x])/b^(1/4)])/(Sqrt[2]*b^(3/4)*c^(5/4)) + ((b*B - A*c)*Log[Sqrt[b] - Sqrt
[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(3/4)*c^(5/4)) - ((b
*B - A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sq
rt[2]*b^(3/4)*c^(5/4))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{3/2}\left(A+Bx^2\right)}{bx^2+cx^4}\,dx = \int \frac{A+Bx^2}{\sqrt{x}\left(b+cx^2\right)}\,dx$$

$$= \frac{2B\sqrt{x}}{c} - \frac{\left(2\left(\frac{bB}{2}-\frac{Ac}{2}\right)\right)\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{c}$$

$$= \frac{2B\sqrt{x}}{c} - \frac{\left(4\left(\frac{bB}{2}-\frac{Ac}{2}\right)\right)\mathrm{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c}$$

$$= \frac{2B\sqrt{x}}{c} - \frac{(bB-Ac)\,\mathrm{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{b}c} - \frac{(bB-Ac)\,\mathrm{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{b}c}$$

$$= \frac{2B\sqrt{x}}{c} - \frac{(bB-Ac)\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{b}c^{3/2}} - \frac{(bB-Ac)\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \cdots\right)}{2\sqrt{b}c^{3/2}}$$

$$= \frac{2B\sqrt{x}}{c} + \frac{(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{3/4}c^{5/4}} - \frac{(bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{3/4}c^{5/4}}$$

$$= \frac{2B\sqrt{x}}{c} + \frac{(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{3/4}c^{5/4}} - \frac{(bB-Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{3/4}c^{5/4}} + \frac{(bB-Ac)\log\left(\sqrt{b}-\cdots\right)}{2\sqrt{2}\cdots}$$

Mathematica [A] time = 0.132868, size = 166, normalized size = 0.71

$$\frac{(bB-Ac)\left(\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)-\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)+2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)-2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\cdots\right.\right.}{2\sqrt{2}b^{3/4}c^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (2*B*Sqrt[x])/c + ((b*B - A*c)*(2*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1
/4)] - 2*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] + Log[Sqrt[b] - Sqrt
[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] - Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^
(1/4)*Sqrt[x] + Sqrt[c]*x]))/(2*Sqrt[2]*b^(3/4)*c^(5/4))
```

Maple [A] time = 0.007, size = 277, normalized size = 1.2

$$2\,\frac{B\sqrt{x}}{c}+\frac{\sqrt{2}A}{2\,b}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}-1\right)+\frac{\sqrt{2}A}{4\,b}\sqrt[4]{\frac{b}{c}}\ln\left(\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)+\frac{\sqrt{2}A}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2),x)
```

```
[Out] 2*B*x^(1/2)/c+1/2*(b/c)^(1/4)/b*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2
)-1)+1/4*(b/c)^(1/4)/b*2^(1/2)*A*ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1
/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+1/2*(b/c)^(1/4)/b*2^(1/2)
*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-1/2/c*(b/c)^(1/4)*2^(1/2)*B*arctan
(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-1/4/c*(b/c)^(1/4)*2^(1/2)*B*ln((x+(b/c)^(1/
4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))
)-1/2/c*(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.18009, size = 1347, normalized size = 5.73

$$\frac{4\,c\left(-\frac{B^4b^4-4\,AB^3b^3c+6\,A^2B^2b^2c^2-4\,A^3Bbc^3+A^4c^4}{b^3c^5}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{b^2c^2\sqrt{-\frac{B^4b^4-4\,AB^3b^3c+6\,A^2B^2b^2c^2-4\,A^3Bbc^3+A^4c^4}{b^3c^5}}+\left(B^2b^2-2\,ABbc+A^2c^2\right)x}\,b^2c^4\left(-\frac{B^4b^4-}{}\right.}{B^4b^4-4\,AB^3b^3c}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/2*(4*c*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A
^4*c^4)/(b^3*c^5))^(1/4)*arctan((sqrt(b^2*c^2*sqrt(-(B^4*b^4 - 4*A*B^3*b^3*
c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^5)) + (B^2*b^2 - 2*
A*B*b*c + A^2*c^2)*x)*b^2*c^4*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^
2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^5))^(3/4) + (B*b^3*c^4 - A*b^2*c^5)*sqr
t(x)*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c
^4)/(b^3*c^5))^(3/4))/(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*
B*b*c^3 + A^4*c^4)) + c*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*
A^3*B*b*c^3 + A^4*c^4)/(b^3*c^5))^(1/4)*log(b*c*(-(B^4*b^4 - 4*A*B^3*b^3*c
+ 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^5))^(1/4) - (B*b - A*
c)*sqrt(x)) - c*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*
c^3 + A^4*c^4)/(b^3*c^5))^(1/4)*log(-b*c*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2
*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^5))^(1/4) - (B*b - A*c)*sqrt
(x)) + 4*B*sqrt(x))/c
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19875, size = 339, normalized size = 1.44

$$\frac{2\,B\sqrt{x}}{c}-\frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{1}{4}}Bb-\left(bc^{3}\right)^{\frac{1}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,bc^{2}}-\frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{1}{4}}Bb-\left(bc^{3}\right)^{\frac{1}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,bc^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 2*B*sqrt(x)/c - 1/2*sqrt(2)*((b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*arctan(
1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b*c^2) - 1/2*sq
rt(2)*((b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*
(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b*c^2) - 1/4*sqrt(2)*((b*c^3)^(1/4)*
B*b - (b*c^3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(
b*c^2) + 1/4*sqrt(2)*((b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*log(-sqrt(2)*s
qrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^2)
```

3.189 $\int \frac{\sqrt{x}\left(A+Bx^2\right)}{bx^2+cx^4}\,dx$

Optimal. Leaf size=235

$$\frac{(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{5/4}c^{3/4}}-\frac{(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{5/4}c^{3/4}}-\frac{(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right.}{\sqrt{2}b^{5/4}c^{3/4}}$$

```
[Out] (-2*A)/(b*Sqrt[x]) - ((b*B - A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1
/4)])/(Sqrt[2]*b^(5/4)*c^(3/4)) + ((b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*
Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(5/4)*c^(3/4)) + ((b*B - A*c)*Log[Sqrt[b] - S
qrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(5/4)*c^(3/4)) -
((b*B - A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2
*Sqrt[2]*b^(5/4)*c^(3/4))
```

Rubi [A] time = 0.189378, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {1584, 453, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{5/4}c^{3/4}}-\frac{(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{5/4}c^{3/4}}-\frac{(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right.}{\sqrt{2}b^{5/4}c^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (-2*A)/(b*Sqrt[x]) - ((b*B - A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1
/4)])/(Sqrt[2]*b^(5/4)*c^(3/4)) + ((b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*
Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(5/4)*c^(3/4)) + ((b*B - A*c)*Log[Sqrt[b] - S
qrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(5/4)*c^(3/4)) -
((b*B - A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2
*Sqrt[2]*b^(5/4)*c^(3/4))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{x}\left(A+Bx^2\right)}{bx^2+cx^4}\,dx = \int \frac{A+Bx^2}{x^{3/2}\left(b+cx^2\right)}\,dx$$
$$= -\frac{2A}{b\sqrt{x}} - \frac{\left(2\left(-\frac{bB}{2}+\frac{Ac}{2}\right)\right)\int \frac{\sqrt{x}}{b+cx^2}\,dx}{b}$$
$$= -\frac{2A}{b\sqrt{x}} - \frac{\left(4\left(-\frac{bB}{2}+\frac{Ac}{2}\right)\right)\operatorname{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{b}$$
$$= -\frac{2A}{b\sqrt{x}} - \frac{(bB-Ac)\operatorname{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{b\sqrt{c}} + \frac{(bB-Ac)\operatorname{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{b\sqrt{c}}$$
$$= -\frac{2A}{b\sqrt{x}} + \frac{(bB-Ac)\operatorname{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{2bc} + \frac{(bB-Ac)\operatorname{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{}\right.}{2bc}$$
$$= -\frac{2A}{b\sqrt{x}} + \frac{(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{5/4}c^{3/4}} - \frac{(bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{5/4}c^{3/4}}$$
$$= -\frac{2A}{b\sqrt{x}} - \frac{(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{5/4}c^{3/4}} + \frac{(bB-Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{5/4}c^{3/4}} + \frac{(bB-Ac)\log\left(\sqrt{b}-\right.}{2\sqrt{2}b}$$

Mathematica [A] time = 0.0883225, size = 74, normalized size = 0.31

$$\frac{\frac{(bB-Ac)\left(\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)+\tanh^{-1}\left(\frac{b\sqrt[4]{c}\sqrt{x}}{(-b)^{5/4}}\right)\right)}{\sqrt[4]{-b}c^{3/4}}-\frac{2A}{\sqrt{x}}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] ((-2*A)/Sqrt[x] + ((b*B - A*c)*(ArcTan[(c^(1/4)*Sqrt[x])/(-b)^(1/4)] + ArcT
anh[(b*c^(1/4)*Sqrt[x])/(-b)^(5/4)]))/((-b)^(1/4)*c^(3/4)))/b
```

Maple [A] time = 0.01, size = 277, normalized size = 1.2

$$-\frac{\sqrt{2}A}{2b}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}-\frac{\sqrt{2}A}{2b}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}-1\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}-\frac{\sqrt{2}A}{4b}\ln\left(\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2),x)
```

```
[Out] -1/2/b/(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-1/2/b/(b
/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-1/4/b/(b/c)^(1/4)
*2^(1/2)*A*ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^
(1/2)*2^(1/2)+(b/c)^(1/2)))+1/2/c/(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c
)^(1/4)*x^(1/2)+1)+1/2/c/(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x
^(1/2)-1)+1/4/c/(b/c)^(1/4)*2^(1/2)*B*ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/
c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))-2*A/b/x^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.32999, size = 1735, normalized size = 7.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/2*(4*b*x*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 +
 A^4*c^4)/(b^5*c^3))^(1/4)*arctan((sqrt((B^6*b^6 - 6*A*B^5*b^5*c + 15*A^2*B
^4*b^4*c^2 - 20*A^3*B^3*b^3*c^3 + 15*A^4*B^2*b^2*c^4 - 6*A^5*B*b*c^5 + A^6*
c^6)*x - (B^4*b^7*c - 4*A*B^3*b^6*c^2 + 6*A^2*B^2*b^5*c^3 - 4*A^3*B*b^4*c^4
 + A^4*b^3*c^5)*sqrt(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*
B*b*c^3 + A^4*c^4)/(b^5*c^3)))*b*c*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b
^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^3))^(1/4) + (B^3*b^4*c - 3*A*B^2*b
^3*c^2 + 3*A^2*B*b^2*c^3 - A^3*b*c^4)*sqrt(x)*(-(B^4*b^4 - 4*A*B^3*b^3*c +
6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^3))^(1/4))/(B^4*b^4 - 4
*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)) - b*x*(-(B^4*b
^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^3)
)^(1/4)*log(b^4*c^2*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*
B*b*c^3 + A^4*c^4)/(b^5*c^3))^(3/4) - (B^3*b^3 - 3*A*B^2*b^2*c + 3*A^2*B*b*
c^2 - A^3*c^3)*sqrt(x)) + b*x*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^
2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^3))^(1/4)*log(-b^4*c^2*(-(B^4*b^4 - 4*A
*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^3))^(3/4)
- (B^3*b^3 - 3*A*B^2*b^2*c + 3*A^2*B*b*c^2 - A^3*c^3)*sqrt(x)) - 4*A*sqrt(x
))/(b*x)
```

Sympy [A] time = 85.0817, size = 374, normalized size = 1.59

$$\begin{cases} \tilde{\infty} \left(- \frac{2A}{5x^{\frac{5}{2}}} - \frac{2B}{\sqrt{x}}\right) \\ \frac{- \frac{2A}{5x^{\frac{5}{2}}} - \frac{2B}{\sqrt{x}}}{c} \\ \frac{- \frac{2A}{\sqrt{x}} + \frac{2Bx^{\frac{3}{2}}}{3}}{b} \\ - \frac{2A}{b\sqrt{x}} + \frac{(-1)^{\frac{3}{4}} A \log\left(- \sqrt[4]{-1}\sqrt[4]{b}\sqrt[4]{\frac{1}{c}} + \sqrt{x}\right)}{2b^{\frac{5}{4}} c^2 \left(\frac{1}{c}\right)^{\frac{9}{4}}} - \frac{(-1)^{\frac{3}{4}} A \log\left(\sqrt[4]{-1}\sqrt[4]{b}\sqrt[4]{\frac{1}{c}} + \sqrt{x}\right)}{2b^{\frac{5}{4}} c^2 \left(\frac{1}{c}\right)^{\frac{9}{4}}} - \frac{(-1)^{\frac{3}{4}} A \operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}}\right)}{b^{\frac{5}{4}} c^2 \left(\frac{1}{c}\right)^{\frac{9}{4}}} - \frac{(-1)^{\frac{3}{4}} B \log\left(- \sqrt[4]{-1}\sqrt[4]{b}\sqrt[4]{\frac{1}{c}} + \sqrt{x}\right)}{2\sqrt[4]{b}c^3\left(\frac{1}{c}\right)^{\frac{9}{4}}} + (-1)^{\frac{3}{4}} B \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*x**(1/2)/(c*x**4+b*x**2),x)
```

```
[Out] Piecewise((zoo*(-2*A/(5*x**(5/2)) - 2*B/sqrt(x)), Eq(b, 0) & Eq(c, 0)), ((-
2*A/(5*x**(5/2)) - 2*B/sqrt(x))/c, Eq(b, 0)), ((-2*A/sqrt(x) + 2*B*x**(3/2)
/3)/b, Eq(c, 0)), (-2*A/(b*sqrt(x)) + (-1)**(3/4)*A*log(-(-1)**(1/4)*b**(1/
4)*(1/c)**(1/4) + sqrt(x))/(2*b**(5/4)*c**2*(1/c)**(9/4)) - (-1)**(3/4)*A*l
og((-1)**(1/4)*b**(1/4)*(1/c)**(1/4) + sqrt(x))/(2*b**(5/4)*c**2*(1/c)**(9/
4)) - (-1)**(3/4)*A*atan((-1)**(3/4)*sqrt(x)/(b**(1/4)*(1/c)**(1/4)))/(b**(
5/4)*c**2*(1/c)**(9/4)) - (-1)**(3/4)*B*log(-(-1)**(1/4)*b**(1/4)*(1/c)**(1
/4) + sqrt(x))/(2*b**(1/4)*c**3*(1/c)**(9/4)) + (-1)**(3/4)*B*log((-1)**(1/
4)*b**(1/4)*(1/c)**(1/4) + sqrt(x))/(2*b**(1/4)*c**3*(1/c)**(9/4)) + (-1)**
(3/4)*B*atan((-1)**(3/4)*sqrt(x)/(b**(1/4)*(1/c)**(1/4)))/(b**(1/4)*c**3*(1
/c)**(9/4)), True))
```

Giac [A] time = 1.15761, size = 339, normalized size = 1.44

$$-\frac{2\,A}{b\sqrt{x}}+\frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{3}{4}}Bb-\left(bc^3\right)^{\frac{3}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^2c^3}+\frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{3}{4}}Bb-\left(bc^3\right)^{\frac{3}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -2*A/(b*sqrt(x)) + 1/2*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*arct
an(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^3) + 1
/2*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqr
t(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^3) - 1/4*sqrt(2)*((b*c^3)
^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(
b/c))/(b^2*c^3) + 1/4*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*log(-
sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c^3)
```

3.190 $\int \frac{A+Bx^2}{\sqrt{x}\left(bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=237

$$-\frac{(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{7/4}\sqrt[4]{c}}+\frac{(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{7/4}\sqrt[4]{c}}-\frac{(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{7/4}\sqrt[4]{c}}$$

```
[Out] (-2*A)/(3*b*x^(3/2)) - ((b*B - A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^
(1/4)])/(Sqrt[2]*b^(7/4)*c^(1/4)) + ((b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4
)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(7/4)*c^(1/4)) - ((b*B - A*c)*Log[Sqrt[b] -
 Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(7/4)*c^(1/4))
+ ((b*B - A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/
(2*Sqrt[2]*b^(7/4)*c^(1/4))
```

Rubi [A] time = 0.188393, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {1584, 453, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{7/4}\sqrt[4]{c}}+\frac{(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{7/4}\sqrt[4]{c}}-\frac{(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{7/4}\sqrt[4]{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(Sqrt[x]*(b*x^2 + c*x^4)),x]
```

```
[Out] (-2*A)/(3*b*x^(3/2)) - ((b*B - A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^
(1/4)])/(Sqrt[2]*b^(7/4)*c^(1/4)) + ((b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4
)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(7/4)*c^(1/4)) - ((b*B - A*c)*Log[Sqrt[b] -
 Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(7/4)*c^(1/4))
+ ((b*B - A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/
(2*Sqrt[2]*b^(7/4)*c^(1/4))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+Bx^2}{\sqrt{x}\left(bx^2+cx^4\right)}\,dx = \int \frac{A+Bx^2}{x^{5/2}\left(b+cx^2\right)}\,dx$$

$$= -\frac{2A}{3bx^{3/2}} - \frac{\left(2\left(-\frac{3bB}{2}+\frac{3Ac}{2}\right)\right)\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{3b}$$

$$= -\frac{2A}{3bx^{3/2}} - \frac{\left(4\left(-\frac{3bB}{2}+\frac{3Ac}{2}\right)\right)\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{3b}$$

$$= -\frac{2A}{3bx^{3/2}} + \frac{(bB-Ac)\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^{3/2}} + \frac{(bB-Ac)\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^{3/2}}$$

$$= -\frac{2A}{3bx^{3/2}} + \frac{(bB-Ac)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^{3/2}\sqrt{c}} + \frac{(bB-Ac)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x\right.}{2b^{3/2}\sqrt{c}}$$

$$= -\frac{2A}{3bx^{3/2}} - \frac{(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{7/4}\sqrt[4]{c}} + \frac{(bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right.}{2\sqrt{2}b^{7/4}\sqrt[4]{c}}$$

$$= -\frac{2A}{3bx^{3/2}} - \frac{(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{7/4}\sqrt[4]{c}} + \frac{(bB-Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{7/4}\sqrt[4]{c}} - \frac{(bB-Ac)\log\left(\sqrt{\right.}{2\ldots}$$

Mathematica [A] time = 0.135793, size = 168, normalized size = 0.71

$$-\frac{(bB-Ac)\left(\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)-\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)+2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)-2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\ldots}{\sqrt[4]{b}}\right.\right.}{2\sqrt{2}b^{7/4}\sqrt[4]{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(Sqrt[x]*(b*x^2 + c*x^4)),x]
```

```
[Out] (-2*A)/(3*b*x^(3/2)) - ((b*B - A*c)*(2*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])
/b^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] + Log[Sqrt[b] -
 Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] - Log[Sqrt[b] + Sqrt[2]*b^(1/
4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]))/(2*Sqrt[2]*b^(7/4)*c^(1/4))
```

Maple [A] time = 0.008, size = 280, normalized size = 1.2

$$-\frac{2A}{3b}x^{-\frac{3}{2}}-\frac{\sqrt{2}Ac}{2b^2}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)-\frac{\sqrt{2}Ac}{2b^2}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}-1\right)-\frac{\sqrt{2}Ac}{4b^2}\sqrt[4]{\frac{b}{c}}\ln\left(\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(c*x^4+b*x^2)/x^(1/2),x)
```

```
[Out] -2/3*A/b/x^(3/2)-1/2/b^2*(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x
^(1/2)+1)*c-1/2/b^2*(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2
)-1)*c-1/4/b^2*(b/c)^(1/4)*2^(1/2)*A*ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c
)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))*c+1/2/b*(b/c)^(1/4)*2
^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+1/2/b*(b/c)^(1/4)*2^(1/2)*B*
arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)+1/4/b*(b/c)^(1/4)*2^(1/2)*B*ln((x+(b/
c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^
(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.29509, size = 1347, normalized size = 5.68

$$-\frac{12\,bx^2\left(-\frac{B^4b^4-4\,AB^3b^3c+6\,A^2B^2b^2c^2-4\,A^3Bbc^3+A^4c^4}{b^7c}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{b^4\sqrt{-\frac{B^4b^4-4\,AB^3b^3c+6\,A^2B^2b^2c^2-4\,A^3Bbc^3+A^4c^4}{b^7c}}+\left(B^2b^2-2\,ABbc+A^2c^2\right)xb^5c\left(-\frac{B^4}{}\right.}}{B^4b^4-4\,AB}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(12*b*x^2*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c
^3 + A^4*c^4)/(b^7*c))^(1/4)*arctan((sqrt(b^4*sqrt(-(B^4*b^4 - 4*A*B^3*b^3*
c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^7*c)) + (B^2*b^2 - 2*A*
B*b*c + A^2*c^2)*x)*b^5*c*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 -
4*A^3*B*b*c^3 + A^4*c^4)/(b^7*c))^(3/4) + (B*b^6*c - A*b^5*c^2)*sqrt(x)*(-(
B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^7
*c))^(3/4))/(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 +
A^4*c^4)) + 3*b*x^2*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*
B*b*c^3 + A^4*c^4)/(b^7*c))^(1/4)*log(b^2*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^
2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^7*c))^(1/4) - (B*b - A*c)*sqrt(
x)) - 3*b*x^2*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^
3 + A^4*c^4)/(b^7*c))^(1/4)*log(-b^2*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2
*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^7*c))^(1/4) - (B*b - A*c)*sqrt(x)) +
 4*A*sqrt(x))/(b*x^2)
```

Sympy [A] time = 160.822, size = 374, normalized size = 1.58

$$\begin{cases} \tilde{\infty}\left(-\frac{2A}{7x^{\frac{7}{2}}}-\frac{2B}{3x^{\frac{3}{2}}}\right) \\ \frac{-\frac{2A}{7x^{\frac{7}{2}}}-\frac{2B}{3x^{\frac{3}{2}}}}{c} \\ \frac{-\frac{2A}{3x^{\frac{3}{2}}}+2B\sqrt{x}}{b} \\ -\frac{2A}{3bx^{\frac{3}{2}}}+\frac{\sqrt[4]{-1}A\log\left(-\sqrt[4]{-1}\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}+\sqrt{x}\right)}{2b^{\frac{7}{4}}c\left(\frac{1}{c}\right)^{\frac{7}{4}}}-\frac{\sqrt[4]{-1}A\log\left(\sqrt[4]{-1}\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}+\sqrt{x}\right)}{2b^{\frac{7}{4}}c\left(\frac{1}{c}\right)^{\frac{7}{4}}}+\frac{\sqrt[4]{-1}A\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}}\right)}{b^{\frac{7}{4}}c\left(\frac{1}{c}\right)^{\frac{7}{4}}}-\frac{\sqrt[4]{-1}B\log\left(-\sqrt[4]{-1}\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}+\sqrt{x}\right)}{2b^{\frac{3}{4}}c^2\left(\frac{1}{c}\right)^{\frac{7}{4}}}+\frac{\sqrt[4]{-1}B\log\left(\right.}{2b^{\frac{3}{}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2)/x**(1/2),x)
```

```
[Out] Piecewise((zoo*(-2*A/(7*x**(7/2)) - 2*B/(3*x**(3/2))), Eq(b, 0) & Eq(c, 0))
, ((-2*A/(7*x**(7/2)) - 2*B/(3*x**(3/2)))/c, Eq(b, 0)), ((-2*A/(3*x**(3/2))
 + 2*B*sqrt(x))/b, Eq(c, 0)), (-2*A/(3*b*x**(3/2)) + (-1)**(1/4)*A*log(-(-1
)**(1/4)*b**(1/4)*(1/c)**(1/4) + sqrt(x))/(2*b**(7/4)*c*(1/c)**(7/4)) - (-1
)**(1/4)*A*log((-1)**(1/4)*b**(1/4)*(1/c)**(1/4) + sqrt(x))/(2*b**(7/4)*c*(
1/c)**(7/4)) + (-1)**(1/4)*A*atan((-1)**(3/4)*sqrt(x)/(b**(1/4)*(1/c)**(1/4
)))/(b**(7/4)*c*(1/c)**(7/4)) - (-1)**(1/4)*B*log(-(-1)**(1/4)*b**(1/4)*(1/
c)**(1/4) + sqrt(x))/(2*b**(3/4)*c**2*(1/c)**(7/4)) + (-1)**(1/4)*B*log((-1
)**(1/4)*b**(1/4)*(1/c)**(1/4) + sqrt(x))/(2*b**(3/4)*c**2*(1/c)**(7/4)) -
(-1)**(1/4)*B*atan((-1)**(3/4)*sqrt(x)/(b**(1/4)*(1/c)**(1/4)))/(b**(3/4)*c
**2*(1/c)**(7/4)), True))
```

Giac [A] time = 1.18459, size = 339, normalized size = 1.43

$$\frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{1}{4}}Bb-\left(bc^{3}\right)^{\frac{1}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^{2}c}+\frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{1}{4}}Bb-\left(bc^{3}\right)^{\frac{1}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^{2}c}+\frac{\sqrt{2}\left(\left(b\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)/x^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt(2)*(sqr
t(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^2*c) + 1/2*sqrt(2)*((b*c^3)^(
1/4)*B*b - (b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*
sqrt(x))/(b/c)^(1/4))/(b^2*c) + 1/4*sqrt(2)*((b*c^3)^(1/4)*B*b - (b*c^3)^(1
/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c) - 1/4*sqr
t(2)*((b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/
4) + x + sqrt(b/c))/(b^2*c) - 2/3*A/(b*x^(3/2))
```

3.191 $\int \frac{A+Bx^2}{x^{3/2}(bx^2+cx^4)}\,dx$

Optimal. Leaf size=255

$$-\frac{2(bB-Ac)}{b^2\sqrt{x}}-\frac{\sqrt[4]{c}(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{9/4}}+\frac{\sqrt[4]{c}(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{9/4}}+\frac{\sqrt[4]{c}(bB}{}$$

```
[Out] (-2*A)/(5*b*x^(5/2)) - (2*(b*B - A*c))/(b^2*Sqrt[x]) + (c^(1/4)*(b*B - A*c)
*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(9/4)) - (c^(1/4
)*(b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(9/
4)) - (c^(1/4)*(b*B - A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] +
Sqrt[c]*x])/(2*Sqrt[2]*b^(9/4)) + (c^(1/4)*(b*B - A*c)*Log[Sqrt[b] + Sqrt[2
]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(9/4))
```

Rubi [A] time = 0.214583, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1584, 453, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{2(bB-Ac)}{b^2\sqrt{x}}-\frac{\sqrt[4]{c}(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{9/4}}+\frac{\sqrt[4]{c}(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{9/4}}+\frac{\sqrt[4]{c}(bB}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(3/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] (-2*A)/(5*b*x^(5/2)) - (2*(b*B - A*c))/(b^2*Sqrt[x]) + (c^(1/4)*(b*B - A*c)
*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(9/4)) - (c^(1/4
)*(b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(9/
4)) - (c^(1/4)*(b*B - A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] +
Sqrt[c]*x])/(2*Sqrt[2]*b^(9/4)) + (c^(1/4)*(b*B - A*c)*Log[Sqrt[b] + Sqrt[2
]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(9/4))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)}\,dx = \int \frac{A+Bx^2}{x^{7/2}\left(b+cx^2\right)}\,dx$$

$$= -\frac{2A}{5bx^{5/2}} - \frac{\left(2\left(-\frac{5bB}{2}+\frac{5Ac}{2}\right)\right)\int \frac{1}{x^{3/2}\left(b+cx^2\right)}\,dx}{5b}$$

$$= -\frac{2A}{5bx^{5/2}} - \frac{2(bB-Ac)}{b^2\sqrt{x}} - \frac{(c(bB-Ac))\int \frac{\sqrt{x}}{b+cx^2}\,dx}{b^2}$$

$$= -\frac{2A}{5bx^{5/2}} - \frac{2(bB-Ac)}{b^2\sqrt{x}} - \frac{(2c(bB-Ac))\,\mathrm{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^2}$$

$$= -\frac{2A}{5bx^{5/2}} - \frac{2(bB-Ac)}{b^2\sqrt{x}} + \frac{\left(\sqrt{c}(bB-Ac)\right)\mathrm{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^2} - \frac{\left(\sqrt{c}(bB-Ac)\right)\mathrm{Subst}\left(\int\right.}{b^2}$$

$$= -\frac{2A}{5bx^{5/2}} - \frac{2(bB-Ac)}{b^2\sqrt{x}} - \frac{(bB-Ac)\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^2} - \frac{(bB-Ac)\,\mathrm{Subst}\left(\int \frac{}{\frac{\sqrt{b}}{\sqrt{c}}+}\right.}{2b^2}$$

$$= -\frac{2A}{5bx^{5/2}} - \frac{2(bB-Ac)}{b^2\sqrt{x}} - \frac{\sqrt[4]{c}(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{9/4}} + \frac{\sqrt[4]{c}(bB-Ac)\log\left(\sqrt{b}+\right.}{2\sqrt{2}b}$$

$$= -\frac{2A}{5bx^{5/2}} - \frac{2(bB-Ac)}{b^2\sqrt{x}} + \frac{\sqrt[4]{c}(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{9/4}} - \frac{\sqrt[4]{c}(bB-Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{9/4}} -$$

Mathematica [C] time = 0.0200648, size = 46, normalized size = 0.18

$$-\frac{2\left(5x^2(bB-Ac)\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\frac{cx^2}{b}\right)+Ab\right)}{5b^2x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(3/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] (-2*(A*b + 5*(b*B - A*c)*x^2*Hypergeometric2F1[-1/4, 1, 3/4, -((c*x^2)/b)])
)/(5*b^2*x^(5/2))
```

Maple [A] time = 0.011, size = 299, normalized size = 1.2

$$-\frac{2A}{5b}x^{-\frac{5}{2}}+2\frac{Ac}{b^2\sqrt{x}}-2\frac{B}{b\sqrt{x}}+\frac{\sqrt{2}Ac}{2b^2}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}-1\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}+\frac{\sqrt{2}Ac}{4b^2}\ln\left(\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2),x)
```

```
[Out] -2/5*A/b/x^(5/2)+2/b^2/x^(1/2)*A*c-2/b/x^(1/2)*B+1/2/b^2/(b/c)^(1/4)*2^(1/2
)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)*c+1/4/b^2/(b/c)^(1/4)*2^(1/2)*A*l
n((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2
)+(b/c)^(1/2)))*c+1/2/b^2/(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*
x^(1/2)+1)*c-1/2/b/(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)
-1)-1/4/b/(b/c)^(1/4)*2^(1/2)*B*ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/
2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))-1/2/b/(b/c)^(1/4)*2^(1/2)*
B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.2636, size = 1793, normalized size = 7.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] -1/10*(20*b^2*x^3*(-(B^4*b^4*c - 4*A*B^3*b^3*c^2 + 6*A^2*B^2*b^2*c^3 - 4*A^
3*B*b*c^4 + A^4*c^5)/b^9)^(1/4)*arctan((sqrt((B^6*b^6*c^2 - 6*A*B^5*b^5*c^3
```

```
 + 15*A^2*B^4*b^4*c^4 - 20*A^3*B^3*b^3*c^5 + 15*A^4*B^2*b^2*c^6 - 6*A^5*B*b
*c^7 + A^6*c^8)*x - (B^4*b^9*c - 4*A*B^3*b^8*c^2 + 6*A^2*B^2*b^7*c^3 - 4*A^
3*B*b^6*c^4 + A^4*b^5*c^5)*sqrt(-(B^4*b^4*c - 4*A*B^3*b^3*c^2 + 6*A^2*B^2*b
^2*c^3 - 4*A^3*B*b*c^4 + A^4*c^5)/b^9))*b^2*(-(B^4*b^4*c - 4*A*B^3*b^3*c^2
+ 6*A^2*B^2*b^2*c^3 - 4*A^3*B*b*c^4 + A^4*c^5)/b^9)^(1/4) + (B^3*b^5*c - 3*
A*B^2*b^4*c^2 + 3*A^2*B*b^3*c^3 - A^3*b^2*c^4)*sqrt(x)*(-(B^4*b^4*c - 4*A*B
^3*b^3*c^2 + 6*A^2*B^2*b^2*c^3 - 4*A^3*B*b*c^4 + A^4*c^5)/b^9)^(1/4))/(B^4*
b^4*c - 4*A*B^3*b^3*c^2 + 6*A^2*B^2*b^2*c^3 - 4*A^3*B*b*c^4 + A^4*c^5)) - 5
*b^2*x^3*(-(B^4*b^4*c - 4*A*B^3*b^3*c^2 + 6*A^2*B^2*b^2*c^3 - 4*A^3*B*b*c^4
 + A^4*c^5)/b^9)^(1/4)*log(b^7*(-(B^4*b^4*c - 4*A*B^3*b^3*c^2 + 6*A^2*B^2*b
^2*c^3 - 4*A^3*B*b*c^4 + A^4*c^5)/b^9)^(3/4) - (B^3*b^3*c - 3*A*B^2*b^2*c^2
 + 3*A^2*B*b*c^3 - A^3*c^4)*sqrt(x)) + 5*b^2*x^3*(-(B^4*b^4*c - 4*A*B^3*b^3
*c^2 + 6*A^2*B^2*b^2*c^3 - 4*A^3*B*b*c^4 + A^4*c^5)/b^9)^(1/4)*log(-b^7*(-(
B^4*b^4*c - 4*A*B^3*b^3*c^2 + 6*A^2*B^2*b^2*c^3 - 4*A^3*B*b*c^4 + A^4*c^5)/
b^9)^(3/4) - (B^3*b^3*c - 3*A*B^2*b^2*c^2 + 3*A^2*B*b*c^3 - A^3*c^4)*sqrt(x
)) + 4*(5*(B*b - A*c)*x^2 + A*b)*sqrt(x))/(b^2*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(3/2)/(c*x**4+b*x**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17888, size = 362, normalized size = 1.42

$$-\frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{3}{4}}Bb-\left(bc^3\right)^{\frac{3}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^3c^2}-\frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{3}{4}}Bb-\left(bc^3\right)^{\frac{3}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^3c^2}+\frac{\sqrt{2}\left(\left(\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2)*(sq
rt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^3*c^2) - 1/2*sqrt(2)*((b*c^3
)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) -
 2*sqrt(x))/(b/c)^(1/4))/(b^3*c^2) + 1/4*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^
3)^(3/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c^2) -
1/4*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b
/c)^(1/4) + x + sqrt(b/c))/(b^3*c^2) - 2/5*(5*B*b*x^2 - 5*A*c*x^2 + A*b)/(b
^2*x^(5/2))
```

3.192 $\int \frac{A+Bx^2}{x^{5/2}\left(bx^2+cx^4\right)} dx$

Optimal. Leaf size=257

$$\frac{c^{3/4}(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{11/4}}-\frac{c^{3/4}(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{11/4}}+\frac{c^{3/4}(bB-Ac)\tan^{-1}\left(1-\right.}{\sqrt{2}b^{11/4}}$$

```
[Out] (-2*A)/(7*b*x^(7/2)) - (2*(b*B - A*c))/(3*b^2*x^(3/2)) + (c^(3/4)*(b*B - A*
c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(11/4)) - (c^(
3/4)*(b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^
(11/4)) + (c^(3/4)*(b*B - A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x
] + Sqrt[c]*x])/(2*Sqrt[2]*b^(11/4)) - (c^(3/4)*(b*B - A*c)*Log[Sqrt[b] + S
qrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(11/4))
```

Rubi [A] time = 0.213796, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1584, 453, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{c^{3/4}(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{11/4}}-\frac{c^{3/4}(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{11/4}}+\frac{c^{3/4}(bB-Ac)\tan^{-1}\left(1-\right.}{\sqrt{2}b^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(5/2)*(b*x^2 + c*x^4)),x]
```


```
[Out] (-2*A)/(7*b*x^(7/2)) - (2*(b*B - A*c))/(3*b^2*x^(3/2)) + (c^(3/4)*(b*B - A*
c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(11/4)) - (c^(
3/4)*(b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^
(11/4)) + (c^(3/4)*(b*B - A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x
] + Sqrt[c]*x])/(2*Sqrt[2]*b^(11/4)) - (c^(3/4)*(b*B - A*c)*Log[Sqrt[b] + S
qrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(11/4))
```


Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^2}{x^{5/2}\left(bx^2+cx^4\right)}\,dx &= \int \frac{A+Bx^2}{x^{9/2}\left(b+cx^2\right)}\,dx \\
&= -\frac{2A}{7bx^{7/2}} - \frac{\left(2\left(-\frac{7bB}{2}+\frac{7Ac}{2}\right)\right)\int \frac{1}{x^{5/2}\left(b+cx^2\right)}\,dx}{7b} \\
&= -\frac{2A}{7bx^{7/2}} - \frac{2(bB-Ac)}{3b^2x^{3/2}} - \frac{(c(bB-Ac))\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{b^2} \\
&= -\frac{2A}{7bx^{7/2}} - \frac{2(bB-Ac)}{3b^2x^{3/2}} - \frac{(2c(bB-Ac))\,\mathrm{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^2} \\
&= -\frac{2A}{7bx^{7/2}} - \frac{2(bB-Ac)}{3b^2x^{3/2}} - \frac{(c(bB-Ac))\,\mathrm{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^{5/2}} - \frac{(c(bB-Ac))\,\mathrm{Subst}\left(\int \frac{\sqrt{}}{}\right.}{b^{5/2}} \\
&= -\frac{2A}{7bx^{7/2}} - \frac{2(bB-Ac)}{3b^2x^{3/2}} - \frac{\left(\sqrt{c}(bB-Ac)\right)\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^{5/2}} - \frac{\left(\sqrt{c}(bB-Ac)\right)\mathrm{Su}}{} \\
&= -\frac{2A}{7bx^{7/2}} - \frac{2(bB-Ac)}{3b^2x^{3/2}} + \frac{c^{3/4}(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{11/4}} - \frac{c^{3/4}(bB-Ac)\log\left(\sqrt{b}\right.}{2\sqrt{}} \\
&= -\frac{2A}{7bx^{7/2}} - \frac{2(bB-Ac)}{3b^2x^{3/2}} + \frac{c^{3/4}(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{11/4}} - \frac{c^{3/4}(bB-Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right.}{\sqrt{2}b^{11/4}}
\end{aligned}
$$

Mathematica [C] time = 0.0178511, size = 47, normalized size = 0.18

$$\frac{14x^2(Ac-bB)\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\frac{cx^2}{b}\right)-6Ab}{21b^2x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(5/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] (-6*A*b + 14*(-(b*B) + A*c)*x^2*Hypergeometric2F1[-3/4, 1, 1/4, -((c*x^2)/b
)])/(21*b^2*x^(7/2))
```

Maple [A] time = 0.011, size = 308, normalized size = 1.2

$$-\frac{2A}{7b}x^{-\frac{7}{2}}+\frac{2Ac}{3b^2}x^{-\frac{3}{2}}-\frac{2B}{3b}x^{-\frac{3}{2}}+\frac{c^2\sqrt{2}A}{4b^3}\sqrt[4]{\frac{b}{c}}\ln\left(\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)+\frac{c^2\sqrt{2}A}{2b^3}\sqrt[4]{\frac{b}{c}}\,\mathrm{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2),x)
```

```
[Out] -2/7*A/b/x^(7/2)+2/3/b^2/x^(3/2)*A*c-2/3/b/x^(3/2)*B+1/4*c^2/b^3*(b/c)^(1/4
)*2^(1/2)*A*ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x
^(1/2)*2^(1/2)+(b/c)^(1/2)))+1/2*c^2/b^3*(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/
2)/(b/c)^(1/4)*x^(1/2)+1)+1/2*c^2/b^3*(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/
(b/c)^(1/4)*x^(1/2)-1)-1/4*c/b^2*(b/c)^(1/4)*2^(1/2)*B*ln((x+(b/c)^(1/4)*x^
(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))-1/2
*c/b^2*(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-1/2*c/b^
2*(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.31124, size = 1453, normalized size = 5.65

$$\frac{84\,b^2x^4\left(-\frac{B^4b^4c^3-4\,AB^3b^3c^4+6\,A^2B^2b^2c^5-4\,A^3Bbc^6+A^4c^7}{b^{11}}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{b^6\sqrt{-\frac{B^4b^4c^3-4\,AB^3b^3c^4+6\,A^2B^2b^2c^5-4\,A^3Bbc^6+A^4c^7}{b^{11}}}+\left(B^2b^2c^2-2\,ABbc^3+A^2c^4\right)xb^8}}{B^4b^4c^3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/42*(84*b^2*x^4*(-(B^4*b^4*c^3 - 4*A*B^3*b^3*c^4 + 6*A^2*B^2*b^2*c^5 - 4*A
^3*B*b*c^6 + A^4*c^7)/b^11)^(1/4)*arctan((sqrt(b^6*sqrt(-(B^4*b^4*c^3 - 4*A
*B^3*b^3*c^4 + 6*A^2*B^2*b^2*c^5 - 4*A^3*B*b*c^6 + A^4*c^7)/b^11) + (B^2*b^
2*c^2 - 2*A*B*b*c^3 + A^2*c^4)*x)*b^8*(-(B^4*b^4*c^3 - 4*A*B^3*b^3*c^4 + 6*
A^2*B^2*b^2*c^5 - 4*A^3*B*b*c^6 + A^4*c^7)/b^11)^(3/4) + (B*b^9*c - A*b^8*c
^2)*sqrt(x)*(-(B^4*b^4*c^3 - 4*A*B^3*b^3*c^4 + 6*A^2*B^2*b^2*c^5 - 4*A^3*B*
b*c^6 + A^4*c^7)/b^11)^(3/4))/(B^4*b^4*c^3 - 4*A*B^3*b^3*c^4 + 6*A^2*B^2*b^
2*c^5 - 4*A^3*B*b*c^6 + A^4*c^7)) + 21*b^2*x^4*(-(B^4*b^4*c^3 - 4*A*B^3*b^3
*c^4 + 6*A^2*B^2*b^2*c^5 - 4*A^3*B*b*c^6 + A^4*c^7)/b^11)^(1/4)*log(b^3*(-(
B^4*b^4*c^3 - 4*A*B^3*b^3*c^4 + 6*A^2*B^2*b^2*c^5 - 4*A^3*B*b*c^6 + A^4*c^7
)/b^11)^(1/4) - (B*b*c - A*c^2)*sqrt(x)) - 21*b^2*x^4*(-(B^4*b^4*c^3 - 4*A*
B^3*b^3*c^4 + 6*A^2*B^2*b^2*c^5 - 4*A^3*B*b*c^6 + A^4*c^7)/b^11)^(1/4)*log(
-b^3*(-(B^4*b^4*c^3 - 4*A*B^3*b^3*c^4 + 6*A^2*B^2*b^2*c^5 - 4*A^3*B*b*c^6 +
 A^4*c^7)/b^11)^(1/4) - (B*b*c - A*c^2)*sqrt(x)) - 4*(7*(B*b - A*c)*x^2 + 3
*A*b)*sqrt(x))/(b^2*x^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(5/2)/(c*x**4+b*x**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18617, size = 347, normalized size = 1.35

$$-\frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{1}{4}}Bb-\left(bc^{3}\right)^{\frac{1}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2b^{3}}-\frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{1}{4}}Bb-\left(bc^{3}\right)^{\frac{1}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2b^{3}}-\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*((b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt(2)*(sq
rt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/b^3 - 1/2*sqrt(2)*((b*c^3)^(1/4
)*B*b - (b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqr
t(x))/(b/c)^(1/4))/b^3 - 1/4*sqrt(2)*((b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c
)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^3 + 1/4*sqrt(2)*((b*c^
3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sq
rt(b/c))/b^3 - 2/21*(7*B*b*x^2 - 7*A*c*x^2 + 3*A*b)/(b^2*x^(7/2))
```

3.193 $\int \frac{A+Bx^2}{x^{7/2}\left(bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=276

$$\frac{c^{5/4}(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{13/4}}-\frac{c^{5/4}(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{13/4}}-\frac{c^{5/4}(bB-Ac)\tan^{-1}\left(1\right.}{\sqrt{2}b^{13/4}}$$

```
[Out] (-2*A)/(9*b*x^(9/2)) - (2*(b*B - A*c))/(5*b^2*x^(5/2)) + (2*c*(b*B - A*c))/
(b^3*Sqrt[x]) - (c^(5/4)*(b*B - A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b
^(1/4)])/(Sqrt[2]*b^(13/4)) + (c^(5/4)*(b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1
/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(13/4)) + (c^(5/4)*(b*B - A*c)*Log[Sqrt[b
] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(13/4)) - (c
^(5/4)*(b*B - A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*
x])/(2*Sqrt[2]*b^(13/4))
```

Rubi [A] time = 0.244658, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1584, 453, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{c^{5/4}(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{13/4}}-\frac{c^{5/4}(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{13/4}}-\frac{c^{5/4}(bB-Ac)\tan^{-1}\left(1\right.}{\sqrt{2}b^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(7/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] (-2*A)/(9*b*x^(9/2)) - (2*(b*B - A*c))/(5*b^2*x^(5/2)) + (2*c*(b*B - A*c))/
(b^3*Sqrt[x]) - (c^(5/4)*(b*B - A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b
^(1/4)])/(Sqrt[2]*b^(13/4)) + (c^(5/4)*(b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1
/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(13/4)) + (c^(5/4)*(b*B - A*c)*Log[Sqrt[b
] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(13/4)) - (c
^(5/4)*(b*B - A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*
x])/(2*Sqrt[2]*b^(13/4))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^2}{x^{7/2}\left(bx^2+cx^4\right)}\,dx &= \int \frac{A+Bx^2}{x^{11/2}\left(b+cx^2\right)}\,dx \\
&= -\frac{2A}{9bx^{9/2}} - \frac{\left(2\left(-\frac{9bB}{2}+\frac{9Ac}{2}\right)\right)\int \frac{1}{x^{7/2}\left(b+cx^2\right)}\,dx}{9b} \\
&= -\frac{2A}{9bx^{9/2}} - \frac{2(bB-Ac)}{5b^2x^{5/2}} - \frac{(c(bB-Ac))\int \frac{1}{x^{3/2}\left(b+cx^2\right)}\,dx}{b^2} \\
&= -\frac{2A}{9bx^{9/2}} - \frac{2(bB-Ac)}{5b^2x^{5/2}} + \frac{2c(bB-Ac)}{b^3\sqrt{x}} + \frac{\left(c^2(bB-Ac)\right)\int \frac{\sqrt{x}}{b+cx^2}\,dx}{b^3} \\
&= -\frac{2A}{9bx^{9/2}} - \frac{2(bB-Ac)}{5b^2x^{5/2}} + \frac{2c(bB-Ac)}{b^3\sqrt{x}} + \frac{\left(2c^2(bB-Ac)\right)\mathrm{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{b^3} \\
&= -\frac{2A}{9bx^{9/2}} - \frac{2(bB-Ac)}{5b^2x^{5/2}} + \frac{2c(bB-Ac)}{b^3\sqrt{x}} - \frac{\left(c^{3/2}(bB-Ac)\right)\mathrm{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{b^3} + \\
&= -\frac{2A}{9bx^{9/2}} - \frac{2(bB-Ac)}{5b^2x^{5/2}} + \frac{2c(bB-Ac)}{b^3\sqrt{x}} + \frac{(c(bB-Ac))\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{2b^3} + \\
&= -\frac{2A}{9bx^{9/2}} - \frac{2(bB-Ac)}{5b^2x^{5/2}} + \frac{2c(bB-Ac)}{b^3\sqrt{x}} + \frac{c^{5/4}(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{13/4}} - \\
&= -\frac{2A}{9bx^{9/2}} - \frac{2(bB-Ac)}{5b^2x^{5/2}} + \frac{2c(bB-Ac)}{b^3\sqrt{x}} - \frac{c^{5/4}(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{13/4}} +
\end{aligned}
$$

Mathematica [C] time = 0.0166331, size = 47, normalized size = 0.17

$$
\frac{2\left(9x^2(Ac-bB)\,{}_2F_1\left(-\frac{5}{4},1;-\frac{1}{4};-\frac{cx^2}{b}\right)-5Ab\right)}{45b^2x^{9/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(7/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] (2*(-5*A*b + 9*(-(b*B) + A*c)*x^2*Hypergeometric2F1[-5/4, 1, -1/4, -((c*x^2
)/b)]))/(45*b^2*x^(9/2))
```

Maple [A] time = 0.012, size = 330, normalized size = 1.2

$$-\frac{2A}{9b}x^{-\frac{9}{2}}+\frac{2Ac}{5b^{2}}x^{-\frac{5}{2}}-\frac{2B}{5b}x^{-\frac{5}{2}}-2\frac{Ac^{2}}{b^{3}\sqrt{x}}+2\frac{Bc}{b^{2}\sqrt{x}}-\frac{c^{2}\sqrt{2}A}{2b^{3}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}-1\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}-\frac{c^{2}\sqrt{2}A}{4b^{3}}\ln\left(\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(7/2)/(c*x^4+b*x^2),x)
```

```
[Out] -2/9*A/b/x^(9/2)+2/5/b^2/x^(5/2)*A*c-2/5/b/x^(5/2)*B-2/b^3*c^2/x^(1/2)*A+2/
b^2*c/x^(1/2)*B-1/2*c^2/b^3/(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4
)*x^(1/2)-1)-1/4*c^2/b^3/(b/c)^(1/4)*2^(1/2)*A*ln((x-(b/c)^(1/4)*x^(1/2)*2^
(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))-1/2*c^2/b^3
/(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+1/2*c/b^2/(b/c
)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)+1/4*c/b^2/(b/c)^(1/
4)*2^(1/2)*B*ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*
x^(1/2)*2^(1/2)+(b/c)^(1/2)))+1/2*c/b^2/(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2
)/(b/c)^(1/4)*x^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(7/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.63926, size = 1898, normalized size = 6.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(7/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/90*(180*b^3*x^5*(-(B^4*b^4*c^5 - 4*A*B^3*b^3*c^6 + 6*A^2*B^2*b^2*c^7 - 4*
A^3*B*b*c^8 + A^4*c^9)/b^13)^(1/4)*arctan((sqrt((B^6*b^6*c^8 - 6*A*B^5*b^5*
c^9 + 15*A^2*B^4*b^4*c^10 - 20*A^3*B^3*b^3*c^11 + 15*A^4*B^2*b^2*c^12 - 6*A
^5*B*b*c^13 + A^6*c^14)*x - (B^4*b^11*c^5 - 4*A*B^3*b^10*c^6 + 6*A^2*B^2*b^
9*c^7 - 4*A^3*B*b^8*c^8 + A^4*b^7*c^9)*sqrt(-(B^4*b^4*c^5 - 4*A*B^3*b^3*c^6
 + 6*A^2*B^2*b^2*c^7 - 4*A^3*B*b*c^8 + A^4*c^9)/b^13))*b^3*(-(B^4*b^4*c^5 -
 4*A*B^3*b^3*c^6 + 6*A^2*B^2*b^2*c^7 - 4*A^3*B*b*c^8 + A^4*c^9)/b^13)^(1/4)
 + (B^3*b^6*c^4 - 3*A*B^2*b^5*c^5 + 3*A^2*B*b^4*c^6 - A^3*b^3*c^7)*sqrt(x)*
(-(B^4*b^4*c^5 - 4*A*B^3*b^3*c^6 + 6*A^2*B^2*b^2*c^7 - 4*A^3*B*b*c^8 + A^4*
c^9)/b^13)^(1/4))/(B^4*b^4*c^5 - 4*A*B^3*b^3*c^6 + 6*A^2*B^2*b^2*c^7 - 4*A^
3*B*b*c^8 + A^4*c^9)) - 45*b^3*x^5*(-(B^4*b^4*c^5 - 4*A*B^3*b^3*c^6 + 6*A^2
*B^2*b^2*c^7 - 4*A^3*B*b*c^8 + A^4*c^9)/b^13)^(1/4)*log(b^10*(-(B^4*b^4*c^5
 - 4*A*B^3*b^3*c^6 + 6*A^2*B^2*b^2*c^7 - 4*A^3*B*b*c^8 + A^4*c^9)/b^13)^(3/
4) - (B^3*b^3*c^4 - 3*A*B^2*b^2*c^5 + 3*A^2*B*b*c^6 - A^3*c^7)*sqrt(x)) + 4
5*b^3*x^5*(-(B^4*b^4*c^5 - 4*A*B^3*b^3*c^6 + 6*A^2*B^2*b^2*c^7 - 4*A^3*B*b*
c^8 + A^4*c^9)/b^13)^(1/4)*log(-b^10*(-(B^4*b^4*c^5 - 4*A*B^3*b^3*c^6 + 6*A
^2*B^2*b^2*c^7 - 4*A^3*B*b*c^8 + A^4*c^9)/b^13)^(3/4) - (B^3*b^3*c^4 - 3*A*
B^2*b^2*c^5 + 3*A^2*B*b*c^6 - A^3*c^7)*sqrt(x)) + 4*(45*(B*b*c - A*c^2)*x^4
 - 5*A*b^2 - 9*(B*b^2 - A*b*c)*x^2)*sqrt(x))/(b^3*x^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(7/2)/(c*x**4+b*x**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19111, size = 393, normalized size = 1.42

$$\frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{3}{4}}Bb-\left(bc^3\right)^{\frac{3}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^4c}+\frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{3}{4}}Bb-\left(bc^3\right)^{\frac{3}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^4c}-\frac{\sqrt{2}\left(\left(b\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(7/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2)*(sqr
t(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^4*c) + 1/2*sqrt(2)*((b*c^3)^(
3/4)*B*b - (b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*
sqrt(x))/(b/c)^(1/4))/(b^4*c) - 1/4*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3
/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^4*c) + 1/4*sqr
t(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/
4) + x + sqrt(b/c))/(b^4*c) + 2/45*(45*B*b*c*x^4 - 45*A*c^2*x^4 - 9*B*b^2*x
^2 + 9*A*b*c*x^2 - 5*A*b^2)/(b^3*x^(9/2))
```

3.194 $\int \frac{A+Bx^2}{x^{9/2}\left(bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=278

$$-\frac{c^{7/4}(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{15/4}}+\frac{c^{7/4}(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{15/4}}-\frac{c^{7/4}(bB-Ac)\tan^{-1}\left(1\right.}{\sqrt{2}b^{15/4}}$$

```
[Out] (-2*A)/(11*b*x^(11/2)) - (2*(b*B - A*c))/(7*b^2*x^(7/2)) + (2*c*(b*B - A*c)
)/(3*b^3*x^(3/2)) - (c^(7/4)*(b*B - A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x
])/b^(1/4)])/(Sqrt[2]*b^(15/4)) + (c^(7/4)*(b*B - A*c)*ArcTan[1 + (Sqrt[2]*
c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(15/4)) - (c^(7/4)*(b*B - A*c)*Log[Sq
rt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(15/4))
+ (c^(7/4)*(b*B - A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt
[c]*x])/(2*Sqrt[2]*b^(15/4))
```

Rubi [A] time = 0.241391, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1584, 453, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{c^{7/4}(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{15/4}}+\frac{c^{7/4}(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{15/4}}-\frac{c^{7/4}(bB-Ac)\tan^{-1}\left(1\right.}{\sqrt{2}b^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(9/2)*(b*x^2 + c*x^4)),x]
```


```
[Out] (-2*A)/(11*b*x^(11/2)) - (2*(b*B - A*c))/(7*b^2*x^(7/2)) + (2*c*(b*B - A*c)
)/(3*b^3*x^(3/2)) - (c^(7/4)*(b*B - A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x
])/b^(1/4)])/(Sqrt[2]*b^(15/4)) + (c^(7/4)*(b*B - A*c)*ArcTan[1 + (Sqrt[2]*
c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(15/4)) - (c^(7/4)*(b*B - A*c)*Log[Sq
rt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(15/4))
+ (c^(7/4)*(b*B - A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt
[c]*x])/(2*Sqrt[2]*b^(15/4))
```


Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+Bx^2}{x^{9/2}\left(bx^2+cx^4\right)}\,dx = \int \frac{A+Bx^2}{x^{13/2}\left(b+cx^2\right)}\,dx$$
$$= -\frac{2A}{11bx^{11/2}} - \frac{\left(2\left(-\frac{11bB}{2}+\frac{11Ac}{2}\right)\right)\int \frac{1}{x^{9/2}\left(b+cx^2\right)}\,dx}{11b}$$
$$= -\frac{2A}{11bx^{11/2}} - \frac{2(bB-Ac)}{7b^2x^{7/2}} - \frac{(c(bB-Ac))\int \frac{1}{x^{5/2}\left(b+cx^2\right)}\,dx}{b^2}$$
$$= -\frac{2A}{11bx^{11/2}} - \frac{2(bB-Ac)}{7b^2x^{7/2}} + \frac{2c(bB-Ac)}{3b^3x^{3/2}} + \frac{\left(c^2(bB-Ac)\right)\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{b^3}$$
$$= -\frac{2A}{11bx^{11/2}} - \frac{2(bB-Ac)}{7b^2x^{7/2}} + \frac{2c(bB-Ac)}{3b^3x^{3/2}} + \frac{\left(2c^2(bB-Ac)\right)\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^3}$$
$$= -\frac{2A}{11bx^{11/2}} - \frac{2(bB-Ac)}{7b^2x^{7/2}} + \frac{2c(bB-Ac)}{3b^3x^{3/2}} + \frac{\left(c^2(bB-Ac)\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^{7/2}} + \frac{\left(c^2(bB\right.}{}$$
$$= -\frac{2A}{11bx^{11/2}} - \frac{2(bB-Ac)}{7b^2x^{7/2}} + \frac{2c(bB-Ac)}{3b^3x^{3/2}} + \frac{\left(c^{3/2}(bB-Ac)\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^{7/2}} +$$
$$= -\frac{2A}{11bx^{11/2}} - \frac{2(bB-Ac)}{7b^2x^{7/2}} + \frac{2c(bB-Ac)}{3b^3x^{3/2}} - \frac{c^{7/4}(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{15/4}} + \frac{c^{7/4}(}{}$$
$$= -\frac{2A}{11bx^{11/2}} - \frac{2(bB-Ac)}{7b^2x^{7/2}} + \frac{2c(bB-Ac)}{3b^3x^{3/2}} - \frac{c^{7/4}(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{15/4}} + \frac{c^{7/4}(bB-Ac)\,\text{ta}}{\sqrt{}}$$

Mathematica [C] time = 0.0164283, size = 47, normalized size = 0.17

$$\frac{-22x^2(bB-Ac)\,{}_2F_1\left(-\frac{7}{4},1;-\frac{3}{4};-\frac{cx^2}{b}\right)-14Ab}{77b^2x^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(9/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] (-14*A*b - 22*(b*B - A*c)*x^2*Hypergeometric2F1[-7/4, 1, -3/4, -((c*x^2)/b)
])/(77*b^2*x^(11/2))
```

Maple [A] time = 0.011, size = 336, normalized size = 1.2

$$-\frac{2A}{11b}x^{-\frac{11}{2}}+\frac{2Ac}{7b^2}x^{-\frac{7}{2}}-\frac{2B}{7b}x^{-\frac{7}{2}}-\frac{2Ac^2}{3b^3}x^{-\frac{3}{2}}+\frac{2Bc}{3b^2}x^{-\frac{3}{2}}-\frac{c^3\sqrt{2}A}{2b^4}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)-\frac{c^3\sqrt{2}A}{2b^4}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(9/2)/(c*x^4+b*x^2),x)
```

```
[Out] -2/11*A/b/x^(11/2)+2/7/b^2/x^(7/2)*A*c-2/7/b/x^(7/2)*B-2/3/b^3*c^2/x^(3/2)*
A+2/3/b^2*c/x^(3/2)*B-1/2*c^3/b^4*(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c
)^(1/4)*x^(1/2)+1)-1/2*c^3/b^4*(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(
1/4)*x^(1/2)-1)-1/4*c^3/b^4*(b/c)^(1/4)*2^(1/2)*A*ln((x+(b/c)^(1/4)*x^(1/2)
*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+1/2*c^2/
b^3*(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+1/2*c^2/b^3
*(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)+1/4*c^2/b^3*(b
/c)^(1/4)*2^(1/2)*B*ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)
^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(9/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.52287, size = 1540, normalized size = 5.54

$$-\frac{924\,b^3x^6\left(-\frac{B^4b^4c^7-4\,AB^3b^3c^8+6\,A^2B^2b^2c^9-4\,A^3Bbc^{10}+A^4c^{11}}{b^{15}}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{b^8\sqrt{-\frac{B^4b^4c^7-4\,AB^3b^3c^8+6\,A^2B^2b^2c^9-4\,A^3Bbc^{10}+A^4c^{11}}{b^{15}}}+\left(B^2b^2c^4-2\,ABbc^5+A\right.}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(9/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] -1/462*(924*b^3*x^6*(-(B^4*b^4*c^7 - 4*A*B^3*b^3*c^8 + 6*A^2*B^2*b^2*c^9 -
4*A^3*B*b*c^10 + A^4*c^11)/b^15)^(1/4)*arctan((sqrt(b^8*sqrt(-(B^4*b^4*c^7
- 4*A*B^3*b^3*c^8 + 6*A^2*B^2*b^2*c^9 - 4*A^3*B*b*c^10 + A^4*c^11)/b^15) +
(B^2*b^2*c^4 - 2*A*B*b*c^5 + A^2*c^6)*x)*b^11*(-(B^4*b^4*c^7 - 4*A*B^3*b^3*
c^8 + 6*A^2*B^2*b^2*c^9 - 4*A^3*B*b*c^10 + A^4*c^11)/b^15)^(3/4) + (B*b^12*
c^2 - A*b^11*c^3)*sqrt(x)*(-(B^4*b^4*c^7 - 4*A*B^3*b^3*c^8 + 6*A^2*B^2*b^2*
c^9 - 4*A^3*B*b*c^10 + A^4*c^11)/b^15)^(3/4))/(B^4*b^4*c^7 - 4*A*B^3*b^3*c^
8 + 6*A^2*B^2*b^2*c^9 - 4*A^3*B*b*c^10 + A^4*c^11)) + 231*b^3*x^6*(-(B^4*b^
4*c^7 - 4*A*B^3*b^3*c^8 + 6*A^2*B^2*b^2*c^9 - 4*A^3*B*b*c^10 + A^4*c^11)/b^
15)^(1/4)*log(b^4*(-(B^4*b^4*c^7 - 4*A*B^3*b^3*c^8 + 6*A^2*B^2*b^2*c^9 - 4*
A^3*B*b*c^10 + A^4*c^11)/b^15)^(1/4) - (B*b*c^2 - A*c^3)*sqrt(x)) - 231*b^3
*x^6*(-(B^4*b^4*c^7 - 4*A*B^3*b^3*c^8 + 6*A^2*B^2*b^2*c^9 - 4*A^3*B*b*c^10
+ A^4*c^11)/b^15)^(1/4)*log(-b^4*(-(B^4*b^4*c^7 - 4*A*B^3*b^3*c^8 + 6*A^2*B
^2*b^2*c^9 - 4*A^3*B*b*c^10 + A^4*c^11)/b^15)^(1/4) - (B*b*c^2 - A*c^3)*sqr
t(x)) - 4*(77*(B*b*c - A*c^2)*x^4 - 21*A*b^2 - 33*(B*b^2 - A*b*c)*x^2)*sqrt
(x))/(b^3*x^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(9/2)/(c*x**4+b*x**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21939, size = 393, normalized size = 1.41

$$\frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{1}{4}} Bbc - \left(bc^3\right)^{\frac{1}{4}} Ac^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2b^4} + \frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{1}{4}} Bbc - \left(bc^3\right)^{\frac{1}{4}} Ac^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2b^4} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(9/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((b*c^3)^(1/4)*B*b*c - (b*c^3)^(1/4)*A*c^2)*arctan(1/2*sqrt(2)*
(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/b^4 + 1/2*sqrt(2)*((b*c^3)^(
1/4)*B*b*c - (b*c^3)^(1/4)*A*c^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4)
- 2*sqrt(x))/(b/c)^(1/4))/b^4 + 1/4*sqrt(2)*((b*c^3)^(1/4)*B*b*c - (b*c^3)^
(1/4)*A*c^2)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^4 - 1/4*sqr
t(2)*((b*c^3)^(1/4)*B*b*c - (b*c^3)^(1/4)*A*c^2)*log(-sqrt(2)*sqrt(x)*(b/c)
^(1/4) + x + sqrt(b/c))/b^4 + 2/231*(77*B*b*c*x^4 - 77*A*c^2*x^4 - 33*B*b^2
*x^2 + 33*A*b*c*x^2 - 21*A*b^2)/(b^3*x^(11/2))
```

3.195 $\int \frac{x^{19/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=332

$$\frac{b^{5/4}(13bB-9Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{17/4}}-\frac{b^{5/4}(13bB-9Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{17/4}}+\frac{b^{5/4}(13bB-9A}{}$$

```
[Out] (b*(13*b*B - 9*A*c)*Sqrt[x])/(2*c^4) - ((13*b*B - 9*A*c)*x^(5/2))/(10*c^3)
+ ((13*b*B - 9*A*c)*x^(9/2))/(18*b*c^2) - ((b*B - A*c)*x^(13/2))/(2*b*c*(b
+ c*x^2)) + (b^(5/4)*(13*b*B - 9*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/
b^(1/4)])/(4*Sqrt[2]*c^(17/4)) - (b^(5/4)*(13*b*B - 9*A*c)*ArcTan[1 + (Sqrt
[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*c^(17/4)) + (b^(5/4)*(13*b*B - 9*
A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]
*c^(17/4)) - (b^(5/4)*(13*b*B - 9*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4
)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*c^(17/4))
```

Rubi [A] time = 0.277151, antiderivative size = 332, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1584, 457, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{b^{5/4}(13bB-9Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{17/4}}-\frac{b^{5/4}(13bB-9Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{17/4}}+\frac{b^{5/4}(13bB-9A}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(19/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (b*(13*b*B - 9*A*c)*Sqrt[x])/(2*c^4) - ((13*b*B - 9*A*c)*x^(5/2))/(10*c^3)
+ ((13*b*B - 9*A*c)*x^(9/2))/(18*b*c^2) - ((b*B - A*c)*x^(13/2))/(2*b*c*(b
+ c*x^2)) + (b^(5/4)*(13*b*B - 9*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/
b^(1/4)])/(4*Sqrt[2]*c^(17/4)) - (b^(5/4)*(13*b*B - 9*A*c)*ArcTan[1 + (Sqrt
[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*c^(17/4)) + (b^(5/4)*(13*b*B - 9*
A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]
*c^(17/4)) - (b^(5/4)*(13*b*B - 9*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4
)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*c^(17/4))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{19/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx = \int \frac{x^{11/2}\left(A+Bx^2\right)}{\left(b+cx^2\right)^2}\,dx$$

$$= -\frac{(bB-Ac)x^{13/2}}{2bc\left(b+cx^2\right)} + \frac{\left(\frac{13bB}{2}-\frac{9Ac}{2}\right)\int \frac{x^{11/2}}{b+cx^2}\,dx}{2bc}$$

$$= \frac{(13bB-9Ac)x^{9/2}}{18bc^2} - \frac{(bB-Ac)x^{13/2}}{2bc\left(b+cx^2\right)} - \frac{(13bB-9Ac)\int \frac{x^{7/2}}{b+cx^2}\,dx}{4c^2}$$

$$= -\frac{(13bB-9Ac)x^{5/2}}{10c^3} + \frac{(13bB-9Ac)x^{9/2}}{18bc^2} - \frac{(bB-Ac)x^{13/2}}{2bc\left(b+cx^2\right)} + \frac{(b(13bB-9Ac))\int \frac{x^{3/2}}{b+cx^2}\,dx}{4c^3}$$

$$= \frac{b(13bB-9Ac)\sqrt{x}}{2c^4} - \frac{(13bB-9Ac)x^{5/2}}{10c^3} + \frac{(13bB-9Ac)x^{9/2}}{18bc^2} - \frac{(bB-Ac)x^{13/2}}{2bc\left(b+cx^2\right)} - \frac{\left(b^2(13bB-9Ac)\right.}{4c}$$

$$= \frac{b(13bB-9Ac)\sqrt{x}}{2c^4} - \frac{(13bB-9Ac)x^{5/2}}{10c^3} + \frac{(13bB-9Ac)x^{9/2}}{18bc^2} - \frac{(bB-Ac)x^{13/2}}{2bc\left(b+cx^2\right)} - \frac{\left(b^2(13bB-9Ac)\right.}{}$$

$$= \frac{b(13bB-9Ac)\sqrt{x}}{2c^4} - \frac{(13bB-9Ac)x^{5/2}}{10c^3} + \frac{(13bB-9Ac)x^{9/2}}{18bc^2} - \frac{(bB-Ac)x^{13/2}}{2bc\left(b+cx^2\right)} - \frac{\left(b^{3/2}(13bB-9A\right.}{}$$

$$= \frac{b(13bB-9Ac)\sqrt{x}}{2c^4} - \frac{(13bB-9Ac)x^{5/2}}{10c^3} + \frac{(13bB-9Ac)x^{9/2}}{18bc^2} - \frac{(bB-Ac)x^{13/2}}{2bc\left(b+cx^2\right)} - \frac{\left(b^{3/2}(13bB-9A\right.}{}$$

$$= \frac{b(13bB-9Ac)\sqrt{x}}{2c^4} - \frac{(13bB-9Ac)x^{5/2}}{10c^3} + \frac{(13bB-9Ac)x^{9/2}}{18bc^2} - \frac{(bB-Ac)x^{13/2}}{2bc\left(b+cx^2\right)} + \frac{b^{5/4}(13bB-9Ac)}{}$$

$$= \frac{b(13bB-9Ac)\sqrt{x}}{2c^4} - \frac{(13bB-9Ac)x^{5/2}}{10c^3} + \frac{(13bB-9Ac)x^{9/2}}{18bc^2} - \frac{(bB-Ac)x^{13/2}}{2bc\left(b+cx^2\right)} + \frac{b^{5/4}(13bB-9Ac)}{4\sqrt{}}$$

Mathematica [A] time = 0.454586, size = 417, normalized size = 1.26

$$\frac{90\sqrt{2}b^{5/4}(13bB-9Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)-90\sqrt{2}b^{5/4}(13bB-9Ac)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)-\frac{360Ab^2c^{5/4}\sqrt{x}}{b+cx^2}-405\sqrt{2}Ab^{5/4}c\,\mathrm{lo}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(19/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (4320*b^2*B*c^(1/4)*Sqrt[x] - 2880*A*b*c^(5/4)*Sqrt[x] - 576*b*B*c^(5/4)*x^
(5/2) + 288*A*c^(9/4)*x^(5/2) + 160*B*c^(9/4)*x^(9/2) + (360*b^3*B*c^(1/4)*
Sqrt[x])/(b + c*x^2) - (360*A*b^2*c^(5/4)*Sqrt[x])/(b + c*x^2) + 90*Sqrt[2]
*b^(5/4)*(13*b*B - 9*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] - 9
0*Sqrt[2]*b^(5/4)*(13*b*B - 9*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(
1/4)] + 585*Sqrt[2]*b^(9/4)*B*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x]
 + Sqrt[c]*x] - 405*Sqrt[2]*A*b^(5/4)*c*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/
4)*Sqrt[x] + Sqrt[c]*x] - 585*Sqrt[2]*b^(9/4)*B*Log[Sqrt[b] + Sqrt[2]*b^(1/
4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] + 405*Sqrt[2]*A*b^(5/4)*c*Log[Sqrt[b] + Sqr
t[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(720*c^(17/4))
```

Maple [A] time = 0.016, size = 372, normalized size = 1.1

$$\frac{2B}{9c^2}x^{\frac{9}{2}}+\frac{2A}{5c^2}x^{\frac{5}{2}}-\frac{4Bb}{5c^3}x^{\frac{5}{2}}-4\frac{Ab\sqrt{x}}{c^3}+6\frac{Bb^2\sqrt{x}}{c^4}-\frac{Ab^2}{2c^3\left(cx^2+b\right)}\sqrt{x}+\frac{Bb^3}{2c^4\left(cx^2+b\right)}\sqrt{x}+\frac{9b\sqrt{2}A}{8c^3}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(19/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x)
```

```
[Out] 2/9/c^2*B*x^(9/2)+2/5/c^2*A*x^(5/2)-4/5/c^3*B*x^(5/2)*b-4/c^3*A*b*x^(1/2)+6
/c^4*B*b^2*x^(1/2)-1/2*b^2/c^3*x^(1/2)/(c*x^2+b)*A+1/2*b^3/c^4*x^(1/2)/(c*x
^2+b)*B+9/8*b/c^3*(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-
1)+9/16*b/c^3*(b/c)^(1/4)*2^(1/2)*A*ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)
^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+9/8*b/c^3*(b/c)^(1/4)*
2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-13/8*b^2/c^4*(b/c)^(1/4)*2^
(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-13/16*b^2/c^4*(b/c)^(1/4)*2^(
1/2)*B*ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2
)*2^(1/2)+(b/c)^(1/2)))-13/8*b^2/c^4*(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(
b/c)^(1/4)*x^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(19/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.42732, size = 1928, normalized size = 5.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(19/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/360*(180*(c^5*x^2 + b*c^4)*(-(28561*B^4*b^9 - 79092*A*B^3*b^8*c + 82134*A
^2*B^2*b^7*c^2 - 37908*A^3*B*b^6*c^3 + 6561*A^4*b^5*c^4)/c^17)^(1/4)*arctan
((sqrt(c^8*sqrt(-(28561*B^4*b^9 - 79092*A*B^3*b^8*c + 82134*A^2*B^2*b^7*c^2
 - 37908*A^3*B*b^6*c^3 + 6561*A^4*b^5*c^4)/c^17) + (169*B^2*b^4 - 234*A*B*b
^3*c + 81*A^2*b^2*c^2)*x)*c^13*(-(28561*B^4*b^9 - 79092*A*B^3*b^8*c + 82134
*A^2*B^2*b^7*c^2 - 37908*A^3*B*b^6*c^3 + 6561*A^4*b^5*c^4)/c^17)^(3/4) + (1
3*B*b^2*c^13 - 9*A*b*c^14)*sqrt(x)*(-(28561*B^4*b^9 - 79092*A*B^3*b^8*c + 8
2134*A^2*B^2*b^7*c^2 - 37908*A^3*B*b^6*c^3 + 6561*A^4*b^5*c^4)/c^17)^(3/4))
/(28561*B^4*b^9 - 79092*A*B^3*b^8*c + 82134*A^2*B^2*b^7*c^2 - 37908*A^3*B*b
^6*c^3 + 6561*A^4*b^5*c^4)) + 45*(c^5*x^2 + b*c^4)*(-(28561*B^4*b^9 - 79092
*A*B^3*b^8*c + 82134*A^2*B^2*b^7*c^2 - 37908*A^3*B*b^6*c^3 + 6561*A^4*b^5*c
^4)/c^17)^(1/4)*log(c^4*(-(28561*B^4*b^9 - 79092*A*B^3*b^8*c + 82134*A^2*B^
2*b^7*c^2 - 37908*A^3*B*b^6*c^3 + 6561*A^4*b^5*c^4)/c^17)^(1/4) - (13*B*b^2
 - 9*A*b*c)*sqrt(x)) - 45*(c^5*x^2 + b*c^4)*(-(28561*B^4*b^9 - 79092*A*B^3*
b^8*c + 82134*A^2*B^2*b^7*c^2 - 37908*A^3*B*b^6*c^3 + 6561*A^4*b^5*c^4)/c^1
7)^(1/4)*log(-c^4*(-(28561*B^4*b^9 - 79092*A*B^3*b^8*c + 82134*A^2*B^2*b^7*
c^2 - 37908*A^3*B*b^6*c^3 + 6561*A^4*b^5*c^4)/c^17)^(1/4) - (13*B*b^2 - 9*A
*b*c)*sqrt(x)) + 4*(20*B*c^3*x^6 - 4*(13*B*b*c^2 - 9*A*c^3)*x^4 + 585*B*b^3
 - 405*A*b^2*c + 36*(13*B*b^2*c - 9*A*b*c^2)*x^2)*sqrt(x))/(c^5*x^2 + b*c^4
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(19/2)*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27709, size = 452, normalized size = 1.36

$$-\frac{\sqrt{2}\left(13\left(bc^{3}\right)^{\frac{1}{4}}Bb^{2}-9\left(bc^{3}\right)^{\frac{1}{4}}Abc\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8c^{5}}-\frac{\sqrt{2}\left(13\left(bc^{3}\right)^{\frac{1}{4}}Bb^{2}-9\left(bc^{3}\right)^{\frac{1}{4}}Abc\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)\right.}{2\left(\frac{b}{c}\right.}\right.}{8c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(19/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*(13*(b*c^3)^(1/4)*B*b^2 - 9*(b*c^3)^(1/4)*A*b*c)*arctan(1/2*sq
rt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/c^5 - 1/8*sqrt(2)*(13*
(b*c^3)^(1/4)*B*b^2 - 9*(b*c^3)^(1/4)*A*b*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(
b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/c^5 - 1/16*sqrt(2)*(13*(b*c^3)^(1/4)*B
*b^2 - 9*(b*c^3)^(1/4)*A*b*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/
c))/c^5 + 1/16*sqrt(2)*(13*(b*c^3)^(1/4)*B*b^2 - 9*(b*c^3)^(1/4)*A*b*c)*log
(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^5 + 1/2*(B*b^3*sqrt(x) - A
*b^2*c*sqrt(x))/((c*x^2 + b)*c^4) + 2/45*(5*B*c^16*x^(9/2) - 18*B*b*c^15*x^
(5/2) + 9*A*c^16*x^(5/2) + 135*B*b^2*c^14*sqrt(x) - 90*A*b*c^15*sqrt(x))/c^
18
```

3.196 $\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=310

$$\frac{b^{3/4}(11bB-7Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{15/4}}-\frac{b^{3/4}(11bB-7Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{15/4}}-\frac{b^{3/4}(11bB-7Ac)}{4\sqrt{}}$$

```
[Out] -((11*b*B - 7*A*c)*x^(3/2))/(6*c^3) + ((11*b*B - 7*A*c)*x^(7/2))/(14*b*c^2)
 - ((b*B - A*c)*x^(11/2))/(2*b*c*(b + c*x^2)) - (b^(3/4)*(11*b*B - 7*A*c)*A
rcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*c^(15/4)) + (b^(3/
4)*(11*b*B - 7*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[
2]*c^(15/4)) + (b^(3/4)*(11*b*B - 7*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1
/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*c^(15/4)) - (b^(3/4)*(11*b*B - 7*A*c)*
Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*c^(1
5/4))
```

Rubi [A] time = 0.244669, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1584, 457, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{b^{3/4}(11bB-7Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{15/4}}-\frac{b^{3/4}(11bB-7Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{15/4}}-\frac{b^{3/4}(11bB-7Ac)}{4\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(17/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -((11*b*B - 7*A*c)*x^(3/2))/(6*c^3) + ((11*b*B - 7*A*c)*x^(7/2))/(14*b*c^2)
 - ((b*B - A*c)*x^(11/2))/(2*b*c*(b + c*x^2)) - (b^(3/4)*(11*b*B - 7*A*c)*A
rcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*c^(15/4)) + (b^(3/
4)*(11*b*B - 7*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[
2]*c^(15/4)) + (b^(3/4)*(11*b*B - 7*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1
/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*c^(15/4)) - (b^(3/4)*(11*b*B - 7*A*c)*
Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*c^(1
5/4))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^{17/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx = \int \frac{x^{9/2}\left(A+Bx^2\right)}{\left(b+cx^2\right)^2}\,dx$$

$$= -\frac{(bB-Ac)x^{11/2}}{2bc\left(b+cx^2\right)} + \frac{\left(\frac{11bB}{2}-\frac{7Ac}{2}\right)\int \frac{x^{9/2}}{b+cx^2}\,dx}{2bc}$$

$$= \frac{(11bB-7Ac)x^{7/2}}{14bc^2} - \frac{(bB-Ac)x^{11/2}}{2bc\left(b+cx^2\right)} - \frac{(11bB-7Ac)\int \frac{x^{5/2}}{b+cx^2}\,dx}{4c^2}$$

$$= -\frac{(11bB-7Ac)x^{3/2}}{6c^3} + \frac{(11bB-7Ac)x^{7/2}}{14bc^2} - \frac{(bB-Ac)x^{11/2}}{2bc\left(b+cx^2\right)} + \frac{(b(11bB-7Ac))\int \frac{\sqrt{x}}{b+cx^2}\,dx}{4c^3}$$

$$= -\frac{(11bB-7Ac)x^{3/2}}{6c^3} + \frac{(11bB-7Ac)x^{7/2}}{14bc^2} - \frac{(bB-Ac)x^{11/2}}{2bc\left(b+cx^2\right)} + \frac{(b(11bB-7Ac))\operatorname{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx\right.}{2c^3}$$

$$= -\frac{(11bB-7Ac)x^{3/2}}{6c^3} + \frac{(11bB-7Ac)x^{7/2}}{14bc^2} - \frac{(bB-Ac)x^{11/2}}{2bc\left(b+cx^2\right)} - \frac{(b(11bB-7Ac))\operatorname{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\right.}{4c^{7/2}}$$

$$= -\frac{(11bB-7Ac)x^{3/2}}{6c^3} + \frac{(11bB-7Ac)x^{7/2}}{14bc^2} - \frac{(bB-Ac)x^{11/2}}{2bc\left(b+cx^2\right)} + \frac{(b(11bB-7Ac))\operatorname{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}}\right.}{8c^4}$$

$$= -\frac{(11bB-7Ac)x^{3/2}}{6c^3} + \frac{(11bB-7Ac)x^{7/2}}{14bc^2} - \frac{(bB-Ac)x^{11/2}}{2bc\left(b+cx^2\right)} + \frac{b^{3/4}(11bB-7Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\right.}{8\sqrt{2}c^{15/4}}$$

$$= -\frac{(11bB-7Ac)x^{3/2}}{6c^3} + \frac{(11bB-7Ac)x^{7/2}}{14bc^2} - \frac{(bB-Ac)x^{11/2}}{2bc\left(b+cx^2\right)} - \frac{b^{3/4}(11bB-7Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{}}{\sqrt[4]{b}}\right.}{4\sqrt{2}c^{15/4}}$$

Mathematica [C] time = 0.220721, size = 154, normalized size = 0.5

$$\frac{2x^{3/2}(Ac-bB)\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{cx^2}{b}\right)}{3c^3} + \frac{2x^{3/2}(Ac-2bB)}{3c^3} - \frac{(-b)^{3/4}(3bB-2Ac)\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)}{c^{15/4}} + \frac{(-b)^{3/4}(3bB-2Ac)\tanh^{-1}}{c^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(17/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*(-2*b*B + A*c)*x^(3/2))/(3*c^3) + (2*B*x^(7/2))/(7*c^2) - ((-b)^(3/4)*(3
*b*B - 2*A*c)*ArcTan[(c^(1/4)*Sqrt[x])/(-b)^(1/4)])/c^(15/4) + ((-b)^(3/4)*
(3*b*B - 2*A*c)*ArcTanh[(c^(1/4)*Sqrt[x])/(-b)^(1/4)])/c^(15/4) + (2*(-(b*B
) + A*c)*x^(3/2)*Hypergeometric2F1[3/4, 2, 7/4, -((c*x^2)/b)])/(3*c^3)
```

Maple [A] time = 0.015, size = 348, normalized size = 1.1

$$\frac{2B}{7c^2}x^{\frac{7}{2}}+\frac{2A}{3c^2}x^{\frac{3}{2}}-\frac{4Bb}{3c^3}x^{\frac{3}{2}}+\frac{Ab}{2c^2\left(cx^2+b\right)}x^{\frac{3}{2}}-\frac{Bb^2}{2c^3\left(cx^2+b\right)}x^{\frac{3}{2}}-\frac{7b\sqrt{2}A}{16c^3}\ln\left(\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x)
```

```
[Out] 2/7/c^2*B*x^(7/2)+2/3/c^2*x^(3/2)*A-4/3/c^3*x^(3/2)*B*b+1/2*b/c^2*x^(3/2)/(
c*x^2+b)*A-1/2*b^2/c^3*x^(3/2)/(c*x^2+b)*B-7/16*b/c^3/(b/c)^(1/4)*2^(1/2)*A
*ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1
/2)+(b/c)^(1/2)))-7/8*b/c^3/(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4
)*x^(1/2)+1)-7/8*b/c^3/(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(
1/2)-1)+11/16*b^2/c^4/(b/c)^(1/4)*2^(1/2)*B*ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/
2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+11/8*b^2/c^4/(
b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+11/8*b^2/c^4/(b/
c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.64511, size = 2390, normalized size = 7.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/168*(84*(c^4*x^2 + b*c^3)*(-(14641*B^4*b^7 - 37268*A*B^3*b^6*c + 35574*A^
2*B^2*b^5*c^2 - 15092*A^3*B*b^4*c^3 + 2401*A^4*b^3*c^4)/c^15)^(1/4)*arctan(
(sqrt((1771561*B^6*b^10 - 6764142*A*B^5*b^9*c + 10761135*A^2*B^4*b^8*c^2 -
9130660*A^3*B^3*b^7*c^3 + 4357815*A^4*B^2*b^6*c^4 - 1109262*A^5*B*b^5*c^5 +
 117649*A^6*b^4*c^6)*x - (14641*B^4*b^7*c^7 - 37268*A*B^3*b^6*c^8 + 35574*A
^2*B^2*b^5*c^9 - 15092*A^3*B*b^4*c^10 + 2401*A^4*b^3*c^11)*sqrt(-(14641*B^4
*b^7 - 37268*A*B^3*b^6*c + 35574*A^2*B^2*b^5*c^2 - 15092*A^3*B*b^4*c^3 + 24
01*A^4*b^3*c^4)/c^15))*c^4*(-(14641*B^4*b^7 - 37268*A*B^3*b^6*c + 35574*A^2
*B^2*b^5*c^2 - 15092*A^3*B*b^4*c^3 + 2401*A^4*b^3*c^4)/c^15)^(1/4) + (1331*
B^3*b^5*c^4 - 2541*A*B^2*b^4*c^5 + 1617*A^2*B*b^3*c^6 - 343*A^3*b^2*c^7)*sq
rt(x)*(-(14641*B^4*b^7 - 37268*A*B^3*b^6*c + 35574*A^2*B^2*b^5*c^2 - 15092*
A^3*B*b^4*c^3 + 2401*A^4*b^3*c^4)/c^15)^(1/4))/(14641*B^4*b^7 - 37268*A*B^3
*b^6*c + 35574*A^2*B^2*b^5*c^2 - 15092*A^3*B*b^4*c^3 + 2401*A^4*b^3*c^4)) -
 21*(c^4*x^2 + b*c^3)*(-(14641*B^4*b^7 - 37268*A*B^3*b^6*c + 35574*A^2*B^2*
b^5*c^2 - 15092*A^3*B*b^4*c^3 + 2401*A^4*b^3*c^4)/c^15)^(1/4)*log(c^11*(-(1
4641*B^4*b^7 - 37268*A*B^3*b^6*c + 35574*A^2*B^2*b^5*c^2 - 15092*A^3*B*b^4*
c^3 + 2401*A^4*b^3*c^4)/c^15)^(3/4) - (1331*B^3*b^5 - 2541*A*B^2*b^4*c + 16
17*A^2*B*b^3*c^2 - 343*A^3*b^2*c^3)*sqrt(x)) + 21*(c^4*x^2 + b*c^3)*(-(1464
1*B^4*b^7 - 37268*A*B^3*b^6*c + 35574*A^2*B^2*b^5*c^2 - 15092*A^3*B*b^4*c^3
 + 2401*A^4*b^3*c^4)/c^15)^(1/4)*log(-c^11*(-(14641*B^4*b^7 - 37268*A*B^3*b
^6*c + 35574*A^2*B^2*b^5*c^2 - 15092*A^3*B*b^4*c^3 + 2401*A^4*b^3*c^4)/c^15
)^(3/4) - (1331*B^3*b^5 - 2541*A*B^2*b^4*c + 1617*A^2*B*b^3*c^2 - 343*A^3*b
^2*c^3)*sqrt(x)) + 4*(12*B*c^2*x^5 - 4*(11*B*b*c - 7*A*c^2)*x^3 - 7*(11*B*b
^2 - 7*A*b*c)*x)*sqrt(x))/(c^4*x^2 + b*c^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(17/2)*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24149, size = 404, normalized size = 1.3

$$-\frac{Bb^2x^{\frac{3}{2}} - Abcx^{\frac{3}{2}}}{2\left(cx^2+b\right)c^3} + \frac{\sqrt{2}\left(11\,\left(bc^3\right)^{\frac{3}{4}}Bb - 7\,\left(bc^3\right)^{\frac{3}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,c^6} + \frac{\sqrt{2}\left(11\,\left(bc^3\right)^{\frac{3}{4}}Bb - 7\,\left(bc^3\right)^{\frac{3}{4}}Ac\right)\text{arcta}}{8\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(B*b^2*x^(3/2) - A*b*c*x^(3/2))/((c*x^2 + b)*c^3) + 1/8*sqrt(2)*(11*(b
*c^3)^(3/4)*B*b - 7*(b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1
/4) + 2*sqrt(x))/(b/c)^(1/4))/c^6 + 1/8*sqrt(2)*(11*(b*c^3)^(3/4)*B*b - 7*(
b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/
c)^(1/4))/c^6 - 1/16*sqrt(2)*(11*(b*c^3)^(3/4)*B*b - 7*(b*c^3)^(3/4)*A*c)*l
og(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^6 + 1/16*sqrt(2)*(11*(b*c
^3)^(3/4)*B*b - 7*(b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x +
 sqrt(b/c))/c^6 + 2/21*(3*B*c^12*x^(7/2) - 14*B*b*c^11*x^(3/2) + 7*A*c^12*x
^(3/2))/c^14
```

3.197 $\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=310

$$\frac{x^{5/2}(9bB-5Ac)}{10bc^2}-\frac{\sqrt{x}(9bB-5Ac)}{2c^3}-\frac{\sqrt[4]{b}(9bB-5Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{13/4}}+\frac{\sqrt[4]{b}(9bB-5Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\right.}{8\sqrt{2}c^{13/4}}$$

```
[Out] -((9*b*B - 5*A*c)*Sqrt[x])/(2*c^3) + ((9*b*B - 5*A*c)*x^(5/2))/(10*b*c^2) -
 ((b*B - A*c)*x^(9/2))/(2*b*c*(b + c*x^2)) - (b^(1/4)*(9*b*B - 5*A*c)*ArcTa
n[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*c^(13/4)) + (b^(1/4)*(
9*b*B - 5*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*c^
(13/4)) - (b^(1/4)*(9*b*B - 5*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sq
rt[x] + Sqrt[c]*x])/(8*Sqrt[2]*c^(13/4)) + (b^(1/4)*(9*b*B - 5*A*c)*Log[Sqr
t[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*c^(13/4))
```

Rubi [A] time = 0.253366, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1584, 457, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{x^{5/2}(9bB-5Ac)}{10bc^2}-\frac{\sqrt{x}(9bB-5Ac)}{2c^3}-\frac{\sqrt[4]{b}(9bB-5Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{13/4}}+\frac{\sqrt[4]{b}(9bB-5Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\right.}{8\sqrt{2}c^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(15/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -((9*b*B - 5*A*c)*Sqrt[x])/(2*c^3) + ((9*b*B - 5*A*c)*x^(5/2))/(10*b*c^2) -
 ((b*B - A*c)*x^(9/2))/(2*b*c*(b + c*x^2)) - (b^(1/4)*(9*b*B - 5*A*c)*ArcTa
n[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*c^(13/4)) + (b^(1/4)*(
9*b*B - 5*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*c^
(13/4)) - (b^(1/4)*(9*b*B - 5*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sq
rt[x] + Sqrt[c]*x])/(8*Sqrt[2]*c^(13/4)) + (b^(1/4)*(9*b*B - 5*A*c)*Log[Sqr
t[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*c^(13/4))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{15/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx=\int \frac{x^{7/2}\left(A+Bx^2\right)}{\left(b+cx^2\right)^2}\,dx$$

$$=-\frac{(bB-Ac)x^{9/2}}{2bc\left(b+cx^2\right)}+\frac{\left(\frac{9bB}{2}-\frac{5Ac}{2}\right)\int \frac{x^{7/2}}{b+cx^2}\,dx}{2bc}$$

$$=\frac{(9bB-5Ac)x^{5/2}}{10bc^2}-\frac{(bB-Ac)x^{9/2}}{2bc\left(b+cx^2\right)}-\frac{(9bB-5Ac)\int \frac{x^{3/2}}{b+cx^2}\,dx}{4c^2}$$

$$=-\frac{(9bB-5Ac)\sqrt{x}}{2c^3}+\frac{(9bB-5Ac)x^{5/2}}{10bc^2}-\frac{(bB-Ac)x^{9/2}}{2bc\left(b+cx^2\right)}+\frac{(b(9bB-5Ac))\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{4c^3}$$

$$=-\frac{(9bB-5Ac)\sqrt{x}}{2c^3}+\frac{(9bB-5Ac)x^{5/2}}{10bc^2}-\frac{(bB-Ac)x^{9/2}}{2bc\left(b+cx^2\right)}+\frac{(b(9bB-5Ac))\,\mathrm{Subst}\left(\int \frac{1}{b+cx^4}\,dx,x,\sqrt{x}\right.}{2c^3}$$

$$=-\frac{(9bB-5Ac)\sqrt{x}}{2c^3}+\frac{(9bB-5Ac)x^{5/2}}{10bc^2}-\frac{(bB-Ac)x^{9/2}}{2bc\left(b+cx^2\right)}+\frac{\left(\sqrt{b}(9bB-5Ac)\right)\mathrm{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx,\right.}{4c^3}$$

$$=-\frac{(9bB-5Ac)\sqrt{x}}{2c^3}+\frac{(9bB-5Ac)x^{5/2}}{10bc^2}-\frac{(bB-Ac)x^{9/2}}{2bc\left(b+cx^2\right)}+\frac{\left(\sqrt{b}(9bB-5Ac)\right)\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\right.}{8c^{7/2}}$$

$$=-\frac{(9bB-5Ac)\sqrt{x}}{2c^3}+\frac{(9bB-5Ac)x^{5/2}}{10bc^2}-\frac{(bB-Ac)x^{9/2}}{2bc\left(b+cx^2\right)}-\frac{\sqrt[4]{b}(9bB-5Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}\right.}{8\sqrt{2}c^{13/4}}$$

$$=-\frac{(9bB-5Ac)\sqrt{x}}{2c^3}+\frac{(9bB-5Ac)x^{5/2}}{10bc^2}-\frac{(bB-Ac)x^{9/2}}{2bc\left(b+cx^2\right)}-\frac{\sqrt[4]{b}(9bB-5Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{13/4}}+$$

Mathematica [A] time = 0.398497, size = 385, normalized size = 1.24

$$\frac{-10\sqrt{2}\sqrt[4]{b}(9bB-5Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)+10\sqrt{2}\sqrt[4]{b}(9bB-5Ac)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)+\frac{40Abc^{5/4}\sqrt{x}}{b+cx^2}+25\sqrt{2}A\sqrt[4]{b}c\log\left(-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(15/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (-320*b*B*c^(1/4)*Sqrt[x] + 160*A*c^(5/4)*Sqrt[x] + 32*B*c^(5/4)*x^(5/2) -
(40*b^2*B*c^(1/4)*Sqrt[x])/(b + c*x^2) + (40*A*b*c^(5/4)*Sqrt[x])/(b + c*x^
2) - 10*Sqrt[2]*b^(1/4)*(9*b*B - 5*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x]
)/b^(1/4)] + 10*Sqrt[2]*b^(1/4)*(9*b*B - 5*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)
*Sqrt[x])/b^(1/4)] - 45*Sqrt[2]*b^(5/4)*B*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(
1/4)*Sqrt[x] + Sqrt[c]*x] + 25*Sqrt[2]*A*b^(1/4)*c*Log[Sqrt[b] - Sqrt[2]*b^
(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] + 45*Sqrt[2]*b^(5/4)*B*Log[Sqrt[b] + Sqr
t[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] - 25*Sqrt[2]*A*b^(1/4)*c*Log[Sqrt
[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(80*c^(13/4))
```

Maple [A] time = 0.013, size = 339, normalized size = 1.1

$$\frac{2B}{5c^2}x^{\frac{5}{2}}+2\frac{A\sqrt{x}}{c^2}-4\frac{Bb\sqrt{x}}{c^3}+\frac{Ab}{2c^2\left(cx^2+b\right)}\sqrt{x}-\frac{Bb^2}{2c^3\left(cx^2+b\right)}\sqrt{x}-\frac{5\sqrt{2}A}{8c^2}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)-\frac{5\sqrt{2}A}{8c^2}\sqrt[4]{\frac{b}{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x)
```

```
[Out] 2/5/c^2*B*x^(5/2)+2/c^2*A*x^(1/2)-4/c^3*B*b*x^(1/2)+1/2*b/c^2*x^(1/2)/(c*x^
2+b)*A-1/2*b^2/c^3*x^(1/2)/(c*x^2+b)*B-5/8/c^2*(b/c)^(1/4)*2^(1/2)*A*arctan
(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-5/8/c^2*(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2
)/(b/c)^(1/4)*x^(1/2)-1)-5/16/c^2*(b/c)^(1/4)*2^(1/2)*A*ln((x+(b/c)^(1/4)*x
^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+9/
8*b/c^3*(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+9/8*b/c
^3*(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)+9/16*b/c^3*(
b/c)^(1/4)*2^(1/2)*B*ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c
)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.46764, size = 1775, normalized size = 5.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/40*(20*(c^4*x^2 + b*c^3)*(-(6561*B^4*b^5 - 14580*A*B^3*b^4*c + 12150*A^2
*B^2*b^3*c^2 - 4500*A^3*B*b^2*c^3 + 625*A^4*b*c^4)/c^13)^(1/4)*arctan((sqrt
(c^6*sqrt(-(6561*B^4*b^5 - 14580*A*B^3*b^4*c + 12150*A^2*B^2*b^3*c^2 - 4500
*A^3*B*b^2*c^3 + 625*A^4*b*c^4)/c^13) + (81*B^2*b^2 - 90*A*B*b*c + 25*A^2*c
^2)*x)*c^10*(-(6561*B^4*b^5 - 14580*A*B^3*b^4*c + 12150*A^2*B^2*b^3*c^2 - 4
500*A^3*B*b^2*c^3 + 625*A^4*b*c^4)/c^13)^(3/4) + (9*B*b*c^10 - 5*A*c^11)*sq
rt(x)*(-(6561*B^4*b^5 - 14580*A*B^3*b^4*c + 12150*A^2*B^2*b^3*c^2 - 4500*A^
3*B*b^2*c^3 + 625*A^4*b*c^4)/c^13)^(3/4))/(6561*B^4*b^5 - 14580*A*B^3*b^4*c
 + 12150*A^2*B^2*b^3*c^2 - 4500*A^3*B*b^2*c^3 + 625*A^4*b*c^4)) + 5*(c^4*x^
2 + b*c^3)*(-(6561*B^4*b^5 - 14580*A*B^3*b^4*c + 12150*A^2*B^2*b^3*c^2 - 45
00*A^3*B*b^2*c^3 + 625*A^4*b*c^4)/c^13)^(1/4)*log(c^3*(-(6561*B^4*b^5 - 145
80*A*B^3*b^4*c + 12150*A^2*B^2*b^3*c^2 - 4500*A^3*B*b^2*c^3 + 625*A^4*b*c^4
)/c^13)^(1/4) - (9*B*b - 5*A*c)*sqrt(x)) - 5*(c^4*x^2 + b*c^3)*(-(6561*B^4*
b^5 - 14580*A*B^3*b^4*c + 12150*A^2*B^2*b^3*c^2 - 4500*A^3*B*b^2*c^3 + 625*
A^4*b*c^4)/c^13)^(1/4)*log(-c^3*(-(6561*B^4*b^5 - 14580*A*B^3*b^4*c + 12150
*A^2*B^2*b^3*c^2 - 4500*A^3*B*b^2*c^3 + 625*A^4*b*c^4)/c^13)^(1/4) - (9*B*b
 - 5*A*c)*sqrt(x)) - 4*(4*B*c^2*x^4 - 45*B*b^2 + 25*A*b*c - 4*(9*B*b*c - 5*
A*c^2)*x^2)*sqrt(x))/(c^4*x^2 + b*c^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(15/2)*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32878, size = 402, normalized size = 1.3

$$\frac{\sqrt{2}\left(9\left(bc^3\right)^{\frac{1}{4}}Bb-5\left(bc^3\right)^{\frac{1}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8c^4}+\frac{\sqrt{2}\left(9\left(bc^3\right)^{\frac{1}{4}}Bb-5\left(bc^3\right)^{\frac{1}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(9*(b*c^3)^(1/4)*B*b - 5*(b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt(2)*
(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/c^4 + 1/8*sqrt(2)*(9*(b*c^3)
^(1/4)*B*b - 5*(b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4)
- 2*sqrt(x))/(b/c)^(1/4))/c^4 + 1/16*sqrt(2)*(9*(b*c^3)^(1/4)*B*b - 5*(b*c^
3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^4 - 1/16*s
qrt(2)*(9*(b*c^3)^(1/4)*B*b - 5*(b*c^3)^(1/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/
c)^(1/4) + x + sqrt(b/c))/c^4 - 1/2*(B*b^2*sqrt(x) - A*b*c*sqrt(x))/((c*x^2
 + b)*c^3) + 2/5*(B*c^8*x^(5/2) - 10*B*b*c^7*sqrt(x) + 5*A*c^8*sqrt(x))/c^1
0
```

3.198 $\int \frac{x^{13/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=289

$$\frac{x^{3/2}(7bB-3Ac)}{6bc^2}-\frac{(7bB-3Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}\sqrt[4]{b}c^{11/4}}+\frac{(7bB-3Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}\sqrt[4]{b}c^{11/4}}+\frac{(7bB-}{}$$

```
[Out] ((7*b*B - 3*A*c)*x^(3/2))/(6*b*c^2) - ((b*B - A*c)*x^(7/2))/(2*b*c*(b + c*x
^2)) + ((7*b*B - 3*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*S
qrt[2]*b^(1/4)*c^(11/4)) - ((7*b*B - 3*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqr
t[x])/b^(1/4)])/(4*Sqrt[2]*b^(1/4)*c^(11/4)) - ((7*b*B - 3*A*c)*Log[Sqrt[b]
 - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(1/4)*c^(11/4
)) + ((7*b*B - 3*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[
c]*x])/(8*Sqrt[2]*b^(1/4)*c^(11/4))
```

Rubi [A] time = 0.229241, antiderivative size = 289, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1584, 457, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{x^{3/2}(7bB-3Ac)}{6bc^2}-\frac{(7bB-3Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}\sqrt[4]{b}c^{11/4}}+\frac{(7bB-3Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}\sqrt[4]{b}c^{11/4}}+\frac{(7bB-}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```


```
[Out] ((7*b*B - 3*A*c)*x^(3/2))/(6*b*c^2) - ((b*B - A*c)*x^(7/2))/(2*b*c*(b + c*x
^2)) + ((7*b*B - 3*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*S
qrt[2]*b^(1/4)*c^(11/4)) - ((7*b*B - 3*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqr
t[x])/b^(1/4)])/(4*Sqrt[2]*b^(1/4)*c^(11/4)) - ((7*b*B - 3*A*c)*Log[Sqrt[b]
 - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(1/4)*c^(11/4
)) + ((7*b*B - 3*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[
c]*x])/(8*Sqrt[2]*b^(1/4)*c^(11/4))
```


Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^{13/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx = \int \frac{x^{5/2}\left(A+Bx^2\right)}{\left(b+cx^2\right)^2}\,dx$$

$$= -\frac{(bB-Ac)x^{7/2}}{2bc\left(b+cx^2\right)} + \frac{\left(\frac{7bB}{2}-\frac{3Ac}{2}\right)\int \frac{x^{5/2}}{b+cx^2}\,dx}{2bc}$$

$$= \frac{(7bB-3Ac)x^{3/2}}{6bc^2} - \frac{(bB-Ac)x^{7/2}}{2bc\left(b+cx^2\right)} - \frac{(7bB-3Ac)\int \frac{\sqrt{x}}{b+cx^2}\,dx}{4c^2}$$

$$= \frac{(7bB-3Ac)x^{3/2}}{6bc^2} - \frac{(bB-Ac)x^{7/2}}{2bc\left(b+cx^2\right)} - \frac{(7bB-3Ac)\,\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{2c^2}$$

$$= \frac{(7bB-3Ac)x^{3/2}}{6bc^2} - \frac{(bB-Ac)x^{7/2}}{2bc\left(b+cx^2\right)} + \frac{(7bB-3Ac)\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4c^{5/2}} - \frac{(7bB-3Ac)\,\text{S}}{}$$

$$= \frac{(7bB-3Ac)x^{3/2}}{6bc^2} - \frac{(bB-Ac)x^{7/2}}{2bc\left(b+cx^2\right)} - \frac{(7bB-3Ac)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{8c^3} - \frac{(7bB-3A}{}$$

$$= \frac{(7bB-3Ac)x^{3/2}}{6bc^2} - \frac{(bB-Ac)x^{7/2}}{2bc\left(b+cx^2\right)} - \frac{(7bB-3Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}\sqrt[4]{b}c^{11/4}} + \frac{(7bB-3Ac}{}$$

$$= \frac{(7bB-3Ac)x^{3/2}}{6bc^2} - \frac{(bB-Ac)x^{7/2}}{2bc\left(b+cx^2\right)} + \frac{(7bB-3Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\sqrt[4]{b}c^{11/4}} - \frac{(7bB-3Ac)\tan^{-1}\left(1-\right.}{4\sqrt{2}\sqrt[4]{b}c^{11/4}}$$

<u>Mathematica</u> [C] time = 0.197619, size = 136, normalized size = 0.47

$$\frac{2x^{3/2}(bB-Ac)\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{cx^2}{b}\right)}{3bc^2} + \frac{(3Ac-6bB)\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)+(6bB-3Ac)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)+2\sqrt[4]{-b}Bc^{3/4}x^{3/2}}{3\sqrt[4]{-b}c^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*(-b)^(1/4)*B*c^(3/4)*x^(3/2) + (-6*b*B + 3*A*c)*ArcTan[(c^(1/4)*Sqrt[x])
/(-b)^(1/4)] + (6*b*B - 3*A*c)*ArcTanh[(c^(1/4)*Sqrt[x])/(-b)^(1/4)])/(3*(-
b)^(1/4)*c^(11/4)) + (2*(b*B - A*c)*x^(3/2)*Hypergeometric2F1[3/4, 2, 7/4,
```

```
-((c*x^2)/b)])/(3*b*c^2)
```

Maple [A] time = 0.013, size = 317, normalized size = 1.1

$$\frac{2B}{3c^2}x^{\frac{3}{2}}-\frac{A}{2c\left(cx^2+b\right)}x^{\frac{3}{2}}+\frac{Bb}{2c^2\left(cx^2+b\right)}x^{\frac{3}{2}}-\frac{7\sqrt{2}Bb}{16c^3}\ln\left(\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}-\frac{7\sqrt{2}}{8c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x)
```

```
[Out] 2/3/c^2*B*x^(3/2)-1/2/c*x^(3/2)/(c*x^2+b)*A+1/2/c^2*x^(3/2)/(c*x^2+b)*B*b-7
/16/c^3/(b/c)^(1/4)*2^(1/2)*B*b*ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/
2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))-7/8/c^3/(b/c)^(1/4)*2^(1/2
)*B*b*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-7/8/c^3/(b/c)^(1/4)*2^(1/2)*B*b
*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)+3/16/c^2/(b/c)^(1/4)*2^(1/2)*A*ln((x
-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b
/c)^(1/2)))+3/8/c^2/(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2
)+1)+3/8/c^2/(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.66426, size = 2141, normalized size = 7.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/24*(12*(c^3*x^2 + b*c^2)*(-(2401*B^4*b^4 - 4116*A*B^3*b^3*c + 2646*A^2*B
^2*b^2*c^2 - 756*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^11))^(1/4)*arctan((sqrt((11
7649*B^6*b^6 - 302526*A*B^5*b^5*c + 324135*A^2*B^4*b^4*c^2 - 185220*A^3*B^3
*b^3*c^3 + 59535*A^4*B^2*b^2*c^4 - 10206*A^5*B*b*c^5 + 729*A^6*c^6)*x - (24
01*B^4*b^5*c^5 - 4116*A*B^3*b^4*c^6 + 2646*A^2*B^2*b^3*c^7 - 756*A^3*B*b^2*
c^8 + 81*A^4*b*c^9)*sqrt(-(2401*B^4*b^4 - 4116*A*B^3*b^3*c + 2646*A^2*B^2*b
^2*c^2 - 756*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^11)))*c^3*(-(2401*B^4*b^4 - 411
6*A*B^3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 756*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^1
1))^(1/4) + (343*B^3*b^3*c^3 - 441*A*B^2*b^2*c^4 + 189*A^2*B*b*c^5 - 27*A^3
*c^6)*sqrt(x)*(-(2401*B^4*b^4 - 4116*A*B^3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 7
56*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^11))^(1/4))/(2401*B^4*b^4 - 4116*A*B^3*b^
3*c + 2646*A^2*B^2*b^2*c^2 - 756*A^3*B*b*c^3 + 81*A^4*c^4)) - 3*(c^3*x^2 +
b*c^2)*(-(2401*B^4*b^4 - 4116*A*B^3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 756*A^3*
B*b*c^3 + 81*A^4*c^4)/(b*c^11))^(1/4)*log(b*c^8*(-(2401*B^4*b^4 - 4116*A*B^
3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 756*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^11))^(3
/4) - (343*B^3*b^3 - 441*A*B^2*b^2*c + 189*A^2*B*b*c^2 - 27*A^3*c^3)*sqrt(x
)) + 3*(c^3*x^2 + b*c^2)*(-(2401*B^4*b^4 - 4116*A*B^3*b^3*c + 2646*A^2*B^2*
b^2*c^2 - 756*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^11))^(1/4)*log(-b*c^8*(-(2401*
B^4*b^4 - 4116*A*B^3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 756*A^3*B*b*c^3 + 81*A^
4*c^4)/(b*c^11))^(3/4) - (343*B^3*b^3 - 441*A*B^2*b^2*c + 189*A^2*B*b*c^2 -
 27*A^3*c^3)*sqrt(x)) - 4*(4*B*c*x^3 + (7*B*b - 3*A*c)*x)*sqrt(x))/(c^3*x^2
 + b*c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(13/2)*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.349, size = 382, normalized size = 1.32

$$\frac{2\,Bx^{\frac{3}{2}}}{3\,c^2}+\frac{Bbx^{\frac{3}{2}}-Acx^{\frac{3}{2}}}{2\left(cx^2+b\right)c^2}-\frac{\sqrt{2}\left(7\left(bc^3\right)^{\frac{3}{4}}Bb-3\left(bc^3\right)^{\frac{3}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,bc^5}-\frac{\sqrt{2}\left(7\left(bc^3\right)^{\frac{3}{4}}Bb-3\left(bc^3\right)^{\frac{3}{4}}Ac\right)\mathrm{ar}}{8\,bc^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 2/3*B*x^(3/2)/c^2 + 1/2*(B*b*x^(3/2) - A*c*x^(3/2))/((c*x^2 + b)*c^2) - 1/8
*sqrt(2)*(7*(b*c^3)^(3/4)*B*b - 3*(b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2)*(sq
rt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b*c^5) - 1/8*sqrt(2)*(7*(b*c^3
)^(3/4)*B*b - 3*(b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4)
 - 2*sqrt(x))/(b/c)^(1/4))/(b*c^5) + 1/16*sqrt(2)*(7*(b*c^3)^(3/4)*B*b - 3*
(b*c^3)^(3/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^5)
 - 1/16*sqrt(2)*(7*(b*c^3)^(3/4)*B*b - 3*(b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sq
rt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^5)
```

3.199 $\int \frac{x^{11/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=289

$$\frac{(5bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{3/4}c^{9/4}}-\frac{(5bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{3/4}c^{9/4}}+\frac{(5bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{\cdots}}{\sqrt[4]{\cdots}}\right.\cdots}{4\sqrt{2}b^{3/4}c^{9/4}}\cdots$$

```
[Out] ((5*b*B - A*c)*Sqrt[x])/(2*b*c^2) - ((b*B - A*c)*x^(5/2))/(2*b*c*(b + c*x^2
)) + ((5*b*B - A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[
2]*b^(3/4)*c^(9/4)) - ((5*b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b
^(1/4)])/(4*Sqrt[2]*b^(3/4)*c^(9/4)) + ((5*b*B - A*c)*Log[Sqrt[b] - Sqrt[2]
*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(3/4)*c^(9/4)) - ((5*b*
B - A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqr
t[2]*b^(3/4)*c^(9/4))
```

Rubi [A] time = 0.233747, antiderivative size = 289, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1584, 457, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{(5bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{3/4}c^{9/4}}-\frac{(5bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{3/4}c^{9/4}}+\frac{(5bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{\cdots}}{\sqrt[4]{\cdots}}\right.\cdots}{4\sqrt{2}b^{3/4}c^{9/4}}\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((5*b*B - A*c)*Sqrt[x])/(2*b*c^2) - ((b*B - A*c)*x^(5/2))/(2*b*c*(b + c*x^2
)) + ((5*b*B - A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[
2]*b^(3/4)*c^(9/4)) - ((5*b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b
^(1/4)])/(4*Sqrt[2]*b^(3/4)*c^(9/4)) + ((5*b*B - A*c)*Log[Sqrt[b] - Sqrt[2]
*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(3/4)*c^(9/4)) - ((5*b*
B - A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqr
t[2]*b^(3/4)*c^(9/4))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{11/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx=\int \frac{x^{3/2}\left(A+Bx^2\right)}{\left(b+cx^2\right)^2}\,dx$$

$$=-\frac{(bB-Ac)x^{5/2}}{2bc\left(b+cx^2\right)}+\frac{\left(\frac{5bB}{2}-\frac{Ac}{2}\right)\int \frac{x^{3/2}}{b+cx^2}\,dx}{2bc}$$

$$=\frac{(5bB-Ac)\sqrt{x}}{2bc^2}-\frac{(bB-Ac)x^{5/2}}{2bc\left(b+cx^2\right)}-\frac{(5bB-Ac)\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{4c^2}$$

$$=\frac{(5bB-Ac)\sqrt{x}}{2bc^2}-\frac{(bB-Ac)x^{5/2}}{2bc\left(b+cx^2\right)}-\frac{(5bB-Ac)\,\mathrm{Subst}\left(\int \frac{1}{b+cx^4}\,dx,x,\sqrt{x}\right)}{2c^2}$$

$$=\frac{(5bB-Ac)\sqrt{x}}{2bc^2}-\frac{(bB-Ac)x^{5/2}}{2bc\left(b+cx^2\right)}-\frac{(5bB-Ac)\,\mathrm{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{4\sqrt{b}c^2}-\frac{(5bB-Ac)\,\mathrm{Subst}\left(\right.}{4\surd}$$

$$=\frac{(5bB-Ac)\sqrt{x}}{2bc^2}-\frac{(bB-Ac)x^{5/2}}{2bc\left(b+cx^2\right)}-\frac{(5bB-Ac)\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{8\sqrt{b}c^{5/2}}-\frac{(5bB-Ac)\,\mathrm{Su}}{}$$

$$=\frac{(5bB-Ac)\sqrt{x}}{2bc^2}-\frac{(bB-Ac)x^{5/2}}{2bc\left(b+cx^2\right)}+\frac{(5bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{3/4}c^{9/4}}-\frac{(5bB-Ac)\log\left(\right.}{8}$$

$$=\frac{(5bB-Ac)\sqrt{x}}{2bc^2}-\frac{(bB-Ac)x^{5/2}}{2bc\left(b+cx^2\right)}+\frac{(5bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{3/4}c^{9/4}}-\frac{(5bB-Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{b}}\right.}{4\sqrt{2}b^{3/4}c^{9/4}}$$

Mathematica [A] time = 0.390944, size = 354, normalized size = 1.22

$$\frac{\frac{2\sqrt{2}(5bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{b^{3/4}}-\frac{2\sqrt{2}(5bB-Ac)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{b^{3/4}}-\frac{\sqrt{2}Ac\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{b^{3/4}}+\frac{\sqrt{2}Ac\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{b^{3/4}}-\frac{8Ac^5}{b+}}{16c^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (32*B*c^(1/4)*Sqrt[x] + (8*b*B*c^(1/4)*Sqrt[x])/(b + c*x^2) - (8*A*c^(5/4)*
Sqrt[x])/(b + c*x^2) + (2*Sqrt[2]*(5*b*B - A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)
*Sqrt[x])/b^(1/4)])/b^(3/4) - (2*Sqrt[2]*(5*b*B - A*c)*ArcTan[1 + (Sqrt[2]*
```

```
c^(1/4)*Sqrt[x])/b^(1/4)])/b^(3/4) + 5*Sqrt[2]*b^(1/4)*B*Log[Sqrt[b] - Sqrt
[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] - (Sqrt[2]*A*c*Log[Sqrt[b] - Sqrt[
2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/b^(3/4) - 5*Sqrt[2]*b^(1/4)*B*Log[
Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] + (Sqrt[2]*A*c*Log[S
qrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/b^(3/4))/(16*c^(9/4)
)
```

Maple [A] time = 0.014, size = 323, normalized size = 1.1

$$2\,\frac{B\sqrt{x}}{c^2}-\frac{A}{2\,c\left(cx^2+b\right)}\sqrt{x}+\frac{Bb}{2\,c^2\left(cx^2+b\right)}\sqrt{x}+\frac{\sqrt{2}A}{8\,bc}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)+\frac{\sqrt{2}A}{8\,bc}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x)
```

```
[Out] 2*B/c^2*x^(1/2)-1/2/c*x^(1/2)/(c*x^2+b)*A+1/2/c^2*x^(1/2)/(c*x^2+b)*B*b+1/8
/c*(b/c)^(1/4)/b*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+1/8/c*(b/c
)^(1/4)/b*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)+1/16/c*(b/c)^(1/4
)/b*2^(1/2)*A*ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)
*x^(1/2)*2^(1/2)+(b/c)^(1/2)))-5/8/c^2*(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)
/(b/c)^(1/4)*x^(1/2)+1)-5/8/c^2*(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^
(1/4)*x^(1/2)-1)-5/16/c^2*(b/c)^(1/4)*2^(1/2)*B*ln((x+(b/c)^(1/4)*x^(1/2)*2
^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.52275, size = 1594, normalized size = 5.52

$$\frac{4\left(c^3x^2+bc^2\right)\left(-\frac{625\,B^4b^4-500\,AB^3b^3c+150\,A^2B^2b^2c^2-20\,A^3Bbc^3+A^4c^4}{b^3c^9}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{b^2c^4\sqrt{-\frac{625\,B^4b^4-500\,AB^3b^3c+150\,A^2B^2b^2c^2-20\,A^3Bbc^3+A^4c^4}{b^3c^9}}+(25\,B}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*(c^3*x^2 + b*c^2)*(-(625*B^4*b^4 - 500*A*B^3*b^3*c + 150*A^2*B^2*b^2
*c^2 - 20*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^9))^(1/4)*arctan((sqrt(b^2*c^4*sqrt
(-(625*B^4*b^4 - 500*A*B^3*b^3*c + 150*A^2*B^2*b^2*c^2 - 20*A^3*B*b*c^3 + A
^4*c^4)/(b^3*c^9)) + (25*B^2*b^2 - 10*A*B*b*c + A^2*c^2)*x)*b^2*c^7*(-(625*
B^4*b^4 - 500*A*B^3*b^3*c + 150*A^2*B^2*b^2*c^2 - 20*A^3*B*b*c^3 + A^4*c^4)
/(b^3*c^9))^(3/4) + (5*B*b^3*c^7 - A*b^2*c^8)*sqrt(x)*(-(625*B^4*b^4 - 500*
A*B^3*b^3*c + 150*A^2*B^2*b^2*c^2 - 20*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^9))^(3
/4))/(625*B^4*b^4 - 500*A*B^3*b^3*c + 150*A^2*B^2*b^2*c^2 - 20*A^3*B*b*c^3
+ A^4*c^4)) + (c^3*x^2 + b*c^2)*(-(625*B^4*b^4 - 500*A*B^3*b^3*c + 150*A^2*
B^2*b^2*c^2 - 20*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^9))^(1/4)*log(b*c^2*(-(625*B
^4*b^4 - 500*A*B^3*b^3*c + 150*A^2*B^2*b^2*c^2 - 20*A^3*B*b*c^3 + A^4*c^4)/
(b^3*c^9))^(1/4) - (5*B*b - A*c)*sqrt(x)) - (c^3*x^2 + b*c^2)*(-(625*B^4*b^
4 - 500*A*B^3*b^3*c + 150*A^2*B^2*b^2*c^2 - 20*A^3*B*b*c^3 + A^4*c^4)/(b^3*
c^9))^(1/4)*log(-b*c^2*(-(625*B^4*b^4 - 500*A*B^3*b^3*c + 150*A^2*B^2*b^2*c
^2 - 20*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^9))^(1/4) - (5*B*b - A*c)*sqrt(x)) +
4*(4*B*c*x^2 + 5*B*b - A*c)*sqrt(x))/(c^3*x^2 + b*c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(11/2)*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22107, size = 382, normalized size = 1.32

$$\frac{2 B\sqrt{x}}{c^2} - \frac{\sqrt{2}\left(5 \left(bc^3\right)^{\frac{1}{4}} Bb - \left(bc^3\right)^{\frac{1}{4}} Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\, bc^3} - \frac{\sqrt{2}\left(5 \left(bc^3\right)^{\frac{1}{4}} Bb - \left(bc^3\right)^{\frac{1}{4}} Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-\right.}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.}{8\, bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 2*B*sqrt(x)/c^2 - 1/8*sqrt(2)*(5*(b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*arc
tan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b*c^3) - 1/
8*sqrt(2)*(5*(b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sq
rt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b*c^3) - 1/16*sqrt(2)*(5*(b*c^
3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqr
t(b/c))/(b*c^3) + 1/16*sqrt(2)*(5*(b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*lo
g(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^3) + 1/2*(B*b*sqrt(x)
- A*c*sqrt(x))/((c*x^2 + b)*c^2)
```

3.200 $\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=261

$$\frac{(Ac+3bB)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{5/4}c^{7/4}}-\frac{(Ac+3bB)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{5/4}c^{7/4}}-\frac{(Ac+3bB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{b}}\right.}{4\sqrt{2}b^{5/4}c^{7/4}}$$

```
[Out] -((b*B - A*c)*x^(3/2))/(2*b*c*(b + c*x^2)) - ((3*b*B + A*c)*ArcTan[1 - (Sqr
t[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(5/4)*c^(7/4)) + ((3*b*B + A*c
)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(5/4)*c^(7/4)
) + ((3*b*B + A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*
x])/(8*Sqrt[2]*b^(5/4)*c^(7/4)) - ((3*b*B + A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1
/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(5/4)*c^(7/4))
```

Rubi [A] time = 0.19996, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {1584, 457, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{(Ac+3bB)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{5/4}c^{7/4}}-\frac{(Ac+3bB)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{5/4}c^{7/4}}-\frac{(Ac+3bB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{b}}\right.}{4\sqrt{2}b^{5/4}c^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```


```
[Out] -((b*B - A*c)*x^(3/2))/(2*b*c*(b + c*x^2)) - ((3*b*B + A*c)*ArcTan[1 - (Sqr
t[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(5/4)*c^(7/4)) + ((3*b*B + A*c
)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(5/4)*c^(7/4)
) + ((3*b*B + A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*
x])/(8*Sqrt[2]*b^(5/4)*c^(7/4)) - ((3*b*B + A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1
/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(5/4)*c^(7/4))
```


Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^{9/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx=\int \frac{\sqrt{x}\left(A+Bx^2\right)}{\left(b+cx^2\right)^2}\,dx$$

$$=-\frac{(bB-Ac)x^{3/2}}{2bc\left(b+cx^2\right)}+\frac{\left(\frac{3bB}{2}+\frac{Ac}{2}\right)\int \frac{\sqrt{x}}{b+cx^2}\,dx}{2bc}$$

$$=-\frac{(bB-Ac)x^{3/2}}{2bc\left(b+cx^2\right)}+\frac{\left(\frac{3bB}{2}+\frac{Ac}{2}\right)\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{bc}$$

$$=-\frac{(bB-Ac)x^{3/2}}{2bc\left(b+cx^2\right)}-\frac{(3bB+Ac)\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{4bc^{3/2}}+\frac{(3bB+Ac)\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx,x,\right.}{4bc^{3/2}}$$

$$=-\frac{(bB-Ac)x^{3/2}}{2bc\left(b+cx^2\right)}+\frac{(3bB+Ac)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{8bc^2}+\frac{(3bB+Ac)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}\right.}{8bc^2}$$

$$=-\frac{(bB-Ac)x^{3/2}}{2bc\left(b+cx^2\right)}+\frac{(3bB+Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{5/4}c^{7/4}}-\frac{(3bB+Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\right.}{8\sqrt{2}b^{5/4}c^{7/4}}$$

$$=-\frac{(bB-Ac)x^{3/2}}{2bc\left(b+cx^2\right)}-\frac{(3bB+Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{5/4}c^{7/4}}+\frac{(3bB+Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{5/4}c^{7/4}}+\frac{(3bB+A}{}$$

Mathematica [C] time = 0.127734, size = 95, normalized size = 0.36

$$\frac{2x^{3/2}(Ac-bB)\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{cx^2}{b}\right)}{3b^2c}+\frac{B\left(\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)+\tanh^{-1}\left(\frac{b\sqrt[4]{c}\sqrt{x}}{(-b)^{5/4}}\right)\right)}{\sqrt[4]{-b}c^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (B*(ArcTan[(c^(1/4)*Sqrt[x])/(-b)^(1/4)] + ArcTanh[(b*c^(1/4)*Sqrt[x])/(-b)
^(5/4)]))/((-b)^(1/4)*c^(7/4)) + (2*(-(b*B) + A*c)*x^(3/2)*Hypergeometric2F
1[3/4, 2, 7/4, -((c*x^2)/b)])/(3*b^2*c)
```

Maple [A] time = 0.013, size = 305, normalized size = 1.2

$$\frac{Ac-Bb}{2\,bc\left(cx^2+b\right)}x^{\frac{3}{2}}+\frac{\sqrt{2}A}{16\,bc}\ln\left(\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}+\frac{\sqrt{2}A}{8\,bc}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/2*(A*c-B*b)/b/c*x^(3/2)/(c*x^2+b)+1/16/b/c/(b/c)^(1/4)*2^(1/2)*A*ln((x-(b
/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)
^(1/2)))+1/8/b/c/(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1
)+1/8/b/c/(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)+3/16/
c^2/(b/c)^(1/4)*2^(1/2)*B*ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x
+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+3/8/c^2/(b/c)^(1/4)*2^(1/2)*B*ar
ctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+3/8/c^2/(b/c)^(1/4)*2^(1/2)*B*arctan(2^
(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.56223, size = 1968, normalized size = 7.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*(B*b - A*c)*x^(3/2) + 4*(b*c^2*x^2 + b^2*c)*(-(81*B^4*b^4 + 108*A*B
^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 12*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^7))^(1/4)*
arctan((sqrt((729*B^6*b^6 + 1458*A*B^5*b^5*c + 1215*A^2*B^4*b^4*c^2 + 540*A
^3*B^3*b^3*c^3 + 135*A^4*B^2*b^2*c^4 + 18*A^5*B*b*c^5 + A^6*c^6)*x - (81*B^
4*b^7*c^3 + 108*A*B^3*b^6*c^4 + 54*A^2*B^2*b^5*c^5 + 12*A^3*B*b^4*c^6 + A^4
*b^3*c^7)*sqrt(-(81*B^4*b^4 + 108*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 12*A^3
*B*b*c^3 + A^4*c^4)/(b^5*c^7)))*b*c^2*(-(81*B^4*b^4 + 108*A*B^3*b^3*c + 54*
A^2*B^2*b^2*c^2 + 12*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^7))^(1/4) - (27*B^3*b^4*
c^2 + 27*A*B^2*b^3*c^3 + 9*A^2*B*b^2*c^4 + A^3*b*c^5)*sqrt(x)*(-(81*B^4*b^4
 + 108*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 12*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^
7))^(1/4))/(81*B^4*b^4 + 108*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 12*A^3*B*b*
c^3 + A^4*c^4)) - (b*c^2*x^2 + b^2*c)*(-(81*B^4*b^4 + 108*A*B^3*b^3*c + 54*
A^2*B^2*b^2*c^2 + 12*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^7))^(1/4)*log(b^4*c^5*(-
(81*B^4*b^4 + 108*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 12*A^3*B*b*c^3 + A^4*c
^4)/(b^5*c^7))^(3/4) + (27*B^3*b^3 + 27*A*B^2*b^2*c + 9*A^2*B*b*c^2 + A^3*c
^3)*sqrt(x)) + (b*c^2*x^2 + b^2*c)*(-(81*B^4*b^4 + 108*A*B^3*b^3*c + 54*A^2
*B^2*b^2*c^2 + 12*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^7))^(1/4)*log(-b^4*c^5*(-(8
1*B^4*b^4 + 108*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 12*A^3*B*b*c^3 + A^4*c^4
)/(b^5*c^7))^(3/4) + (27*B^3*b^3 + 27*A*B^2*b^2*c + 9*A^2*B*b*c^2 + A^3*c^3
)*sqrt(x)))/(b*c^2*x^2 + b^2*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2076, size = 369, normalized size = 1.41

$$-\frac{Bbx^{\frac{3}{2}} - Acx^{\frac{3}{2}}}{2\left(cx^2+b\right)bc} + \frac{\sqrt{2}\left(3\left(bc^3\right)^{\frac{3}{4}}Bb + \left(bc^3\right)^{\frac{3}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^2c^4} + \frac{\sqrt{2}\left(3\left(bc^3\right)^{\frac{3}{4}}Bb + \left(bc^3\right)^{\frac{3}{4}}Ac\right)\arctan\left(-\cdots\right.}{8\,b^2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(B*b*x^(3/2) - A*c*x^(3/2))/((c*x^2 + b)*b*c) + 1/8*sqrt(2)*(3*(b*c^3)
^(3/4)*B*b + (b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2
*sqrt(x))/(b/c)^(1/4))/(b^2*c^4) + 1/8*sqrt(2)*(3*(b*c^3)^(3/4)*B*b + (b*c^
3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(
1/4))/(b^2*c^4) - 1/16*sqrt(2)*(3*(b*c^3)^(3/4)*B*b + (b*c^3)^(3/4)*A*c)*lo
g(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c^4) + 1/16*sqrt(2)*(3*
(b*c^3)^(3/4)*B*b + (b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x
 + sqrt(b/c))/(b^2*c^4)
```

3.201 $\int \frac{x^{7/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2} \, dx$

Optimal. Leaf size=261

$$-\frac{(3Ac+bB)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{7/4}c^{5/4}}+\frac{(3Ac+bB)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{7/4}c^{5/4}}-\frac{(3Ac+bB)\tan^{-1}\left(1-\right.}{4\sqrt{2}b^{7/4}c^{5/4}}$$

```
[Out] -((b*B - A*c)*Sqrt[x])/(2*b*c*(b + c*x^2)) - ((b*B + 3*A*c)*ArcTan[1 - (Sqr
t[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(7/4)*c^(5/4)) + ((b*B + 3*A*c
)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(7/4)*c^(5/4)
) - ((b*B + 3*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*
x])/(8*Sqrt[2]*b^(7/4)*c^(5/4)) + ((b*B + 3*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1
/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(7/4)*c^(5/4))
```

Rubi [A] time = 0.200101, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {1584, 457, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(3Ac+bB)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{7/4}c^{5/4}}+\frac{(3Ac+bB)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{7/4}c^{5/4}}-\frac{(3Ac+bB)\tan^{-1}\left(1-\right.}{4\sqrt{2}b^{7/4}c^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -((b*B - A*c)*Sqrt[x])/(2*b*c*(b + c*x^2)) - ((b*B + 3*A*c)*ArcTan[1 - (Sqr
t[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(7/4)*c^(5/4)) + ((b*B + 3*A*c
)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(7/4)*c^(5/4)
) - ((b*B + 3*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*
x])/(8*Sqrt[2]*b^(7/4)*c^(5/4)) + ((b*B + 3*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1
/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(7/4)*c^(5/4))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{7/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx=\int \frac{A+Bx^2}{\sqrt{x}\left(b+cx^2\right)^2}\,dx$$

$$=-\frac{(bB-Ac)\sqrt{x}}{2bc\left(b+cx^2\right)}+\frac{\left(\frac{bB}{2}+\frac{3Ac}{2}\right)\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{2bc}$$

$$=-\frac{(bB-Ac)\sqrt{x}}{2bc\left(b+cx^2\right)}+\frac{\left(\frac{bB}{2}+\frac{3Ac}{2}\right)\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx,x,\sqrt{x}\right)}{bc}$$

$$=-\frac{(bB-Ac)\sqrt{x}}{2bc\left(b+cx^2\right)}+\frac{(bB+3Ac)\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{4b^{3/2}c}+\frac{(bB+3Ac)\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx,x,\right.}{4b^{3/2}c}$$

$$=-\frac{(bB-Ac)\sqrt{x}}{2bc\left(b+cx^2\right)}+\frac{(bB+3Ac)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{8b^{3/2}c^{3/2}}+\frac{(bB+3Ac)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}\right.}{8b^{3/2}c^{3/2}}$$

$$=-\frac{(bB-Ac)\sqrt{x}}{2bc\left(b+cx^2\right)}-\frac{(bB+3Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{7/4}c^{5/4}}+\frac{(bB+3Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{\right.}{8\sqrt{2}b^{7/4}c^{5/4}}$$

$$=-\frac{(bB-Ac)\sqrt{x}}{2bc\left(b+cx^2\right)}-\frac{(bB+3Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{7/4}c^{5/4}}+\frac{(bB+3Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{7/4}c^{5/4}}-\frac{(bB+3A}{}$$

Mathematica [A] time = 0.242958, size = 203, normalized size = 0.78

$$\frac{\frac{(3Ac+bB)\left(8b^{3/4}\sqrt[4]{c}\sqrt{x}-3\sqrt{2}(b+cx^2)\left(\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)-\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)+2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)-2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)\right)\right)}{b^{7/4}\sqrt[4]{c}}-32B\sqrt{x}}{48c\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (-32*B*Sqrt[x] + ((b*B + 3*A*c)*(8*b^(3/4)*c^(1/4)*Sqrt[x] - 3*Sqrt[2]*(b +
 c*x^2)*(2*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] - 2*ArcTan[1 + (Sq
rt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] + Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqr
t[x] + Sqrt[c]*x] - Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]
*x])))/(b^(7/4)*c^(1/4)))/(48*c*(b + c*x^2))
```

Maple [A] time = 0.013, size = 305, normalized size = 1.2

$$\frac{Ac-Bb}{2\,bc\left(cx^2+b\right)}\sqrt{x}+\frac{3\sqrt{2}A}{8\,b^2}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)+\frac{3\sqrt{2}A}{8\,b^2}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}-1\right)+\frac{3\sqrt{2}A}{16\,b^2}\sqrt[4]{\frac{b}{c}}\ln\left(\left(x+\sqrt[4]{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/2*(A*c-B*b)/b/c*x^(1/2)/(c*x^2+b)+3/8/b^2*(b/c)^(1/4)*2^(1/2)*A*arctan(2^
(1/2)/(b/c)^(1/4)*x^(1/2)+1)+3/8/b^2*(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(
b/c)^(1/4)*x^(1/2)-1)+3/16/b^2*(b/c)^(1/4)*2^(1/2)*A*ln((x+(b/c)^(1/4)*x^(1
/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+1/8/b
/c*(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+1/8/b/c*(b/c
)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)+1/16/b/c*(b/c)^(1/4
)*2^(1/2)*B*ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x
^(1/2)*2^(1/2)+(b/c)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.47528, size = 1559, normalized size = 5.97

$$\frac{4\left(bc^2x^2+b^2c\right)\left(-\frac{B^4b^4+12\,AB^3b^3c+54\,A^2B^2b^2c^2+108\,A^3Bbc^3+81\,A^4c^4}{b^7c^5}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{b^4c^2\sqrt{-\frac{B^4b^4+12\,AB^3b^3c+54\,A^2B^2b^2c^2+108\,A^3Bbc^3+81\,A^4c^4}{b^7c^5}}+\left(B^2b^2+\right.}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*(b*c^2*x^2 + b^2*c)*(-(B^4*b^4 + 12*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2
 + 108*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^5))^(1/4)*arctan((sqrt(b^4*c^2*sqrt
(-(B^4*b^4 + 12*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 108*A^3*B*b*c^3 + 81*A^4
*c^4)/(b^7*c^5)) + (B^2*b^2 + 6*A*B*b*c + 9*A^2*c^2)*x)*b^5*c^4*(-(B^4*b^4
+ 12*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 108*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*
c^5))^(3/4) - (B*b^6*c^4 + 3*A*b^5*c^5)*sqrt(x)*(-(B^4*b^4 + 12*A*B^3*b^3*c
 + 54*A^2*B^2*b^2*c^2 + 108*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^5))^(3/4))/(B^
4*b^4 + 12*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 108*A^3*B*b*c^3 + 81*A^4*c^4)
) + (b*c^2*x^2 + b^2*c)*(-(B^4*b^4 + 12*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 +
108*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^5))^(1/4)*log(b^2*c*(-(B^4*b^4 + 12*A*
B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 108*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^5))^(
1/4) + (B*b + 3*A*c)*sqrt(x)) - (b*c^2*x^2 + b^2*c)*(-(B^4*b^4 + 12*A*B^3*b
^3*c + 54*A^2*B^2*b^2*c^2 + 108*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^5))^(1/4)*
log(-b^2*c*(-(B^4*b^4 + 12*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 108*A^3*B*b*c
^3 + 81*A^4*c^4)/(b^7*c^5))^(1/4) + (B*b + 3*A*c)*sqrt(x)) - 4*(B*b - A*c)*
sqrt(x))/(b*c^2*x^2 + b^2*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19109, size = 369, normalized size = 1.41

$$\frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{1}{4}} Bb + 3\left(bc^3\right)^{\frac{1}{4}} Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}} + 2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^2c^2} + \frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{1}{4}} Bb + 3\left(bc^3\right)^{\frac{1}{4}} Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}} - 2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^2c^2} + \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*((b*c^3)^(1/4)*B*b + 3*(b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt(2)*(s
qrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^2) + 1/8*sqrt(2)*((b*c^
3)^(1/4)*B*b + 3*(b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4
) - 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^2) + 1/16*sqrt(2)*((b*c^3)^(1/4)*B*b + 3
*(b*c^3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c
^2) - 1/16*sqrt(2)*((b*c^3)^(1/4)*B*b + 3*(b*c^3)^(1/4)*A*c)*log(-sqrt(2)*s
qrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c^2) - 1/2*(B*b*sqrt(x) - A*c*sqrt
(x))/((c*x^2 + b)*b*c)
```

3.202 $\int \frac{x^{5/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=284

$$\frac{(bB-5Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{cx}\right)}{8\sqrt{2}b^{9/4}c^{3/4}}-\frac{(bB-5Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{cx}\right)}{8\sqrt{2}b^{9/4}c^{3/4}}-\frac{(bB-5Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{b}}\right.}{4\sqrt{2}b^{9/4}c^{3/4}}$$

```
[Out] (b*B - 5*A*c)/(2*b^2*c*Sqrt[x]) - (b*B - A*c)/(2*b*c*Sqrt[x]*(b + c*x^2)) -
 ((b*B - 5*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b
^(9/4)*c^(3/4)) + ((b*B - 5*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/
4)])/(4*Sqrt[2]*b^(9/4)*c^(3/4)) + ((b*B - 5*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(
1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(9/4)*c^(3/4)) - ((b*B - 5*
A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]
*b^(9/4)*c^(3/4))
```

Rubi [A] time = 0.230192, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1584, 457, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{(bB-5Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{cx}\right)}{8\sqrt{2}b^{9/4}c^{3/4}}-\frac{(bB-5Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{cx}\right)}{8\sqrt{2}b^{9/4}c^{3/4}}-\frac{(bB-5Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{b}}\right.}{4\sqrt{2}b^{9/4}c^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (b*B - 5*A*c)/(2*b^2*c*Sqrt[x]) - (b*B - A*c)/(2*b*c*Sqrt[x]*(b + c*x^2)) -
 ((b*B - 5*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b
^(9/4)*c^(3/4)) + ((b*B - 5*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/
4)])/(4*Sqrt[2]*b^(9/4)*c^(3/4)) + ((b*B - 5*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(
1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(9/4)*c^(3/4)) - ((b*B - 5*
A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]
*b^(9/4)*c^(3/4))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^{5/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx = \int \frac{A+Bx^2}{x^{3/2}\left(b+cx^2\right)^2}\,dx$$

$$= -\frac{bB-Ac}{2bc\sqrt{x}\left(b+cx^2\right)} + \frac{\left(-\frac{bB}{2}+\frac{5Ac}{2}\right)\int \frac{1}{x^{3/2}\left(b+cx^2\right)}\,dx}{2bc}$$

$$= \frac{bB-5Ac}{2b^2c\sqrt{x}} - \frac{bB-Ac}{2bc\sqrt{x}\left(b+cx^2\right)} + \frac{(bB-5Ac)\int \frac{\sqrt{x}}{b+cx^2}\,dx}{4b^2}$$

$$= \frac{bB-5Ac}{2b^2c\sqrt{x}} - \frac{bB-Ac}{2bc\sqrt{x}\left(b+cx^2\right)} + \frac{(bB-5Ac)\,\mathrm{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{2b^2}$$

$$= \frac{bB-5Ac}{2b^2c\sqrt{x}} - \frac{bB-Ac}{2bc\sqrt{x}\left(b+cx^2\right)} - \frac{(bB-5Ac)\,\mathrm{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4b^2\sqrt{c}} + \frac{(bB-5Ac)\,\mathrm{Subst}\left(\right.}{4b^2}$$

$$= \frac{bB-5Ac}{2b^2c\sqrt{x}} - \frac{bB-Ac}{2bc\sqrt{x}\left(b+cx^2\right)} + \frac{(bB-5Ac)\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{8b^2c} + \frac{(bB-5Ac)\,\mathrm{Sub}}{}$$

$$= \frac{bB-5Ac}{2b^2c\sqrt{x}} - \frac{bB-Ac}{2bc\sqrt{x}\left(b+cx^2\right)} + \frac{(bB-5Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{9/4}c^{3/4}} - \frac{(bB-5Ac)\log\left(\right.}{8}$$

$$= \frac{bB-5Ac}{2b^2c\sqrt{x}} - \frac{bB-Ac}{2bc\sqrt{x}\left(b+cx^2\right)} - \frac{(bB-5Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{9/4}c^{3/4}} + \frac{(bB-5Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{b}}\right.}{4\sqrt{2}b^{9/4}c^{3/4}}$$

Mathematica [C] time = 0.213537, size = 117, normalized size = 0.41

$$\frac{2x^{3/2}(bB-Ac)\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{cx^2}{b}\right)+3A\left((-b)^{3/4}\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)-(-b)^{3/4}\sqrt[4]{c}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)-\frac{2b}{\sqrt{x}}\right)}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (3*A*((-2*b)/Sqrt[x] + (-b)^(3/4)*c^(1/4)*ArcTan[(c^(1/4)*Sqrt[x])/(-b)^(1/
4)] - (-b)^(3/4)*c^(1/4)*ArcTanh[(c^(1/4)*Sqrt[x])/(-b)^(1/4)]) + 2*(b*B -
A*c)*x^(3/2)*Hypergeometric2F1[3/4, 2, 7/4, -((c*x^2)/b)])/(3*b^3)
```

Maple [A] time = 0.016, size = 323, normalized size = 1.1

$$-2\frac{A}{b^2\sqrt{x}}-\frac{Ac}{2b^2\left(cx^2+b\right)}x^{\frac{3}{2}}+\frac{B}{2b\left(cx^2+b\right)}x^{\frac{3}{2}}-\frac{5\sqrt{2}A}{16b^2}\ln\left(\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}-\frac{5}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x)
```

```
[Out] -2*A/b^2/x^(1/2)-1/2/b^2*x^(3/2)/(c*x^2+b)*A*c+1/2/b*x^(3/2)/(c*x^2+b)*B-5/
16/b^2/(b/c)^(1/4)*2^(1/2)*A*ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))
/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))-5/8/b^2/(b/c)^(1/4)*2^(1/2)*A
*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-5/8/b^2/(b/c)^(1/4)*2^(1/2)*A*arctan
(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)+1/16/b/c/(b/c)^(1/4)*2^(1/2)*B*ln((x-(b/c)^
(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/
2)))+1/8/b/c/(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+1/
8/b/c/(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.56586, size = 2026, normalized size = 7.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*(b^2*c*x^3 + b^3*x)*(-(B^4*b^4 - 20*A*B^3*b^3*c + 150*A^2*B^2*b^2*c^
2 - 500*A^3*B*b*c^3 + 625*A^4*c^4)/(b^9*c^3))^(1/4)*arctan((sqrt((B^6*b^6 -
 30*A*B^5*b^5*c + 375*A^2*B^4*b^4*c^2 - 2500*A^3*B^3*b^3*c^3 + 9375*A^4*B^2
*b^2*c^4 - 18750*A^5*B*b*c^5 + 15625*A^6*c^6)*x - (B^4*b^9*c - 20*A*B^3*b^8
*c^2 + 150*A^2*B^2*b^7*c^3 - 500*A^3*B*b^6*c^4 + 625*A^4*b^5*c^5)*sqrt(-(B^
4*b^4 - 20*A*B^3*b^3*c + 150*A^2*B^2*b^2*c^2 - 500*A^3*B*b*c^3 + 625*A^4*c^
4)/(b^9*c^3)))*b^2*c*(-(B^4*b^4 - 20*A*B^3*b^3*c + 150*A^2*B^2*b^2*c^2 - 50
0*A^3*B*b*c^3 + 625*A^4*c^4)/(b^9*c^3))^(1/4) + (B^3*b^5*c - 15*A*B^2*b^4*c
^2 + 75*A^2*B*b^3*c^3 - 125*A^3*b^2*c^4)*sqrt(x)*(-(B^4*b^4 - 20*A*B^3*b^3*
c + 150*A^2*B^2*b^2*c^2 - 500*A^3*B*b*c^3 + 625*A^4*c^4)/(b^9*c^3))^(1/4))/
(B^4*b^4 - 20*A*B^3*b^3*c + 150*A^2*B^2*b^2*c^2 - 500*A^3*B*b*c^3 + 625*A^4
*c^4)) - (b^2*c*x^3 + b^3*x)*(-(B^4*b^4 - 20*A*B^3*b^3*c + 150*A^2*B^2*b^2*
c^2 - 500*A^3*B*b*c^3 + 625*A^4*c^4)/(b^9*c^3))^(1/4)*log(b^7*c^2*(-(B^4*b^
4 - 20*A*B^3*b^3*c + 150*A^2*B^2*b^2*c^2 - 500*A^3*B*b*c^3 + 625*A^4*c^4)/(
b^9*c^3))^(3/4) - (B^3*b^3 - 15*A*B^2*b^2*c + 75*A^2*B*b*c^2 - 125*A^3*c^3)
*sqrt(x)) + (b^2*c*x^3 + b^3*x)*(-(B^4*b^4 - 20*A*B^3*b^3*c + 150*A^2*B^2*b
^2*c^2 - 500*A^3*B*b*c^3 + 625*A^4*c^4)/(b^9*c^3))^(1/4)*log(-b^7*c^2*(-(B^
4*b^4 - 20*A*B^3*b^3*c + 150*A^2*B^2*b^2*c^2 - 500*A^3*B*b*c^3 + 625*A^4*c^
4)/(b^9*c^3))^(3/4) - (B^3*b^3 - 15*A*B^2*b^2*c + 75*A^2*B*b*c^2 - 125*A^3*
c^3)*sqrt(x)) + 4*((B*b - 5*A*c)*x^2 - 4*A*b)*sqrt(x))/(b^2*c*x^3 + b^3*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27823, size = 375, normalized size = 1.32

$$\frac{Bbx^2 - 5\,Acx^2 - 4\,Ab}{2\left(cx^{\frac{5}{2}} + b\sqrt{x}\right)b^2} + \frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{3}{4}} Bb - 5\left(bc^3\right)^{\frac{3}{4}} Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}} + 2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^3c^3} + \frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{3}{4}} Bb - 5\left(bc^3\right)^{\frac{3}{4}} Ac\right)\mathrm{arcta}}{8\,b^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(B*b*x^2 - 5*A*c*x^2 - 4*A*b)/((c*x^(5/2) + b*sqrt(x))*b^2) + 1/8*sqrt(
2)*((b*c^3)^(3/4)*B*b - 5*(b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*(b
/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^3*c^3) + 1/8*sqrt(2)*((b*c^3)^(3/4)*
B*b - 5*(b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqr
t(x))/(b/c)^(1/4))/(b^3*c^3) - 1/16*sqrt(2)*((b*c^3)^(3/4)*B*b - 5*(b*c^3)^
(3/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c^3) + 1/1
6*sqrt(2)*((b*c^3)^(3/4)*B*b - 5*(b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b
/c)^(1/4) + x + sqrt(b/c))/(b^3*c^3)
```

3.203 $\int \frac{x^{3/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=289

$$\frac{3bB-7Ac}{6b^2cx^{3/2}}-\frac{(3bB-7Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{11/4}\sqrt[4]{c}}+\frac{(3bB-7Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{11/4}\sqrt[4]{c}}-\frac{(3bB-7A}{}$$

```
[Out] (3*b*B - 7*A*c)/(6*b^2*c*x^(3/2)) - (b*B - A*c)/(2*b*c*x^(3/2)*(b + c*x^2))
 - ((3*b*B - 7*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[
2]*b^(11/4)*c^(1/4)) + ((3*b*B - 7*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x]
)/b^(1/4)])/(4*Sqrt[2]*b^(11/4)*c^(1/4)) - ((3*b*B - 7*A*c)*Log[Sqrt[b] - S
qrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(11/4)*c^(1/4)) +
 ((3*b*B - 7*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x
])/(8*Sqrt[2]*b^(11/4)*c^(1/4))
```

Rubi [A] time = 0.225164, antiderivative size = 289, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1584, 457, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{3bB-7Ac}{6b^2cx^{3/2}}-\frac{(3bB-7Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{11/4}\sqrt[4]{c}}+\frac{(3bB-7Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{11/4}\sqrt[4]{c}}-\frac{(3bB-7A}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```


```
[Out] (3*b*B - 7*A*c)/(6*b^2*c*x^(3/2)) - (b*B - A*c)/(2*b*c*x^(3/2)*(b + c*x^2))
 - ((3*b*B - 7*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[
2]*b^(11/4)*c^(1/4)) + ((3*b*B - 7*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x]
)/b^(1/4)])/(4*Sqrt[2]*b^(11/4)*c^(1/4)) - ((3*b*B - 7*A*c)*Log[Sqrt[b] - S
qrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(11/4)*c^(1/4)) +
 ((3*b*B - 7*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x
])/(8*Sqrt[2]*b^(11/4)*c^(1/4))
```


Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{3/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx = \int \frac{A+Bx^2}{x^{5/2}\left(b+cx^2\right)^2}\,dx$$

$$= -\frac{bB-Ac}{2bcx^{3/2}\left(b+cx^2\right)} + \frac{\left(-\frac{3bB}{2}+\frac{7Ac}{2}\right)\int \frac{1}{x^{5/2}\left(b+cx^2\right)}\,dx}{2bc}$$

$$= \frac{3bB-7Ac}{6b^2cx^{3/2}} - \frac{bB-Ac}{2bcx^{3/2}\left(b+cx^2\right)} + \frac{(3bB-7Ac)\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{4b^2}$$

$$= \frac{3bB-7Ac}{6b^2cx^{3/2}} - \frac{bB-Ac}{2bcx^{3/2}\left(b+cx^2\right)} + \frac{(3bB-7Ac)\,\mathrm{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{2b^2}$$

$$= \frac{3bB-7Ac}{6b^2cx^{3/2}} - \frac{bB-Ac}{2bcx^{3/2}\left(b+cx^2\right)} + \frac{(3bB-7Ac)\,\mathrm{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4b^{5/2}} + \frac{(3bB-7Ac)\,\mathrm{Subst}}{4}$$

$$= \frac{3bB-7Ac}{6b^2cx^{3/2}} - \frac{bB-Ac}{2bcx^{3/2}\left(b+cx^2\right)} + \frac{(3bB-7Ac)\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{8b^{5/2}\sqrt{c}} + \frac{(3bB-7Ac)\,\mathrm{S}}{}$$

$$= \frac{3bB-7Ac}{6b^2cx^{3/2}} - \frac{bB-Ac}{2bcx^{3/2}\left(b+cx^2\right)} - \frac{(3bB-7Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{11/4}\sqrt[4]{c}} + \frac{(3bB-7Ac)\log}{8}$$

$$= \frac{3bB-7Ac}{6b^2cx^{3/2}} - \frac{bB-Ac}{2bcx^{3/2}\left(b+cx^2\right)} - \frac{(3bB-7Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{11/4}\sqrt[4]{c}} + \frac{(3bB-7Ac)\tan^{-1}\left(1+\frac{\sqrt{2}}{}\right.}{4\sqrt{2}b^{11/4}\sqrt[4]{c}}$$

Mathematica [A] time = 0.423149, size = 355, normalized size = 1.23

$$\frac{-\frac{24Ab^{3/4}c\sqrt{x}}{b+cx^2} - \frac{32Ab^{3/4}}{x^{3/2}} + \frac{6\sqrt{2}(7Ac-3bB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt[4]{c}} + \frac{6\sqrt{2}(3bB-7Ac)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt[4]{c}} + 21\sqrt{2}Ac^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\right.}{48}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-32*A*b^(3/4))/x^(3/2) + (24*b^(7/4)*B*Sqrt[x])/(b + c*x^2) - (24*A*b^(3/
4)*c*Sqrt[x])/(b + c*x^2) + (6*Sqrt[2]*(-3*b*B + 7*A*c)*ArcTan[1 - (Sqrt[2]
```

```
*c^(1/4)*Sqrt[x])/b^(1/4)])/c^(1/4) + (6*Sqrt[2]*(3*b*B - 7*A*c)*ArcTan[1 +
 (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/c^(1/4) - (9*Sqrt[2]*b*B*Log[Sqrt[b] -
 Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/c^(1/4) + 21*Sqrt[2]*A*c^(3/
4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] + (9*Sqrt[2]*
b*B*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/c^(1/4) - 2
1*Sqrt[2]*A*c^(3/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]
*x])/(48*b^(11/4))
```

Maple [A] time = 0.015, size = 317, normalized size = 1.1

$$-\frac{2A}{3b^2}x^{-\frac{3}{2}} - \frac{Ac}{2b^2\left(cx^2+b\right)}\sqrt{x} + \frac{B}{2b\left(cx^2+b\right)}\sqrt{x} - \frac{7\sqrt{2}Ac}{8b^3}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}-1\right) - \frac{7\sqrt{2}Ac}{16b^3}\sqrt[4]{\frac{b}{c}}\ln\left(\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x)
```

```
[Out] -2/3*A/b^2/x^(3/2)-1/2/b^2*x^(1/2)/(c*x^2+b)*A*c+1/2/b*x^(1/2)/(c*x^2+b)*B-
7/8/b^3*(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)*c-7/16/
b^3*(b/c)^(1/4)*2^(1/2)*A*ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x
-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))*c-7/8/b^3*(b/c)^(1/4)*2^(1/2)*A*
arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)*c+3/8/b^2*(b/c)^(1/4)*2^(1/2)*B*arcta
n(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)+3/16/b^2*(b/c)^(1/4)*2^(1/2)*B*ln((x+(b/c)
^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1
/2)))+3/8/b^2*(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.4927, size = 1700, normalized size = 5.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/24*(12*(b^2*c*x^4 + b^3*x^2)*(-(81*B^4*b^4 - 756*A*B^3*b^3*c + 2646*A^2*
B^2*b^2*c^2 - 4116*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^11*c))^(1/4)*arctan((sqrt
(b^6*sqrt(-(81*B^4*b^4 - 756*A*B^3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 4116*A^3*
B*b*c^3 + 2401*A^4*c^4)/(b^11*c)) + (9*B^2*b^2 - 42*A*B*b*c + 49*A^2*c^2)*x
)*b^8*c*(-(81*B^4*b^4 - 756*A*B^3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 4116*A^3*B
*b*c^3 + 2401*A^4*c^4)/(b^11*c))^(3/4) + (3*B*b^9*c - 7*A*b^8*c^2)*sqrt(x)*
(-(81*B^4*b^4 - 756*A*B^3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 4116*A^3*B*b*c^3 +
 2401*A^4*c^4)/(b^11*c))^(3/4))/(81*B^4*b^4 - 756*A*B^3*b^3*c + 2646*A^2*B^
2*b^2*c^2 - 4116*A^3*B*b*c^3 + 2401*A^4*c^4)) + 3*(b^2*c*x^4 + b^3*x^2)*(-(
81*B^4*b^4 - 756*A*B^3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 4116*A^3*B*b*c^3 + 24
01*A^4*c^4)/(b^11*c))^(1/4)*log(b^3*(-(81*B^4*b^4 - 756*A*B^3*b^3*c + 2646*
A^2*B^2*b^2*c^2 - 4116*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^11*c))^(1/4) - (3*B*b
 - 7*A*c)*sqrt(x)) - 3*(b^2*c*x^4 + b^3*x^2)*(-(81*B^4*b^4 - 756*A*B^3*b^3*
c + 2646*A^2*B^2*b^2*c^2 - 4116*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^11*c))^(1/4)
*log(-b^3*(-(81*B^4*b^4 - 756*A*B^3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 4116*A^3
*B*b*c^3 + 2401*A^4*c^4)/(b^11*c))^(1/4) - (3*B*b - 7*A*c)*sqrt(x)) - 4*((3
*B*b - 7*A*c)*x^2 - 4*A*b)*sqrt(x))/(b^2*c*x^4 + b^3*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26993, size = 382, normalized size = 1.32

$$\frac{\sqrt{2}\left(3\left(bc^{3}\right)^{\frac{1}{4}}Bb-7\left(bc^{3}\right)^{\frac{1}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^{3}c}+\frac{\sqrt{2}\left(3\left(bc^{3}\right)^{\frac{1}{4}}Bb-7\left(bc^{3}\right)^{\frac{1}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^{3}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(3*(b*c^3)^(1/4)*B*b - 7*(b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt(2)*
(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^3*c) + 1/8*sqrt(2)*(3*(b*
c^3)^(1/4)*B*b - 7*(b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1
/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^3*c) + 1/16*sqrt(2)*(3*(b*c^3)^(1/4)*B*b -
 7*(b*c^3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3
*c) - 1/16*sqrt(2)*(3*(b*c^3)^(1/4)*B*b - 7*(b*c^3)^(1/4)*A*c)*log(-sqrt(2)
*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c) + 1/2*(B*b*sqrt(x) - A*c*sqrt
(x))/((c*x^2 + b)*b^2) - 2/3*A/(b^2*x^(3/2))
```

3.204 $\int \frac{\sqrt{x}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=310

$$\frac{5bB-9Ac}{10b^2cx^{5/2}}-\frac{5bB-9Ac}{2b^3\sqrt{x}}-\frac{\sqrt[4]{c}(5bB-9Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{13/4}}+\frac{\sqrt[4]{c}(5bB-9Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\right.}{8\sqrt{2}b^{13/4}}$$

```
[Out] (5*b*B - 9*A*c)/(10*b^2*c*x^(5/2)) - (5*b*B - 9*A*c)/(2*b^3*Sqrt[x]) - (b*B
 - A*c)/(2*b*c*x^(5/2)*(b + c*x^2)) + (c^(1/4)*(5*b*B - 9*A*c)*ArcTan[1 - (
Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(13/4)) - (c^(1/4)*(5*b*B -
 9*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(13/4))
 - (c^(1/4)*(5*b*B - 9*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] +
 Sqrt[c]*x])/(8*Sqrt[2]*b^(13/4)) + (c^(1/4)*(5*b*B - 9*A*c)*Log[Sqrt[b] +
Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(13/4))
```

Rubi [A] time = 0.271474, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1584, 457, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{5bB-9Ac}{10b^2cx^{5/2}}-\frac{5bB-9Ac}{2b^3\sqrt{x}}-\frac{\sqrt[4]{c}(5bB-9Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{13/4}}+\frac{\sqrt[4]{c}(5bB-9Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\right.}{8\sqrt{2}b^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```


```
[Out] (5*b*B - 9*A*c)/(10*b^2*c*x^(5/2)) - (5*b*B - 9*A*c)/(2*b^3*Sqrt[x]) - (b*B
 - A*c)/(2*b*c*x^(5/2)*(b + c*x^2)) + (c^(1/4)*(5*b*B - 9*A*c)*ArcTan[1 - (
Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(13/4)) - (c^(1/4)*(5*b*B -
 9*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(13/4))
 - (c^(1/4)*(5*b*B - 9*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] +
 Sqrt[c]*x])/(8*Sqrt[2]*b^(13/4)) + (c^(1/4)*(5*b*B - 9*A*c)*Log[Sqrt[b] +
Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(13/4))
```


Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{x}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx = \int \frac{A+Bx^2}{x^{7/2}\left(b+cx^2\right)^2}\,dx$$
$$= -\frac{bB-Ac}{2bcx^{5/2}\left(b+cx^2\right)} + \frac{\left(-\frac{5bB}{2}+\frac{9Ac}{2}\right)\int \frac{1}{x^{7/2}\left(b+cx^2\right)}\,dx}{2bc}$$
$$= \frac{5bB-9Ac}{10b^2cx^{5/2}} - \frac{bB-Ac}{2bcx^{5/2}\left(b+cx^2\right)} + \frac{(5bB-9Ac)\int \frac{1}{x^{3/2}\left(b+cx^2\right)}\,dx}{4b^2}$$
$$= \frac{5bB-9Ac}{10b^2cx^{5/2}} - \frac{5bB-9Ac}{2b^3\sqrt{x}} - \frac{bB-Ac}{2bcx^{5/2}\left(b+cx^2\right)} - \frac{(c(5bB-9Ac))\int \frac{\sqrt{x}}{b+cx^2}\,dx}{4b^3}$$
$$= \frac{5bB-9Ac}{10b^2cx^{5/2}} - \frac{5bB-9Ac}{2b^3\sqrt{x}} - \frac{bB-Ac}{2bcx^{5/2}\left(b+cx^2\right)} - \frac{(c(5bB-9Ac))\,\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{2b^3}$$
$$= \frac{5bB-9Ac}{10b^2cx^{5/2}} - \frac{5bB-9Ac}{2b^3\sqrt{x}} - \frac{bB-Ac}{2bcx^{5/2}\left(b+cx^2\right)} + \frac{\left(\sqrt{c}(5bB-9Ac)\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{4b^3} -$$
$$= \frac{5bB-9Ac}{10b^2cx^{5/2}} - \frac{5bB-9Ac}{2b^3\sqrt{x}} - \frac{bB-Ac}{2bcx^{5/2}\left(b+cx^2\right)} - \frac{(5bB-9Ac)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{8b^3} -$$
$$= \frac{5bB-9Ac}{10b^2cx^{5/2}} - \frac{5bB-9Ac}{2b^3\sqrt{x}} - \frac{bB-Ac}{2bcx^{5/2}\left(b+cx^2\right)} - \frac{\sqrt[4]{c}(5bB-9Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{13/4}} +$$
$$= \frac{5bB-9Ac}{10b^2cx^{5/2}} - \frac{5bB-9Ac}{2b^3\sqrt{x}} - \frac{bB-Ac}{2bcx^{5/2}\left(b+cx^2\right)} + \frac{\sqrt[4]{c}(5bB-9Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{13/4}} - \frac{\sqrt[4]{c}(5bB-}{}$$

Mathematica [C] time = 0.44617, size = 151, normalized size = 0.49

$$\frac{2cx^{3/2}(Ac-bB)\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{cx^2}{b}\right)}{3b^4} + \frac{4Ac-2bB}{b^3\sqrt{x}} - \frac{2A}{5b^2x^{5/2}} + \frac{\sqrt[4]{c}(bB-2Ac)\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)}{(-b)^{13/4}} + \frac{b\sqrt[4]{c}(bB-2Ac)\tanh^{-1}\left(\ldots\right)}{(-b)^{17/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (-2*A)/(5*b^2*x^(5/2)) + (-2*b*B + 4*A*c)/(b^3*Sqrt[x]) + (c^(1/4)*(b*B - 2
*A*c)*ArcTan[(c^(1/4)*Sqrt[x])/(-b)^(1/4)])/(-b)^(13/4) + (b*c^(1/4)*(b*B -
 2*A*c)*ArcTanh[(c^(1/4)*Sqrt[x])/(-b)^(1/4)])/(-b)^(17/4) + (2*c*(-(b*B) +
 A*c)*x^(3/2)*Hypergeometric2F1[3/4, 2, 7/4, -((c*x^2)/b)])/(3*b^4)
```

Maple [A] time = 0.017, size = 339, normalized size = 1.1

$$-\frac{2A}{5b^2}x^{-\frac{5}{2}}+4\frac{Ac}{b^3\sqrt{x}}-2\frac{B}{b^2\sqrt{x}}+\frac{Ac^2}{2b^3\left(cx^2+b\right)}x^{\frac{3}{2}}-\frac{Bc}{2b^2\left(cx^2+b\right)}x^{\frac{3}{2}}+\frac{9c\sqrt{2}A}{16b^3}\ln\left(\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^2,x)
```

```
[Out] -2/5*A/b^2/x^(5/2)+4/b^3/x^(1/2)*A*c-2/b^2/x^(1/2)*B+1/2/b^3*c^2*x^(3/2)/(c
*x^2+b)*A-1/2/b^2*c*x^(3/2)/(c*x^2+b)*B+9/16/b^3*c/(b/c)^(1/4)*2^(1/2)*A*ln
((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)
+(b/c)^(1/2)))+9/8/b^3*c/(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x
^(1/2)+1)+9/8/b^3*c/(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2
)-1)-5/16/b^2/(b/c)^(1/4)*2^(1/2)*B*ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)
^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))-5/8/b^2/(b/c)^(1/4)*2^
(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-5/8/b^2/(b/c)^(1/4)*2^(1/2)*B
*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.65527, size = 2310, normalized size = 7.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/40*(20*(b^3*c*x^5 + b^4*x^3)*(-(625*B^4*b^4*c - 4500*A*B^3*b^3*c^2 + 121
50*A^2*B^2*b^2*c^3 - 14580*A^3*B*b*c^4 + 6561*A^4*c^5)/b^13)^(1/4)*arctan((
sqrt((15625*B^6*b^6*c^2 - 168750*A*B^5*b^5*c^3 + 759375*A^2*B^4*b^4*c^4 - 1
822500*A^3*B^3*b^3*c^5 + 2460375*A^4*B^2*b^2*c^6 - 1771470*A^5*B*b*c^7 + 53
1441*A^6*c^8)*x - (625*B^4*b^11*c - 4500*A*B^3*b^10*c^2 + 12150*A^2*B^2*b^9
*c^3 - 14580*A^3*B*b^8*c^4 + 6561*A^4*b^7*c^5)*sqrt(-(625*B^4*b^4*c - 4500*
A*B^3*b^3*c^2 + 12150*A^2*B^2*b^2*c^3 - 14580*A^3*B*b*c^4 + 6561*A^4*c^5)/b
^13))*b^3*(-(625*B^4*b^4*c - 4500*A*B^3*b^3*c^2 + 12150*A^2*B^2*b^2*c^3 - 1
4580*A^3*B*b*c^4 + 6561*A^4*c^5)/b^13)^(1/4) + (125*B^3*b^6*c - 675*A*B^2*b
^5*c^2 + 1215*A^2*B*b^4*c^3 - 729*A^3*b^3*c^4)*sqrt(x)*(-(625*B^4*b^4*c - 4
500*A*B^3*b^3*c^2 + 12150*A^2*B^2*b^2*c^3 - 14580*A^3*B*b*c^4 + 6561*A^4*c^
5)/b^13)^(1/4))/(625*B^4*b^4*c - 4500*A*B^3*b^3*c^2 + 12150*A^2*B^2*b^2*c^3
 - 14580*A^3*B*b*c^4 + 6561*A^4*c^5)) - 5*(b^3*c*x^5 + b^4*x^3)*(-(625*B^4*
b^4*c - 4500*A*B^3*b^3*c^2 + 12150*A^2*B^2*b^2*c^3 - 14580*A^3*B*b*c^4 + 65
61*A^4*c^5)/b^13)^(1/4)*log(b^10*(-(625*B^4*b^4*c - 4500*A*B^3*b^3*c^2 + 12
150*A^2*B^2*b^2*c^3 - 14580*A^3*B*b*c^4 + 6561*A^4*c^5)/b^13)^(3/4) - (125*
B^3*b^3*c - 675*A*B^2*b^2*c^2 + 1215*A^2*B*b*c^3 - 729*A^3*c^4)*sqrt(x)) +
5*(b^3*c*x^5 + b^4*x^3)*(-(625*B^4*b^4*c - 4500*A*B^3*b^3*c^2 + 12150*A^2*B
^2*b^2*c^3 - 14580*A^3*B*b*c^4 + 6561*A^4*c^5)/b^13)^(1/4)*log(-b^10*(-(625
*B^4*b^4*c - 4500*A*B^3*b^3*c^2 + 12150*A^2*B^2*b^2*c^3 - 14580*A^3*B*b*c^4
 + 6561*A^4*c^5)/b^13)^(3/4) - (125*B^3*b^3*c - 675*A*B^2*b^2*c^2 + 1215*A^
2*B*b*c^3 - 729*A^3*c^4)*sqrt(x)) + 4*(5*(5*B*b*c - 9*A*c^2)*x^4 + 4*A*b^2
+ 4*(5*B*b^2 - 9*A*b*c)*x^2)*sqrt(x))/(b^3*c*x^5 + b^4*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*x**(1/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.35634, size = 409, normalized size = 1.32

$$-\frac{Bbcx^{\frac{3}{2}} - Ac^2x^{\frac{3}{2}}}{2\left(cx^2+b\right)b^3} - \frac{\sqrt{2}\left(5\left(bc^3\right)^{\frac{3}{4}}Bb - 9\left(bc^3\right)^{\frac{3}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^4c^2} - \frac{\sqrt{2}\left(5\left(bc^3\right)^{\frac{3}{4}}Bb - 9\left(bc^3\right)^{\frac{3}{4}}Ac\right)\arctan}{8\,b^4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(B*b*c*x^(3/2) - A*c^2*x^(3/2))/((c*x^2 + b)*b^3) - 1/8*sqrt(2)*(5*(b*
c^3)^(3/4)*B*b - 9*(b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/
4) + 2*sqrt(x))/(b/c)^(1/4))/(b^4*c^2) - 1/8*sqrt(2)*(5*(b*c^3)^(3/4)*B*b -
 9*(b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))
/(b/c)^(1/4))/(b^4*c^2) + 1/16*sqrt(2)*(5*(b*c^3)^(3/4)*B*b - 9*(b*c^3)^(3/
4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^4*c^2) - 1/16*s
qrt(2)*(5*(b*c^3)^(3/4)*B*b - 9*(b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/
c)^(1/4) + x + sqrt(b/c))/(b^4*c^2) - 2/5*(5*B*b*x^2 - 10*A*c*x^2 + A*b)/(b
^3*x^(5/2))
```

3.205 $\int \frac{A+Bx^2}{\sqrt{x}\left(bx^2+cx^4\right)^2} dx$

Optimal. Leaf size=310

$$\frac{c^{3/4}(7bB-11Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{15/4}}-\frac{c^{3/4}(7bB-11Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{15/4}}+\frac{c^{3/4}(7bB-11A}{4}$$

```
[Out] (7*b*B - 11*A*c)/(14*b^2*c*x^(7/2)) - (7*b*B - 11*A*c)/(6*b^3*x^(3/2)) - (b
*B - A*c)/(2*b*c*x^(7/2)*(b + c*x^2)) + (c^(3/4)*(7*b*B - 11*A*c)*ArcTan[1
- (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(15/4)) - (c^(3/4)*(7*b*
B - 11*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(15
/4)) + (c^(3/4)*(7*b*B - 11*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt
[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(15/4)) - (c^(3/4)*(7*b*B - 11*A*c)*Log[Sqrt
[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(15/4))
```

Rubi [A] time = 0.255063, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1584, 457, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{c^{3/4}(7bB-11Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{15/4}}-\frac{c^{3/4}(7bB-11Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{15/4}}+\frac{c^{3/4}(7bB-11A}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(Sqrt[x]*(b*x^2 + c*x^4)^2),x]
```


```
[Out] (7*b*B - 11*A*c)/(14*b^2*c*x^(7/2)) - (7*b*B - 11*A*c)/(6*b^3*x^(3/2)) - (b
*B - A*c)/(2*b*c*x^(7/2)*(b + c*x^2)) + (c^(3/4)*(7*b*B - 11*A*c)*ArcTan[1
- (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(15/4)) - (c^(3/4)*(7*b*
B - 11*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(15
/4)) + (c^(3/4)*(7*b*B - 11*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt
[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(15/4)) - (c^(3/4)*(7*b*B - 11*A*c)*Log[Sqrt
[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(15/4))
```


Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+Bx^2}{\sqrt{x}\left(bx^2+cx^4\right)^2}\,dx = \int \frac{A+Bx^2}{x^{9/2}\left(b+cx^2\right)^2}\,dx$$

$$= -\frac{bB-Ac}{2bcx^{7/2}\left(b+cx^2\right)} + \frac{\left(-\frac{7bB}{2}+\frac{11Ac}{2}\right)\int \frac{1}{x^{9/2}\left(b+cx^2\right)}\,dx}{2bc}$$

$$= \frac{7bB-11Ac}{14b^2cx^{7/2}} - \frac{bB-Ac}{2bcx^{7/2}\left(b+cx^2\right)} + \frac{(7bB-11Ac)\int \frac{1}{x^{5/2}\left(b+cx^2\right)}\,dx}{4b^2}$$

$$= \frac{7bB-11Ac}{14b^2cx^{7/2}} - \frac{7bB-11Ac}{6b^3x^{3/2}} - \frac{bB-Ac}{2bcx^{7/2}\left(b+cx^2\right)} - \frac{(c(7bB-11Ac))\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{4b^3}$$

$$= \frac{7bB-11Ac}{14b^2cx^{7/2}} - \frac{7bB-11Ac}{6b^3x^{3/2}} - \frac{bB-Ac}{2bcx^{7/2}\left(b+cx^2\right)} - \frac{(c(7bB-11Ac))\,\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{2b^3}$$

$$= \frac{7bB-11Ac}{14b^2cx^{7/2}} - \frac{7bB-11Ac}{6b^3x^{3/2}} - \frac{bB-Ac}{2bcx^{7/2}\left(b+cx^2\right)} - \frac{(c(7bB-11Ac))\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4b^{7/2}}$$

$$= \frac{7bB-11Ac}{14b^2cx^{7/2}} - \frac{7bB-11Ac}{6b^3x^{3/2}} - \frac{bB-Ac}{2bcx^{7/2}\left(b+cx^2\right)} - \frac{\left(\sqrt{c}(7bB-11Ac)\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x\right.}{8b^{7/2}}$$

$$= \frac{7bB-11Ac}{14b^2cx^{7/2}} - \frac{7bB-11Ac}{6b^3x^{3/2}} - \frac{bB-Ac}{2bcx^{7/2}\left(b+cx^2\right)} + \frac{c^{3/4}(7bB-11Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\right.}{8\sqrt{2}b^{15/4}}$$

$$= \frac{7bB-11Ac}{14b^2cx^{7/2}} - \frac{7bB-11Ac}{6b^3x^{3/2}} - \frac{bB-Ac}{2bcx^{7/2}\left(b+cx^2\right)} + \frac{c^{3/4}(7bB-11Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{15/4}} - \frac{c^{3/4}(7}{}$$

Mathematica [A] time = 0.492923, size = 385, normalized size = 1.24

$$\frac{\frac{168Ab^{3/4}c^2\sqrt{x}}{b+cx^2}+\frac{448Ab^{3/4}c}{x^{3/2}}-\frac{96Ab^{7/4}}{x^{7/2}}+42\sqrt{2}c^{3/4}(7bB-11Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)+42\sqrt{2}c^{3/4}(11Ac-7bB)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(Sqrt[x]*(b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-96*A*b^(7/4))/x^(7/2) - (224*b^(7/4)*B)/x^(3/2) + (448*A*b^(3/4)*c)/x^(3
/2) - (168*b^(7/4)*B*c*Sqrt[x])/(b + c*x^2) + (168*A*b^(3/4)*c^2*Sqrt[x])/(
b + c*x^2) + 42*Sqrt[2]*c^(3/4)*(7*b*B - 11*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4
)*Sqrt[x])/b^(1/4)] + 42*Sqrt[2]*c^(3/4)*(-7*b*B + 11*A*c)*ArcTan[1 + (Sqrt
[2]*c^(1/4)*Sqrt[x])/b^(1/4)] + 147*Sqrt[2]*b*B*c^(3/4)*Log[Sqrt[b] - Sqrt[
2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] - 231*Sqrt[2]*A*c^(7/4)*Log[Sqrt[b]
 - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] - 147*Sqrt[2]*b*B*c^(3/4)*L
og[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] + 231*Sqrt[2]*A*c
^(7/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(336*b^(
15/4))
```

Maple [A] time = 0.019, size = 348, normalized size = 1.1

$$-\frac{2A}{7b^2}x^{-\frac{7}{2}}+\frac{4Ac}{3b^3}x^{-\frac{3}{2}}-\frac{2B}{3b^2}x^{-\frac{3}{2}}+\frac{Ac^2}{2b^3\left(cx^2+b\right)}\sqrt{x}-\frac{Bc}{2b^2\left(cx^2+b\right)}\sqrt{x}+\frac{11c^2\sqrt{2}A}{8b^4}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)+\frac{11c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(c*x^4+b*x^2)^2/x^(1/2),x)
```

```
[Out] -2/7*A/b^2/x^(7/2)+4/3/b^3/x^(3/2)*A*c-2/3/b^2/x^(3/2)*B+1/2/b^3*c^2*x^(1/2
)/(c*x^2+b)*A-1/2/b^2*c*x^(1/2)/(c*x^2+b)*B+11/8/b^4*c^2*(b/c)^(1/4)*2^(1/2
)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+11/8/b^4*c^2*(b/c)^(1/4)*2^(1/2)*
A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)+11/16/b^4*c^2*(b/c)^(1/4)*2^(1/2)*A
*ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1
/2)+(b/c)^(1/2)))-7/8/b^3*c*(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4
)*x^(1/2)+1)-7/8/b^3*c*(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(
1/2)-1)-7/16/b^3*c*(b/c)^(1/4)*2^(1/2)*B*ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+
(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.45904, size = 1898, normalized size = 6.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/168*(84*(b^3*c*x^6 + b^4*x^4)*(-(2401*B^4*b^4*c^3 - 15092*A*B^3*b^3*c^4 +
 35574*A^2*B^2*b^2*c^5 - 37268*A^3*B*b*c^6 + 14641*A^4*c^7)/b^15)^(1/4)*arc
tan((sqrt(b^8*sqrt(-(2401*B^4*b^4*c^3 - 15092*A*B^3*b^3*c^4 + 35574*A^2*B^2
*b^2*c^5 - 37268*A^3*B*b*c^6 + 14641*A^4*c^7)/b^15) + (49*B^2*b^2*c^2 - 154
*A*B*b*c^3 + 121*A^2*c^4)*x)*b^11*(-(2401*B^4*b^4*c^3 - 15092*A*B^3*b^3*c^4
 + 35574*A^2*B^2*b^2*c^5 - 37268*A^3*B*b*c^6 + 14641*A^4*c^7)/b^15)^(3/4) +
 (7*B*b^12*c - 11*A*b^11*c^2)*sqrt(x)*(-(2401*B^4*b^4*c^3 - 15092*A*B^3*b^3
*c^4 + 35574*A^2*B^2*b^2*c^5 - 37268*A^3*B*b*c^6 + 14641*A^4*c^7)/b^15)^(3/
4))/(2401*B^4*b^4*c^3 - 15092*A*B^3*b^3*c^4 + 35574*A^2*B^2*b^2*c^5 - 37268
*A^3*B*b*c^6 + 14641*A^4*c^7)) + 21*(b^3*c*x^6 + b^4*x^4)*(-(2401*B^4*b^4*c
^3 - 15092*A*B^3*b^3*c^4 + 35574*A^2*B^2*b^2*c^5 - 37268*A^3*B*b*c^6 + 1464
1*A^4*c^7)/b^15)^(1/4)*log(b^4*(-(2401*B^4*b^4*c^3 - 15092*A*B^3*b^3*c^4 +
35574*A^2*B^2*b^2*c^5 - 37268*A^3*B*b*c^6 + 14641*A^4*c^7)/b^15)^(1/4) - (7
*B*b*c - 11*A*c^2)*sqrt(x)) - 21*(b^3*c*x^6 + b^4*x^4)*(-(2401*B^4*b^4*c^3
- 15092*A*B^3*b^3*c^4 + 35574*A^2*B^2*b^2*c^5 - 37268*A^3*B*b*c^6 + 14641*A
^4*c^7)/b^15)^(1/4)*log(-b^4*(-(2401*B^4*b^4*c^3 - 15092*A*B^3*b^3*c^4 + 35
574*A^2*B^2*b^2*c^5 - 37268*A^3*B*b*c^6 + 14641*A^4*c^7)/b^15)^(1/4) - (7*B
*b*c - 11*A*c^2)*sqrt(x)) - 4*(7*(7*B*b*c - 11*A*c^2)*x^4 + 12*A*b^2 + 4*(7
*B*b^2 - 11*A*b*c)*x^2)*sqrt(x))/(b^3*c*x^6 + b^4*x^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2)**2/x**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.34121, size = 394, normalized size = 1.27

$$-\frac{\sqrt{2}\left(7\left(bc^3\right)^{\frac{1}{4}}Bb-11\left(bc^3\right)^{\frac{1}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^4}-\frac{\sqrt{2}\left(7\left(bc^3\right)^{\frac{1}{4}}Bb-11\left(bc^3\right)^{\frac{1}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*(7*(b*c^3)^(1/4)*B*b - 11*(b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt(2
)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/b^4 - 1/8*sqrt(2)*(7*(b*c^
3)^(1/4)*B*b - 11*(b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/
4) - 2*sqrt(x))/(b/c)^(1/4))/b^4 - 1/16*sqrt(2)*(7*(b*c^3)^(1/4)*B*b - 11*(
b*c^3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^4 + 1/
16*sqrt(2)*(7*(b*c^3)^(1/4)*B*b - 11*(b*c^3)^(1/4)*A*c)*log(-sqrt(2)*sqrt(x
)*(b/c)^(1/4) + x + sqrt(b/c))/b^4 - 1/2*(B*b*c*sqrt(x) - A*c^2*sqrt(x))/((
c*x^2 + b)*b^3) - 2/21*(7*B*b*x^2 - 14*A*c*x^2 + 3*A*b)/(b^3*x^(7/2))
```

3.206 $\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=332

$$\frac{c^{5/4}(9bB-13Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{cx}\right)}{8\sqrt{2}b^{17/4}}-\frac{c^{5/4}(9bB-13Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{cx}\right)}{8\sqrt{2}b^{17/4}}-\frac{c^{5/4}(9bB-13Ac}{4\sqrt{}}$$

```
[Out] (9*b*B - 13*A*c)/(18*b^2*c*x^(9/2)) - (9*b*B - 13*A*c)/(10*b^3*x^(5/2)) + (
c*(9*b*B - 13*A*c))/(2*b^4*Sqrt[x]) - (b*B - A*c)/(2*b*c*x^(9/2)*(b + c*x^2
)) - (c^(5/4)*(9*b*B - 13*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)
])/(4*Sqrt[2]*b^(17/4)) + (c^(5/4)*(9*b*B - 13*A*c)*ArcTan[1 + (Sqrt[2]*c^(
1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(17/4)) + (c^(5/4)*(9*b*B - 13*A*c)*Lo
g[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(17/
4)) - (c^(5/4)*(9*b*B - 13*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[
x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(17/4))
```

Rubi [A] time = 0.28566, antiderivative size = 332, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1584, 457, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{c^{5/4}(9bB-13Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{cx}\right)}{8\sqrt{2}b^{17/4}}-\frac{c^{5/4}(9bB-13Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{cx}\right)}{8\sqrt{2}b^{17/4}}-\frac{c^{5/4}(9bB-13Ac}{4\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(3/2)*(b*x^2 + c*x^4)^2),x]
```

```
[Out] (9*b*B - 13*A*c)/(18*b^2*c*x^(9/2)) - (9*b*B - 13*A*c)/(10*b^3*x^(5/2)) + (
c*(9*b*B - 13*A*c))/(2*b^4*Sqrt[x]) - (b*B - A*c)/(2*b*c*x^(9/2)*(b + c*x^2
)) - (c^(5/4)*(9*b*B - 13*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)
])/(4*Sqrt[2]*b^(17/4)) + (c^(5/4)*(9*b*B - 13*A*c)*ArcTan[1 + (Sqrt[2]*c^(
1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(17/4)) + (c^(5/4)*(9*b*B - 13*A*c)*Lo
g[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(17/
4)) - (c^(5/4)*(9*b*B - 13*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[
x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(17/4))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)^2}\,dx = \int \frac{A+Bx^2}{x^{11/2}\left(b+cx^2\right)^2}\,dx$$

$$= -\frac{bB-Ac}{2bcx^{9/2}\left(b+cx^2\right)} + \frac{\left(-\frac{9bB}{2}+\frac{13Ac}{2}\right)\int \frac{1}{x^{11/2}\left(b+cx^2\right)}\,dx}{2bc}$$

$$= \frac{9bB-13Ac}{18b^2cx^{9/2}} - \frac{bB-Ac}{2bcx^{9/2}\left(b+cx^2\right)} + \frac{(9bB-13Ac)\int \frac{1}{x^{7/2}\left(b+cx^2\right)}\,dx}{4b^2}$$

$$= \frac{9bB-13Ac}{18b^2cx^{9/2}} - \frac{9bB-13Ac}{10b^3x^{5/2}} - \frac{bB-Ac}{2bcx^{9/2}\left(b+cx^2\right)} - \frac{(c(9bB-13Ac))\int \frac{1}{x^{3/2}\left(b+cx^2\right)}\,dx}{4b^3}$$

$$= \frac{9bB-13Ac}{18b^2cx^{9/2}} - \frac{9bB-13Ac}{10b^3x^{5/2}} + \frac{c(9bB-13Ac)}{2b^4\sqrt{x}} - \frac{bB-Ac}{2bcx^{9/2}\left(b+cx^2\right)} + \frac{\left(c^2(9bB-13Ac)\right)\int \frac{\sqrt{x}}{b+cx^2}}{4b^4}$$

$$= \frac{9bB-13Ac}{18b^2cx^{9/2}} - \frac{9bB-13Ac}{10b^3x^{5/2}} + \frac{c(9bB-13Ac)}{2b^4\sqrt{x}} - \frac{bB-Ac}{2bcx^{9/2}\left(b+cx^2\right)} + \frac{\left(c^2(9bB-13Ac)\right)\text{Subst}\left(\right.}{2b^4}$$

$$= \frac{9bB-13Ac}{18b^2cx^{9/2}} - \frac{9bB-13Ac}{10b^3x^{5/2}} + \frac{c(9bB-13Ac)}{2b^4\sqrt{x}} - \frac{bB-Ac}{2bcx^{9/2}\left(b+cx^2\right)} - \frac{\left(c^{3/2}(9bB-13Ac)\right)\text{Subst}}{4b^4}$$

$$= \frac{9bB-13Ac}{18b^2cx^{9/2}} - \frac{9bB-13Ac}{10b^3x^{5/2}} + \frac{c(9bB-13Ac)}{2b^4\sqrt{x}} - \frac{bB-Ac}{2bcx^{9/2}\left(b+cx^2\right)} + \frac{(c(9bB-13Ac))\text{Subst}\left(\int\right.}{8b}$$

$$= \frac{9bB-13Ac}{18b^2cx^{9/2}} - \frac{9bB-13Ac}{10b^3x^{5/2}} + \frac{c(9bB-13Ac)}{2b^4\sqrt{x}} - \frac{bB-Ac}{2bcx^{9/2}\left(b+cx^2\right)} + \frac{c^{5/4}(9bB-13Ac)\log\left(\sqrt{b}\right.}{8\sqrt{2}b}$$

$$= \frac{9bB-13Ac}{18b^2cx^{9/2}} - \frac{9bB-13Ac}{10b^3x^{5/2}} + \frac{c(9bB-13Ac)}{2b^4\sqrt{x}} - \frac{bB-Ac}{2bcx^{9/2}\left(b+cx^2\right)} - \frac{c^{5/4}(9bB-13Ac)\tan^{-1}\left(1\right.}{4\sqrt{2}b^{17/4}}$$

Mathematica [C] time = 0.4141, size = 176, normalized size = 0.53

$$\frac{2c^2x^{3/2}(bB-Ac)\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{cx^2}{b}\right)}{3b^5} - \frac{2(bB-2Ac)}{5b^3x^{5/2}} + \frac{2c(2bB-3Ac)}{b^4\sqrt{x}} - \frac{2A}{9b^2x^{9/2}} + \frac{c^{5/4}(2bB-3Ac)\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)}{(-b)^{17/4}} + \frac{c^{5/4}(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(3/2)*(b*x^2 + c*x^4)^2),x]
```

```
[Out] (-2*A)/(9*b^2*x^(9/2)) - (2*(b*B - 2*A*c))/(5*b^3*x^(5/2)) + (2*c*(2*b*B -
3*A*c))/(b^4*Sqrt[x]) + (c^(5/4)*(2*b*B - 3*A*c)*ArcTan[(c^(1/4)*Sqrt[x])/(
-b)^(1/4)])/(-b)^(17/4) + (c^(5/4)*(-2*b*B + 3*A*c)*ArcTanh[(c^(1/4)*Sqrt[x
])/(-b)^(1/4)])/(-b)^(17/4) + (2*c^2*(b*B - A*c)*x^(3/2)*Hypergeometric2F1[
3/4, 2, 7/4, -((c*x^2)/b)])/(3*b^5)
```

Maple [A] time = 0.018, size = 372, normalized size = 1.1

$$-\frac{2\,A}{9\,b^{2}}x^{-\frac{9}{2}}+\frac{4\,Ac}{5\,b^{3}}x^{-\frac{5}{2}}-\frac{2\,B}{5\,b^{2}}x^{-\frac{5}{2}}-6\,\frac{Ac^{2}}{b^{4}\sqrt{x}}+4\,\frac{Bc}{b^{3}\sqrt{x}}-\frac{Ac^{3}}{2\,b^{4}\left(cx^{2}+b\right)}x^{\frac{3}{2}}+\frac{c^{2}B}{2\,b^{3}\left(cx^{2}+b\right)}x^{\frac{3}{2}}-\frac{13\,c^{2}\sqrt{2}A}{16\,b^{4}}\ln\left(\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^2,x)
```

```
[Out] -2/9*A/b^2/x^(9/2)+4/5/b^3/x^(5/2)*A*c-2/5/b^2/x^(5/2)*B-6*c^2/b^4/x^(1/2)*
A+4*c/b^3/x^(1/2)*B-1/2/b^4*c^3*x^(3/2)/(c*x^2+b)*A+1/2/b^3*c^2*x^(3/2)/(c*
x^2+b)*B-13/16/b^4*c^2/(b/c)^(1/4)*2^(1/2)*A*ln((x-(b/c)^(1/4)*x^(1/2)*2^(1
/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))-13/8/b^4*c^2/
(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-13/8/b^4*c^2/(b
/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)+9/16/b^3*c/(b/c)^
(1/4)*2^(1/2)*B*ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/
4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+9/8/b^3*c/(b/c)^(1/4)*2^(1/2)*B*arctan(2^(
1/2)/(b/c)^(1/4)*x^(1/2)+1)+9/8/b^3*c/(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/
(b/c)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.63345, size = 2475, normalized size = 7.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/360*(180*(b^4*c*x^7 + b^5*x^5)*(-(6561*B^4*b^4*c^5 - 37908*A*B^3*b^3*c^6
+ 82134*A^2*B^2*b^2*c^7 - 79092*A^3*B*b*c^8 + 28561*A^4*c^9)/b^17)^(1/4)*ar
ctan((sqrt((531441*B^6*b^6*c^8 - 4605822*A*B^5*b^5*c^9 + 16632135*A^2*B^4*b
^4*c^10 - 32032260*A^3*B^3*b^3*c^11 + 34701615*A^4*B^2*b^2*c^12 - 20049822*
A^5*B*b*c^13 + 4826809*A^6*c^14)*x - (6561*B^4*b^13*c^5 - 37908*A*B^3*b^12*
c^6 + 82134*A^2*B^2*b^11*c^7 - 79092*A^3*B*b^10*c^8 + 28561*A^4*b^9*c^9)*sq
rt(-(6561*B^4*b^4*c^5 - 37908*A*B^3*b^3*c^6 + 82134*A^2*B^2*b^2*c^7 - 79092
*A^3*B*b*c^8 + 28561*A^4*c^9)/b^17))*b^4*(-(6561*B^4*b^4*c^5 - 37908*A*B^3*
b^3*c^6 + 82134*A^2*B^2*b^2*c^7 - 79092*A^3*B*b*c^8 + 28561*A^4*c^9)/b^17)^
(1/4) + (729*B^3*b^7*c^4 - 3159*A*B^2*b^6*c^5 + 4563*A^2*B*b^5*c^6 - 2197*A
^3*b^4*c^7)*sqrt(x)*(-(6561*B^4*b^4*c^5 - 37908*A*B^3*b^3*c^6 + 82134*A^2*B
^2*b^2*c^7 - 79092*A^3*B*b*c^8 + 28561*A^4*c^9)/b^17)^(1/4))/(6561*B^4*b^4*
c^5 - 37908*A*B^3*b^3*c^6 + 82134*A^2*B^2*b^2*c^7 - 79092*A^3*B*b*c^8 + 285
61*A^4*c^9)) - 45*(b^4*c*x^7 + b^5*x^5)*(-(6561*B^4*b^4*c^5 - 37908*A*B^3*b
^3*c^6 + 82134*A^2*B^2*b^2*c^7 - 79092*A^3*B*b*c^8 + 28561*A^4*c^9)/b^17)^(
1/4)*log(b^13*(-(6561*B^4*b^4*c^5 - 37908*A*B^3*b^3*c^6 + 82134*A^2*B^2*b^2
*c^7 - 79092*A^3*B*b*c^8 + 28561*A^4*c^9)/b^17)^(3/4) - (729*B^3*b^3*c^4 -
3159*A*B^2*b^2*c^5 + 4563*A^2*B*b*c^6 - 2197*A^3*c^7)*sqrt(x)) + 45*(b^4*c*
x^7 + b^5*x^5)*(-(6561*B^4*b^4*c^5 - 37908*A*B^3*b^3*c^6 + 82134*A^2*B^2*b^
2*c^7 - 79092*A^3*B*b*c^8 + 28561*A^4*c^9)/b^17)^(1/4)*log(-b^13*(-(6561*B^
4*b^4*c^5 - 37908*A*B^3*b^3*c^6 + 82134*A^2*B^2*b^2*c^7 - 79092*A^3*B*b*c^8
 + 28561*A^4*c^9)/b^17)^(3/4) - (729*B^3*b^3*c^4 - 3159*A*B^2*b^2*c^5 + 456
3*A^2*B*b*c^6 - 2197*A^3*c^7)*sqrt(x)) + 4*(45*(9*B*b*c^2 - 13*A*c^3)*x^6 +
 36*(9*B*b^2*c - 13*A*b*c^2)*x^4 - 20*A*b^3 - 4*(9*B*b^3 - 13*A*b^2*c)*x^2)
*sqrt(x))/(b^4*c*x^7 + b^5*x^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(3/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.37606, size = 443, normalized size = 1.33

$$\frac{\sqrt{2}\left(9\left(bc^3\right)^{\frac{3}{4}}Bb-13\left(bc^3\right)^{\frac{3}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^5c}+\frac{\sqrt{2}\left(9\left(bc^3\right)^{\frac{3}{4}}Bb-13\left(bc^3\right)^{\frac{3}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.}{8\,b^5c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(9*(b*c^3)^(3/4)*B*b - 13*(b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2)
*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^5*c) + 1/8*sqrt(2)*(9*(b
*c^3)^(3/4)*B*b - 13*(b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^
(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^5*c) - 1/16*sqrt(2)*(9*(b*c^3)^(3/4)*B*b
 - 13*(b*c^3)^(3/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(
b^5*c) + 1/16*sqrt(2)*(9*(b*c^3)^(3/4)*B*b - 13*(b*c^3)^(3/4)*A*c)*log(-sqr
t(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^5*c) + 1/2*(B*b*c^2*x^(3/2) -
A*c^3*x^(3/2))/((c*x^2 + b)*b^4) + 2/45*(90*B*b*c*x^4 - 135*A*c^2*x^4 - 9*B
*b^2*x^2 + 18*A*b*c*x^2 - 5*A*b^2)/(b^4*x^(9/2))
```

3.207 $\int \frac{x^{23/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=343

$$-\frac{x^{9/2}(13bB-5Ac)}{16bc^2(b+cx^2)}+\frac{9x^{5/2}(13bB-5Ac)}{80bc^3}-\frac{9\sqrt{x}(13bB-5Ac)}{16c^4}-\frac{9\sqrt[4]{b}(13bB-5Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}c^{17/4}}+\cdots$$

```
[Out] (-9*(13*b*B - 5*A*c)*Sqrt[x])/(16*c^4) + (9*(13*b*B - 5*A*c)*x^(5/2))/(80*b
*c^3) - ((b*B - A*c)*x^(13/2))/(4*b*c*(b + c*x^2)^2) - ((13*b*B - 5*A*c)*x^
(9/2))/(16*b*c^2*(b + c*x^2)) - (9*b^(1/4)*(13*b*B - 5*A*c)*ArcTan[1 - (Sqr
t[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*c^(17/4)) + (9*b^(1/4)*(13*b*B
- 5*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*c^(17/4
)) - (9*b^(1/4)*(13*b*B - 5*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt
[x] + Sqrt[c]*x])/(64*Sqrt[2]*c^(17/4)) + (9*b^(1/4)*(13*b*B - 5*A*c)*Log[S
qrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*c^(17/4)
)
```

Rubi [A] time = 0.278437, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {1584, 457, 288, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{x^{9/2}(13bB-5Ac)}{16bc^2(b+cx^2)}+\frac{9x^{5/2}(13bB-5Ac)}{80bc^3}-\frac{9\sqrt{x}(13bB-5Ac)}{16c^4}-\frac{9\sqrt[4]{b}(13bB-5Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}c^{17/4}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(x^(23/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```


```
[Out] (-9*(13*b*B - 5*A*c)*Sqrt[x])/(16*c^4) + (9*(13*b*B - 5*A*c)*x^(5/2))/(80*b
*c^3) - ((b*B - A*c)*x^(13/2))/(4*b*c*(b + c*x^2)^2) - ((13*b*B - 5*A*c)*x^
(9/2))/(16*b*c^2*(b + c*x^2)) - (9*b^(1/4)*(13*b*B - 5*A*c)*ArcTan[1 - (Sqr
t[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*c^(17/4)) + (9*b^(1/4)*(13*b*B
- 5*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*c^(17/4
)) - (9*b^(1/4)*(13*b*B - 5*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt
[x] + Sqrt[c]*x])/(64*Sqrt[2]*c^(17/4)) + (9*b^(1/4)*(13*b*B - 5*A*c)*Log[S
qrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*c^(17/4)
)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{23/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{x^{11/2}\left(A+Bx^2\right)}{\left(b+cx^2\right)^3}\,dx$$

$$= -\frac{(bB-Ac)x^{13/2}}{4bc\left(b+cx^2\right)^2} + \frac{\left(\frac{13bB}{2}-\frac{5Ac}{2}\right)\int \frac{x^{11/2}}{\left(b+cx^2\right)^2}\,dx}{4bc}$$

$$= -\frac{(bB-Ac)x^{13/2}}{4bc\left(b+cx^2\right)^2} - \frac{(13bB-5Ac)x^{9/2}}{16bc^2\left(b+cx^2\right)} + \frac{(9(13bB-5Ac))\int \frac{x^{7/2}}{b+cx^2}\,dx}{32bc^2}$$

$$= \frac{9(13bB-5Ac)x^{5/2}}{80bc^3} - \frac{(bB-Ac)x^{13/2}}{4bc\left(b+cx^2\right)^2} - \frac{(13bB-5Ac)x^{9/2}}{16bc^2\left(b+cx^2\right)} - \frac{(9(13bB-5Ac))\int \frac{x^{3/2}}{b+cx^2}\,dx}{32c^3}$$

$$= -\frac{9(13bB-5Ac)\sqrt{x}}{16c^4} + \frac{9(13bB-5Ac)x^{5/2}}{80bc^3} - \frac{(bB-Ac)x^{13/2}}{4bc\left(b+cx^2\right)^2} - \frac{(13bB-5Ac)x^{9/2}}{16bc^2\left(b+cx^2\right)} + \frac{(9b(13bB-5\ldots}{\ldots}$$

$$= -\frac{9(13bB-5Ac)\sqrt{x}}{16c^4} + \frac{9(13bB-5Ac)x^{5/2}}{80bc^3} - \frac{(bB-Ac)x^{13/2}}{4bc\left(b+cx^2\right)^2} - \frac{(13bB-5Ac)x^{9/2}}{16bc^2\left(b+cx^2\right)} + \frac{(9b(13bB-5\ldots}{\ldots}$$

$$= -\frac{9(13bB-5Ac)\sqrt{x}}{16c^4} + \frac{9(13bB-5Ac)x^{5/2}}{80bc^3} - \frac{(bB-Ac)x^{13/2}}{4bc\left(b+cx^2\right)^2} - \frac{(13bB-5Ac)x^{9/2}}{16bc^2\left(b+cx^2\right)} + \frac{\left(9\sqrt{b}(13bB-\ldots\right.}{\ldots}$$

$$= -\frac{9(13bB-5Ac)\sqrt{x}}{16c^4} + \frac{9(13bB-5Ac)x^{5/2}}{80bc^3} - \frac{(bB-Ac)x^{13/2}}{4bc\left(b+cx^2\right)^2} - \frac{(13bB-5Ac)x^{9/2}}{16bc^2\left(b+cx^2\right)} + \frac{\left(9\sqrt{b}(13bB-\ldots\right.}{\ldots}$$

$$= -\frac{9(13bB-5Ac)\sqrt{x}}{16c^4} + \frac{9(13bB-5Ac)x^{5/2}}{80bc^3} - \frac{(bB-Ac)x^{13/2}}{4bc\left(b+cx^2\right)^2} - \frac{(13bB-5Ac)x^{9/2}}{16bc^2\left(b+cx^2\right)} - \frac{9\sqrt[4]{b}(13bB-5\ldots}{\ldots}$$

$$= -\frac{9(13bB-5Ac)\sqrt{x}}{16c^4} + \frac{9(13bB-5Ac)x^{5/2}}{80bc^3} - \frac{(bB-Ac)x^{13/2}}{4bc\left(b+cx^2\right)^2} - \frac{(13bB-5Ac)x^{9/2}}{16bc^2\left(b+cx^2\right)} - \frac{9\sqrt[4]{b}(13bB-5\ldots}{\ldots}$$

Mathematica [A] time = 0.47518, size = 435, normalized size = 1.27

$$-\frac{160Ab^2c^{5/4}\sqrt{x}}{\left(b+cx^2\right)^2} - 90\sqrt{2}\sqrt[4]{b}(13bB-5Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right) + 90\sqrt{2}\sqrt[4]{b}(13bB-5Ac)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right) + \frac{680Abc^{5/4}\sqrt{x}}{b+cx^2} + 22\ldots$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(23/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (-3840*b*B*c^(1/4)*Sqrt[x] + 1280*A*c^(5/4)*Sqrt[x] + 256*B*c^(5/4)*x^(5/2)
 + (160*b^3*B*c^(1/4)*Sqrt[x])/(b + c*x^2)^2 - (160*A*b^2*c^(5/4)*Sqrt[x])/
(b + c*x^2)^2 - (1000*b^2*B*c^(1/4)*Sqrt[x])/(b + c*x^2) + (680*A*b*c^(5/4)
*Sqrt[x])/(b + c*x^2) - 90*Sqrt[2]*b^(1/4)*(13*b*B - 5*A*c)*ArcTan[1 - (Sqr
t[2]*c^(1/4)*Sqrt[x])/b^(1/4)] + 90*Sqrt[2]*b^(1/4)*(13*b*B - 5*A*c)*ArcTan
[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] - 585*Sqrt[2]*b^(5/4)*B*Log[Sqrt[b]
 - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] + 225*Sqrt[2]*A*b^(1/4)*c*L
og[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] + 585*Sqrt[2]*b^(
5/4)*B*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] - 225*Sqr
t[2]*A*b^(1/4)*c*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]
)/(640*c^(17/4))
```

Maple [A] time = 0.019, size = 381, normalized size = 1.1

$$\frac{2B}{5c^3}x^{\frac{5}{2}} + 2\frac{A\sqrt{x}}{c^3} - 6\frac{Bb\sqrt{x}}{c^4} + \frac{17\,Ab}{16\,c^2\left(cx^2+b\right)^2}x^{\frac{5}{2}} - \frac{25\,Bb^2}{16\,c^3\left(cx^2+b\right)^2}x^{\frac{5}{2}} + \frac{13\,Ab^2}{16\,c^3\left(cx^2+b\right)^2}\sqrt{x} - \frac{21\,Bb^3}{16\,c^4\left(cx^2+b\right)^2}\sqrt{x} - \frac{45}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(23/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 2/5/c^3*B*x^(5/2)+2/c^3*A*x^(1/2)-6/c^4*B*b*x^(1/2)+17/16*b/c^2/(c*x^2+b)^2
*x^(5/2)*A-25/16*b^2/c^3/(c*x^2+b)^2*x^(5/2)*B+13/16*b^2/c^3/(c*x^2+b)^2*A*
x^(1/2)-21/16*b^3/c^4/(c*x^2+b)^2*B*x^(1/2)-45/64/c^3*(b/c)^(1/4)*2^(1/2)*A
*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-45/64/c^3*(b/c)^(1/4)*2^(1/2)*A*arct
an(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-45/128/c^3*(b/c)^(1/4)*2^(1/2)*A*ln((x+(b
/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)
^(1/2)))+117/64*b/c^4*(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1
/2)+1)+117/64*b/c^4*(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2
)-1)+117/128*b/c^4*(b/c)^(1/4)*2^(1/2)*B*ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+
(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(23/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.57405, size = 1955, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(23/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/320*(180*(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4)*(-(28561*B^4*b^5 - 43940*A*B^
3*b^4*c + 25350*A^2*B^2*b^3*c^2 - 6500*A^3*B*b^2*c^3 + 625*A^4*b*c^4)/c^17)
^(1/4)*arctan((sqrt(c^8*sqrt(-(28561*B^4*b^5 - 43940*A*B^3*b^4*c + 25350*A^
2*B^2*b^3*c^2 - 6500*A^3*B*b^2*c^3 + 625*A^4*b*c^4)/c^17) + (169*B^2*b^2 -
130*A*B*b*c + 25*A^2*c^2)*x)*c^13*(-(28561*B^4*b^5 - 43940*A*B^3*b^4*c + 25
350*A^2*B^2*b^3*c^2 - 6500*A^3*B*b^2*c^3 + 625*A^4*b*c^4)/c^17)^(3/4) + (13
*B*b*c^13 - 5*A*c^14)*sqrt(x)*(-(28561*B^4*b^5 - 43940*A*B^3*b^4*c + 25350*
A^2*B^2*b^3*c^2 - 6500*A^3*B*b^2*c^3 + 625*A^4*b*c^4)/c^17)^(3/4))/(28561*B
^4*b^5 - 43940*A*B^3*b^4*c + 25350*A^2*B^2*b^3*c^2 - 6500*A^3*B*b^2*c^3 + 6
25*A^4*b*c^4)) + 45*(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4)*(-(28561*B^4*b^5 - 43
940*A*B^3*b^4*c + 25350*A^2*B^2*b^3*c^2 - 6500*A^3*B*b^2*c^3 + 625*A^4*b*c^
4)/c^17)^(1/4)*log(9*c^4*(-(28561*B^4*b^5 - 43940*A*B^3*b^4*c + 25350*A^2*B
^2*b^3*c^2 - 6500*A^3*B*b^2*c^3 + 625*A^4*b*c^4)/c^17)^(1/4) - 9*(13*B*b -
5*A*c)*sqrt(x)) - 45*(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4)*(-(28561*B^4*b^5 - 4
3940*A*B^3*b^4*c + 25350*A^2*B^2*b^3*c^2 - 6500*A^3*B*b^2*c^3 + 625*A^4*b*c
^4)/c^17)^(1/4)*log(-9*c^4*(-(28561*B^4*b^5 - 43940*A*B^3*b^4*c + 25350*A^2
*B^2*b^3*c^2 - 6500*A^3*B*b^2*c^3 + 625*A^4*b*c^4)/c^17)^(1/4) - 9*(13*B*b
- 5*A*c)*sqrt(x)) - 4*(32*B*c^3*x^6 - 32*(13*B*b*c^2 - 5*A*c^3)*x^4 - 585*B
*b^3 + 225*A*b^2*c - 81*(13*B*b^2*c - 5*A*b*c^2)*x^2)*sqrt(x))/(c^6*x^4 + 2
*b*c^5*x^2 + b^2*c^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(23/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31172, size = 433, normalized size = 1.26

$$\frac{9\sqrt{2}\left(13\left(bc^{3}\right)^{\frac{1}{4}}Bb-5\left(bc^{3}\right)^{\frac{1}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,c^{5}}+\frac{9\sqrt{2}\left(13\left(bc^{3}\right)^{\frac{1}{4}}Bb-5\left(bc^{3}\right)^{\frac{1}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-\right.}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.}{64\,c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(23/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 9/64*sqrt(2)*(13*(b*c^3)^(1/4)*B*b - 5*(b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt(2
)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/c^5 + 9/64*sqrt(2)*(13*(b*
c^3)^(1/4)*B*b - 5*(b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1
/4) - 2*sqrt(x))/(b/c)^(1/4))/c^5 + 9/128*sqrt(2)*(13*(b*c^3)^(1/4)*B*b - 5
*(b*c^3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^5 -
9/128*sqrt(2)*(13*(b*c^3)^(1/4)*B*b - 5*(b*c^3)^(1/4)*A*c)*log(-sqrt(2)*sqr
t(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^5 - 1/16*(25*B*b^2*c*x^(5/2) - 17*A*b*c
^2*x^(5/2) + 21*B*b^3*sqrt(x) - 13*A*b^2*c*sqrt(x))/((c*x^2 + b)^2*c^4) + 2
/5*(B*c^12*x^(5/2) - 15*B*b*c^11*sqrt(x) + 5*A*c^12*sqrt(x))/c^15
```

3.208 $\int \frac{x^{21/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=322

$$-\frac{x^{7/2}(11bB-3Ac)}{16bc^2\left(b+cx^2\right)}+\frac{7x^{3/2}(11bB-3Ac)}{48bc^3}-\frac{7(11bB-3Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}\sqrt[4]{b}c^{15/4}}+\frac{7(11bB-3Ac)\log\left(\sqrt{2}\sqrt[4]{b}\right.}{64\sqrt{2}\sqrt[4]{b}c^{1}}$$

```
[Out] (7*(11*b*B - 3*A*c)*x^(3/2))/(48*b*c^3) - ((b*B - A*c)*x^(11/2))/(4*b*c*(b
+ c*x^2)^2) - ((11*b*B - 3*A*c)*x^(7/2))/(16*b*c^2*(b + c*x^2)) + (7*(11*b*
B - 3*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(1/
4)*c^(15/4)) - (7*(11*b*B - 3*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(
1/4)])/(32*Sqrt[2]*b^(1/4)*c^(15/4)) - (7*(11*b*B - 3*A*c)*Log[Sqrt[b] - Sq
rt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(1/4)*c^(15/4)) +
 (7*(11*b*B - 3*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c
]*x])/(64*Sqrt[2]*b^(1/4)*c^(15/4))
```

Rubi [A] time = 0.246662, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {1584, 457, 288, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{x^{7/2}(11bB-3Ac)}{16bc^2\left(b+cx^2\right)}+\frac{7x^{3/2}(11bB-3Ac)}{48bc^3}-\frac{7(11bB-3Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}\sqrt[4]{b}c^{15/4}}+\frac{7(11bB-3Ac)\log\left(\sqrt{2}\sqrt[4]{b}\right.}{64\sqrt{2}\sqrt[4]{b}c^{1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(21/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```


```
[Out] (7*(11*b*B - 3*A*c)*x^(3/2))/(48*b*c^3) - ((b*B - A*c)*x^(11/2))/(4*b*c*(b
+ c*x^2)^2) - ((11*b*B - 3*A*c)*x^(7/2))/(16*b*c^2*(b + c*x^2)) + (7*(11*b*
B - 3*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(1/
4)*c^(15/4)) - (7*(11*b*B - 3*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(
1/4)])/(32*Sqrt[2]*b^(1/4)*c^(15/4)) - (7*(11*b*B - 3*A*c)*Log[Sqrt[b] - Sq
rt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(1/4)*c^(15/4)) +
 (7*(11*b*B - 3*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c
]*x])/(64*Sqrt[2]*b^(1/4)*c^(15/4))
```


Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^{21/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx=\int \frac{x^{9/2}\left(A+Bx^2\right)}{\left(b+cx^2\right)^3}\,dx$$

$$=-\frac{(bB-Ac)x^{11/2}}{4bc\left(b+cx^2\right)^2}+\frac{\left(\frac{11bB}{2}-\frac{3Ac}{2}\right)\int \frac{x^{9/2}}{\left(b+cx^2\right)^2}\,dx}{4bc}$$

$$=-\frac{(bB-Ac)x^{11/2}}{4bc\left(b+cx^2\right)^2}-\frac{(11bB-3Ac)x^{7/2}}{16bc^2\left(b+cx^2\right)}+\frac{(7(11bB-3Ac))\int \frac{x^{5/2}}{b+cx^2}\,dx}{32bc^2}$$

$$=\frac{7(11bB-3Ac)x^{3/2}}{48bc^3}-\frac{(bB-Ac)x^{11/2}}{4bc\left(b+cx^2\right)^2}-\frac{(11bB-3Ac)x^{7/2}}{16bc^2\left(b+cx^2\right)}-\frac{(7(11bB-3Ac))\int \frac{\sqrt{x}}{b+cx^2}\,dx}{32c^3}$$

$$=\frac{7(11bB-3Ac)x^{3/2}}{48bc^3}-\frac{(bB-Ac)x^{11/2}}{4bc\left(b+cx^2\right)^2}-\frac{(11bB-3Ac)x^{7/2}}{16bc^2\left(b+cx^2\right)}-\frac{(7(11bB-3Ac))\,\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx,\right.}{16c^3}$$

$$=\frac{7(11bB-3Ac)x^{3/2}}{48bc^3}-\frac{(bB-Ac)x^{11/2}}{4bc\left(b+cx^2\right)^2}-\frac{(11bB-3Ac)x^{7/2}}{16bc^2\left(b+cx^2\right)}+\frac{(7(11bB-3Ac))\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\right.}{32c^{7/2}}$$

$$=\frac{7(11bB-3Ac)x^{3/2}}{48bc^3}-\frac{(bB-Ac)x^{11/2}}{4bc\left(b+cx^2\right)^2}-\frac{(11bB-3Ac)x^{7/2}}{16bc^2\left(b+cx^2\right)}-\frac{(7(11bB-3Ac))\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}}\right.}{64c^4}$$

$$=\frac{7(11bB-3Ac)x^{3/2}}{48bc^3}-\frac{(bB-Ac)x^{11/2}}{4bc\left(b+cx^2\right)^2}-\frac{(11bB-3Ac)x^{7/2}}{16bc^2\left(b+cx^2\right)}-\frac{7(11bB-3Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\right.}{64\sqrt{2}\sqrt[4]{b}c^{15/4}}$$

$$=\frac{7(11bB-3Ac)x^{3/2}}{48bc^3}-\frac{(bB-Ac)x^{11/2}}{4bc\left(b+cx^2\right)^2}-\frac{(11bB-3Ac)x^{7/2}}{16bc^2\left(b+cx^2\right)}+\frac{7(11bB-3Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\sqrt[4]{b}c^{15/4}}$$

Mathematica [C] time = 0.316034, size = 176, normalized size = 0.55

$$\frac{\frac{2c^{3/4}x^{3/2}(3bB-2Ac)\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{cx^2}{b}\right)}{b}+\frac{2c^{3/4}x^{3/2}(Ac-bB)\,{}_2F_1\left(\frac{3}{4},3;\frac{7}{4};-\frac{cx^2}{b}\right)}{b}+\frac{(3Ac-9bB)\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)}{\sqrt[4]{-b}}+\frac{(9bB-3Ac)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)}{\sqrt[4]{-b}}+2Bc^{3/4}x^{3/2}}{3c^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(21/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*B*c^(3/4)*x^(3/2) + ((-9*b*B + 3*A*c)*ArcTan[(c^(1/4)*Sqrt[x])/(-b)^(1/4
)])/(-b)^(1/4) + ((9*b*B - 3*A*c)*ArcTanh[(c^(1/4)*Sqrt[x])/(-b)^(1/4)])/(-
b)^(1/4) + (2*c^(3/4)*(3*b*B - 2*A*c)*x^(3/2)*Hypergeometric2F1[3/4, 2, 7/4
, -((c*x^2)/b)])/b + (2*c^(3/4)*(-(b*B) + A*c)*x^(3/2)*Hypergeometric2F1[3/
4, 3, 7/4, -((c*x^2)/b)])/b)/(3*c^(15/4))
```

Maple [A] time = 0.018, size = 357, normalized size = 1.1

$$\frac{2B}{3c^3}x^{\frac{3}{2}}-\frac{11A}{16c\left(cx^2+b\right)^2}x^{\frac{7}{2}}+\frac{19Bb}{16c^2\left(cx^2+b\right)^2}x^{\frac{7}{2}}-\frac{7Ab}{16c^2\left(cx^2+b\right)^2}x^{\frac{3}{2}}+\frac{15Bb^2}{16c^3\left(cx^2+b\right)^2}x^{\frac{3}{2}}+\frac{21\sqrt{2}A}{128c^3}\ln\left(\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(21/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 2/3*B*x^(3/2)/c^3-11/16/c/(c*x^2+b)^2*A*x^(7/2)+19/16/c^2/(c*x^2+b)^2*B*x^(
7/2)*b-7/16/c^2/(c*x^2+b)^2*x^(3/2)*A*b+15/16/c^3/(c*x^2+b)^2*x^(3/2)*B*b^2
+21/128/c^3/(b/c)^(1/4)*2^(1/2)*A*ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(
1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+21/64/c^3/(b/c)^(1/4)*2^
(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+21/64/c^3/(b/c)^(1/4)*2^(1/2)
*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-77/128/c^4/(b/c)^(1/4)*2^(1/2)*B*b
*ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1
/2)+(b/c)^(1/2)))-77/64/c^4/(b/c)^(1/4)*2^(1/2)*B*b*arctan(2^(1/2)/(b/c)^(1
/4)*x^(1/2)+1)-77/64/c^4/(b/c)^(1/4)*2^(1/2)*B*b*arctan(2^(1/2)/(b/c)^(1/4)
*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(21/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.81858, size = 2367, normalized size = 7.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(21/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/192*(84*(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3)*(-(14641*B^4*b^4 - 15972*A*B^3
*b^3*c + 6534*A^2*B^2*b^2*c^2 - 1188*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^15))^(1
/4)*arctan((sqrt((1771561*B^6*b^6 - 2898918*A*B^5*b^5*c + 1976535*A^2*B^4*b
^4*c^2 - 718740*A^3*B^3*b^3*c^3 + 147015*A^4*B^2*b^2*c^4 - 16038*A^5*B*b*c^
5 + 729*A^6*c^6)*x - (14641*B^4*b^5*c^7 - 15972*A*B^3*b^4*c^8 + 6534*A^2*B^
2*b^3*c^9 - 1188*A^3*B*b^2*c^10 + 81*A^4*b*c^11)*sqrt(-(14641*B^4*b^4 - 159
72*A*B^3*b^3*c + 6534*A^2*B^2*b^2*c^2 - 1188*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c
^15)))*c^4*(-(14641*B^4*b^4 - 15972*A*B^3*b^3*c + 6534*A^2*B^2*b^2*c^2 - 11
88*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^15))^(1/4) + (1331*B^3*b^3*c^4 - 1089*A*B
^2*b^2*c^5 + 297*A^2*B*b*c^6 - 27*A^3*c^7)*sqrt(x)*(-(14641*B^4*b^4 - 15972
*A*B^3*b^3*c + 6534*A^2*B^2*b^2*c^2 - 1188*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^1
5))^(1/4))/(14641*B^4*b^4 - 15972*A*B^3*b^3*c + 6534*A^2*B^2*b^2*c^2 - 1188
*A^3*B*b*c^3 + 81*A^4*c^4)) - 21*(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3)*(-(14641
*B^4*b^4 - 15972*A*B^3*b^3*c + 6534*A^2*B^2*b^2*c^2 - 1188*A^3*B*b*c^3 + 81
*A^4*c^4)/(b*c^15))^(1/4)*log(343*b*c^11*(-(14641*B^4*b^4 - 15972*A*B^3*b^3
*c + 6534*A^2*B^2*b^2*c^2 - 1188*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^15))^(3/4)
- 343*(1331*B^3*b^3 - 1089*A*B^2*b^2*c + 297*A^2*B*b*c^2 - 27*A^3*c^3)*sqrt
(x)) + 21*(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3)*(-(14641*B^4*b^4 - 15972*A*B^3*
b^3*c + 6534*A^2*B^2*b^2*c^2 - 1188*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^15))^(1/
4)*log(-343*b*c^11*(-(14641*B^4*b^4 - 15972*A*B^3*b^3*c + 6534*A^2*B^2*b^2*
c^2 - 1188*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^15))^(3/4) - 343*(1331*B^3*b^3 -
1089*A*B^2*b^2*c + 297*A^2*B*b*c^2 - 27*A^3*c^3)*sqrt(x)) - 4*(32*B*c^2*x^5
 + 11*(11*B*b*c - 3*A*c^2)*x^3 + 7*(11*B*b^2 - 3*A*b*c)*x)*sqrt(x))/(c^5*x^
4 + 2*b*c^4*x^2 + b^2*c^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(21/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18073, size = 410, normalized size = 1.27

$$\frac{2\,Bx^{\frac{3}{2}}}{3\,c^3}+\frac{19\,Bbcx^{\frac{7}{2}}-11\,Ac^2x^{\frac{7}{2}}+15\,Bb^2x^{\frac{3}{2}}-7\,Abcx^{\frac{3}{2}}}{16\left(cx^2+b\right)^2c^3}-\frac{7\sqrt{2}\left(11\left(bc^3\right)^{\frac{3}{4}}Bb-3\left(bc^3\right)^{\frac{3}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,bc^6}-\frac{7}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(21/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 2/3*B*x^(3/2)/c^3 + 1/16*(19*B*b*c*x^(7/2) - 11*A*c^2*x^(7/2) + 15*B*b^2*x^
(3/2) - 7*A*b*c*x^(3/2))/((c*x^2 + b)^2*c^3) - 7/64*sqrt(2)*(11*(b*c^3)^(3/
4)*B*b - 3*(b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*s
qrt(x))/(b/c)^(1/4))/(b*c^6) - 7/64*sqrt(2)*(11*(b*c^3)^(3/4)*B*b - 3*(b*c^
3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(
1/4))/(b*c^6) + 7/128*sqrt(2)*(11*(b*c^3)^(3/4)*B*b - 3*(b*c^3)^(3/4)*A*c)*
log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^6) - 7/128*sqrt(2)*(1
1*(b*c^3)^(3/4)*B*b - 3*(b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4)
 + x + sqrt(b/c))/(b*c^6)
```

3.209 $\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^3} dx$

Optimal. Leaf size=322

$$\frac{5(9bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{3/4}c^{13/4}}-\frac{5(9bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{3/4}c^{13/4}}+\frac{5(9bB-Ac)\tan^{-1}\left(1-\right.}{32\sqrt{2}b^{3/4}c^{13/4}}$$

```
[Out] (5*(9*b*B - A*c)*Sqrt[x])/(16*b*c^3) - ((b*B - A*c)*x^(9/2))/(4*b*c*(b + c*
x^2)^2) - ((9*b*B - A*c)*x^(5/2))/(16*b*c^2*(b + c*x^2)) + (5*(9*b*B - A*c)
*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(3/4)*c^(13/4
)) - (5*(9*b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sq
rt[2]*b^(3/4)*c^(13/4)) + (5*(9*b*B - A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^
(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(3/4)*c^(13/4)) - (5*(9*b*B - A*c
)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b
^(3/4)*c^(13/4))
```

Rubi [A] time = 0.253757, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {1584, 457, 288, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{5(9bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{3/4}c^{13/4}}-\frac{5(9bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{3/4}c^{13/4}}+\frac{5(9bB-Ac)\tan^{-1}\left(1-\right.}{32\sqrt{2}b^{3/4}c^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(19/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```


```
[Out] (5*(9*b*B - A*c)*Sqrt[x])/(16*b*c^3) - ((b*B - A*c)*x^(9/2))/(4*b*c*(b + c*
x^2)^2) - ((9*b*B - A*c)*x^(5/2))/(16*b*c^2*(b + c*x^2)) + (5*(9*b*B - A*c)
*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(3/4)*c^(13/4
)) - (5*(9*b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sq
rt[2]*b^(3/4)*c^(13/4)) + (5*(9*b*B - A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^
(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(3/4)*c^(13/4)) - (5*(9*b*B - A*c
)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b
^(3/4)*c^(13/4))
```


Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{19/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx=\int \frac{x^{7/2}\left(A+Bx^2\right)}{\left(b+cx^2\right)^3}\,dx$$

$$=-\frac{(bB-Ac)x^{9/2}}{4bc\left(b+cx^2\right)^2}+\frac{\left(\frac{9bB}{2}-\frac{Ac}{2}\right)\int \frac{x^{7/2}}{\left(b+cx^2\right)^2}\,dx}{4bc}$$

$$=-\frac{(bB-Ac)x^{9/2}}{4bc\left(b+cx^2\right)^2}-\frac{(9bB-Ac)x^{5/2}}{16bc^2\left(b+cx^2\right)}+\frac{(5(9bB-Ac))\int \frac{x^{3/2}}{b+cx^2}\,dx}{32bc^2}$$

$$=\frac{5(9bB-Ac)\sqrt{x}}{16bc^3}-\frac{(bB-Ac)x^{9/2}}{4bc\left(b+cx^2\right)^2}-\frac{(9bB-Ac)x^{5/2}}{16bc^2\left(b+cx^2\right)}-\frac{(5(9bB-Ac))\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{32c^3}$$

$$=\frac{5(9bB-Ac)\sqrt{x}}{16bc^3}-\frac{(bB-Ac)x^{9/2}}{4bc\left(b+cx^2\right)^2}-\frac{(9bB-Ac)x^{5/2}}{16bc^2\left(b+cx^2\right)}-\frac{(5(9bB-Ac))\,\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx,x,\sqrt{x}\right)}{16c^3}$$

$$=\frac{5(9bB-Ac)\sqrt{x}}{16bc^3}-\frac{(bB-Ac)x^{9/2}}{4bc\left(b+cx^2\right)^2}-\frac{(9bB-Ac)x^{5/2}}{16bc^2\left(b+cx^2\right)}-\frac{(5(9bB-Ac))\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx,x,\sqrt{x}\right.}{32\sqrt{b}c^3}$$

$$=\frac{5(9bB-Ac)\sqrt{x}}{16bc^3}-\frac{(bB-Ac)x^{9/2}}{4bc\left(b+cx^2\right)^2}-\frac{(9bB-Ac)x^{5/2}}{16bc^2\left(b+cx^2\right)}-\frac{(5(9bB-Ac))\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x\right.}{64\sqrt{b}c^{7/2}}$$

$$=\frac{5(9bB-Ac)\sqrt{x}}{16bc^3}-\frac{(bB-Ac)x^{9/2}}{4bc\left(b+cx^2\right)^2}-\frac{(9bB-Ac)x^{5/2}}{16bc^2\left(b+cx^2\right)}+\frac{5(9bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\right.}{64\sqrt{2}b^{3/4}c^{13/4}}$$

$$=\frac{5(9bB-Ac)\sqrt{x}}{16bc^3}-\frac{(bB-Ac)x^{9/2}}{4bc\left(b+cx^2\right)^2}-\frac{(9bB-Ac)x^{5/2}}{16bc^2\left(b+cx^2\right)}+\frac{5(9bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{3/4}c^{13/4}}-\frac{5(9bB}{}$$

Mathematica [A] time = 0.457858, size = 403, normalized size = 1.25

$$\frac{\frac{10\sqrt{2}(9bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{b^{3/4}}-\frac{10\sqrt{2}(9bB-Ac)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{b^{3/4}}-\frac{5\sqrt{2}Ac\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{b^{3/4}}+\frac{5\sqrt{2}Ac\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{b^{3/4}}+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(19/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (256*B*c^(1/4)*Sqrt[x] - (32*b^2*B*c^(1/4)*Sqrt[x])/(b + c*x^2)^2 + (32*A*b
*c^(5/4)*Sqrt[x])/(b + c*x^2)^2 + (136*b*B*c^(1/4)*Sqrt[x])/(b + c*x^2) - (
72*A*c^(5/4)*Sqrt[x])/(b + c*x^2) + (10*Sqrt[2]*(9*b*B - A*c)*ArcTan[1 - (S
qrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/b^(3/4) - (10*Sqrt[2]*(9*b*B - A*c)*ArcTa
n[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/b^(3/4) + 45*Sqrt[2]*b^(1/4)*B*Lo
g[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] - (5*Sqrt[2]*A*c*L
og[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/b^(3/4) - 45*Sqr
t[2]*b^(1/4)*B*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] +
 (5*Sqrt[2]*A*c*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])
/b^(3/4))/(128*c^(13/4))
```

Maple [A] time = 0.016, size = 363, normalized size = 1.1

$$2\,{\frac{B\sqrt{x}}{{c}^{3}}}-{\frac{9\,A}{16\,c \left(c{x}^{2}+b \right) ^{2}}{x}^{{\frac{5}{2}}}}+{\frac{17\,Bb}{16\,{c}^{2} \left(c{x}^{2}+b \right) ^{2}}{x}^{{\frac{5}{2}}}}-{\frac{5\,Ab}{16\,{c}^{2} \left(c{x}^{2}+b \right) ^{2}}\sqrt{x}}+{\frac{13\,B{b}^{2}}{16\,{c}^{3} \left(c{x}^{2}+b \right) ^{2}}\sqrt{x}}+{\frac{5\,\sqrt{2}A}{64\,b{c}^{2}}\sqrt[4]{{\frac{b}{c}}}\arctan \left(\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(19/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 2*B/c^3*x^(1/2)-9/16/c/(c*x^2+b)^2*x^(5/2)*A+17/16/c^2/(c*x^2+b)^2*x^(5/2)*
B*b-5/16/c^2/(c*x^2+b)^2*A*x^(1/2)*b+13/16/c^3/(c*x^2+b)^2*B*x^(1/2)*b^2+5/
64/c^2*(b/c)^(1/4)/b*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)+5/128/
c^2*(b/c)^(1/4)/b*2^(1/2)*A*ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/
(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+5/64/c^2*(b/c)^(1/4)/b*2^(1/2)
*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-45/64/c^3*(b/c)^(1/4)*2^(1/2)*B*ar
ctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-45/128/c^3*(b/c)^(1/4)*2^(1/2)*B*ln((x+
(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/
c)^(1/2)))-45/64/c^3*(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/
2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(19/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.60046, size = 1787, normalized size = 5.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(19/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(20*(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3)*(-(6561*B^4*b^4 - 2916*A*B^3*b^3
*c + 486*A^2*B^2*b^2*c^2 - 36*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^13))^(1/4)*arct
an((sqrt(b^2*c^6*sqrt(-(6561*B^4*b^4 - 2916*A*B^3*b^3*c + 486*A^2*B^2*b^2*c
^2 - 36*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^13)) + (81*B^2*b^2 - 18*A*B*b*c + A^2
*c^2)*x)*b^2*c^10*(-(6561*B^4*b^4 - 2916*A*B^3*b^3*c + 486*A^2*B^2*b^2*c^2
- 36*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^13))^(3/4) + (9*B*b^3*c^10 - A*b^2*c^11)
*sqrt(x)*(-(6561*B^4*b^4 - 2916*A*B^3*b^3*c + 486*A^2*B^2*b^2*c^2 - 36*A^3*
B*b*c^3 + A^4*c^4)/(b^3*c^13))^(3/4))/(6561*B^4*b^4 - 2916*A*B^3*b^3*c + 48
6*A^2*B^2*b^2*c^2 - 36*A^3*B*b*c^3 + A^4*c^4)) + 5*(c^5*x^4 + 2*b*c^4*x^2 +
 b^2*c^3)*(-(6561*B^4*b^4 - 2916*A*B^3*b^3*c + 486*A^2*B^2*b^2*c^2 - 36*A^3
*B*b*c^3 + A^4*c^4)/(b^3*c^13))^(1/4)*log(5*b*c^3*(-(6561*B^4*b^4 - 2916*A*
B^3*b^3*c + 486*A^2*B^2*b^2*c^2 - 36*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^13))^(1/
4) - 5*(9*B*b - A*c)*sqrt(x)) - 5*(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3)*(-(6561
*B^4*b^4 - 2916*A*B^3*b^3*c + 486*A^2*B^2*b^2*c^2 - 36*A^3*B*b*c^3 + A^4*c^
4)/(b^3*c^13))^(1/4)*log(-5*b*c^3*(-(6561*B^4*b^4 - 2916*A*B^3*b^3*c + 486*
A^2*B^2*b^2*c^2 - 36*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^13))^(1/4) - 5*(9*B*b -
A*c)*sqrt(x)) + 4*(32*B*c^2*x^4 + 45*B*b^2 - 5*A*b*c + 9*(9*B*b*c - A*c^2)*
x^2)*sqrt(x))/(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(19/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

[Out] Timed out

Giac [A] time = 1.25843, size = 410, normalized size = 1.27

$$\frac{2B\sqrt{x}}{c^3} - \frac{5\sqrt{2}\left(9\left(bc^3\right)^{\frac{1}{4}}Bb - \left(bc^3\right)^{\frac{1}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,bc^4} - \frac{5\sqrt{2}\left(9\left(bc^3\right)^{\frac{1}{4}}Bb - \left(bc^3\right)^{\frac{1}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right.\right.}{2\left(\right.}\right.}{64\,bc^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(19/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 2*B*sqrt(x)/c^3 - 5/64*sqrt(2)*(9*(b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*ar
ctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b*c^4) - 5
/64*sqrt(2)*(9*(b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(
sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b*c^4) - 5/128*sqrt(2)*(9*(b
*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x +
sqrt(b/c))/(b*c^4) + 5/128*sqrt(2)*(9*(b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c
)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^4) + 1/16*(17*B*b*
c*x^(5/2) - 9*A*c^2*x^(5/2) + 13*B*b^2*sqrt(x) - 5*A*b*c*sqrt(x))/((c*x^2 +
 b)^2*c^3)
```

3.210 $\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=293

$$\frac{3(Ac+7bB)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{5/4}c^{11/4}}-\frac{3(Ac+7bB)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{5/4}c^{11/4}}-\frac{3(Ac+7bB)\tan^{-1}\left(1-\cdots\right.}{32\sqrt{2}b^{5/4}c^{11/4}}$$

```
[Out] -((b*B - A*c)*x^(7/2))/(4*b*c*(b + c*x^2)^2) - ((7*b*B + A*c)*x^(3/2))/(16*
b*c^2*(b + c*x^2)) - (3*(7*b*B + A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/
b^(1/4)])/(32*Sqrt[2]*b^(5/4)*c^(11/4)) + (3*(7*b*B + A*c)*ArcTan[1 + (Sqrt
[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(5/4)*c^(11/4)) + (3*(7*b*B +
A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2
]*b^(5/4)*c^(11/4)) - (3*(7*b*B + A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4
)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(5/4)*c^(11/4))
```

Rubi [A] time = 0.223192, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1584, 457, 288, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{3(Ac+7bB)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{5/4}c^{11/4}}-\frac{3(Ac+7bB)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{5/4}c^{11/4}}-\frac{3(Ac+7bB)\tan^{-1}\left(1-\cdots\right.}{32\sqrt{2}b^{5/4}c^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(17/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```


```
[Out] -((b*B - A*c)*x^(7/2))/(4*b*c*(b + c*x^2)^2) - ((7*b*B + A*c)*x^(3/2))/(16*
b*c^2*(b + c*x^2)) - (3*(7*b*B + A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/
b^(1/4)])/(32*Sqrt[2]*b^(5/4)*c^(11/4)) + (3*(7*b*B + A*c)*ArcTan[1 + (Sqrt
[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(5/4)*c^(11/4)) + (3*(7*b*B +
A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2
]*b^(5/4)*c^(11/4)) - (3*(7*b*B + A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4
)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(5/4)*c^(11/4))
```


Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^{17/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{x^{5/2}\left(A+Bx^2\right)}{\left(b+cx^2\right)^3}\,dx$$

$$= -\frac{(bB-Ac)x^{7/2}}{4bc\left(b+cx^2\right)^2} + \frac{\left(\frac{7bB}{2}+\frac{Ac}{2}\right)\int \frac{x^{5/2}}{\left(b+cx^2\right)^2}\,dx}{4bc}$$

$$= -\frac{(bB-Ac)x^{7/2}}{4bc\left(b+cx^2\right)^2} - \frac{(7bB+Ac)x^{3/2}}{16bc^2\left(b+cx^2\right)} + \frac{(3(7bB+Ac))\int \frac{\sqrt{x}}{b+cx^2}\,dx}{32bc^2}$$

$$= -\frac{(bB-Ac)x^{7/2}}{4bc\left(b+cx^2\right)^2} - \frac{(7bB+Ac)x^{3/2}}{16bc^2\left(b+cx^2\right)} + \frac{(3(7bB+Ac))\,\mathrm{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{16bc^2}$$

$$= -\frac{(bB-Ac)x^{7/2}}{4bc\left(b+cx^2\right)^2} - \frac{(7bB+Ac)x^{3/2}}{16bc^2\left(b+cx^2\right)} - \frac{(3(7bB+Ac))\,\mathrm{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{32bc^{5/2}} + \frac{(3(7bB+A}{}$$

$$= -\frac{(bB-Ac)x^{7/2}}{4bc\left(b+cx^2\right)^2} - \frac{(7bB+Ac)x^{3/2}}{16bc^2\left(b+cx^2\right)} + \frac{(3(7bB+Ac))\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{64bc^3} + \frac{(3(7bB}{}$$

$$= -\frac{(bB-Ac)x^{7/2}}{4bc\left(b+cx^2\right)^2} - \frac{(7bB+Ac)x^{3/2}}{16bc^2\left(b+cx^2\right)} + \frac{3(7bB+Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{5/4}c^{11/4}} - \frac{3(7bB+A}{}$$

$$= -\frac{(bB-Ac)x^{7/2}}{4bc\left(b+cx^2\right)^2} - \frac{(7bB+Ac)x^{3/2}}{16bc^2\left(b+cx^2\right)} - \frac{3(7bB+Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{5/4}c^{11/4}} + \frac{3(7bB+Ac)\tan^{-1}\left(1}{32\sqrt{2}b^{5/4}c^{11}}$$

Mathematica [C] time = 0.224424, size = 137, normalized size = 0.47

$$\frac{2c^{3/4}x^{3/2}(Ac-2bB)\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{cx^2}{b}\right)+2c^{3/4}x^{3/2}(bB-Ac)\,{}_2F_1\left(\frac{3}{4},3;\frac{7}{4};-\frac{cx^2}{b}\right)+3(-b)^{7/4}B\left(\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)+\tanh^{-1}\left(\frac{b\sqrt[4]{c}}{(-b)}\right.\right.}{3b^2c^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(17/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (3*(-b)^(7/4)*B*(ArcTan[(c^(1/4)*Sqrt[x])/(-b)^(1/4)] + ArcTanh[(b*c^(1/4)*
Sqrt[x])/(-b)^(5/4)]) + 2*c^(3/4)*(-2*b*B + A*c)*x^(3/2)*Hypergeometric2F1[
```

```
3/4, 2, 7/4, -((c*x^2)/b)] + 2*c^(3/4)*(b*B - A*c)*x^(3/2)*Hypergeometric2F
1[3/4, 3, 7/4, -((c*x^2)/b)])/(3*b^2*c^(11/4))
```

Maple [A] time = 0.015, size = 325, normalized size = 1.1

$$2\,\frac{1}{\left(cx^2+b\right)^2}\left(1/32\,\frac{(3\,Ac-11\,Bb)\,x^{7/2}}{bc}-1/32\,\frac{(Ac+7\,Bb)\,x^{3/2}}{c^2}\right)+\frac{3\,\sqrt{2}A}{64\,bc^2}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}-1\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}+\frac{3\,\sqrt{2}A}{128\,bc^2}\ln\left(\left(x-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 2*(1/32*(3*A*c-11*B*b)/b/c*x^(7/2)-1/32*(A*c+7*B*b)/c^2*x^(3/2))/(c*x^2+b)^
2+3/64/c^2/b/(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)+3/
128/c^2/b/(b/c)^(1/4)*2^(1/2)*A*ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/
2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+3/64/c^2/b/(b/c)^(1/4)*2^(
1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+21/64/c^3/(b/c)^(1/4)*2^(1/2)*
B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)+21/128/c^3/(b/c)^(1/4)*2^(1/2)*B*ln
((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)
+(b/c)^(1/2)))+21/64/c^3/(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x
^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.66435, size = 2226, normalized size = 7.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(12*(b*c^4*x^4 + 2*b^2*c^3*x^2 + b^3*c^2)*(-(2401*B^4*b^4 + 1372*A*B^
3*b^3*c + 294*A^2*B^2*b^2*c^2 + 28*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^11))^(1/4)
*arctan((sqrt((117649*B^6*b^6 + 100842*A*B^5*b^5*c + 36015*A^2*B^4*b^4*c^2
+ 6860*A^3*B^3*b^3*c^3 + 735*A^4*B^2*b^2*c^4 + 42*A^5*B*b*c^5 + A^6*c^6)*x
- (2401*B^4*b^7*c^5 + 1372*A*B^3*b^6*c^6 + 294*A^2*B^2*b^5*c^7 + 28*A^3*B*b
^4*c^8 + A^4*b^3*c^9)*sqrt(-(2401*B^4*b^4 + 1372*A*B^3*b^3*c + 294*A^2*B^2*
b^2*c^2 + 28*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^11)))*b*c^3*(-(2401*B^4*b^4 + 13
72*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 28*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^11)
)^(1/4) - (343*B^3*b^4*c^3 + 147*A*B^2*b^3*c^4 + 21*A^2*B*b^2*c^5 + A^3*b*c
^6)*sqrt(x)*(-(2401*B^4*b^4 + 1372*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 28*A
^3*B*b*c^3 + A^4*c^4)/(b^5*c^11))^(1/4))/(2401*B^4*b^4 + 1372*A*B^3*b^3*c +
 294*A^2*B^2*b^2*c^2 + 28*A^3*B*b*c^3 + A^4*c^4)) - 3*(b*c^4*x^4 + 2*b^2*c^
3*x^2 + b^3*c^2)*(-(2401*B^4*b^4 + 1372*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 +
 28*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^11))^(1/4)*log(27*b^4*c^8*(-(2401*B^4*b^4
 + 1372*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 28*A^3*B*b*c^3 + A^4*c^4)/(b^5*
c^11))^(3/4) + 27*(343*B^3*b^3 + 147*A*B^2*b^2*c + 21*A^2*B*b*c^2 + A^3*c^3
)*sqrt(x)) + 3*(b*c^4*x^4 + 2*b^2*c^3*x^2 + b^3*c^2)*(-(2401*B^4*b^4 + 1372
*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 28*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^11))^
(1/4)*log(-27*b^4*c^8*(-(2401*B^4*b^4 + 1372*A*B^3*b^3*c + 294*A^2*B^2*b^2*
c^2 + 28*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^11))^(3/4) + 27*(343*B^3*b^3 + 147*A
*B^2*b^2*c + 21*A^2*B*b*c^2 + A^3*c^3)*sqrt(x)) + 4*((11*B*b*c - 3*A*c^2)*x
^3 + (7*B*b^2 + A*b*c)*x)*sqrt(x))/(b*c^4*x^4 + 2*b^2*c^3*x^2 + b^3*c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(17/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32774, size = 396, normalized size = 1.35

$$-\frac{11\,Bbcx^{\frac{7}{2}}-3\,Ac^{2}x^{\frac{7}{2}}+7\,Bb^{2}x^{\frac{3}{2}}+Abcx^{\frac{3}{2}}}{16\,\left(cx^{2}+b\right)^{2}bc^{2}}+\frac{3\,\sqrt{2}\left(7\,\left(bc^{3}\right)^{\frac{3}{4}}Bb+\left(bc^{3}\right)^{\frac{3}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^{2}c^{5}}+\frac{3\,\sqrt{2}\left(7\,\left(bc^{3}\right)^{\frac{3}{4}}\,B\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -1/16*(11*B*b*c*x^(7/2) - 3*A*c^2*x^(7/2) + 7*B*b^2*x^(3/2) + A*b*c*x^(3/2)
)/((c*x^2 + b)^2*b*c^2) + 3/64*sqrt(2)*(7*(b*c^3)^(3/4)*B*b + (b*c^3)^(3/4)
*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^
2*c^5) + 3/64*sqrt(2)*(7*(b*c^3)^(3/4)*B*b + (b*c^3)^(3/4)*A*c)*arctan(-1/2
*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^5) - 3/128*s
qrt(2)*(7*(b*c^3)^(3/4)*B*b + (b*c^3)^(3/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^
(1/4) + x + sqrt(b/c))/(b^2*c^5) + 3/128*sqrt(2)*(7*(b*c^3)^(3/4)*B*b + (b*
c^3)^(3/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c^5)
```

3.211 $\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=298

$$-\frac{(3Ac+5bB)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{7/4}c^{9/4}}+\frac{(3Ac+5bB)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{7/4}c^{9/4}}-\frac{(3Ac+5bB)\tan^{-1}\left(1\right.}{32\sqrt{2}b^{7/4}c^{9/4}}$$

```
[Out] -((b*B - A*c)*x^(5/2))/(4*b*c*(b + c*x^2)^2) - ((5*b*B + 3*A*c)*Sqrt[x])/(1
6*b*c^2*(b + c*x^2)) - ((5*b*B + 3*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x]
)/b^(1/4)])/(32*Sqrt[2]*b^(7/4)*c^(9/4)) + ((5*b*B + 3*A*c)*ArcTan[1 + (Sqr
t[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(7/4)*c^(9/4)) - ((5*b*B + 3*
A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2
]*b^(7/4)*c^(9/4)) + ((5*b*B + 3*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)
*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(7/4)*c^(9/4))
```

Rubi [A] time = 0.235897, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1584, 457, 288, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(3Ac+5bB)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{7/4}c^{9/4}}+\frac{(3Ac+5bB)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{7/4}c^{9/4}}-\frac{(3Ac+5bB)\tan^{-1}\left(1\right.}{32\sqrt{2}b^{7/4}c^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(15/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```


```
[Out] -((b*B - A*c)*x^(5/2))/(4*b*c*(b + c*x^2)^2) - ((5*b*B + 3*A*c)*Sqrt[x])/(1
6*b*c^2*(b + c*x^2)) - ((5*b*B + 3*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x]
)/b^(1/4)])/(32*Sqrt[2]*b^(7/4)*c^(9/4)) + ((5*b*B + 3*A*c)*ArcTan[1 + (Sqr
t[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(7/4)*c^(9/4)) - ((5*b*B + 3*
A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2
]*b^(7/4)*c^(9/4)) + ((5*b*B + 3*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)
*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(7/4)*c^(9/4))
```


Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{15/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx=\int \frac{x^{3/2}\left(A+Bx^2\right)}{\left(b+cx^2\right)^3}\,dx$$

$$=-\frac{(bB-Ac)x^{5/2}}{4bc\left(b+cx^2\right)^2}+\frac{\left(\frac{5bB}{2}+\frac{3Ac}{2}\right)\int \frac{x^{3/2}}{\left(b+cx^2\right)^2}\,dx}{4bc}$$

$$=-\frac{(bB-Ac)x^{5/2}}{4bc\left(b+cx^2\right)^2}-\frac{(5bB+3Ac)\sqrt{x}}{16bc^2\left(b+cx^2\right)}+\frac{(5bB+3Ac)\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{32bc^2}$$

$$=-\frac{(bB-Ac)x^{5/2}}{4bc\left(b+cx^2\right)^2}-\frac{(5bB+3Ac)\sqrt{x}}{16bc^2\left(b+cx^2\right)}+\frac{(5bB+3Ac)\,\mathrm{Subst}\left(\int \frac{1}{b+cx^4}\,dx,x,\sqrt{x}\right)}{16bc^2}$$

$$=-\frac{(bB-Ac)x^{5/2}}{4bc\left(b+cx^2\right)^2}-\frac{(5bB+3Ac)\sqrt{x}}{16bc^2\left(b+cx^2\right)}+\frac{(5bB+3Ac)\,\mathrm{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{32b^{3/2}c^2}+\frac{(5bB+3Ac)\,\mathrm{S}}{}$$

$$=-\frac{(bB-Ac)x^{5/2}}{4bc\left(b+cx^2\right)^2}-\frac{(5bB+3Ac)\sqrt{x}}{16bc^2\left(b+cx^2\right)}+\frac{(5bB+3Ac)\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{64b^{3/2}c^{5/2}}+\frac{(5bB+3.}{}$$

$$=-\frac{(bB-Ac)x^{5/2}}{4bc\left(b+cx^2\right)^2}-\frac{(5bB+3Ac)\sqrt{x}}{16bc^2\left(b+cx^2\right)}-\frac{(5bB+3Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{7/4}c^{9/4}}+\frac{(5bB+3Ac}{}$$

$$=-\frac{(bB-Ac)x^{5/2}}{4bc\left(b+cx^2\right)^2}-\frac{(5bB+3Ac)\sqrt{x}}{16bc^2\left(b+cx^2\right)}-\frac{(5bB+3Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{7/4}c^{9/4}}+\frac{(5bB+3Ac)\tan^{-1}\left(1\right.}{32\sqrt{2}b^{7/4}c^{9/4}}$$

Mathematica [A] time = 0.446242, size = 389, normalized size = 1.31

$$\frac{-\frac{2\sqrt{2}(3Ac+5bB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{b^{7/4}}+\frac{2\sqrt{2}(3Ac+5bB)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{b^{7/4}}+\frac{8Ac^{5/4}\sqrt{x}}{b^2+bcx^2}-\frac{3\sqrt{2}Ac\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{b^{7/4}}+\frac{3\sqrt{2}Ac\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{}\right.}{b^{7/4}}}{128c^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(15/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((32*b*B*c^(1/4)*Sqrt[x])/(b + c*x^2)^2 - (32*A*c^(5/4)*Sqrt[x])/(b + c*x^2
)^2 - (72*B*c^(1/4)*Sqrt[x])/(b + c*x^2) + (8*A*c^(5/4)*Sqrt[x])/(b^2 + b*c
*x^2) - (2*Sqrt[2]*(5*b*B + 3*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(
1/4)])/b^(7/4) + (2*Sqrt[2]*(5*b*B + 3*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqr
t[x])/b^(1/4)])/b^(7/4) - (5*Sqrt[2]*B*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4
)*Sqrt[x] + Sqrt[c]*x])/b^(3/4) - (3*Sqrt[2]*A*c*Log[Sqrt[b] - Sqrt[2]*b^(1
/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/b^(7/4) + (5*Sqrt[2]*B*Log[Sqrt[b] + Sqrt
[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/b^(3/4) + (3*Sqrt[2]*A*c*Log[Sqrt
[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/b^(7/4))/(128*c^(9/4))
```

Maple [A] time = 0.015, size = 334, normalized size = 1.1

$$2\,\frac{1}{\left(cx^2+b\right)^2}\left(1/32\,\frac{(Ac-9\,Bb)\,x^{5/2}}{bc}-1/32\,\frac{(3\,Ac+5\,Bb)\,\sqrt{x}}{c^2}\right)+\frac{3\,\sqrt{2}A}{64\,b^2c}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}-1\right)+\frac{3\,\sqrt{2}A}{128\,b^2c}\sqrt[4]{\frac{b}{c}}\ln\left(\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 2*(1/32*(A*c-9*B*b)/b/c*x^(5/2)-1/32*(3*A*c+5*B*b)/c^2*x^(1/2))/(c*x^2+b)^2
+3/64/c/b^2*(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)+3/1
28/c/b^2*(b/c)^(1/4)*2^(1/2)*A*ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2
))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+3/64/c/b^2*(b/c)^(1/4)*2^(1
/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+5/64/c^2/b*(b/c)^(1/4)*2^(1/2)*
B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)+5/128/c^2/b*(b/c)^(1/4)*2^(1/2)*B*l
n((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2
)+(b/c)^(1/2)))+5/64/c^2/b*(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)
*x^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.60415, size = 1817, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(4*(b*c^4*x^4 + 2*b^2*c^3*x^2 + b^3*c^2)*(-(625*B^4*b^4 + 1500*A*B^3*b
^3*c + 1350*A^2*B^2*b^2*c^2 + 540*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^9))^(1/4
)*arctan((sqrt(b^4*c^4*sqrt(-(625*B^4*b^4 + 1500*A*B^3*b^3*c + 1350*A^2*B^2
*b^2*c^2 + 540*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^9)) + (25*B^2*b^2 + 30*A*B*
b*c + 9*A^2*c^2)*x)*b^5*c^7*(-(625*B^4*b^4 + 1500*A*B^3*b^3*c + 1350*A^2*B^
2*b^2*c^2 + 540*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^9))^(3/4) - (5*B*b^6*c^7 +
 3*A*b^5*c^8)*sqrt(x)*(-(625*B^4*b^4 + 1500*A*B^3*b^3*c + 1350*A^2*B^2*b^2*
c^2 + 540*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^9))^(3/4))/(625*B^4*b^4 + 1500*A
*B^3*b^3*c + 1350*A^2*B^2*b^2*c^2 + 540*A^3*B*b*c^3 + 81*A^4*c^4)) + (b*c^4
*x^4 + 2*b^2*c^3*x^2 + b^3*c^2)*(-(625*B^4*b^4 + 1500*A*B^3*b^3*c + 1350*A^
2*B^2*b^2*c^2 + 540*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^9))^(1/4)*log(b^2*c^2*
(-(625*B^4*b^4 + 1500*A*B^3*b^3*c + 1350*A^2*B^2*b^2*c^2 + 540*A^3*B*b*c^3
+ 81*A^4*c^4)/(b^7*c^9))^(1/4) + (5*B*b + 3*A*c)*sqrt(x)) - (b*c^4*x^4 + 2*
b^2*c^3*x^2 + b^3*c^2)*(-(625*B^4*b^4 + 1500*A*B^3*b^3*c + 1350*A^2*B^2*b^2
*c^2 + 540*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^9))^(1/4)*log(-b^2*c^2*(-(625*B
^4*b^4 + 1500*A*B^3*b^3*c + 1350*A^2*B^2*b^2*c^2 + 540*A^3*B*b*c^3 + 81*A^4
*c^4)/(b^7*c^9))^(1/4) + (5*B*b + 3*A*c)*sqrt(x)) - 4*(5*B*b^2 + 3*A*b*c +
(9*B*b*c - A*c^2)*x^2)*sqrt(x))/(b*c^4*x^4 + 2*b^2*c^3*x^2 + b^3*c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(15/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29353, size = 402, normalized size = 1.35

$$\frac{\sqrt{2}\left(5\left(bc^3\right)^{\frac{1}{4}}Bb+3\left(bc^3\right)^{\frac{1}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^2c^3}+\frac{\sqrt{2}\left(5\left(bc^3\right)^{\frac{1}{4}}Bb+3\left(bc^3\right)^{\frac{1}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/64*sqrt(2)*(5*(b*c^3)^(1/4)*B*b + 3*(b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt(2)
*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^3) + 1/64*sqrt(2)*(5
*(b*c^3)^(1/4)*B*b + 3*(b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c
)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^3) + 1/128*sqrt(2)*(5*(b*c^3)^(1/4
)*B*b + 3*(b*c^3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c
))/(b^2*c^3) - 1/128*sqrt(2)*(5*(b*c^3)^(1/4)*B*b + 3*(b*c^3)^(1/4)*A*c)*lo
g(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c^3) - 1/16*(9*B*b*c*x
^(5/2) - A*c^2*x^(5/2) + 5*B*b^2*sqrt(x) + 3*A*b*c*sqrt(x))/((c*x^2 + b)^2*
b*c^2)
```

3.212 $\int \frac{x^{13/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=298

$$\frac{(5Ac+3bB)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{9/4}c^{7/4}}-\frac{(5Ac+3bB)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{9/4}c^{7/4}}-\frac{(5Ac+3bB)\tan^{-1}\left(1-\right.}{32\sqrt{2}b^{9/4}c^{7/4}}$$

```
[Out] -((b*B - A*c)*x^(3/2))/(4*b*c*(b + c*x^2)^2) + ((3*b*B + 5*A*c)*x^(3/2))/(1
6*b^2*c*(b + c*x^2)) - ((3*b*B + 5*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x]
)/b^(1/4)])/(32*Sqrt[2]*b^(9/4)*c^(7/4)) + ((3*b*B + 5*A*c)*ArcTan[1 + (Sqr
t[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(9/4)*c^(7/4)) + ((3*b*B + 5*
A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2
]*b^(9/4)*c^(7/4)) - ((3*b*B + 5*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)
*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(9/4)*c^(7/4))
```

Rubi [A] time = 0.22876, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1584, 457, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{(5Ac+3bB)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{9/4}c^{7/4}}-\frac{(5Ac+3bB)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{9/4}c^{7/4}}-\frac{(5Ac+3bB)\tan^{-1}\left(1-\right.}{32\sqrt{2}b^{9/4}c^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -((b*B - A*c)*x^(3/2))/(4*b*c*(b + c*x^2)^2) + ((3*b*B + 5*A*c)*x^(3/2))/(1
6*b^2*c*(b + c*x^2)) - ((3*b*B + 5*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x]
)/b^(1/4)])/(32*Sqrt[2]*b^(9/4)*c^(7/4)) + ((3*b*B + 5*A*c)*ArcTan[1 + (Sqr
t[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(9/4)*c^(7/4)) + ((3*b*B + 5*
A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2
]*b^(9/4)*c^(7/4)) - ((3*b*B + 5*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)
*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(9/4)*c^(7/4))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^{13/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{\sqrt{x}\left(A+Bx^2\right)}{\left(b+cx^2\right)^3}\,dx$$

$$= -\frac{(bB-Ac)x^{3/2}}{4bc\left(b+cx^2\right)^2} + \frac{\left(\frac{3bB}{2}+\frac{5Ac}{2}\right)\int \frac{\sqrt{x}}{\left(b+cx^2\right)^2}\,dx}{4bc}$$

$$= -\frac{(bB-Ac)x^{3/2}}{4bc\left(b+cx^2\right)^2} + \frac{(3bB+5Ac)x^{3/2}}{16b^2c\left(b+cx^2\right)} + \frac{(3bB+5Ac)\int \frac{\sqrt{x}}{b+cx^2}\,dx}{32b^2c}$$

$$= -\frac{(bB-Ac)x^{3/2}}{4bc\left(b+cx^2\right)^2} + \frac{(3bB+5Ac)x^{3/2}}{16b^2c\left(b+cx^2\right)} + \frac{(3bB+5Ac)\,\mathrm{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{16b^2c}$$

$$= -\frac{(bB-Ac)x^{3/2}}{4bc\left(b+cx^2\right)^2} + \frac{(3bB+5Ac)x^{3/2}}{16b^2c\left(b+cx^2\right)} - \frac{(3bB+5Ac)\,\mathrm{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{32b^2c^{3/2}} + \frac{(3bB+5Ac \cdots}{\cdots}$$

$$= -\frac{(bB-Ac)x^{3/2}}{4bc\left(b+cx^2\right)^2} + \frac{(3bB+5Ac)x^{3/2}}{16b^2c\left(b+cx^2\right)} + \frac{(3bB+5Ac)\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{64b^2c^2} + \frac{(3bB+\cdots}{\cdots}$$

$$= -\frac{(bB-Ac)x^{3/2}}{4bc\left(b+cx^2\right)^2} + \frac{(3bB+5Ac)x^{3/2}}{16b^2c\left(b+cx^2\right)} + \frac{(3bB+5Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{9/4}c^{7/4}} - \frac{(3bB+5\cdots}{\cdots}$$

$$= -\frac{(bB-Ac)x^{3/2}}{4bc\left(b+cx^2\right)^2} + \frac{(3bB+5Ac)x^{3/2}}{16b^2c\left(b+cx^2\right)} - \frac{(3bB+5Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{9/4}c^{7/4}} + \frac{(3bB+5Ac)\tan^{-1}\left(\cdots\right.}{32\sqrt{2}b^{9/4}c\cdots}$$

<u>Mathematica</u> [C] time = 0.054874, size = 62, normalized size = 0.21

$$\frac{2x^{3/2}\left((Ac-bB)\,{}_2F_1\left(\frac{3}{4},3;\frac{7}{4};-\frac{cx^2}{b}\right)+bB\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{cx^2}{b}\right)\right)}{3b^3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*x^(3/2)*(b*B*Hypergeometric2F1[3/4, 2, 7/4, -((c*x^2)/b)] + (-(b*B) + A*
c)*Hypergeometric2F1[3/4, 3, 7/4, -((c*x^2)/b)]))/(3*b^3*c)
```

Maple [A] time = 0.016, size = 335, normalized size = 1.1

$$2\frac{1}{(cx^2+b)^2}\left(1/32\frac{(5Ac+3Bb)x^{7/2}}{b^2}+1/32\frac{(9Ac-Bb)x^{3/2}}{bc}\right)+\frac{5\sqrt{2}A}{64b^2c}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}+\frac{5\sqrt{2}A}{64b^2c}\arctan\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 2*(1/32*(5*A*c+3*B*b)/b^2*x^(7/2)+1/32*(9*A*c-B*b)/b/c*x^(3/2))/(c*x^2+b)^2
+5/64/b^2/c/(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+5/6
4/b^2/c/(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)+5/128/b
^2/c/(b/c)^(1/4)*2^(1/2)*A*ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(
x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+3/64/b/c^2/(b/c)^(1/4)*2^(1/2)*
B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+3/64/b/c^2/(b/c)^(1/4)*2^(1/2)*B*ar
ctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)+3/128/b/c^2/(b/c)^(1/4)*2^(1/2)*B*ln((x
-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b
/c)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.76955, size = 2276, normalized size = 7.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(4*(b^2*c^3*x^4 + 2*b^3*c^2*x^2 + b^4*c)*(-(81*B^4*b^4 + 540*A*B^3*b^
3*c + 1350*A^2*B^2*b^2*c^2 + 1500*A^3*B*b*c^3 + 625*A^4*c^4)/(b^9*c^7))^(1/
4)*arctan((sqrt((729*B^6*b^6 + 7290*A*B^5*b^5*c + 30375*A^2*B^4*b^4*c^2 + 6
7500*A^3*B^3*b^3*c^3 + 84375*A^4*B^2*b^2*c^4 + 56250*A^5*B*b*c^5 + 15625*A^
6*c^6)*x - (81*B^4*b^9*c^3 + 540*A*B^3*b^8*c^4 + 1350*A^2*B^2*b^7*c^5 + 150
0*A^3*B*b^6*c^6 + 625*A^4*b^5*c^7)*sqrt(-(81*B^4*b^4 + 540*A*B^3*b^3*c + 13
50*A^2*B^2*b^2*c^2 + 1500*A^3*B*b*c^3 + 625*A^4*c^4)/(b^9*c^7)))*b^2*c^2*(-
(81*B^4*b^4 + 540*A*B^3*b^3*c + 1350*A^2*B^2*b^2*c^2 + 1500*A^3*B*b*c^3 + 6
25*A^4*c^4)/(b^9*c^7))^(1/4) - (27*B^3*b^5*c^2 + 135*A*B^2*b^4*c^3 + 225*A^
2*B*b^3*c^4 + 125*A^3*b^2*c^5)*sqrt(x)*(-(81*B^4*b^4 + 540*A*B^3*b^3*c + 13
50*A^2*B^2*b^2*c^2 + 1500*A^3*B*b*c^3 + 625*A^4*c^4)/(b^9*c^7))^(1/4))/(81*
B^4*b^4 + 540*A*B^3*b^3*c + 1350*A^2*B^2*b^2*c^2 + 1500*A^3*B*b*c^3 + 625*A
^4*c^4)) - (b^2*c^3*x^4 + 2*b^3*c^2*x^2 + b^4*c)*(-(81*B^4*b^4 + 540*A*B^3*
b^3*c + 1350*A^2*B^2*b^2*c^2 + 1500*A^3*B*b*c^3 + 625*A^4*c^4)/(b^9*c^7))^(
1/4)*log(b^7*c^5*(-(81*B^4*b^4 + 540*A*B^3*b^3*c + 1350*A^2*B^2*b^2*c^2 + 1
500*A^3*B*b*c^3 + 625*A^4*c^4)/(b^9*c^7))^(3/4) + (27*B^3*b^3 + 135*A*B^2*b
^2*c + 225*A^2*B*b*c^2 + 125*A^3*c^3)*sqrt(x)) + (b^2*c^3*x^4 + 2*b^3*c^2*x
^2 + b^4*c)*(-(81*B^4*b^4 + 540*A*B^3*b^3*c + 1350*A^2*B^2*b^2*c^2 + 1500*A
^3*B*b*c^3 + 625*A^4*c^4)/(b^9*c^7))^(1/4)*log(-b^7*c^5*(-(81*B^4*b^4 + 540
*A*B^3*b^3*c + 1350*A^2*B^2*b^2*c^2 + 1500*A^3*B*b*c^3 + 625*A^4*c^4)/(b^9*
c^7))^(3/4) + (27*B^3*b^3 + 135*A*B^2*b^2*c + 225*A^2*B*b*c^2 + 125*A^3*c^3
)*sqrt(x)) - 4*((3*B*b*c + 5*A*c^2)*x^3 - (B*b^2 - 9*A*b*c)*x)*sqrt(x))/(b^
2*c^3*x^4 + 2*b^3*c^2*x^2 + b^4*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(13/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.39314, size = 402, normalized size = 1.35

$$\frac{3\,Bbcx^{\frac{7}{2}}+5\,Ac^2x^{\frac{7}{2}}-Bb^2x^{\frac{3}{2}}+9\,Abcx^{\frac{3}{2}}}{16\left(cx^2+b\right)^2b^2c}+\frac{\sqrt{2}\left(3\,\left(bc^3\right)^{\frac{3}{4}}Bb+5\,\left(bc^3\right)^{\frac{3}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^3c^4}+\frac{\sqrt{2}\left(3\,\left(bc^3\right)^{\frac{3}{4}}Bb+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/16*(3*B*b*c*x^(7/2) + 5*A*c^2*x^(7/2) - B*b^2*x^(3/2) + 9*A*b*c*x^(3/2))/
((c*x^2 + b)^2*b^2*c) + 1/64*sqrt(2)*(3*(b*c^3)^(3/4)*B*b + 5*(b*c^3)^(3/4)
*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^
3*c^4) + 1/64*sqrt(2)*(3*(b*c^3)^(3/4)*B*b + 5*(b*c^3)^(3/4)*A*c)*arctan(-1
/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^3*c^4) - 1/128
*sqrt(2)*(3*(b*c^3)^(3/4)*B*b + 5*(b*c^3)^(3/4)*A*c)*log(sqrt(2)*sqrt(x)*(b
/c)^(1/4) + x + sqrt(b/c))/(b^3*c^4) + 1/128*sqrt(2)*(3*(b*c^3)^(3/4)*B*b +
 5*(b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^
3*c^4)
```

3.213 $\int \frac{x^{11/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=293

$$-\frac{3(7Ac+bB)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{11/4}c^{5/4}}+\frac{3(7Ac+bB)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{11/4}c^{5/4}}-\frac{3(7Ac+bB)\tan^{-1}\left(1\right.}{32\sqrt{2}b^{11/4}c^{5/}}$$

```
[Out] -((b*B - A*c)*Sqrt[x])/(4*b*c*(b + c*x^2)^2) + ((b*B + 7*A*c)*Sqrt[x])/(16*
b^2*c*(b + c*x^2)) - (3*(b*B + 7*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/
b^(1/4)])/(32*Sqrt[2]*b^(11/4)*c^(5/4)) + (3*(b*B + 7*A*c)*ArcTan[1 + (Sqrt
[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(11/4)*c^(5/4)) - (3*(b*B + 7*
A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2
]*b^(11/4)*c^(5/4)) + (3*(b*B + 7*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4
)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(11/4)*c^(5/4))
```

Rubi [A] time = 0.230511, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1584, 457, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{3(7Ac+bB)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{11/4}c^{5/4}}+\frac{3(7Ac+bB)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{11/4}c^{5/4}}-\frac{3(7Ac+bB)\tan^{-1}\left(1\right.}{32\sqrt{2}b^{11/4}c^{5/}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -((b*B - A*c)*Sqrt[x])/(4*b*c*(b + c*x^2)^2) + ((b*B + 7*A*c)*Sqrt[x])/(16*
b^2*c*(b + c*x^2)) - (3*(b*B + 7*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/
b^(1/4)])/(32*Sqrt[2]*b^(11/4)*c^(5/4)) + (3*(b*B + 7*A*c)*ArcTan[1 + (Sqrt
[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(11/4)*c^(5/4)) - (3*(b*B + 7*
A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2
]*b^(11/4)*c^(5/4)) + (3*(b*B + 7*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4
)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(11/4)*c^(5/4))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{11/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx=\int \frac{A+Bx^2}{\sqrt{x}\left(b+cx^2\right)^3}\,dx$$

$$=-\frac{(bB-Ac)\sqrt{x}}{4bc\left(b+cx^2\right)^2}+\frac{\left(\frac{bB}{2}+\frac{7Ac}{2}\right)\int \frac{1}{\sqrt{x}\left(b+cx^2\right)^2}\,dx}{4bc}$$

$$=-\frac{(bB-Ac)\sqrt{x}}{4bc\left(b+cx^2\right)^2}+\frac{(bB+7Ac)\sqrt{x}}{16b^2c\left(b+cx^2\right)}+\frac{(3(bB+7Ac))\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{32b^2c}$$

$$=-\frac{(bB-Ac)\sqrt{x}}{4bc\left(b+cx^2\right)^2}+\frac{(bB+7Ac)\sqrt{x}}{16b^2c\left(b+cx^2\right)}+\frac{(3(bB+7Ac))\,\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx,x,\sqrt{x}\right)}{16b^2c}$$

$$=-\frac{(bB-Ac)\sqrt{x}}{4bc\left(b+cx^2\right)^2}+\frac{(bB+7Ac)\sqrt{x}}{16b^2c\left(b+cx^2\right)}+\frac{(3(bB+7Ac))\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{32b^{5/2}c}+\frac{(3(bB+7Ac}{}$$

$$=-\frac{(bB-Ac)\sqrt{x}}{4bc\left(b+cx^2\right)^2}+\frac{(bB+7Ac)\sqrt{x}}{16b^2c\left(b+cx^2\right)}+\frac{(3(bB+7Ac))\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{64b^{5/2}c^{3/2}}+\frac{(3(bB+}{}$$

$$=-\frac{(bB-Ac)\sqrt{x}}{4bc\left(b+cx^2\right)^2}+\frac{(bB+7Ac)\sqrt{x}}{16b^2c\left(b+cx^2\right)}-\frac{3(bB+7Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{11/4}c^{5/4}}+\frac{3(bB+7Ac}{}$$

$$=-\frac{(bB-Ac)\sqrt{x}}{4bc\left(b+cx^2\right)^2}+\frac{(bB+7Ac)\sqrt{x}}{16b^2c\left(b+cx^2\right)}-\frac{3(bB+7Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{11/4}c^{5/4}}+\frac{3(bB+7Ac)\tan^{-1}\left(1+\right.}{32\sqrt{2}b^{11/4}c^{5/4}}$$

Mathematica [A] time = 0.297919, size = 230, normalized size = 0.78

$$\frac{\frac{(7Ac+bB)\left(7\left(b+cx^2\right)\left(8b^{3/4}\sqrt[4]{c}\sqrt{x}-3\sqrt{2}\left(b+cx^2\right)\left(\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)-\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)+2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)-2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)\right)\right)+32b^7}{b^{11/4}\sqrt[4]{c}}}{896c\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (-256*B*Sqrt[x] + ((b*B + 7*A*c)*(32*b^(7/4)*c^(1/4)*Sqrt[x] + 7*(b + c*x^2
)*(8*b^(3/4)*c^(1/4)*Sqrt[x] - 3*Sqrt[2]*(b + c*x^2)*(2*ArcTan[1 - (Sqrt[2]
*c^(1/4)*Sqrt[x])/b^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)
] + Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] - Log[Sqrt[b
] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]))))/(b^(11/4)*c^(1/4)))/(8
96*c*(b + c*x^2)^2)
```

Maple [A] time = 0.015, size = 325, normalized size = 1.1

$$2\,{\frac {1}{ \left(c{x}^{2}+b \right) ^{2}} \left(1/32\,{\frac { \left(7\,Ac+Bb \right) {x}^{5/2}}{{b}^{2}}}+1/32\,{\frac { \left(11\,Ac-3\,Bb \right) \sqrt {x}}{bc}} \right) }+{\frac {21\,\sqrt {2}A}{64\,{b}^{3}}\sqrt [4]{{\frac {b}{c}}}\arctan \left(\sqrt {2}\sqrt {x}{\frac {1}{\sqrt [4]{{\frac {b}{c}}}}}-1 \right) }+{\frac {21\,\sqrt {2}A}{128\,{b}^{3}}\sqrt [4]{{\frac {b}{c}}}\ln }$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 2*(1/32*(7*A*c+B*b)/b^2*x^(5/2)+1/32*(11*A*c-3*B*b)/b/c*x^(1/2))/(c*x^2+b)^
2+21/64/b^3*(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)+21/
128/b^3*(b/c)^(1/4)*2^(1/2)*A*ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)
)/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+21/64/b^3*(b/c)^(1/4)*2^(1/2
)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+3/64/b^2/c*(b/c)^(1/4)*2^(1/2)*B*
arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)+3/128/b^2/c*(b/c)^(1/4)*2^(1/2)*B*ln(
(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+
(b/c)^(1/2)))+3/64/b^2/c*(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x
^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.54239, size = 1782, normalized size = 6.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(12*(b^2*c^3*x^4 + 2*b^3*c^2*x^2 + b^4*c)*(-(B^4*b^4 + 28*A*B^3*b^3*c
+ 294*A^2*B^2*b^2*c^2 + 1372*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^11*c^5))^(1/4)*
arctan((sqrt(b^6*c^2*sqrt(-(B^4*b^4 + 28*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2
+ 1372*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^11*c^5)) + (B^2*b^2 + 14*A*B*b*c + 49
*A^2*c^2)*x)*b^8*c^4*(-(B^4*b^4 + 28*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 13
72*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^11*c^5))^(3/4) - (B*b^9*c^4 + 7*A*b^8*c^5
)*sqrt(x)*(-(B^4*b^4 + 28*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 1372*A^3*B*b*
c^3 + 2401*A^4*c^4)/(b^11*c^5))^(3/4))/(B^4*b^4 + 28*A*B^3*b^3*c + 294*A^2*
B^2*b^2*c^2 + 1372*A^3*B*b*c^3 + 2401*A^4*c^4)) + 3*(b^2*c^3*x^4 + 2*b^3*c^
2*x^2 + b^4*c)*(-(B^4*b^4 + 28*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 1372*A^3
*B*b*c^3 + 2401*A^4*c^4)/(b^11*c^5))^(1/4)*log(3*b^3*c*(-(B^4*b^4 + 28*A*B^
3*b^3*c + 294*A^2*B^2*b^2*c^2 + 1372*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^11*c^5)
)^(1/4) + 3*(B*b + 7*A*c)*sqrt(x)) - 3*(b^2*c^3*x^4 + 2*b^3*c^2*x^2 + b^4*c
)*(-(B^4*b^4 + 28*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 1372*A^3*B*b*c^3 + 24
01*A^4*c^4)/(b^11*c^5))^(1/4)*log(-3*b^3*c*(-(B^4*b^4 + 28*A*B^3*b^3*c + 29
4*A^2*B^2*b^2*c^2 + 1372*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^11*c^5))^(1/4) + 3*
(B*b + 7*A*c)*sqrt(x)) - 4*(3*B*b^2 - 11*A*b*c - (B*b*c + 7*A*c^2)*x^2)*sqr
t(x))/(b^2*c^3*x^4 + 2*b^3*c^2*x^2 + b^4*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(11/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31727, size = 396, normalized size = 1.35

$$\frac{3\sqrt{2}\left(\left(bc^3\right)^{\frac{1}{4}}Bb+7\left(bc^3\right)^{\frac{1}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^3c^2}+\frac{3\sqrt{2}\left(\left(bc^3\right)^{\frac{1}{4}}Bb+7\left(bc^3\right)^{\frac{1}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 3/64*sqrt(2)*((b*c^3)^(1/4)*B*b + 7*(b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt(2)*(
sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^3*c^2) + 3/64*sqrt(2)*((b*
c^3)^(1/4)*B*b + 7*(b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1
/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^3*c^2) + 3/128*sqrt(2)*((b*c^3)^(1/4)*B*b
+ 7*(b*c^3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^
3*c^2) - 3/128*sqrt(2)*((b*c^3)^(1/4)*B*b + 7*(b*c^3)^(1/4)*A*c)*log(-sqrt(
2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c^2) + 1/16*(B*b*c*x^(5/2) + 7
*A*c^2*x^(5/2) - 3*B*b^2*sqrt(x) + 11*A*b*c*sqrt(x))/((c*x^2 + b)^2*b^2*c)
```

3.214 $\int \frac{x^{9/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=316

$$\frac{5(bB-9Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{13/4}c^{3/4}}-\frac{5(bB-9Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{13/4}c^{3/4}}-\frac{5(bB-9Ac)\tan^{-1}\left(1-\sqrt{\cdots}\right.}{32\sqrt{2}b^{13/4}c^{3/4}}$$

```
[Out] (5*(b*B - 9*A*c))/(16*b^3*c*Sqrt[x]) - (b*B - A*c)/(4*b*c*Sqrt[x]*(b + c*x^
2)^2) - (b*B - 9*A*c)/(16*b^2*c*Sqrt[x]*(b + c*x^2)) - (5*(b*B - 9*A*c)*Arc
Tan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(13/4)*c^(3/4)) +
 (5*(b*B - 9*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2
]*b^(13/4)*c^(3/4)) + (5*(b*B - 9*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4
)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(13/4)*c^(3/4)) - (5*(b*B - 9*A*c)*Lo
g[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(13
/4)*c^(3/4))
```

Rubi [A] time = 0.255955, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {1584, 457, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{5(bB-9Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{13/4}c^{3/4}}-\frac{5(bB-9Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{13/4}c^{3/4}}-\frac{5(bB-9Ac)\tan^{-1}\left(1-\sqrt{\cdots}\right.}{32\sqrt{2}b^{13/4}c^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (5*(b*B - 9*A*c))/(16*b^3*c*Sqrt[x]) - (b*B - A*c)/(4*b*c*Sqrt[x]*(b + c*x^
2)^2) - (b*B - 9*A*c)/(16*b^2*c*Sqrt[x]*(b + c*x^2)) - (5*(b*B - 9*A*c)*Arc
Tan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(13/4)*c^(3/4)) +
 (5*(b*B - 9*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2
]*b^(13/4)*c^(3/4)) + (5*(b*B - 9*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4
)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(13/4)*c^(3/4)) - (5*(b*B - 9*A*c)*Lo
g[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(13
/4)*c^(3/4))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^{9/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{A+Bx^2}{x^{3/2}\left(b+cx^2\right)^3}\,dx$$

$$= -\frac{bB-Ac}{4bc\sqrt{x}\left(b+cx^2\right)^2} + \frac{\left(-\frac{bB}{2}+\frac{9Ac}{2}\right)\int \frac{1}{x^{3/2}\left(b+cx^2\right)^2}\,dx}{4bc}$$

$$= -\frac{bB-Ac}{4bc\sqrt{x}\left(b+cx^2\right)^2} - \frac{bB-9Ac}{16b^2c\sqrt{x}\left(b+cx^2\right)} - \frac{(5(bB-9Ac))\int \frac{1}{x^{3/2}\left(b+cx^2\right)}\,dx}{32b^2c}$$

$$= \frac{5(bB-9Ac)}{16b^3c\sqrt{x}} - \frac{bB-Ac}{4bc\sqrt{x}\left(b+cx^2\right)^2} - \frac{bB-9Ac}{16b^2c\sqrt{x}\left(b+cx^2\right)} + \frac{(5(bB-9Ac))\int \frac{\sqrt{x}}{b+cx^2}\,dx}{32b^3}$$

$$= \frac{5(bB-9Ac)}{16b^3c\sqrt{x}} - \frac{bB-Ac}{4bc\sqrt{x}\left(b+cx^2\right)^2} - \frac{bB-9Ac}{16b^2c\sqrt{x}\left(b+cx^2\right)} + \frac{(5(bB-9Ac))\,\mathrm{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{\right.}{16b^3}$$

$$= \frac{5(bB-9Ac)}{16b^3c\sqrt{x}} - \frac{bB-Ac}{4bc\sqrt{x}\left(b+cx^2\right)^2} - \frac{bB-9Ac}{16b^2c\sqrt{x}\left(b+cx^2\right)} - \frac{(5(bB-9Ac))\,\mathrm{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x,\right.}{32b^3\sqrt{c}}$$

$$= \frac{5(bB-9Ac)}{16b^3c\sqrt{x}} - \frac{bB-Ac}{4bc\sqrt{x}\left(b+cx^2\right)^2} - \frac{bB-9Ac}{16b^2c\sqrt{x}\left(b+cx^2\right)} + \frac{(5(bB-9Ac))\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\right.}{64b^3c}$$

$$= \frac{5(bB-9Ac)}{16b^3c\sqrt{x}} - \frac{bB-Ac}{4bc\sqrt{x}\left(b+cx^2\right)^2} - \frac{bB-9Ac}{16b^2c\sqrt{x}\left(b+cx^2\right)} + \frac{5(bB-9Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\right.}{64\sqrt{2}b^{13/4}c^{3/4}}$$

$$= \frac{5(bB-9Ac)}{16b^3c\sqrt{x}} - \frac{bB-Ac}{4bc\sqrt{x}\left(b+cx^2\right)^2} - \frac{bB-9Ac}{16b^2c\sqrt{x}\left(b+cx^2\right)} - \frac{5(bB-9Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{13/4}c^{3/4}} + \frac{5(}{}$$

Mathematica [C] time = 0.204782, size = 147, normalized size = 0.47

$$\frac{2x^{3/2}(bB-Ac)\,{}_2F_1\left(\frac{3}{4},3;\frac{7}{4};-\frac{cx^2}{b}\right)}{3b^4} - \frac{2Acx^{3/2}\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{cx^2}{b}\right)}{3b^4} - \frac{2A}{b^3\sqrt{x}} + \frac{A\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)}{(-b)^{13/4}} + \frac{Ab\sqrt[4]{c}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)}{(-b)^{17/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (-2*A)/(b^3*Sqrt[x]) + (A*c^(1/4)*ArcTan[(c^(1/4)*Sqrt[x])/(-b)^(1/4)])/(-b
)^(13/4) + (A*b*c^(1/4)*ArcTanh[(c^(1/4)*Sqrt[x])/(-b)^(1/4)])/(-b)^(17/4)
- (2*A*c*x^(3/2)*Hypergeometric2F1[3/4, 2, 7/4, -((c*x^2)/b)])/(3*b^4) + (2
*(b*B - A*c)*x^(3/2)*Hypergeometric2F1[3/4, 3, 7/4, -((c*x^2)/b)])/(3*b^4)
```

Maple [A] time = 0.018, size = 363, normalized size = 1.2

$$-2\,\frac{A}{b^3\sqrt{x}}-\frac{13\,Ac^2}{16\,b^3\left(cx^2+b\right)^2}x^{\frac{7}{2}}+\frac{5\,Bc}{16\,b^2\left(cx^2+b\right)^2}x^{\frac{7}{2}}-\frac{17\,Ac}{16\,b^2\left(cx^2+b\right)^2}x^{\frac{3}{2}}+\frac{9\,B}{16\,b\left(cx^2+b\right)^2}x^{\frac{3}{2}}-\frac{45\,\sqrt{2}A}{128\,b^3}\ln\left(\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x)
```

```
[Out] -2*A/b^3/x^(1/2)-13/16/b^3/(c*x^2+b)^2*x^(7/2)*A*c^2+5/16/b^2/(c*x^2+b)^2*x
^(7/2)*B*c-17/16/b^2/(c*x^2+b)^2*A*x^(3/2)*c+9/16/b/(c*x^2+b)^2*B*x^(3/2)-4
5/128/b^3/(b/c)^(1/4)*2^(1/2)*A*ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/
2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))-45/64/b^3/(b/c)^(1/4)*2^(1
/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-45/64/b^3/(b/c)^(1/4)*2^(1/2)*A
*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)+5/128/b^2/c/(b/c)^(1/4)*2^(1/2)*B*ln
((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)
+(b/c)^(1/2)))+5/64/b^2/c/(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*
x^(1/2)+1)+5/64/b^2/c/(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1
/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.73933, size = 2244, normalized size = 7.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(20*(b^3*c^2*x^5 + 2*b^4*c*x^3 + b^5*x)*(-(B^4*b^4 - 36*A*B^3*b^3*c +
486*A^2*B^2*b^2*c^2 - 2916*A^3*B*b*c^3 + 6561*A^4*c^4)/(b^13*c^3))^(1/4)*ar
ctan((sqrt((B^6*b^6 - 54*A*B^5*b^5*c + 1215*A^2*B^4*b^4*c^2 - 14580*A^3*B^3
*b^3*c^3 + 98415*A^4*B^2*b^2*c^4 - 354294*A^5*B*b*c^5 + 531441*A^6*c^6)*x -
 (B^4*b^11*c - 36*A*B^3*b^10*c^2 + 486*A^2*B^2*b^9*c^3 - 2916*A^3*B*b^8*c^4
 + 6561*A^4*b^7*c^5)*sqrt(-(B^4*b^4 - 36*A*B^3*b^3*c + 486*A^2*B^2*b^2*c^2
- 2916*A^3*B*b*c^3 + 6561*A^4*c^4)/(b^13*c^3)))*b^3*c*(-(B^4*b^4 - 36*A*B^3
*b^3*c + 486*A^2*B^2*b^2*c^2 - 2916*A^3*B*b*c^3 + 6561*A^4*c^4)/(b^13*c^3))
^(1/4) + (B^3*b^6*c - 27*A*B^2*b^5*c^2 + 243*A^2*B*b^4*c^3 - 729*A^3*b^3*c^
4)*sqrt(x)*(-(B^4*b^4 - 36*A*B^3*b^3*c + 486*A^2*B^2*b^2*c^2 - 2916*A^3*B*b
*c^3 + 6561*A^4*c^4)/(b^13*c^3))^(1/4))/(B^4*b^4 - 36*A*B^3*b^3*c + 486*A^2
*B^2*b^2*c^2 - 2916*A^3*B*b*c^3 + 6561*A^4*c^4)) - 5*(b^3*c^2*x^5 + 2*b^4*c
*x^3 + b^5*x)*(-(B^4*b^4 - 36*A*B^3*b^3*c + 486*A^2*B^2*b^2*c^2 - 2916*A^3*
B*b*c^3 + 6561*A^4*c^4)/(b^13*c^3))^(1/4)*log(125*b^10*c^2*(-(B^4*b^4 - 36*
A*B^3*b^3*c + 486*A^2*B^2*b^2*c^2 - 2916*A^3*B*b*c^3 + 6561*A^4*c^4)/(b^13*
c^3))^(3/4) - 125*(B^3*b^3 - 27*A*B^2*b^2*c + 243*A^2*B*b*c^2 - 729*A^3*c^3
)*sqrt(x)) + 5*(b^3*c^2*x^5 + 2*b^4*c*x^3 + b^5*x)*(-(B^4*b^4 - 36*A*B^3*b^
3*c + 486*A^2*B^2*b^2*c^2 - 2916*A^3*B*b*c^3 + 6561*A^4*c^4)/(b^13*c^3))^(1
/4)*log(-125*b^10*c^2*(-(B^4*b^4 - 36*A*B^3*b^3*c + 486*A^2*B^2*b^2*c^2 - 2
916*A^3*B*b*c^3 + 6561*A^4*c^4)/(b^13*c^3))^(3/4) - 125*(B^3*b^3 - 27*A*B^2
*b^2*c + 243*A^2*B*b*c^2 - 729*A^3*c^3)*sqrt(x)) + 4*(5*(B*b*c - 9*A*c^2)*x
^4 - 32*A*b^2 + 9*(B*b^2 - 9*A*b*c)*x^2)*sqrt(x))/(b^3*c^2*x^5 + 2*b^4*c*x^
3 + b^5*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.39101, size = 405, normalized size = 1.28

$$-\frac{2A}{b^3\sqrt{x}}+\frac{5Bbcx^{\frac{7}{2}}-13Ac^2x^{\frac{7}{2}}+9Bb^2x^{\frac{3}{2}}-17Abcx^{\frac{3}{2}}}{16\left(cx^2+b\right)^2b^3}+\frac{5\sqrt{2}\left(\left(bc^3\right)^{\frac{3}{4}}Bb-9\left(bc^3\right)^{\frac{3}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64b^4c^3}+\frac{5\sqrt{2}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -2*A/(b^3*sqrt(x)) + 1/16*(5*B*b*c*x^(7/2) - 13*A*c^2*x^(7/2) + 9*B*b^2*x^(
3/2) - 17*A*b*c*x^(3/2))/((c*x^2 + b)^2*b^3) + 5/64*sqrt(2)*((b*c^3)^(3/4)*
B*b - 9*(b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt
(x))/(b/c)^(1/4))/(b^4*c^3) + 5/64*sqrt(2)*((b*c^3)^(3/4)*B*b - 9*(b*c^3)^(
3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4)
)/(b^4*c^3) - 5/128*sqrt(2)*((b*c^3)^(3/4)*B*b - 9*(b*c^3)^(3/4)*A*c)*log(s
qrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^4*c^3) + 5/128*sqrt(2)*((b*c
^3)^(3/4)*B*b - 9*(b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x +
 sqrt(b/c))/(b^4*c^3)
```

3.215 $\int \frac{x^{7/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=322

$$-\frac{3bB-11Ac}{16b^2cx^{3/2}\left(b+cx^2\right)}+\frac{7(3bB-11Ac)}{48b^3cx^{3/2}}-\frac{7(3bB-11Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{15/4}\sqrt[4]{c}}+\frac{7(3bB-11Ac)\log\left(\sqrt{2}\sqrt[4]{b}\right.}{64\sqrt{2}b^{15/4}}$$

```
[Out] (7*(3*b*B - 11*A*c))/(48*b^3*c*x^(3/2)) - (b*B - A*c)/(4*b*c*x^(3/2)*(b + c
*x^2)^2) - (3*b*B - 11*A*c)/(16*b^2*c*x^(3/2)*(b + c*x^2)) - (7*(3*b*B - 11
*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(15/4)*c
^(1/4)) + (7*(3*b*B - 11*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]
)/(32*Sqrt[2]*b^(15/4)*c^(1/4)) - (7*(3*b*B - 11*A*c)*Log[Sqrt[b] - Sqrt[2]
*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(15/4)*c^(1/4)) + (7*(
3*b*B - 11*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])
/(64*Sqrt[2]*b^(15/4)*c^(1/4))
```

Rubi [A] time = 0.250694, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {1584, 457, 290, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{3bB-11Ac}{16b^2cx^{3/2}\left(b+cx^2\right)}+\frac{7(3bB-11Ac)}{48b^3cx^{3/2}}-\frac{7(3bB-11Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{15/4}\sqrt[4]{c}}+\frac{7(3bB-11Ac)\log\left(\sqrt{2}\sqrt[4]{b}\right.}{64\sqrt{2}b^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (7*(3*b*B - 11*A*c))/(48*b^3*c*x^(3/2)) - (b*B - A*c)/(4*b*c*x^(3/2)*(b + c
*x^2)^2) - (3*b*B - 11*A*c)/(16*b^2*c*x^(3/2)*(b + c*x^2)) - (7*(3*b*B - 11
*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(15/4)*c
^(1/4)) + (7*(3*b*B - 11*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]
)/(32*Sqrt[2]*b^(15/4)*c^(1/4)) - (7*(3*b*B - 11*A*c)*Log[Sqrt[b] - Sqrt[2]
*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(15/4)*c^(1/4)) + (7*(
3*b*B - 11*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])
/(64*Sqrt[2]*b^(15/4)*c^(1/4))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{7/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{A+Bx^2}{x^{5/2}\left(b+cx^2\right)^3}\,dx$$

$$= -\frac{bB-Ac}{4bcx^{3/2}\left(b+cx^2\right)^2} + \frac{\left(-\frac{3bB}{2}+\frac{11Ac}{2}\right)\int \frac{1}{x^{5/2}\left(b+cx^2\right)^2}\,dx}{4bc}$$

$$= -\frac{bB-Ac}{4bcx^{3/2}\left(b+cx^2\right)^2} - \frac{3bB-11Ac}{16b^2cx^{3/2}\left(b+cx^2\right)} - \frac{(7(3bB-11Ac))\int \frac{1}{x^{5/2}\left(b+cx^2\right)}\,dx}{32b^2c}$$

$$= \frac{7(3bB-11Ac)}{48b^3cx^{3/2}} - \frac{bB-Ac}{4bcx^{3/2}\left(b+cx^2\right)^2} - \frac{3bB-11Ac}{16b^2cx^{3/2}\left(b+cx^2\right)} + \frac{(7(3bB-11Ac))\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{32b^3}$$

$$= \frac{7(3bB-11Ac)}{48b^3cx^{3/2}} - \frac{bB-Ac}{4bcx^{3/2}\left(b+cx^2\right)^2} - \frac{3bB-11Ac}{16b^2cx^{3/2}\left(b+cx^2\right)} + \frac{(7(3bB-11Ac))\,\mathrm{Subst}\left(\int \frac{1}{b+cx^4}\,dx,\right.}{16b^3}$$

$$= \frac{7(3bB-11Ac)}{48b^3cx^{3/2}} - \frac{bB-Ac}{4bcx^{3/2}\left(b+cx^2\right)^2} - \frac{3bB-11Ac}{16b^2cx^{3/2}\left(b+cx^2\right)} + \frac{(7(3bB-11Ac))\,\mathrm{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,d\right.}{32b^{7/2}}$$

$$= \frac{7(3bB-11Ac)}{48b^3cx^{3/2}} - \frac{bB-Ac}{4bcx^{3/2}\left(b+cx^2\right)^2} - \frac{3bB-11Ac}{16b^2cx^{3/2}\left(b+cx^2\right)} + \frac{(7(3bB-11Ac))\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}}\right.}{64b^{7/2}\sqrt{c}}$$

$$= \frac{7(3bB-11Ac)}{48b^3cx^{3/2}} - \frac{bB-Ac}{4bcx^{3/2}\left(b+cx^2\right)^2} - \frac{3bB-11Ac}{16b^2cx^{3/2}\left(b+cx^2\right)} - \frac{7(3bB-11Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{\right.}{64\sqrt{2}b^{15/4}\sqrt[4]{c}}$$

$$= \frac{7(3bB-11Ac)}{48b^3cx^{3/2}} - \frac{bB-Ac}{4bcx^{3/2}\left(b+cx^2\right)^2} - \frac{3bB-11Ac}{16b^2cx^{3/2}\left(b+cx^2\right)} - \frac{7(3bB-11Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{15/4}\sqrt[4]{c}} -$$

Mathematica [A] time = 0.447229, size = 400, normalized size = 1.24

$$\frac{-\frac{96Ab^{7/4}c\sqrt{x}}{\left(b+cx^2\right)^2} - \frac{360Ab^{3/4}c\sqrt{x}}{b+cx^2} - \frac{256Ab^{3/4}}{x^{3/2}} + \frac{42\sqrt{2}(11Ac-3bB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt[4]{c}} + \frac{42\sqrt{2}(3bB-11Ac)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt[4]{c}} + 231\sqrt{2}Ac^{3/4}\log\left(-\sqrt{\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-256*A*b^(3/4))/x^(3/2) + (96*b^(11/4)*B*Sqrt[x])/(b + c*x^2)^2 - (96*A*b
^(7/4)*c*Sqrt[x])/(b + c*x^2)^2 + (168*b^(7/4)*B*Sqrt[x])/(b + c*x^2) - (36
0*A*b^(3/4)*c*Sqrt[x])/(b + c*x^2) + (42*Sqrt[2]*(-3*b*B + 11*A*c)*ArcTan[1
 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/c^(1/4) + (42*Sqrt[2]*(3*b*B - 11*A*
c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/c^(1/4) - (63*Sqrt[2]*b*B
*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/c^(1/4) + 231*
Sqrt[2]*A*c^(3/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x
] + (63*Sqrt[2]*b*B*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]
*x])/c^(1/4) - 231*Sqrt[2]*A*c^(3/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*
Sqrt[x] + Sqrt[c]*x])/(384*b^(15/4))
```

Maple [A] time = 0.017, size = 357, normalized size = 1.1

$$-\frac{2A}{3b^3}x^{-\frac{3}{2}} - \frac{15Ac^2}{16b^3(cx^2+b)^2}x^{\frac{5}{2}} + \frac{7Bc}{16b^2(cx^2+b)^2}x^{\frac{5}{2}} - \frac{19Ac}{16b^2(cx^2+b)^2}\sqrt{x} + \frac{11B}{16b(cx^2+b)^2}\sqrt{x} - \frac{77\sqrt{2}Ac}{64b^4}\sqrt[4]{\frac{b}{c}}\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x)
```

```
[Out] -2/3*A/b^3/x^(3/2)-15/16/b^3/(c*x^2+b)^2*x^(5/2)*A*c^2+7/16/b^2/(c*x^2+b)^2
*x^(5/2)*B*c-19/16/b^2/(c*x^2+b)^2*A*x^(1/2)*c+11/16/b/(c*x^2+b)^2*B*x^(1/2
)-77/64/b^4*(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)*c-7
7/64/b^4*(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)*c-77/1
28/b^4*(b/c)^(1/4)*2^(1/2)*A*ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))
/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))*c+21/64/b^3*(b/c)^(1/4)*2^(1/
2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+21/64/b^3*(b/c)^(1/4)*2^(1/2)*B*
arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)+21/128/b^3*(b/c)^(1/4)*2^(1/2)*B*ln((
x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(
b/c)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.75335, size = 1901, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/192*(84*(b^3*c^2*x^6 + 2*b^4*c*x^4 + b^5*x^2)*(-(81*B^4*b^4 - 1188*A*B^3
*b^3*c + 6534*A^2*B^2*b^2*c^2 - 15972*A^3*B*b*c^3 + 14641*A^4*c^4)/(b^15*c)
)^(1/4)*arctan((sqrt(b^8*sqrt(-(81*B^4*b^4 - 1188*A*B^3*b^3*c + 6534*A^2*B^
2*b^2*c^2 - 15972*A^3*B*b*c^3 + 14641*A^4*c^4)/(b^15*c)) + (9*B^2*b^2 - 66*
A*B*b*c + 121*A^2*c^2)*x)*b^11*c*(-(81*B^4*b^4 - 1188*A*B^3*b^3*c + 6534*A^
2*B^2*b^2*c^2 - 15972*A^3*B*b*c^3 + 14641*A^4*c^4)/(b^15*c))^(3/4) + (3*B*b
^12*c - 11*A*b^11*c^2)*sqrt(x)*(-(81*B^4*b^4 - 1188*A*B^3*b^3*c + 6534*A^2*
B^2*b^2*c^2 - 15972*A^3*B*b*c^3 + 14641*A^4*c^4)/(b^15*c))^(3/4))/(81*B^4*b
^4 - 1188*A*B^3*b^3*c + 6534*A^2*B^2*b^2*c^2 - 15972*A^3*B*b*c^3 + 14641*A^
4*c^4)) + 21*(b^3*c^2*x^6 + 2*b^4*c*x^4 + b^5*x^2)*(-(81*B^4*b^4 - 1188*A*B
^3*b^3*c + 6534*A^2*B^2*b^2*c^2 - 15972*A^3*B*b*c^3 + 14641*A^4*c^4)/(b^15*
c))^(1/4)*log(7*b^4*(-(81*B^4*b^4 - 1188*A*B^3*b^3*c + 6534*A^2*B^2*b^2*c^2
 - 15972*A^3*B*b*c^3 + 14641*A^4*c^4)/(b^15*c))^(1/4) - 7*(3*B*b - 11*A*c)*
sqrt(x)) - 21*(b^3*c^2*x^6 + 2*b^4*c*x^4 + b^5*x^2)*(-(81*B^4*b^4 - 1188*A*
B^3*b^3*c + 6534*A^2*B^2*b^2*c^2 - 15972*A^3*B*b*c^3 + 14641*A^4*c^4)/(b^15
*c))^(1/4)*log(-7*b^4*(-(81*B^4*b^4 - 1188*A*B^3*b^3*c + 6534*A^2*B^2*b^2*c
^2 - 15972*A^3*B*b*c^3 + 14641*A^4*c^4)/(b^15*c))^(1/4) - 7*(3*B*b - 11*A*c
)*sqrt(x)) - 4*(7*(3*B*b*c - 11*A*c^2)*x^4 - 32*A*b^2 + 11*(3*B*b^2 - 11*A*
b*c)*x^2)*sqrt(x))/(b^3*c^2*x^6 + 2*b^4*c*x^4 + b^5*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

[Out] Timed out

Giac [A] time = 1.31035, size = 410, normalized size = 1.27

$$\frac{7\sqrt{2}\left(3\left(bc^3\right)^{\frac{1}{4}}Bb-11\left(bc^3\right)^{\frac{1}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^4c}+\frac{7\sqrt{2}\left(3\left(bc^3\right)^{\frac{1}{4}}Bb-11\left(bc^3\right)^{\frac{1}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-\right.}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.}{64\,b^4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 7/64*sqrt(2)*(3*(b*c^3)^(1/4)*B*b - 11*(b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt(2
)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^4*c) + 7/64*sqrt(2)*(3*
(b*c^3)^(1/4)*B*b - 11*(b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c
)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^4*c) + 7/128*sqrt(2)*(3*(b*c^3)^(1/4)*
B*b - 11*(b*c^3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c)
)/(b^4*c) - 7/128*sqrt(2)*(3*(b*c^3)^(1/4)*B*b - 11*(b*c^3)^(1/4)*A*c)*log(
-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^4*c) - 2/3*A/(b^3*x^(3/2))
 + 1/16*(7*B*b*c*x^(5/2) - 15*A*c^2*x^(5/2) + 11*B*b^2*sqrt(x) - 19*A*b*c*s
qrt(x))/((c*x^2 + b)^2*b^3)
```

3.216 $\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=343

$$-\frac{5bB-13Ac}{16b^2cx^{5/2}\left(b+cx^2\right)}+\frac{9(5bB-13Ac)}{80b^3cx^{5/2}}-\frac{9(5bB-13Ac)}{16b^4\sqrt{x}}-\frac{9\sqrt[4]{c}(5bB-13Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{17/4}}+\frac{9\sqrt[4]{c}(5b}{}$$

```
[Out] (9*(5*b*B - 13*A*c))/(80*b^3*c*x^(5/2)) - (9*(5*b*B - 13*A*c))/(16*b^4*Sqrt
[x]) - (b*B - A*c)/(4*b*c*x^(5/2)*(b + c*x^2)^2) - (5*b*B - 13*A*c)/(16*b^2
*c*x^(5/2)*(b + c*x^2)) + (9*c^(1/4)*(5*b*B - 13*A*c)*ArcTan[1 - (Sqrt[2]*c
^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(17/4)) - (9*c^(1/4)*(5*b*B - 13*A*
c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(17/4)) - (
9*c^(1/4)*(5*b*B - 13*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] +
Sqrt[c]*x])/(64*Sqrt[2]*b^(17/4)) + (9*c^(1/4)*(5*b*B - 13*A*c)*Log[Sqrt[b]
 + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(17/4))
```

Rubi [A] time = 0.275441, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {1584, 457, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{5bB-13Ac}{16b^2cx^{5/2}\left(b+cx^2\right)}+\frac{9(5bB-13Ac)}{80b^3cx^{5/2}}-\frac{9(5bB-13Ac)}{16b^4\sqrt{x}}-\frac{9\sqrt[4]{c}(5bB-13Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{17/4}}+\frac{9\sqrt[4]{c}(5b}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```


```
[Out] (9*(5*b*B - 13*A*c))/(80*b^3*c*x^(5/2)) - (9*(5*b*B - 13*A*c))/(16*b^4*Sqrt
[x]) - (b*B - A*c)/(4*b*c*x^(5/2)*(b + c*x^2)^2) - (5*b*B - 13*A*c)/(16*b^2
*c*x^(5/2)*(b + c*x^2)) + (9*c^(1/4)*(5*b*B - 13*A*c)*ArcTan[1 - (Sqrt[2]*c
^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(17/4)) - (9*c^(1/4)*(5*b*B - 13*A*
c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(17/4)) - (
9*c^(1/4)*(5*b*B - 13*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] +
Sqrt[c]*x])/(64*Sqrt[2]*b^(17/4)) + (9*c^(1/4)*(5*b*B - 13*A*c)*Log[Sqrt[b]
 + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(17/4))
```


Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^{5/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx=\int \frac{A+Bx^2}{x^{7/2}\left(b+cx^2\right)^3}\,dx$$

$$=-\frac{bB-Ac}{4bcx^{5/2}\left(b+cx^2\right)^2}+\frac{\left(-\frac{5bB}{2}+\frac{13Ac}{2}\right)\int \frac{1}{x^{7/2}\left(b+cx^2\right)^2}\,dx}{4bc}$$

$$=-\frac{bB-Ac}{4bcx^{5/2}\left(b+cx^2\right)^2}-\frac{5bB-13Ac}{16b^2cx^{5/2}\left(b+cx^2\right)}-\frac{(9(5bB-13Ac))\int \frac{1}{x^{7/2}\left(b+cx^2\right)}\,dx}{32b^2c}$$

$$=\frac{9(5bB-13Ac)}{80b^3cx^{5/2}}-\frac{bB-Ac}{4bcx^{5/2}\left(b+cx^2\right)^2}-\frac{5bB-13Ac}{16b^2cx^{5/2}\left(b+cx^2\right)}+\frac{(9(5bB-13Ac))\int \frac{1}{x^{3/2}\left(b+cx^2\right)}\,dx}{32b^3}$$

$$=\frac{9(5bB-13Ac)}{80b^3cx^{5/2}}-\frac{9(5bB-13Ac)}{16b^4\sqrt{x}}-\frac{bB-Ac}{4bcx^{5/2}\left(b+cx^2\right)^2}-\frac{5bB-13Ac}{16b^2cx^{5/2}\left(b+cx^2\right)}-\frac{(9c(5bB-13Ac)}{32b^4}$$

$$=\frac{9(5bB-13Ac)}{80b^3cx^{5/2}}-\frac{9(5bB-13Ac)}{16b^4\sqrt{x}}-\frac{bB-Ac}{4bcx^{5/2}\left(b+cx^2\right)^2}-\frac{5bB-13Ac}{16b^2cx^{5/2}\left(b+cx^2\right)}-\frac{(9c(5bB-13Ac)}{}$$

$$=\frac{9(5bB-13Ac)}{80b^3cx^{5/2}}-\frac{9(5bB-13Ac)}{16b^4\sqrt{x}}-\frac{bB-Ac}{4bcx^{5/2}\left(b+cx^2\right)^2}-\frac{5bB-13Ac}{16b^2cx^{5/2}\left(b+cx^2\right)}+\frac{\left(9\sqrt{c}(5bB-13A\right.}{}$$

$$=\frac{9(5bB-13Ac)}{80b^3cx^{5/2}}-\frac{9(5bB-13Ac)}{16b^4\sqrt{x}}-\frac{bB-Ac}{4bcx^{5/2}\left(b+cx^2\right)^2}-\frac{5bB-13Ac}{16b^2cx^{5/2}\left(b+cx^2\right)}-\frac{(9(5bB-13Ac))}{}$$

$$=\frac{9(5bB-13Ac)}{80b^3cx^{5/2}}-\frac{9(5bB-13Ac)}{16b^4\sqrt{x}}-\frac{bB-Ac}{4bcx^{5/2}\left(b+cx^2\right)^2}-\frac{5bB-13Ac}{16b^2cx^{5/2}\left(b+cx^2\right)}-\frac{9\sqrt[4]{c}(5bB-13Ac}{}$$

$$=\frac{9(5bB-13Ac)}{80b^3cx^{5/2}}-\frac{9(5bB-13Ac)}{16b^4\sqrt{x}}-\frac{bB-Ac}{4bcx^{5/2}\left(b+cx^2\right)^2}-\frac{5bB-13Ac}{16b^2cx^{5/2}\left(b+cx^2\right)}+\frac{9\sqrt[4]{c}(5bB-13Ac}{32}$$

Mathematica [C] time = 0.477227, size = 189, normalized size = 0.55

$$-\frac{2cx^{3/2}(bB-2Ac)\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{cx^2}{b}\right)}{3b^5}+\frac{2cx^{3/2}(Ac-bB)\,{}_2F_1\left(\frac{3}{4},3;\frac{7}{4};-\frac{cx^2}{b}\right)}{3b^5}+\frac{6Ac-2bB}{b^4\sqrt{x}}-\frac{2A}{5b^3x^{5/2}}+\frac{\sqrt[4]{c}(3Ac-bB)\tan}{(-b)^{17/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (-2*A)/(5*b^3*x^(5/2)) + (-2*b*B + 6*A*c)/(b^4*Sqrt[x]) + (c^(1/4)*(-(b*B)
+ 3*A*c)*ArcTan[(c^(1/4)*Sqrt[x])/(-b)^(1/4)])/(-b)^(17/4) + (c^(1/4)*(b*B
- 3*A*c)*ArcTanh[(c^(1/4)*Sqrt[x])/(-b)^(1/4)])/(-b)^(17/4) - (2*c*(b*B - 2
*A*c)*x^(3/2)*Hypergeometric2F1[3/4, 2, 7/4, -((c*x^2)/b)])/(3*b^5) + (2*c*
(-(b*B) + A*c)*x^(3/2)*Hypergeometric2F1[3/4, 3, 7/4, -((c*x^2)/b)])/(3*b^5
)
```

Maple [A] time = 0.021, size = 381, normalized size = 1.1

$$-\frac{2A}{5b^3}x^{-\frac{5}{2}}+6\frac{Ac}{b^4\sqrt{x}}-2\frac{B}{b^3\sqrt{x}}+\frac{21Ac^3}{16b^4\left(cx^2+b\right)^2}x^{\frac{7}{2}}-\frac{13c^2B}{16b^3\left(cx^2+b\right)^2}x^{\frac{7}{2}}+\frac{25Ac^2}{16b^3\left(cx^2+b\right)^2}x^{\frac{3}{2}}-\frac{17Bc}{16b^2\left(cx^2+b\right)^2}x^{\frac{3}{2}}+11$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x)
```

```
[Out] -2/5*A/b^3/x^(5/2)+6/b^4/x^(1/2)*A*c-2/b^3/x^(1/2)*B+21/16/b^4*c^3/(c*x^2+b
)^2*x^(7/2)*A-13/16/b^3*c^2/(c*x^2+b)^2*x^(7/2)*B+25/16/b^3*c^2/(c*x^2+b)^2
*A*x^(3/2)-17/16/b^2*c/(c*x^2+b)^2*B*x^(3/2)+117/128/b^4*c/(b/c)^(1/4)*2^(1
/2)*A*ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)
*2^(1/2)+(b/c)^(1/2)))+117/64/b^4*c/(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b
/c)^(1/4)*x^(1/2)+1)+117/64/b^4*c/(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c
)^(1/4)*x^(1/2)-1)-45/128/b^3/(b/c)^(1/4)*2^(1/2)*B*ln((x-(b/c)^(1/4)*x^(1/
2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))-45/64/
b^3/(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-45/64/b^3/(
b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.6971, size = 2507, normalized size = 7.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/320*(180*(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^3)*(-(625*B^4*b^4*c - 6500*A
*B^3*b^3*c^2 + 25350*A^2*B^2*b^2*c^3 - 43940*A^3*B*b*c^4 + 28561*A^4*c^5)/b
^17)^(1/4)*arctan((sqrt((15625*B^6*b^6*c^2 - 243750*A*B^5*b^5*c^3 + 1584375
*A^2*B^4*b^4*c^4 - 5492500*A^3*B^3*b^3*c^5 + 10710375*A^4*B^2*b^2*c^6 - 111
38790*A^5*B*b*c^7 + 4826809*A^6*c^8)*x - (625*B^4*b^13*c - 6500*A*B^3*b^12*
c^2 + 25350*A^2*B^2*b^11*c^3 - 43940*A^3*B*b^10*c^4 + 28561*A^4*b^9*c^5)*sq
rt(-(625*B^4*b^4*c - 6500*A*B^3*b^3*c^2 + 25350*A^2*B^2*b^2*c^3 - 43940*A^3
*B*b*c^4 + 28561*A^4*c^5)/b^17))*b^4*(-(625*B^4*b^4*c - 6500*A*B^3*b^3*c^2
+ 25350*A^2*B^2*b^2*c^3 - 43940*A^3*B*b*c^4 + 28561*A^4*c^5)/b^17)^(1/4) +
(125*B^3*b^7*c - 975*A*B^2*b^6*c^2 + 2535*A^2*B*b^5*c^3 - 2197*A^3*b^4*c^4)
*sqrt(x)*(-(625*B^4*b^4*c - 6500*A*B^3*b^3*c^2 + 25350*A^2*B^2*b^2*c^3 - 43
940*A^3*B*b*c^4 + 28561*A^4*c^5)/b^17)^(1/4))/(625*B^4*b^4*c - 6500*A*B^3*b
^3*c^2 + 25350*A^2*B^2*b^2*c^3 - 43940*A^3*B*b*c^4 + 28561*A^4*c^5)) - 45*(
b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^3)*(-(625*B^4*b^4*c - 6500*A*B^3*b^3*c^2
+ 25350*A^2*B^2*b^2*c^3 - 43940*A^3*B*b*c^4 + 28561*A^4*c^5)/b^17)^(1/4)*lo
g(729*b^13*(-(625*B^4*b^4*c - 6500*A*B^3*b^3*c^2 + 25350*A^2*B^2*b^2*c^3 -
43940*A^3*B*b*c^4 + 28561*A^4*c^5)/b^17)^(3/4) - 729*(125*B^3*b^3*c - 975*A
*B^2*b^2*c^2 + 2535*A^2*B*b*c^3 - 2197*A^3*c^4)*sqrt(x)) + 45*(b^4*c^2*x^7
+ 2*b^5*c*x^5 + b^6*x^3)*(-(625*B^4*b^4*c - 6500*A*B^3*b^3*c^2 + 25350*A^2*
B^2*b^2*c^3 - 43940*A^3*B*b*c^4 + 28561*A^4*c^5)/b^17)^(1/4)*log(-729*b^13*
(-(625*B^4*b^4*c - 6500*A*B^3*b^3*c^2 + 25350*A^2*B^2*b^2*c^3 - 43940*A^3*B
*b*c^4 + 28561*A^4*c^5)/b^17)^(3/4) - 729*(125*B^3*b^3*c - 975*A*B^2*b^2*c^
2 + 2535*A^2*B*b*c^3 - 2197*A^3*c^4)*sqrt(x)) + 4*(45*(5*B*b*c^2 - 13*A*c^3
)*x^6 + 81*(5*B*b^2*c - 13*A*b*c^2)*x^4 + 32*A*b^3 + 32*(5*B*b^3 - 13*A*b^2
*c)*x^2)*sqrt(x))/(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.35145, size = 440, normalized size = 1.28

$$-\frac{9\sqrt{2}\left(5\left(bc^3\right)^{\frac{3}{4}}Bb-13\left(bc^3\right)^{\frac{3}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^5c^2}-\frac{9\sqrt{2}\left(5\left(bc^3\right)^{\frac{3}{4}}Bb-13\left(bc^3\right)^{\frac{3}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-\right.}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.}{64\,b^5c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -9/64*sqrt(2)*(5*(b*c^3)^(3/4)*B*b - 13*(b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(
2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^5*c^2) - 9/64*sqrt(2)*
(5*(b*c^3)^(3/4)*B*b - 13*(b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(
b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^5*c^2) + 9/128*sqrt(2)*(5*(b*c^3)^(
3/4)*B*b - 13*(b*c^3)^(3/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt
(b/c))/(b^5*c^2) - 9/128*sqrt(2)*(5*(b*c^3)^(3/4)*B*b - 13*(b*c^3)^(3/4)*A*
c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^5*c^2) - 1/16*(13*B
*b*c^2*x^(7/2) - 21*A*c^3*x^(7/2) + 17*B*b^2*c*x^(3/2) - 25*A*b*c^2*x^(3/2)
)/((c*x^2 + b)^2*b^4) - 2/5*(5*B*b*x^2 - 15*A*c*x^2 + A*b)/(b^4*x^(5/2))
```

3.217 $\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=343

$$\frac{11c^{3/4}(7bB-15Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{19/4}}-\frac{11c^{3/4}(7bB-15Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{19/4}}+\frac{11c^{3/4}(7bB}{}$$

```
[Out] (11*(7*b*B - 15*A*c))/(112*b^3*c*x^(7/2)) - (11*(7*b*B - 15*A*c))/(48*b^4*x
^(3/2)) - (b*B - A*c)/(4*b*c*x^(7/2)*(b + c*x^2)^2) - (7*b*B - 15*A*c)/(16*
b^2*c*x^(7/2)*(b + c*x^2)) + (11*c^(3/4)*(7*b*B - 15*A*c)*ArcTan[1 - (Sqrt[
2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(19/4)) - (11*c^(3/4)*(7*b*B -
15*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(19/4)
) + (11*c^(3/4)*(7*b*B - 15*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt
[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(19/4)) - (11*c^(3/4)*(7*b*B - 15*A*c)*Log[
Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(19/4
))
```

Rubi [A] time = 0.277669, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {1584, 457, 290, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{11c^{3/4}(7bB-15Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{19/4}}-\frac{11c^{3/4}(7bB-15Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{19/4}}+\frac{11c^{3/4}(7bB}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```


```
[Out] (11*(7*b*B - 15*A*c))/(112*b^3*c*x^(7/2)) - (11*(7*b*B - 15*A*c))/(48*b^4*x
^(3/2)) - (b*B - A*c)/(4*b*c*x^(7/2)*(b + c*x^2)^2) - (7*b*B - 15*A*c)/(16*
b^2*c*x^(7/2)*(b + c*x^2)) + (11*c^(3/4)*(7*b*B - 15*A*c)*ArcTan[1 - (Sqrt[
2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(19/4)) - (11*c^(3/4)*(7*b*B -
15*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(19/4)
) + (11*c^(3/4)*(7*b*B - 15*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt
[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(19/4)) - (11*c^(3/4)*(7*b*B - 15*A*c)*Log[
Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(19/4
))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{3/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{A+Bx^2}{x^{9/2}\left(b+cx^2\right)^3}\,dx$$

$$= -\frac{bB-Ac}{4bcx^{7/2}\left(b+cx^2\right)^2} + \frac{\left(-\frac{7bB}{2}+\frac{15Ac}{2}\right)\int \frac{1}{x^{9/2}\left(b+cx^2\right)^2}\,dx}{4bc}$$

$$= -\frac{bB-Ac}{4bcx^{7/2}\left(b+cx^2\right)^2} - \frac{7bB-15Ac}{16b^2cx^{7/2}\left(b+cx^2\right)} - \frac{(11(7bB-15Ac))\int \frac{1}{x^{9/2}\left(b+cx^2\right)}\,dx}{32b^2c}$$

$$= \frac{11(7bB-15Ac)}{112b^3cx^{7/2}} - \frac{bB-Ac}{4bcx^{7/2}\left(b+cx^2\right)^2} - \frac{7bB-15Ac}{16b^2cx^{7/2}\left(b+cx^2\right)} + \frac{(11(7bB-15Ac))\int \frac{1}{x^{5/2}\left(b+cx^2\right)}\,dx}{32b^3}$$

$$= \frac{11(7bB-15Ac)}{112b^3cx^{7/2}} - \frac{11(7bB-15Ac)}{48b^4x^{3/2}} - \frac{bB-Ac}{4bcx^{7/2}\left(b+cx^2\right)^2} - \frac{7bB-15Ac}{16b^2cx^{7/2}\left(b+cx^2\right)} - \frac{(11c(7bB-15Ac\cdots}{32\cdots}$$

$$= \frac{11(7bB-15Ac)}{112b^3cx^{7/2}} - \frac{11(7bB-15Ac)}{48b^4x^{3/2}} - \frac{bB-Ac}{4bcx^{7/2}\left(b+cx^2\right)^2} - \frac{7bB-15Ac}{16b^2cx^{7/2}\left(b+cx^2\right)} - \frac{(11c(7bB-15Ac\cdots}{\cdots}$$

$$= \frac{11(7bB-15Ac)}{112b^3cx^{7/2}} - \frac{11(7bB-15Ac)}{48b^4x^{3/2}} - \frac{bB-Ac}{4bcx^{7/2}\left(b+cx^2\right)^2} - \frac{7bB-15Ac}{16b^2cx^{7/2}\left(b+cx^2\right)} - \frac{(11c(7bB-15Ac\cdots}{\cdots}$$

$$= \frac{11(7bB-15Ac)}{112b^3cx^{7/2}} - \frac{11(7bB-15Ac)}{48b^4x^{3/2}} - \frac{bB-Ac}{4bcx^{7/2}\left(b+cx^2\right)^2} - \frac{7bB-15Ac}{16b^2cx^{7/2}\left(b+cx^2\right)} - \frac{\left(11\sqrt{c}(7bB-15\cdots\right.}{\cdots}$$

$$= \frac{11(7bB-15Ac)}{112b^3cx^{7/2}} - \frac{11(7bB-15Ac)}{48b^4x^{3/2}} - \frac{bB-Ac}{4bcx^{7/2}\left(b+cx^2\right)^2} - \frac{7bB-15Ac}{16b^2cx^{7/2}\left(b+cx^2\right)} + \frac{11c^{3/4}(7bB-15\cdots}{\cdots}$$

$$= \frac{11(7bB-15Ac)}{112b^3cx^{7/2}} - \frac{11(7bB-15Ac)}{48b^4x^{3/2}} - \frac{bB-Ac}{4bcx^{7/2}\left(b+cx^2\right)^2} - \frac{7bB-15Ac}{16b^2cx^{7/2}\left(b+cx^2\right)} + \frac{11c^{3/4}(7bB-15\cdots}{3\cdots}$$

Mathematica [A] time = 0.51115, size = 433, normalized size = 1.26

$$\frac{672Ab^{7/4}c^2\sqrt{x}}{\left(b+cx^2\right)^2} + \frac{3864Ab^{3/4}c^2\sqrt{x}}{b+cx^2} + \frac{5376Ab^{3/4}c}{x^{3/2}} - \frac{768Ab^{7/4}}{x^{7/2}} + 462\sqrt{2}c^{3/4}(7bB-15Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right) + 462\sqrt{2}c^{3/4}(15Ac-7bB\cdots$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-768*A*b^(7/4))/x^(7/2) - (1792*b^(7/4)*B)/x^(3/2) + (5376*A*b^(3/4)*c)/x
^(3/2) - (672*b^(11/4)*B*c*Sqrt[x])/(b + c*x^2)^2 + (672*A*b^(7/4)*c^2*Sqrt
[x])/(b + c*x^2)^2 - (2520*b^(7/4)*B*c*Sqrt[x])/(b + c*x^2) + (3864*A*b^(3/
4)*c^2*Sqrt[x])/(b + c*x^2) + 462*Sqrt[2]*c^(3/4)*(7*b*B - 15*A*c)*ArcTan[1
 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] + 462*Sqrt[2]*c^(3/4)*(-7*b*B + 15*A*
c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] + 1617*Sqrt[2]*b*B*c^(3/4)
*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] - 3465*Sqrt[2]*
A*c^(7/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] - 1617
*Sqrt[2]*b*B*c^(3/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c
]*x] + 3465*Sqrt[2]*A*c^(7/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x]
 + Sqrt[c]*x])/(2688*b^(19/4))
```

Maple [A] time = 0.02, size = 390, normalized size = 1.1

$$-\frac{2A}{7b^3}x^{-\frac{7}{2}}+2\frac{Ac}{b^4x^{3/2}}-\frac{2B}{3b^3}x^{-\frac{3}{2}}+\frac{23\,Ac^3}{16\,b^4\left(cx^2+b\right)^2}x^{\frac{5}{2}}-\frac{15\,c^2B}{16\,b^3\left(cx^2+b\right)^2}x^{\frac{5}{2}}+\frac{27\,Ac^2}{16\,b^3\left(cx^2+b\right)^2}\sqrt{x}-\frac{19\,Bc}{16\,b^2\left(cx^2+b\right)^2}\sqrt{x}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x)
```

```
[Out] -2/7*A/b^3/x^(7/2)+2/b^4/x^(3/2)*A*c-2/3/b^3/x^(3/2)*B+23/16/b^4*c^3/(c*x^2
+b)^2*x^(5/2)*A-15/16/b^3*c^2/(c*x^2+b)^2*x^(5/2)*B+27/16/b^3*c^2/(c*x^2+b)
^2*A*x^(1/2)-19/16/b^2*c/(c*x^2+b)^2*B*x^(1/2)+165/64/b^5*c^2*(b/c)^(1/4)*2
^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)+165/128/b^5*c^2*(b/c)^(1/4)*
2^(1/2)*A*ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(
1/2)*2^(1/2)+(b/c)^(1/2)))+165/64/b^5*c^2*(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1
/2)/(b/c)^(1/4)*x^(1/2)+1)-77/64/b^4*c*(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)
/(b/c)^(1/4)*x^(1/2)-1)-77/128/b^4*c*(b/c)^(1/4)*2^(1/2)*B*ln((x+(b/c)^(1/4
)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))
-77/64/b^4*c*(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.72965, size = 2063, normalized size = 6.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/1344*(924*(b^4*c^2*x^8 + 2*b^5*c*x^6 + b^6*x^4)*(-(2401*B^4*b^4*c^3 - 205
80*A*B^3*b^3*c^4 + 66150*A^2*B^2*b^2*c^5 - 94500*A^3*B*b*c^6 + 50625*A^4*c^
7)/b^19)^(1/4)*arctan((sqrt(b^10*sqrt(-(2401*B^4*b^4*c^3 - 20580*A*B^3*b^3*
c^4 + 66150*A^2*B^2*b^2*c^5 - 94500*A^3*B*b*c^6 + 50625*A^4*c^7)/b^19) + (4
9*B^2*b^2*c^2 - 210*A*B*b*c^3 + 225*A^2*c^4)*x)*b^14*(-(2401*B^4*b^4*c^3 -
20580*A*B^3*b^3*c^4 + 66150*A^2*B^2*b^2*c^5 - 94500*A^3*B*b*c^6 + 50625*A^4
*c^7)/b^19)^(3/4) + (7*B*b^15*c - 15*A*b^14*c^2)*sqrt(x)*(-(2401*B^4*b^4*c^
3 - 20580*A*B^3*b^3*c^4 + 66150*A^2*B^2*b^2*c^5 - 94500*A^3*B*b*c^6 + 50625
*A^4*c^7)/b^19)^(3/4))/(2401*B^4*b^4*c^3 - 20580*A*B^3*b^3*c^4 + 66150*A^2*
B^2*b^2*c^5 - 94500*A^3*B*b*c^6 + 50625*A^4*c^7)) + 231*(b^4*c^2*x^8 + 2*b^
5*c*x^6 + b^6*x^4)*(-(2401*B^4*b^4*c^3 - 20580*A*B^3*b^3*c^4 + 66150*A^2*B^
2*b^2*c^5 - 94500*A^3*B*b*c^6 + 50625*A^4*c^7)/b^19)^(1/4)*log(11*b^5*(-(24
01*B^4*b^4*c^3 - 20580*A*B^3*b^3*c^4 + 66150*A^2*B^2*b^2*c^5 - 94500*A^3*B*
b*c^6 + 50625*A^4*c^7)/b^19)^(1/4) - 11*(7*B*b*c - 15*A*c^2)*sqrt(x)) - 231
*(b^4*c^2*x^8 + 2*b^5*c*x^6 + b^6*x^4)*(-(2401*B^4*b^4*c^3 - 20580*A*B^3*b^
3*c^4 + 66150*A^2*B^2*b^2*c^5 - 94500*A^3*B*b*c^6 + 50625*A^4*c^7)/b^19)^(1
/4)*log(-11*b^5*(-(2401*B^4*b^4*c^3 - 20580*A*B^3*b^3*c^4 + 66150*A^2*B^2*b
^2*c^5 - 94500*A^3*B*b*c^6 + 50625*A^4*c^7)/b^19)^(1/4) - 11*(7*B*b*c - 15*
A*c^2)*sqrt(x)) - 4*(77*(7*B*b*c^2 - 15*A*c^3)*x^6 + 121*(7*B*b^2*c - 15*A*
b*c^2)*x^4 + 96*A*b^3 + 32*(7*B*b^3 - 15*A*b^2*c)*x^2)*sqrt(x))/(b^4*c^2*x^
8 + 2*b^5*c*x^6 + b^6*x^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.3088, size = 425, normalized size = 1.24

$$-\frac{11\sqrt{2}\left(7\left(bc^3\right)^{\frac{1}{4}}Bb-15\left(bc^3\right)^{\frac{1}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^5}-\frac{11\sqrt{2}\left(7\left(bc^3\right)^{\frac{1}{4}}Bb-15\left(bc^3\right)^{\frac{1}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right.\right.}{2\left(\right.}\right.}{64\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -11/64*sqrt(2)*(7*(b*c^3)^(1/4)*B*b - 15*(b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt
(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/b^5 - 11/64*sqrt(2)*(7*(
b*c^3)^(1/4)*B*b - 15*(b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)
^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^5 - 11/128*sqrt(2)*(7*(b*c^3)^(1/4)*B*b
- 15*(b*c^3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^
5 + 11/128*sqrt(2)*(7*(b*c^3)^(1/4)*B*b - 15*(b*c^3)^(1/4)*A*c)*log(-sqrt(2
)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^5 - 1/16*(15*B*b*c^2*x^(5/2) - 23*
A*c^3*x^(5/2) + 19*B*b^2*c*sqrt(x) - 27*A*b*c^2*sqrt(x))/((c*x^2 + b)^2*b^4
) - 2/21*(7*B*b*x^2 - 21*A*c*x^2 + 3*A*b)/(b^4*x^(7/2))
```

3.218 $\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=365

$$\frac{13c^{5/4}(9bB-17Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{21/4}}-\frac{13c^{5/4}(9bB-17Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{21/4}}-\frac{13c^{5/4}(9bB-}{}$$

```
[Out] (13*(9*b*B - 17*A*c))/(144*b^3*c*x^(9/2)) - (13*(9*b*B - 17*A*c))/(80*b^4*x
^(5/2)) + (13*c*(9*b*B - 17*A*c))/(16*b^5*Sqrt[x]) - (b*B - A*c)/(4*b*c*x^(
9/2)*(b + c*x^2)^2) - (9*b*B - 17*A*c)/(16*b^2*c*x^(9/2)*(b + c*x^2)) - (13
*c^(5/4)*(9*b*B - 17*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(3
2*Sqrt[2]*b^(21/4)) + (13*c^(5/4)*(9*b*B - 17*A*c)*ArcTan[1 + (Sqrt[2]*c^(1
/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(21/4)) + (13*c^(5/4)*(9*b*B - 17*A*c)
*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^
(21/4)) - (13*c^(5/4)*(9*b*B - 17*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4
)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(21/4))
```

Rubi [A] time = 0.319722, antiderivative size = 365, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {1584, 457, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{13c^{5/4}(9bB-17Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{21/4}}-\frac{13c^{5/4}(9bB-17Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{21/4}}-\frac{13c^{5/4}(9bB-}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```


```
[Out] (13*(9*b*B - 17*A*c))/(144*b^3*c*x^(9/2)) - (13*(9*b*B - 17*A*c))/(80*b^4*x
^(5/2)) + (13*c*(9*b*B - 17*A*c))/(16*b^5*Sqrt[x]) - (b*B - A*c)/(4*b*c*x^(
9/2)*(b + c*x^2)^2) - (9*b*B - 17*A*c)/(16*b^2*c*x^(9/2)*(b + c*x^2)) - (13
*c^(5/4)*(9*b*B - 17*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(3
2*Sqrt[2]*b^(21/4)) + (13*c^(5/4)*(9*b*B - 17*A*c)*ArcTan[1 + (Sqrt[2]*c^(1
/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(21/4)) + (13*c^(5/4)*(9*b*B - 17*A*c)
*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^
(21/4)) - (13*c^(5/4)*(9*b*B - 17*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4
)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(21/4))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{x}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{A+Bx^2}{x^{11/2}\left(b+cx^2\right)^3}\,dx$$

$$= -\frac{bB-Ac}{4bcx^{9/2}\left(b+cx^2\right)^2} + \frac{\left(-\frac{9bB}{2}+\frac{17Ac}{2}\right)\int \frac{1}{x^{11/2}\left(b+cx^2\right)^2}\,dx}{4bc}$$

$$= -\frac{bB-Ac}{4bcx^{9/2}\left(b+cx^2\right)^2} - \frac{9bB-17Ac}{16b^2cx^{9/2}\left(b+cx^2\right)} - \frac{(13(9bB-17Ac))\int \frac{1}{x^{11/2}\left(b+cx^2\right)}\,dx}{32b^2c}$$

$$= \frac{13(9bB-17Ac)}{144b^3cx^{9/2}} - \frac{bB-Ac}{4bcx^{9/2}\left(b+cx^2\right)^2} - \frac{9bB-17Ac}{16b^2cx^{9/2}\left(b+cx^2\right)} + \frac{(13(9bB-17Ac))\int \frac{1}{x^{7/2}\left(b+cx^2\right)}\,dx}{32b^3}$$

$$= \frac{13(9bB-17Ac)}{144b^3cx^{9/2}} - \frac{13(9bB-17Ac)}{80b^4x^{5/2}} - \frac{bB-Ac}{4bcx^{9/2}\left(b+cx^2\right)^2} - \frac{9bB-17Ac}{16b^2cx^{9/2}\left(b+cx^2\right)} - (13c(9bB-17A$$

$$= \frac{13(9bB-17Ac)}{144b^3cx^{9/2}} - \frac{13(9bB-17Ac)}{80b^4x^{5/2}} + \frac{13c(9bB-17Ac)}{16b^5\sqrt{x}} - \frac{bB-Ac}{4bcx^{9/2}\left(b+cx^2\right)^2} - \frac{9bB-17Ac}{16b^2cx^{9/2}\left(b+cx\right.}$$

$$= \frac{13(9bB-17Ac)}{144b^3cx^{9/2}} - \frac{13(9bB-17Ac)}{80b^4x^{5/2}} + \frac{13c(9bB-17Ac)}{16b^5\sqrt{x}} - \frac{bB-Ac}{4bcx^{9/2}\left(b+cx^2\right)^2} - \frac{9bB-17Ac}{16b^2cx^{9/2}\left(b+cx\right.}$$

$$= \frac{13(9bB-17Ac)}{144b^3cx^{9/2}} - \frac{13(9bB-17Ac)}{80b^4x^{5/2}} + \frac{13c(9bB-17Ac)}{16b^5\sqrt{x}} - \frac{bB-Ac}{4bcx^{9/2}\left(b+cx^2\right)^2} - \frac{9bB-17Ac}{16b^2cx^{9/2}\left(b+cx\right.}$$

$$= \frac{13(9bB-17Ac)}{144b^3cx^{9/2}} - \frac{13(9bB-17Ac)}{80b^4x^{5/2}} + \frac{13c(9bB-17Ac)}{16b^5\sqrt{x}} - \frac{bB-Ac}{4bcx^{9/2}\left(b+cx^2\right)^2} - \frac{9bB-17Ac}{16b^2cx^{9/2}\left(b+cx\right.}$$

$$= \frac{13(9bB-17Ac)}{144b^3cx^{9/2}} - \frac{13(9bB-17Ac)}{80b^4x^{5/2}} + \frac{13c(9bB-17Ac)}{16b^5\sqrt{x}} - \frac{bB-Ac}{4bcx^{9/2}\left(b+cx^2\right)^2} - \frac{9bB-17Ac}{16b^2cx^{9/2}\left(b+cx\right.}$$

$$= \frac{13(9bB-17Ac)}{144b^3cx^{9/2}} - \frac{13(9bB-17Ac)}{80b^4x^{5/2}} + \frac{13c(9bB-17Ac)}{16b^5\sqrt{x}} - \frac{bB-Ac}{4bcx^{9/2}\left(b+cx^2\right)^2} - \frac{9bB-17Ac}{16b^2cx^{9/2}\left(b+cx\right.}$$

Mathematica [C] time = 0.502143, size = 216, normalized size = 0.59

$$\frac{2c^2x^{3/2}(2bB-3Ac)\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{cx^2}{b}\right)}{3b^6}+\frac{2c^2x^{3/2}(bB-Ac)\,{}_2F_1\left(\frac{3}{4},3;\frac{7}{4};-\frac{cx^2}{b}\right)}{3b^6}-\frac{2(bB-3Ac)}{5b^4x^{5/2}}+\frac{6c(bB-2Ac)}{b^5\sqrt{x}}-\frac{2A}{9b^3x^{9/2}}-$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (-2*A)/(9*b^3*x^(9/2)) - (2*(b*B - 3*A*c))/(5*b^4*x^(5/2)) + (6*c*(b*B - 2*
A*c))/(b^5*Sqrt[x]) - (3*c^(5/4)*(b*B - 2*A*c)*ArcTan[(c^(1/4)*Sqrt[x])/(-b
)^(1/4)])/(-b)^(21/4) + (3*c^(5/4)*(b*B - 2*A*c)*ArcTanh[(c^(1/4)*Sqrt[x])/
(-b)^(1/4)])/(-b)^(21/4) + (2*c^2*(2*b*B - 3*A*c)*x^(3/2)*Hypergeometric2F1
[3/4, 2, 7/4, -((c*x^2)/b)])/(3*b^6) + (2*c^2*(b*B - A*c)*x^(3/2)*Hypergeom
etric2F1[3/4, 3, 7/4, -((c*x^2)/b)])/(3*b^6)
```

Maple [A] time = 0.021, size = 414, normalized size = 1.1

$$-\frac{2A}{9b^3}x^{-\frac{9}{2}}+\frac{6Ac}{5b^4}x^{-\frac{5}{2}}-\frac{2B}{5b^3}x^{-\frac{5}{2}}-12\frac{Ac^2}{b^5\sqrt{x}}+6\frac{Bc}{b^4\sqrt{x}}-\frac{29c^4A}{16b^5\left(cx^2+b\right)^2}x^{\frac{7}{2}}+\frac{21c^3B}{16b^4\left(cx^2+b\right)^2}x^{\frac{7}{2}}-\frac{33Ac^3}{16b^4\left(cx^2+b\right)^2}x^{\frac{3}{2}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^3,x)
```

```
[Out] -2/9*A/b^3/x^(9/2)+6/5/b^4/x^(5/2)*A*c-2/5/b^3/x^(5/2)*B-12*c^2/b^5/x^(1/2)
*A+6*c/b^4/x^(1/2)*B-29/16/b^5*c^4/(c*x^2+b)^2*A*x^(7/2)+21/16/b^4*c^3/(c*x
^2+b)^2*B*x^(7/2)-33/16/b^4*c^3/(c*x^2+b)^2*x^(3/2)*A+25/16/b^3*c^2/(c*x^2+
b)^2*x^(3/2)*B-221/128/b^5*c^2/(b/c)^(1/4)*2^(1/2)*A*ln((x-(b/c)^(1/4)*x^(1
/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))-221/6
4/b^5*c^2/(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-221/6
4/b^5*c^2/(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)+117/1
28/b^4*c/(b/c)^(1/4)*2^(1/2)*B*ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2
))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+117/64/b^4*c/(b/c)^(1/4)*2^
(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+117/64/b^4*c/(b/c)^(1/4)*2^(1
/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.72978, size = 2684, normalized size = 7.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/2880*(2340*(b^5*c^2*x^9 + 2*b^6*c*x^7 + b^7*x^5)*(-(6561*B^4*b^4*c^5 - 49
572*A*B^3*b^3*c^6 + 140454*A^2*B^2*b^2*c^7 - 176868*A^3*B*b*c^8 + 83521*A^4
*c^9)/b^21)^(1/4)*arctan((sqrt((531441*B^6*b^6*c^8 - 6022998*A*B^5*b^5*c^9
+ 28441935*A^2*B^4*b^4*c^10 - 71631540*A^3*B^3*b^3*c^11 + 101478015*A^4*B^2
*b^2*c^12 - 76672278*A^5*B*b*c^13 + 24137569*A^6*c^14)*x - (6561*B^4*b^15*c
^5 - 49572*A*B^3*b^14*c^6 + 140454*A^2*B^2*b^13*c^7 - 176868*A^3*B*b^12*c^8
 + 83521*A^4*b^11*c^9)*sqrt(-(6561*B^4*b^4*c^5 - 49572*A*B^3*b^3*c^6 + 1404
54*A^2*B^2*b^2*c^7 - 176868*A^3*B*b*c^8 + 83521*A^4*c^9)/b^21))*b^5*(-(6561
*B^4*b^4*c^5 - 49572*A*B^3*b^3*c^6 + 140454*A^2*B^2*b^2*c^7 - 176868*A^3*B*
b*c^8 + 83521*A^4*c^9)/b^21)^(1/4) + (729*B^3*b^8*c^4 - 4131*A*B^2*b^7*c^5
+ 7803*A^2*B*b^6*c^6 - 4913*A^3*b^5*c^7)*sqrt(x)*(-(6561*B^4*b^4*c^5 - 4957
2*A*B^3*b^3*c^6 + 140454*A^2*B^2*b^2*c^7 - 176868*A^3*B*b*c^8 + 83521*A^4*c
^9)/b^21)^(1/4))/(6561*B^4*b^4*c^5 - 49572*A*B^3*b^3*c^6 + 140454*A^2*B^2*b
^2*c^7 - 176868*A^3*B*b*c^8 + 83521*A^4*c^9)) - 585*(b^5*c^2*x^9 + 2*b^6*c*
x^7 + b^7*x^5)*(-(6561*B^4*b^4*c^5 - 49572*A*B^3*b^3*c^6 + 140454*A^2*B^2*b
^2*c^7 - 176868*A^3*B*b*c^8 + 83521*A^4*c^9)/b^21)^(1/4)*log(2197*b^16*(-(6
561*B^4*b^4*c^5 - 49572*A*B^3*b^3*c^6 + 140454*A^2*B^2*b^2*c^7 - 176868*A^3
*B*b*c^8 + 83521*A^4*c^9)/b^21)^(3/4) - 2197*(729*B^3*b^3*c^4 - 4131*A*B^2*
b^2*c^5 + 7803*A^2*B*b*c^6 - 4913*A^3*c^7)*sqrt(x)) + 585*(b^5*c^2*x^9 + 2*
b^6*c*x^7 + b^7*x^5)*(-(6561*B^4*b^4*c^5 - 49572*A*B^3*b^3*c^6 + 140454*A^2
*B^2*b^2*c^7 - 176868*A^3*B*b*c^8 + 83521*A^4*c^9)/b^21)^(1/4)*log(-2197*b^
16*(-(6561*B^4*b^4*c^5 - 49572*A*B^3*b^3*c^6 + 140454*A^2*B^2*b^2*c^7 - 176
868*A^3*B*b*c^8 + 83521*A^4*c^9)/b^21)^(3/4) - 2197*(729*B^3*b^3*c^4 - 4131
*A*B^2*b^2*c^5 + 7803*A^2*B*b*c^6 - 4913*A^3*c^7)*sqrt(x)) + 4*(585*(9*B*b*
```

```
c^3 - 17*A*c^4)*x^8 + 1053*(9*B*b^2*c^2 - 17*A*b*c^3)*x^6 - 160*A*b^4 + 416
*(9*B*b^3*c - 17*A*b^2*c^2)*x^4 - 32*(9*B*b^4 - 17*A*b^3*c)*x^2)*sqrt(x))/(
b^5*c^2*x^9 + 2*b^6*c*x^7 + b^7*x^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*x**(1/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.38072, size = 474, normalized size = 1.3

$$\frac{13\sqrt{2}\left(9\,(bc^3)^{\frac{3}{4}}Bb-17\,(bc^3)^{\frac{3}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^6c}+\frac{13\sqrt{2}\left(9\,(bc^3)^{\frac{3}{4}}Bb-17\,(bc^3)^{\frac{3}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}\right.}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.}{64\,b^6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 13/64*sqrt(2)*(9*(b*c^3)^(3/4)*B*b - 17*(b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(
2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^6*c) + 13/64*sqrt(2)*(
9*(b*c^3)^(3/4)*B*b - 17*(b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b
/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^6*c) - 13/128*sqrt(2)*(9*(b*c^3)^(3/
4)*B*b - 17*(b*c^3)^(3/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b
/c))/(b^6*c) + 13/128*sqrt(2)*(9*(b*c^3)^(3/4)*B*b - 17*(b*c^3)^(3/4)*A*c)*
log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^6*c) + 1/16*(21*B*b*c^
3*x^(7/2) - 29*A*c^4*x^(7/2) + 25*B*b^2*c^2*x^(3/2) - 33*A*b*c^3*x^(3/2))/(
(c*x^2 + b)^2*b^5) + 2/45*(135*B*b*c*x^4 - 270*A*c^2*x^4 - 9*B*b^2*x^2 + 27
*A*b*c*x^2 - 5*A*b^2)/(b^5*x^(9/2))
```

3.219 $\int \frac{A+Bx^2}{\sqrt{x}\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=365

$$-\frac{15c^{7/4}(11bB-19Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{23/4}}+\frac{15c^{7/4}(11bB-19Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{23/4}}-\frac{15c^{7/4}(}{}$$

```
[Out] (15*(11*b*B - 19*A*c))/(176*b^3*c*x^(11/2)) - (15*(11*b*B - 19*A*c))/(112*b
^4*x^(7/2)) + (5*c*(11*b*B - 19*A*c))/(16*b^5*x^(3/2)) - (b*B - A*c)/(4*b*c
*x^(11/2)*(b + c*x^2)^2) - (11*b*B - 19*A*c)/(16*b^2*c*x^(11/2)*(b + c*x^2)
) - (15*c^(7/4)*(11*b*B - 19*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1
/4)])/(32*Sqrt[2]*b^(23/4)) + (15*c^(7/4)*(11*b*B - 19*A*c)*ArcTan[1 + (Sqr
t[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(23/4)) - (15*c^(7/4)*(11*b*B
 - 19*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*
Sqrt[2]*b^(23/4)) + (15*c^(7/4)*(11*b*B - 19*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(
1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(23/4))
```

Rubi [A] time = 0.320727, antiderivative size = 365, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {1584, 457, 290, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{15c^{7/4}(11bB-19Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{23/4}}+\frac{15c^{7/4}(11bB-19Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{23/4}}-\frac{15c^{7/4}(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(Sqrt[x]*(b*x^2 + c*x^4)^3),x]
```


```
[Out] (15*(11*b*B - 19*A*c))/(176*b^3*c*x^(11/2)) - (15*(11*b*B - 19*A*c))/(112*b
^4*x^(7/2)) + (5*c*(11*b*B - 19*A*c))/(16*b^5*x^(3/2)) - (b*B - A*c)/(4*b*c
*x^(11/2)*(b + c*x^2)^2) - (11*b*B - 19*A*c)/(16*b^2*c*x^(11/2)*(b + c*x^2)
) - (15*c^(7/4)*(11*b*B - 19*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1
/4)])/(32*Sqrt[2]*b^(23/4)) + (15*c^(7/4)*(11*b*B - 19*A*c)*ArcTan[1 + (Sqr
t[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(23/4)) - (15*c^(7/4)*(11*b*B
 - 19*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*
Sqrt[2]*b^(23/4)) + (15*c^(7/4)*(11*b*B - 19*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(
1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(23/4))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+Bx^2}{\sqrt{x}\left(bx^2+cx^4\right)^3}\,dx = \int \frac{A+Bx^2}{x^{13/2}\left(b+cx^2\right)^3}\,dx$$

$$= -\frac{bB-Ac}{4bcx^{11/2}\left(b+cx^2\right)^2} + \frac{\left(-\frac{11bB}{2}+\frac{19Ac}{2}\right)\int \frac{1}{x^{13/2}\left(b+cx^2\right)^2}\,dx}{4bc}$$

$$= -\frac{bB-Ac}{4bcx^{11/2}\left(b+cx^2\right)^2} - \frac{11bB-19Ac}{16b^2cx^{11/2}\left(b+cx^2\right)} - \frac{(15(11bB-19Ac))\int \frac{1}{x^{13/2}\left(b+cx^2\right)}\,dx}{32b^2c}$$

$$= \frac{15(11bB-19Ac)}{176b^3cx^{11/2}} - \frac{bB-Ac}{4bcx^{11/2}\left(b+cx^2\right)^2} - \frac{11bB-19Ac}{16b^2cx^{11/2}\left(b+cx^2\right)} + \frac{(15(11bB-19Ac))\int \frac{1}{x^{9/2}\left(b+cx^2\right)}}{32b^3}$$

$$= \frac{15(11bB-19Ac)}{176b^3cx^{11/2}} - \frac{15(11bB-19Ac)}{112b^4x^{7/2}} - \frac{bB-Ac}{4bcx^{11/2}\left(b+cx^2\right)^2} - \frac{11bB-19Ac}{16b^2cx^{11/2}\left(b+cx^2\right)} - \frac{(15c(11bB}{}$$

$$= \frac{15(11bB-19Ac)}{176b^3cx^{11/2}} - \frac{15(11bB-19Ac)}{112b^4x^{7/2}} + \frac{5c(11bB-19Ac)}{16b^5x^{3/2}} - \frac{bB-Ac}{4bcx^{11/2}\left(b+cx^2\right)^2} - \frac{11bB-19}{16b^2cx^{11/2}\left(b\right.}$$

$$= \frac{15(11bB-19Ac)}{176b^3cx^{11/2}} - \frac{15(11bB-19Ac)}{112b^4x^{7/2}} + \frac{5c(11bB-19Ac)}{16b^5x^{3/2}} - \frac{bB-Ac}{4bcx^{11/2}\left(b+cx^2\right)^2} - \frac{11bB-19}{16b^2cx^{11/2}\left(b\right.}$$

$$= \frac{15(11bB-19Ac)}{176b^3cx^{11/2}} - \frac{15(11bB-19Ac)}{112b^4x^{7/2}} + \frac{5c(11bB-19Ac)}{16b^5x^{3/2}} - \frac{bB-Ac}{4bcx^{11/2}\left(b+cx^2\right)^2} - \frac{11bB-19}{16b^2cx^{11/2}\left(b\right.}$$

$$= \frac{15(11bB-19Ac)}{176b^3cx^{11/2}} - \frac{15(11bB-19Ac)}{112b^4x^{7/2}} + \frac{5c(11bB-19Ac)}{16b^5x^{3/2}} - \frac{bB-Ac}{4bcx^{11/2}\left(b+cx^2\right)^2} - \frac{11bB-19}{16b^2cx^{11/2}\left(b\right.}$$

$$= \frac{15(11bB-19Ac)}{176b^3cx^{11/2}} - \frac{15(11bB-19Ac)}{112b^4x^{7/2}} + \frac{5c(11bB-19Ac)}{16b^5x^{3/2}} - \frac{bB-Ac}{4bcx^{11/2}\left(b+cx^2\right)^2} - \frac{11bB-19}{16b^2cx^{11/2}\left(b\right.}$$

$$= \frac{15(11bB-19Ac)}{176b^3cx^{11/2}} - \frac{15(11bB-19Ac)}{112b^4x^{7/2}} + \frac{5c(11bB-19Ac)}{16b^5x^{3/2}} - \frac{bB-Ac}{4bcx^{11/2}\left(b+cx^2\right)^2} - \frac{11bB-19}{16b^2cx^{11/2}\left(b\right.}$$

Mathematica [A] time = 0.581209, size = 467, normalized size = 1.28

$$-\frac{19096Ab^{3/4}c^3\sqrt{x}}{b+cx^2}-\frac{2464Ab^{7/4}c^3\sqrt{x}}{\left(b+cx^2\right)^2}-\frac{39424Ab^{3/4}c^2}{x^{3/2}}+\frac{8448Ab^{7/4}c}{x^{7/2}}-\frac{1792Ab^{11/4}}{x^{11/2}}+2310\sqrt{2}c^{7/4}(19Ac-11bB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)+2$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(Sqrt[x]*(b*x^2 + c*x^4)^3),x]
```

```
[Out] ((-1792*A*b^(11/4))/x^(11/2) - (2816*b^(11/4)*B)/x^(7/2) + (8448*A*b^(7/4)*
c)/x^(7/2) + (19712*b^(7/4)*B*c)/x^(3/2) - (39424*A*b^(3/4)*c^2)/x^(3/2) +
(2464*b^(11/4)*B*c^2*Sqrt[x])/(b + c*x^2)^2 - (2464*A*b^(7/4)*c^3*Sqrt[x])/
(b + c*x^2)^2 + (14168*b^(7/4)*B*c^2*Sqrt[x])/(b + c*x^2) - (19096*A*b^(3/4
)*c^3*Sqrt[x])/(b + c*x^2) + 2310*Sqrt[2]*c^(7/4)*(-11*b*B + 19*A*c)*ArcTan
[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] + 2310*Sqrt[2]*c^(7/4)*(11*b*B - 19
*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] - 12705*Sqrt[2]*b*B*c^(
7/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] + 21945*Sqr
t[2]*A*c^(11/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]
+ 12705*Sqrt[2]*b*B*c^(7/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] +
 Sqrt[c]*x] - 21945*Sqrt[2]*A*c^(11/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4
)*Sqrt[x] + Sqrt[c]*x])/(9856*b^(23/4))
```

Maple [A] time = 0.022, size = 420, normalized size = 1.2

$$-\frac{2A}{11b^3}x^{-\frac{11}{2}}+\frac{6Ac}{7b^4}x^{-\frac{7}{2}}-\frac{2B}{7b^3}x^{-\frac{7}{2}}-4\frac{Ac^2}{b^5x^{3/2}}+2\frac{Bc}{b^4x^{3/2}}-\frac{31c^4A}{16b^5\left(cx^2+b\right)^2}x^{\frac{5}{2}}+\frac{23c^3B}{16b^4\left(cx^2+b\right)^2}x^{\frac{5}{2}}-\frac{35Ac^3}{16b^4\left(cx^2+b\right)^2}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(c*x^4+b*x^2)^3/x^(1/2),x)
```

```
[Out] -2/11*A/b^3/x^(11/2)+6/7/b^4/x^(7/2)*A*c-2/7/b^3/x^(7/2)*B-4*c^2/b^5/x^(3/2
)*A+2*c/b^4/x^(3/2)*B-31/16/b^5*c^4/(c*x^2+b)^2*x^(5/2)*A+23/16/b^4*c^3/(c*
x^2+b)^2*x^(5/2)*B-35/16/b^4*c^3/(c*x^2+b)^2*A*x^(1/2)+27/16/b^3*c^2/(c*x^2
+b)^2*B*x^(1/2)-285/64/b^6*c^3*(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(
1/4)*x^(1/2)+1)-285/64/b^6*c^3*(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(
1/4)*x^(1/2)-1)-285/128/b^6*c^3*(b/c)^(1/4)*2^(1/2)*A*ln((x+(b/c)^(1/4)*x^(
1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+165/
64/b^5*c^2*(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+165/
64/b^5*c^2*(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)+165/
```

```
128/b^5*c^2*(b/c)^(1/4)*2^(1/2)*B*ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(
1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.5948, size = 2233, normalized size = 6.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4928*(4620*(b^5*c^2*x^10 + 2*b^6*c*x^8 + b^7*x^6)*(-(14641*B^4*b^4*c^7 -
 101156*A*B^3*b^3*c^8 + 262086*A^2*B^2*b^2*c^9 - 301796*A^3*B*b*c^10 + 1303
21*A^4*c^11)/b^23)^(1/4)*arctan((sqrt(b^12*sqrt(-(14641*B^4*b^4*c^7 - 10115
6*A*B^3*b^3*c^8 + 262086*A^2*B^2*b^2*c^9 - 301796*A^3*B*b*c^10 + 130321*A^4
*c^11)/b^23) + (121*B^2*b^2*c^4 - 418*A*B*b*c^5 + 361*A^2*c^6)*x)*b^17*(-(1
4641*B^4*b^4*c^7 - 101156*A*B^3*b^3*c^8 + 262086*A^2*B^2*b^2*c^9 - 301796*A
^3*B*b*c^10 + 130321*A^4*c^11)/b^23)^(3/4) + (11*B*b^18*c^2 - 19*A*b^17*c^3
)*sqrt(x)*(-(14641*B^4*b^4*c^7 - 101156*A*B^3*b^3*c^8 + 262086*A^2*B^2*b^2*
c^9 - 301796*A^3*B*b*c^10 + 130321*A^4*c^11)/b^23)^(3/4))/(14641*B^4*b^4*c^
7 - 101156*A*B^3*b^3*c^8 + 262086*A^2*B^2*b^2*c^9 - 301796*A^3*B*b*c^10 + 1
30321*A^4*c^11)) + 1155*(b^5*c^2*x^10 + 2*b^6*c*x^8 + b^7*x^6)*(-(14641*B^4
*b^4*c^7 - 101156*A*B^3*b^3*c^8 + 262086*A^2*B^2*b^2*c^9 - 301796*A^3*B*b*c
^10 + 130321*A^4*c^11)/b^23)^(1/4)*log(15*b^6*(-(14641*B^4*b^4*c^7 - 101156
*A*B^3*b^3*c^8 + 262086*A^2*B^2*b^2*c^9 - 301796*A^3*B*b*c^10 + 130321*A^4*
c^11)/b^23)^(1/4) - 15*(11*B*b*c^2 - 19*A*c^3)*sqrt(x)) - 1155*(b^5*c^2*x^1
0 + 2*b^6*c*x^8 + b^7*x^6)*(-(14641*B^4*b^4*c^7 - 101156*A*B^3*b^3*c^8 + 26
2086*A^2*B^2*b^2*c^9 - 301796*A^3*B*b*c^10 + 130321*A^4*c^11)/b^23)^(1/4)*l
og(-15*b^6*(-(14641*B^4*b^4*c^7 - 101156*A*B^3*b^3*c^8 + 262086*A^2*B^2*b^2
*c^9 - 301796*A^3*B*b*c^10 + 130321*A^4*c^11)/b^23)^(1/4) - 15*(11*B*b*c^2
```

```
- 19*A*c^3)*sqrt(x)) - 4*(385*(11*B*b*c^3 - 19*A*c^4)*x^8 + 605*(11*B*b^2*c
^2 - 19*A*b*c^3)*x^6 - 224*A*b^4 + 160*(11*B*b^3*c - 19*A*b^2*c^2)*x^4 - 32
*(11*B*b^4 - 19*A*b^3*c)*x^2)*sqrt(x))/(b^5*c^2*x^10 + 2*b^6*c*x^8 + b^7*x^
6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2)**3/x**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.39349, size = 474, normalized size = 1.3

$$\frac{15\sqrt{2}\left(11\left(bc^3\right)^{\frac{1}{4}}Bbc-19\left(bc^3\right)^{\frac{1}{4}}Ac^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^6}+\frac{15\sqrt{2}\left(11\left(bc^3\right)^{\frac{1}{4}}Bbc-19\left(bc^3\right)^{\frac{1}{4}}Ac^2\right)\arctan\left(-\frac{\sqrt{2}}{}\right.}{64\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 15/64*sqrt(2)*(11*(b*c^3)^(1/4)*B*b*c - 19*(b*c^3)^(1/4)*A*c^2)*arctan(1/2*
sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/b^6 + 15/64*sqrt(2)*
(11*(b*c^3)^(1/4)*B*b*c - 19*(b*c^3)^(1/4)*A*c^2)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^6 + 15/128*sqrt(2)*(11*(b*c^3)^
(1/4)*B*b*c - 19*(b*c^3)^(1/4)*A*c^2)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x +
 sqrt(b/c))/b^6 - 15/128*sqrt(2)*(11*(b*c^3)^(1/4)*B*b*c - 19*(b*c^3)^(1/4)
*A*c^2)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^6 + 1/16*(23*B*
b*c^3*x^(5/2) - 31*A*c^4*x^(5/2) + 27*B*b^2*c^2*sqrt(x) - 35*A*b*c^3*sqrt(x
))/((c*x^2 + b)^2*b^5) + 2/77*(77*B*b*c*x^4 - 154*A*c^2*x^4 - 11*B*b^2*x^2
+ 33*A*b*c*x^2 - 7*A*b^2)/(b^5*x^(11/2))
```

3.220 $\int x^{5/2}\left(A+Bx^2\right)\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=243

$$-\frac{2b^{11/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(3bB-5Ac)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{231c^{13/4}\sqrt{bx^2+cx^4}}+\frac{4b^2\sqrt{bx^2+cx^4}(3bB-5Ac)}{231c^3\sqrt{x}}-\frac{4bx^{3/2}\sqrt{bx^2}}{}$$

```
[Out] (4*b^2*(3*b*B - 5*A*c)*Sqrt[b*x^2 + c*x^4])/(231*c^3*Sqrt[x]) - (4*b*(3*b*B
 - 5*A*c)*x^(3/2)*Sqrt[b*x^2 + c*x^4])/(385*c^2) - (2*(3*b*B - 5*A*c)*x^(7/
2)*Sqrt[b*x^2 + c*x^4])/(55*c) + (2*B*x^(3/2)*(b*x^2 + c*x^4)^(3/2))/(15*c)
 - (2*b^(11/4)*(3*b*B - 5*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sq
rt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/
(231*c^(13/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.390354, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2039, 2021, 2024, 2032, 329, 220}

$$\frac{4b^2\sqrt{bx^2+cx^4}(3bB-5Ac)}{231c^3\sqrt{x}}-\frac{2b^{11/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(3bB-5Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{231c^{13/4}\sqrt{bx^2+cx^4}}-\frac{4bx^{3/2}\sqrt{bx^2+cx^4}(3bB}{385c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```



```
[Out] (4*b^2*(3*b*B - 5*A*c)*Sqrt[b*x^2 + c*x^4])/(231*c^3*Sqrt[x]) - (4*b*(3*b*B
 - 5*A*c)*x^(3/2)*Sqrt[b*x^2 + c*x^4])/(385*c^2) - (2*(3*b*B - 5*A*c)*x^(7/
2)*Sqrt[b*x^2 + c*x^4])/(55*c) + (2*B*x^(3/2)*(b*x^2 + c*x^4)^(3/2))/(15*c)
 - (2*b^(11/4)*(3*b*B - 5*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sq
rt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/
(231*c^(13/4)*Sqrt[b*x^2 + c*x^4])
```


Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
```

```
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int x^{5/2}\left(A+Bx^2\right)\sqrt{bx^2+cx^4}\,dx = \frac{2Bx^{3/2}\left(bx^2+cx^4\right)^{3/2}}{15c} - \frac{\left(2\left(\frac{9bB}{2}-\frac{15Ac}{2}\right)\right)\int x^{5/2}\sqrt{bx^2+cx^4}\,dx}{15c}$$

$$= -\frac{2(3bB-5Ac)x^{7/2}\sqrt{bx^2+cx^4}}{55c} + \frac{2Bx^{3/2}\left(bx^2+cx^4\right)^{3/2}}{15c} - \frac{(2b(3bB-5Ac))\int \frac{x^{9/2}}{\sqrt{bx^2+cx^4}}\,d}{55c}$$

$$= -\frac{4b(3bB-5Ac)x^{3/2}\sqrt{bx^2+cx^4}}{385c^2} - \frac{2(3bB-5Ac)x^{7/2}\sqrt{bx^2+cx^4}}{55c} + \frac{2Bx^{3/2}\left(bx^2+cx^4\right)^{3}}{15c}$$

$$= \frac{4b^2(3bB-5Ac)\sqrt{bx^2+cx^4}}{231c^3\sqrt{x}} - \frac{4b(3bB-5Ac)x^{3/2}\sqrt{bx^2+cx^4}}{385c^2} - \frac{2(3bB-5Ac)x^{7/2}\sqrt{bx^2}}{55c}$$

$$= \frac{4b^2(3bB-5Ac)\sqrt{bx^2+cx^4}}{231c^3\sqrt{x}} - \frac{4b(3bB-5Ac)x^{3/2}\sqrt{bx^2+cx^4}}{385c^2} - \frac{2(3bB-5Ac)x^{7/2}\sqrt{bx^2}}{55c}$$

$$= \frac{4b^2(3bB-5Ac)\sqrt{bx^2+cx^4}}{231c^3\sqrt{x}} - \frac{4b(3bB-5Ac)x^{3/2}\sqrt{bx^2+cx^4}}{385c^2} - \frac{2(3bB-5Ac)x^{7/2}\sqrt{bx^2}}{55c}$$

$$= \frac{4b^2(3bB-5Ac)\sqrt{bx^2+cx^4}}{231c^3\sqrt{x}} - \frac{4b(3bB-5Ac)x^{3/2}\sqrt{bx^2+cx^4}}{385c^2} - \frac{2(3bB-5Ac)x^{7/2}\sqrt{bx^2}}{55c}$$

Mathematica [C] time = 0.161978, size = 136, normalized size = 0.56

$$\frac{2\sqrt{x^2\left(b+cx^2\right)}\left(\left(b+cx^2\right)\sqrt{\frac{cx^2}{b}+1}\left(-3bc\left(25A+21Bx^2\right)+7c^2x^2\left(15A+11Bx^2\right)+45b^2B\right)+15b^2(5Ac-3bB)\,{}_2F_1\left(-\frac{1}{2},\right.\right.}{1155c^3\sqrt{x}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[x^2*(b + c*x^2)]*((b + c*x^2)*Sqrt[1 + (c*x^2)/b]*(45*b^2*B + 7*c^2
*x^2*(15*A + 11*B*x^2) - 3*b*c*(25*A + 21*B*x^2)) + 15*b^2*(-3*b*B + 5*A*c)
*Hypergeometric2F1[-1/2, 1/4, 5/4, -((c*x^2)/b)]))/(1155*c^3*Sqrt[x]*Sqrt[1
 + (c*x^2)/b])
```

Maple [A] time = 0.053, size = 307, normalized size = 1.3

$$\frac{2}{\left(1155\,c x^{2}+1155\,b\right) c^{4}}\sqrt{c x^{4}+b x^{2}}\left(77\,B x^{9} c^{5}+105\,A x^{7} c^{5}+91\,B x^{7} b c^{4}+25\,A\sqrt{-b c}\sqrt{\frac{c x+\sqrt{-b c}}{\sqrt{-b c}}}\sqrt{2}\sqrt{\frac{-c x+\sqrt{-b c}}{\sqrt{-b c}}}\sqrt{-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 2/1155*(c*x^4+b*x^2)^(1/2)/x^(3/2)/(c*x^2+b)*(77*B*x^9*c^5+105*A*x^7*c^5+91
*B*x^7*b*c^4+25*A*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1
/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*Elli
pticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^3*c+135*A*x^5*
b*c^4-15*B*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-
c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF((
(c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^4-4*B*x^5*b^2*c^3-20*
A*x^3*b^2*c^3+12*B*x^3*b^3*c^2-50*A*x*b^3*c^2+30*B*x*b^4*c)/c^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c x^{4}+b x^{2}}\left(B x^{2}+A\right) x^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*x^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(B x^{4}+A x^{2}\right)\sqrt{c x^{4}+b x^{2}}\sqrt{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^4 + A*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(B*x**2+A)*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4 + bx^2} (Bx^2 + A) x^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*x^(5/2), x)
```

3.221 $\int x^{3/2}\left(A+Bx^2\right)\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=369

$$\frac{2b^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(7bB-13Ac)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{195c^{11/4}\sqrt{bx^2+cx^4}}+\frac{4b^2x^{3/2}\left(b+cx^2\right)(7bB-13Ac)}{195c^{5/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{4b^{9/4}x\left(\right.}{}$$

```
[Out] (4*b^2*(7*b*B - 13*A*c)*x^(3/2)*(b + c*x^2))/(195*c^(5/2)*(Sqrt[b] + Sqrt[c
]*x)*Sqrt[b*x^2 + c*x^4]) - (4*b*(7*b*B - 13*A*c)*Sqrt[x]*Sqrt[b*x^2 + c*x^
4])/(585*c^2) - (2*(7*b*B - 13*A*c)*x^(5/2)*Sqrt[b*x^2 + c*x^4])/(117*c) +
(2*B*Sqrt[x]*(b*x^2 + c*x^4)^(3/2))/(13*c) - (4*b^(9/4)*(7*b*B - 13*A*c)*x*
(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2
*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(195*c^(11/4)*Sqrt[b*x^2 + c*x^4]
) + (2*b^(9/4)*(7*b*B - 13*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(S
qrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])
/(195*c^(11/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.435648, antiderivative size = 369, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2039, 2021, 2024, 2032, 329, 305, 220, 1196}

$$\frac{4b^2x^{3/2}\left(b+cx^2\right)(7bB-13Ac)}{195c^{5/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{2b^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(7bB-13Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{195c^{11/4}\sqrt{bx^2+cx^4}}-\frac{4b^{9/4}x\left(\sqrt{b}+\sqrt{c}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```



```
[Out] (4*b^2*(7*b*B - 13*A*c)*x^(3/2)*(b + c*x^2))/(195*c^(5/2)*(Sqrt[b] + Sqrt[c
]*x)*Sqrt[b*x^2 + c*x^4]) - (4*b*(7*b*B - 13*A*c)*Sqrt[x]*Sqrt[b*x^2 + c*x^
4])/(585*c^2) - (2*(7*b*B - 13*A*c)*x^(5/2)*Sqrt[b*x^2 + c*x^4])/(117*c) +
(2*B*Sqrt[x]*(b*x^2 + c*x^4)^(3/2))/(13*c) - (4*b^(9/4)*(7*b*B - 13*A*c)*x*
(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2
*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(195*c^(11/4)*Sqrt[b*x^2 + c*x^4]
) + (2*b^(9/4)*(7*b*B - 13*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(S
qrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])
/(195*c^(11/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
```

```
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int x^{3/2}\left(A+Bx^2\right)\sqrt{bx^2+cx^4}\,dx = \frac{2B\sqrt{x}\left(bx^2+cx^4\right)^{3/2}}{13c} - \frac{\left(2\left(\frac{7bB}{2}-\frac{13Ac}{2}\right)\right)\int x^{3/2}\sqrt{bx^2+cx^4}\,dx}{13c}$$

$$= -\frac{2(7bB-13Ac)x^{5/2}\sqrt{bx^2+cx^4}}{117c} + \frac{2B\sqrt{x}\left(bx^2+cx^4\right)^{3/2}}{13c} - \frac{(2b(7bB-13Ac))\int \frac{x^{7/2}}{\sqrt{bx^2+c\cdots}}}{117c}$$

$$= -\frac{4b(7bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{585c^2} - \frac{2(7bB-13Ac)x^{5/2}\sqrt{bx^2+cx^4}}{117c} + \frac{2B\sqrt{x}\left(bx^2+cx\cdots\right.}{13c}$$

$$= -\frac{4b(7bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{585c^2} - \frac{2(7bB-13Ac)x^{5/2}\sqrt{bx^2+cx^4}}{117c} + \frac{2B\sqrt{x}\left(bx^2+cx\cdots\right.}{13c}$$

$$= -\frac{4b(7bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{585c^2} - \frac{2(7bB-13Ac)x^{5/2}\sqrt{bx^2+cx^4}}{117c} + \frac{2B\sqrt{x}\left(bx^2+cx\cdots\right.}{13c}$$

$$= -\frac{4b(7bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{585c^2} - \frac{2(7bB-13Ac)x^{5/2}\sqrt{bx^2+cx^4}}{117c} + \frac{2B\sqrt{x}\left(bx^2+cx\cdots\right.}{13c}$$

$$= \frac{4b^2(7bB-13Ac)x^{3/2}\left(b+cx^2\right)}{195c^{5/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{4b(7bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{585c^2} - \frac{2(7bB-13Ac)x\cdots}{117\cdots}$$

Mathematica [C] time = 0.128747, size = 111, normalized size = 0.3

$$\frac{2\sqrt{x}\sqrt{x^2\left(b+cx^2\right)}\left(b(7bB-13Ac)\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)-\left(b+cx^2\right)\sqrt{\frac{cx^2}{b}+1}\left(-13Ac+7bB-9Bcx^2\right)\right)}{117c^2\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[x]*Sqrt[x^2*(b + c*x^2)]*(-((b + c*x^2)*Sqrt[1 + (c*x^2)/b]*(7*b*B
- 13*A*c - 9*B*c*x^2)) + b*(7*b*B - 13*A*c)*Hypergeometric2F1[-1/2, 3/4, 7/
4, -((c*x^2)/b)]))/(117*c^2*Sqrt[1 + (c*x^2)/b])
```

Maple [A] time = 0.03, size = 446, normalized size = 1.2

$$-\frac{2}{\left(585\,cx^2+585\,b\right)c^3}\sqrt{cx^4+bx^2}\left(-45\,Bx^8c^4-65\,Ax^6c^4-55\,Bx^6bc^3+78\,A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\mathrm{El}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -2/585*(c*x^4+b*x^2)^(1/2)/x^(3/2)/(c*x^2+b)/c^3*(-45*B*x^8*c^4-65*A*x^6*c^
4-55*B*x^6*b*c^3+78*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*
x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c
*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^3*c-39*A*((c*x+(-b*c)^(
1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*
(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)
,1/2*2^(1/2))*b^3*c-42*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((
-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(
((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^4+21*B*((c*x+(-b*c)^
(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)
*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2
),1/2*2^(1/2))*b^4-91*A*x^4*b*c^3+4*B*x^4*b^2*c^2-26*A*x^2*b^2*c^2+14*B*x^2
*b^3*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4 + bx^2}(Bx^2 + A)x^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*x^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{cx^4 + bx^2}(Bx^3 + Ax)\sqrt{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(B*x^3 + A*x)*sqrt(x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(B*x**2+A)*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4 + bx^2}(Bx^2 + A)x^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*x^(3/2), x)
```

3.222 $\int \sqrt{x}\left(A+Bx^2\right)\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=204

$$\frac{2b^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(5bB-11Ac)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{231c^{9/4}\sqrt{bx^2+cx^4}}-\frac{4b\sqrt{bx^2+cx^4}(5bB-11Ac)}{231c^2\sqrt{x}}-\frac{2x^{3/2}\sqrt{bx^2 -}}{}$$

```
[Out] (-4*b*(5*b*B - 11*A*c)*Sqrt[b*x^2 + c*x^4])/(231*c^2*Sqrt[x]) - (2*(5*b*B -
 11*A*c)*x^(3/2)*Sqrt[b*x^2 + c*x^4])/(77*c) + (2*B*(b*x^2 + c*x^4)^(3/2))/
(11*c*Sqrt[x]) + (2*b^(7/4)*(5*b*B - 11*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(
b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^
(1/4)], 1/2])/(231*c^(9/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.29968, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2039, 2021, 2024, 2032, 329, 220}

$$\frac{2b^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(5bB-11Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{231c^{9/4}\sqrt{bx^2+cx^4}}-\frac{4b\sqrt{bx^2+cx^4}(5bB-11Ac)}{231c^2\sqrt{x}}-\frac{2x^{3/2}\sqrt{bx^2+cx^4}(5bI}{77c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```


```
[Out] (-4*b*(5*b*B - 11*A*c)*Sqrt[b*x^2 + c*x^4])/(231*c^2*Sqrt[x]) - (2*(5*b*B -
 11*A*c)*x^(3/2)*Sqrt[b*x^2 + c*x^4])/(77*c) + (2*B*(b*x^2 + c*x^4)^(3/2))/
(11*c*Sqrt[x]) + (2*b^(7/4)*(5*b*B - 11*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(
b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^
(1/4)], 1/2])/(231*c^(9/4)*Sqrt[b*x^2 + c*x^4])
```


Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
```

```
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \sqrt{x}\left(A+Bx^2\right)\sqrt{bx^2+cx^4}\,dx = \frac{2B\left(bx^2+cx^4\right)^{3/2}}{11c\sqrt{x}} - \frac{\left(2\left(\frac{5bB}{2}-\frac{11Ac}{2}\right)\right)\int\sqrt{x}\sqrt{bx^2+cx^4}\,dx}{11c}$$

$$= -\frac{2(5bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c} + \frac{2B\left(bx^2+cx^4\right)^{3/2}}{11c\sqrt{x}} - \frac{(2b(5bB-11Ac))\int\frac{x^{5/2}}{\sqrt{bx^2+cx^4}}\,d}{77c}$$

$$= -\frac{4b(5bB-11Ac)\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}} - \frac{2(5bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c} + \frac{2B\left(bx^2+cx^4\right)^{3/2}}{11c\sqrt{x}} +$$

$$= -\frac{4b(5bB-11Ac)\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}} - \frac{2(5bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c} + \frac{2B\left(bx^2+cx^4\right)^{3/2}}{11c\sqrt{x}} +$$

$$= -\frac{4b(5bB-11Ac)\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}} - \frac{2(5bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c} + \frac{2B\left(bx^2+cx^4\right)^{3/2}}{11c\sqrt{x}} +$$

$$= -\frac{4b(5bB-11Ac)\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}} - \frac{2(5bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c} + \frac{2B\left(bx^2+cx^4\right)^{3/2}}{11c\sqrt{x}} +$$

Mathematica [C] time = 0.129684, size = 111, normalized size = 0.54

$$\frac{2\sqrt{x^2\left(b+cx^2\right)}\left(b(5bB-11Ac)\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{cx^2}{b}\right)-\left(b+cx^2\right)\sqrt{\frac{cx^2}{b}+1}\left(-11Ac+5bB-7Bcx^2\right)\right)}{77c^2\sqrt{x}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[x^2*(b + c*x^2)]*(-((b + c*x^2)*Sqrt[1 + (c*x^2)/b]*(5*b*B - 11*A*c
 - 7*B*c*x^2)) + b*(5*b*B - 11*A*c)*Hypergeometric2F1[-1/2, 1/4, 5/4, -((c*
x^2)/b)]))/(77*c^2*Sqrt[x]*Sqrt[1 + (c*x^2)/b])
```

Maple [A] time = 0.023, size = 283, normalized size = 1.4

$$-\frac{2}{\left(231\,cx^2+231\,b\right)c^3}\sqrt{cx^4+bx^2}\left(-21\,Bx^7c^4+11\,A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*x^(1/2)*(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -2/231*(c*x^4+b*x^2)^(1/2)/x^(3/2)/(c*x^2+b)*(-21*B*x^7*c^4+11*A*((c*x+(-b*
c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1
/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(
1/2),1/2*2^(1/2))*(-b*c)^(1/2)*b^2*c-33*A*x^5*c^4-5*B*((c*x+(-b*c)^(1/2))/(
-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(
-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^
(1/2))*(-b*c)^(1/2)*b^3-27*B*x^5*b*c^3-55*A*x^3*b*c^3+4*B*x^3*b^2*c^2-22*A*
x*b^2*c^2+10*B*x*b^3*c)/c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4 + bx^2}\left(Bx^2 + A\right)\sqrt{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*sqrt(x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^4 + bx^2}\left(Bx^2 + A\right)\sqrt{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*sqrt(x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x}\sqrt{x^2\left(b + cx^2\right)}\left(A + Bx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*x**(1/2)*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(x)*sqrt(x**2*(b + c*x**2))*(A + B*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4 + bx^2}\left(Bx^2 + A\right)\sqrt{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*sqrt(x), x)
```

3.223 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx$

Optimal. Leaf size=326

$$-\frac{2b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(bB-3Ac)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{15c^{7/4}\sqrt{bx^2+cx^4}}+\frac{4b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(bB-3Ac)E}{15c^{7/4}\sqrt{bx^2+cx^4}}$$

```
[Out] (-4*b*(b*B - 3*A*c)*x^(3/2)*(b + c*x^2))/(15*c^(3/2)*(Sqrt[b] + Sqrt[c]*x)*
Sqrt[b*x^2 + c*x^4]) - (2*(b*B - 3*A*c)*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(15*c)
 + (2*B*(b*x^2 + c*x^4)^(3/2))/(9*c*x^(3/2)) + (4*b^(5/4)*(b*B - 3*A*c)*x*(
Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*
ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*c^(7/4)*Sqrt[b*x^2 + c*x^4]) -
 (2*b^(5/4)*(b*B - 3*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b]
 + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*c
^(7/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.374683, antiderivative size = 326, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2039, 2021, 2032, 329, 305, 220, 1196}

$$-\frac{2b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(bB-3Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{7/4}\sqrt{bx^2+cx^4}}+\frac{4b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(bB-3Ac)E\left(2\tan^{-1}\right.}{15c^{7/4}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/Sqrt[x],x]
```


```
[Out] (-4*b*(b*B - 3*A*c)*x^(3/2)*(b + c*x^2))/(15*c^(3/2)*(Sqrt[b] + Sqrt[c]*x)*
Sqrt[b*x^2 + c*x^4]) - (2*(b*B - 3*A*c)*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(15*c)
 + (2*B*(b*x^2 + c*x^4)^(3/2))/(9*c*x^(3/2)) + (4*b^(5/4)*(b*B - 3*A*c)*x*(
Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*
ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*c^(7/4)*Sqrt[b*x^2 + c*x^4]) -
 (2*b^(5/4)*(b*B - 3*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b]
 + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*c
^(7/4)*Sqrt[b*x^2 + c*x^4])
```


Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\left(A+Bx^2\right)\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx = \frac{2B\left(bx^2+cx^4\right)^{3/2}}{9cx^{3/2}} - \frac{\left(2\left(\frac{3bB}{2}-\frac{9Ac}{2}\right)\right)\int \frac{\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx}{9c}$$

$$= -\frac{2(bB-3Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{15c} + \frac{2B\left(bx^2+cx^4\right)^{3/2}}{9cx^{3/2}} - \frac{(2b(bB-3Ac))\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{15c}$$

$$= -\frac{2(bB-3Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{15c} + \frac{2B\left(bx^2+cx^4\right)^{3/2}}{9cx^{3/2}} - \frac{\left(2b(bB-3Ac)x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{15c\sqrt{bx^2+cx^4}}$$

$$= -\frac{2(bB-3Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{15c} + \frac{2B\left(bx^2+cx^4\right)^{3/2}}{9cx^{3/2}} - \frac{\left(4b(bB-3Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int\right.}{15c\sqrt{bx^2+cx^4}}$$

$$= -\frac{2(bB-3Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{15c} + \frac{2B\left(bx^2+cx^4\right)^{3/2}}{9cx^{3/2}} - \frac{\left(4b^{3/2}(bB-3Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\right.}{15c^{3/2}\sqrt{bx^2+cx^4}}$$

$$= -\frac{4b(bB-3Ac)x^{3/2}\left(b+cx^2\right)}{15c^{3/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{2(bB-3Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{15c} + \frac{2B\left(bx^2+cx^4\right)^{3/2}}{9cx^{3/2}} + \frac{4b^{5/}}{}$$

Mathematica [C] time = 0.0823903, size = 94, normalized size = 0.29

$$\frac{2\sqrt{x}\sqrt{x^2\left(b+cx^2\right)}\left((3Ac-bB)\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)+B\sqrt{\frac{cx^2}{b}+1}\left(b+cx^2\right)\right)}{9c\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*Sqrt[x^2*(b + c*x^2)]*(B*(b + c*x^2)*Sqrt[1 + (c*x^2)/b] + (-(b*
B) + 3*A*c)*Hypergeometric2F1[-1/2, 3/4, 7/4, -((c*x^2)/b)]))/(9*c*Sqrt[1 +
```

```
(c*x^2)/b])
```

Maple [A] time = 0.021, size = 422, normalized size = 1.3

$$\frac{2}{\left(45\,c x^2+45\,b\right)c^2}\sqrt{c x^4+b x^2}\left(5\,B c^3 x^6+18\,A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\,\sqrt{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(1/2),x)
```

```
[Out] 2/45*(c*x^4+b*x^2)^(1/2)/x^(3/2)/(c*x^2+b)/c^2*(5*B*c^3*x^6+18*A*((c*x+(-b*
c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1
/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(
1/2),1/2*2^(1/2))*b^2*c-9*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)
*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*Ellipti
cF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c-6*B*((c*x+(-b
*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(
1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^
(1/2),1/2*2^(1/2))*b^3+3*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*
((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*Elliptic
F(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^3+9*A*x^4*c^3+7*B*
x^4*b*c^2+9*A*x^2*b*c^2+2*B*x^2*b^2*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c x^4+b x^2}\left(B x^2+A\right)}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/sqrt(x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\left(Bx^2+A\right)}{\sqrt{x}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/sqrt(x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x^2\left(b+cx^2\right)}\left(A+Bx^2\right)}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**(1/2),x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/sqrt(x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{cx^4+bx^2}\left(Bx^2+A\right)}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/sqrt(x), x)
```

3.224 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{3/2}} dx$

Optimal. Leaf size=165

$$-\frac{2b^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(bB-7Ac)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{21c^{5/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}(bB-7Ac)}{21c\sqrt{x}}+\frac{2B\left(bx^2+cx^4\right)^{3/2}}{7cx^{5/2}}$$

```
[Out] (-2*(b*B - 7*A*c)*Sqrt[b*x^2 + c*x^4])/(21*c*Sqrt[x]) + (2*B*(b*x^2 + c*x^4
)^(3/2))/(7*c*x^(5/2)) - (2*b^(3/4)*(b*B - 7*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*S
qrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x
])/b^(1/4)], 1/2])/(21*c^(5/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.254143, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2039, 2021, 2032, 329, 220}

$$-\frac{2b^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(bB-7Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21c^{5/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}(bB-7Ac)}{21c\sqrt{x}}+\frac{2B\left(bx^2+cx^4\right)^{3/2}}{7cx^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(3/2),x]
```


```
[Out] (-2*(b*B - 7*A*c)*Sqrt[b*x^2 + c*x^4])/(21*c*Sqrt[x]) + (2*B*(b*x^2 + c*x^4
)^(3/2))/(7*c*x^(5/2)) - (2*b^(3/4)*(b*B - 7*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*S
qrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x
])/b^(1/4)], 1/2])/(21*c^(5/4)*Sqrt[b*x^2 + c*x^4])
```


Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx = \frac{2B(bx^2+cx^4)^{3/2}}{7cx^{5/2}} - \frac{\left(2\left(\frac{bB}{2}-\frac{7Ac}{2}\right)\right)\int \frac{\sqrt{bx^2+cx^4}}{x^{3/2}}dx}{7c}$$

$$= -\frac{2(bB-7Ac)\sqrt{bx^2+cx^4}}{21c\sqrt{x}} + \frac{2B(bx^2+cx^4)^{3/2}}{7cx^{5/2}} - \frac{(2b(bB-7Ac))\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}dx}{21c}$$

$$= -\frac{2(bB-7Ac)\sqrt{bx^2+cx^4}}{21c\sqrt{x}} + \frac{2B(bx^2+cx^4)^{3/2}}{7cx^{5/2}} - \frac{\left(2b(bB-7Ac)x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}d}{21c\sqrt{bx^2+cx^4}}$$

$$= -\frac{2(bB-7Ac)\sqrt{bx^2+cx^4}}{21c\sqrt{x}} + \frac{2B(bx^2+cx^4)^{3/2}}{7cx^{5/2}} - \frac{\left(4b(bB-7Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{}{\sqrt{b}}\right.}{21c\sqrt{bx^2+cx^4}}$$

$$= -\frac{2(bB-7Ac)\sqrt{bx^2+cx^4}}{21c\sqrt{x}} + \frac{2B(bx^2+cx^4)^{3/2}}{7cx^{5/2}} - \frac{2b^{3/4}(bB-7Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{(\sqrt{b}+\sqrt{c}}}}{21c^{5/4}\sqrt{bx^2+cx}}$$

Mathematica [C] time = 0.0503193, size = 94, normalized size = 0.57

$$\frac{2\sqrt{x^2(b+cx^2)}\left((7Ac-bB)\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{cx^2}{b}\right)+B\sqrt{\frac{cx^2}{b}+1}\,(b+cx^2)\right)}{7c\sqrt{x}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(3/2),x]
```

```
[Out] (2*Sqrt[x^2*(b + c*x^2)]*(B*(b + c*x^2)*Sqrt[1 + (c*x^2)/b] + (-(b*B) + 7*A
*c)*Hypergeometric2F1[-1/2, 1/4, 5/4, -((c*x^2)/b)]))/(7*c*Sqrt[x]*Sqrt[1 +
 (c*x^2)/b])
```

Maple [A] time = 0.02, size = 257, normalized size = 1.6

$$\frac{2}{(21\,cx^2+21\,b)\,c^2}\sqrt{cx^4+bx^2}\left(7\,A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\text{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)\sqrt{-bc}bc-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(3/2),x)
```

```
[Out] 2/21*(c*x^4+b*x^2)^(1/2)/x^(3/2)/(c*x^2+b)*(7*A*((c*x+(-b*c)^(1/2))/(-b*c)^
(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^
(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))
*(-b*c)^(1/2)*b*c-B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+
(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x
+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(-b*c)^(1/2)*b^2+3*B*c^3*x^
5+7*A*x^3*c^3+5*B*x^3*b*c^2+7*A*b*c^2*x+2*B*b^2*c*x)/c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2}\left(Bx^2 + A\right)}{x^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2}\left(Bx^2 + A\right)}{x^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2\left(b + cx^2\right)}\left(A + Bx^2\right)}{x^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**(3/2),x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2}\left(Bx^2 + A\right)}{x^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(3/2), x)
```

3.225 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{5/2}}\,dx$

Optimal. Leaf size=323

$$\frac{2\sqrt[4]{b}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(5Ac+bB)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5c^{3/4}\sqrt{bx^2+cx^4}}-\frac{4\sqrt[4]{b}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(5Ac+bB)E\left(2\right.}{5c^{3/4}\sqrt{bx^2+cx^4}}$$

```
[Out] (4*(b*B + 5*A*c)*x^(3/2)*(b + c*x^2))/(5*Sqrt[c]*(Sqrt[b] + Sqrt[c]*x)*Sqrt
[b*x^2 + c*x^4]) + (2*(b*B + 5*A*c)*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(5*b) - (2
*A*(b*x^2 + c*x^4)^(3/2))/(b*x^(7/2)) - (4*b^(1/4)*(b*B + 5*A*c)*x*(Sqrt[b]
 + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[
(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*c^(3/4)*Sqrt[b*x^2 + c*x^4]) + (2*b^(1
/4)*(b*B + 5*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[
c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*c^(3/4)*Sq
rt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.361928, antiderivative size = 323, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2038, 2021, 2032, 329, 305, 220, 1196}

$$\frac{2\sqrt[4]{b}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(5Ac+bB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{3/4}\sqrt{bx^2+cx^4}}-\frac{4\sqrt[4]{b}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(5Ac+bB)E\left(2\tan^{-1}\left(\right.\right.}{5c^{3/4}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(5/2),x]
```

```
[Out] (4*(b*B + 5*A*c)*x^(3/2)*(b + c*x^2))/(5*Sqrt[c]*(Sqrt[b] + Sqrt[c]*x)*Sqrt
[b*x^2 + c*x^4]) + (2*(b*B + 5*A*c)*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(5*b) - (2
*A*(b*x^2 + c*x^4)^(3/2))/(b*x^(7/2)) - (4*b^(1/4)*(b*B + 5*A*c)*x*(Sqrt[b]
 + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[
(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*c^(3/4)*Sqrt[b*x^2 + c*x^4]) + (2*b^(1
/4)*(b*B + 5*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[
c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*c^(3/4)*Sq
rt[b*x^2 + c*x^4])
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\left(A+Bx^2\right)\sqrt{bx^2+cx^4}}{x^{5/2}}\,dx = -\frac{2A\left(bx^2+cx^4\right)^{3/2}}{bx^{7/2}} - \frac{\left(2\left(-\frac{bB}{2}-\frac{5Ac}{2}\right)\right)\int \frac{\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx}{b}$$

$$= \frac{2(bB+5Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5b} - \frac{2A\left(bx^2+cx^4\right)^{3/2}}{bx^{7/2}} + \frac{1}{5}(2(bB+5Ac))\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx$$

$$= \frac{2(bB+5Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5b} - \frac{2A\left(bx^2+cx^4\right)^{3/2}}{bx^{7/2}} + \frac{\left(2(bB+5Ac)x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{5\sqrt{bx^2+cx^4}}$$

$$= \frac{2(bB+5Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5b} - \frac{2A\left(bx^2+cx^4\right)^{3/2}}{bx^{7/2}} + \frac{\left(4(bB+5Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{}{\sqrt{b}}\right.}{5\sqrt{bx^2+cx^4}}$$

$$= \frac{2(bB+5Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5b} - \frac{2A\left(bx^2+cx^4\right)^{3/2}}{bx^{7/2}} + \frac{\left(4\sqrt{b}(bB+5Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int\right.}{5\sqrt{c}\sqrt{bx^2+cx^4}}$$

$$= \frac{4(bB+5Ac)x^{3/2}\left(b+cx^2\right)}{5\sqrt{c}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} + \frac{2(bB+5Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5b} - \frac{2A\left(bx^2+cx^4\right)^{3/2}}{bx^{7/2}} - \frac{4\sqrt[4]{b}(b}{}$$

Mathematica [C] time = 0.0434003, size = 97, normalized size = 0.3

$$\frac{2\sqrt{x^2\left(b+cx^2\right)}\left(x^2(5Ac+bB)\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)-3A\left(b+cx^2\right)\sqrt{\frac{cx^2}{b}+1}\right)}{3bx^{3/2}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(5/2),x]
```

```
[Out] (2*Sqrt[x^2*(b + c*x^2)]*(-3*A*(b + c*x^2)*Sqrt[1 + (c*x^2)/b] + (b*B + 5*A
*c)*x^2*Hypergeometric2F1[-1/2, 3/4, 7/4, -((c*x^2)/b)]))/(3*b*x^(3/2)*Sqrt
[1 + (c*x^2)/b])
```

Maple [A] time = 0.03, size = 399, normalized size = 1.2

$$\frac{2}{\left(5\,c x^{2}+5\,b\right) c}\sqrt{c x^{4}+b x^{2}}\left(10\,A\sqrt{\frac{c x+\sqrt{-b c}}{\sqrt{-b c}}}\sqrt{2}\sqrt{\frac{-c x+\sqrt{-b c}}{\sqrt{-b c}}}\sqrt{-\frac{c x}{\sqrt{-b c}}}\mathrm{EllipticE}\left(\sqrt{\frac{c x+\sqrt{-b c}}{\sqrt{-b c}}},1/2\,\sqrt{2}\right) b c-5\,A\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(5/2),x)
```

```
[Out] 2/5*(c*x^4+b*x^2)^(1/2)/x^(3/2)/(c*x^2+b)*(10*A*((c*x+(-b*c)^(1/2))/(-b*c)^
(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^
(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))
*b*c-5*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2
))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/
2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*c+2*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/
2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/
2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^
2-B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-
b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(
-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^2+B*c^2*x^4-5*A*x^2*c^2+B*x^2*b*c-5*A*b*c
)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c x^{4}+b x^{2}}\left(B x^{2}+A\right)}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\left(Bx^2+A\right)}{x^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(5/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2\left(b+cx^2\right)}\left(A+Bx^2\right)}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**(5/2),x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4+bx^2}\left(Bx^2+A\right)}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(5/2), x)
```

3.226 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{7/2}}\,dx$

Optimal. Leaf size=163

$$\frac{2x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(Ac+bB)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt[4]{c}\sqrt{bx^2+cx^4}}+\frac{2\sqrt{bx^2+cx^4}(Ac+bB)}{3b\sqrt{x}}-\frac{2A\left(bx^2+cx^4\right)^{3/2}}{3bx^{9/2}}$$

```
[Out] (2*(b*B + A*c)*Sqrt[b*x^2 + c*x^4])/(3*b*Sqrt[x]) - (2*A*(b*x^2 + c*x^4)^(3
/2))/(3*b*x^(9/2)) + (2*(b*B + A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2
)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1
/2])/(3*b^(1/4)*c^(1/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.242626, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2038, 2021, 2032, 329, 220}

$$\frac{2\sqrt{bx^2+cx^4}(Ac+bB)}{3b\sqrt{x}}+\frac{2x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(Ac+bB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt[4]{c}\sqrt{bx^2+cx^4}}-\frac{2A\left(bx^2+cx^4\right)^{3/2}}{3bx^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(7/2),x]
```

```
[Out] (2*(b*B + A*c)*Sqrt[b*x^2 + c*x^4])/(3*b*Sqrt[x]) - (2*A*(b*x^2 + c*x^4)^(3
/2))/(3*b*x^(9/2)) + (2*(b*B + A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2
)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1
/2])/(3*b^(1/4)*c^(1/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
```

```
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{\left(A+Bx^2\right)\sqrt{bx^2+cx^4}}{x^{7/2}}\,dx = -\frac{2A\left(bx^2+cx^4\right)^{3/2}}{3bx^{9/2}} + -\frac{\left(2\left(-\frac{3bB}{2}-\frac{3Ac}{2}\right)\right)\int \frac{\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx}{3b}$$
$$= \frac{2(bB+Ac)\sqrt{bx^2+cx^4}}{3b\sqrt{x}} - \frac{2A\left(bx^2+cx^4\right)^{3/2}}{3bx^{9/2}} + \frac{1}{3}(2(bB+Ac))\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx$$
$$= \frac{2(bB+Ac)\sqrt{bx^2+cx^4}}{3b\sqrt{x}} - \frac{2A\left(bx^2+cx^4\right)^{3/2}}{3bx^{9/2}} + \frac{\left(2(bB+Ac)x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{3\sqrt{bx^2+cx^4}}$$
$$= \frac{2(bB+Ac)\sqrt{bx^2+cx^4}}{3b\sqrt{x}} - \frac{2A\left(bx^2+cx^4\right)^{3/2}}{3bx^{9/2}} + \frac{\left(4(bB+Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\right.}{3\sqrt{bx^2+cx^4}}$$
$$= \frac{2(bB+Ac)\sqrt{bx^2+cx^4}}{3b\sqrt{x}} - \frac{2A\left(bx^2+cx^4\right)^{3/2}}{3bx^{9/2}} + \frac{2(bB+Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\right.}{3\sqrt[4]{b}\sqrt[4]{c}\sqrt{bx^2+cx^4}}$$

Mathematica [C] time = 0.0435645, size = 97, normalized size = 0.6

$$\frac{2\sqrt{x^2\left(b+cx^2\right)}\left(3x^2(Ac+bB)\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{cx^2}{b}\right)-A\left(b+cx^2\right)\sqrt{\frac{cx^2}{b}+1}\right)}{3bx^{5/2}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(7/2),x]
```

```
[Out] (2*Sqrt[x^2*(b + c*x^2)]*(-(A*(b + c*x^2)*Sqrt[1 + (c*x^2)/b]) + 3*(b*B + A
*c)*x^2*Hypergeometric2F1[-1/2, 1/4, 5/4, -((c*x^2)/b)]))/(3*b*x^(5/2)*Sqrt
[1 + (c*x^2)/b])
```

Maple [A] time = 0.025, size = 239, normalized size = 1.5

$$\frac{2}{\left(3\,cx^2+3\,b\right)c}\sqrt{cx^4+bx^2}\left(A\sqrt{\left(cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}}\sqrt{2}\sqrt{\left(-cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}}\sqrt{-cx\frac{1}{\sqrt{-bc}}}\text{EllipticF}\left(\sqrt{\left(cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(7/2),x)
```

```
[Out] 2/3*(c*x^4+b*x^2)^(1/2)/x^(5/2)/(c*x^2+b)*(A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/
2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/
2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(-
b*c)^(1/2)*x*c+B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b
*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-
b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(-b*c)^(1/2)*x*b+B*c^2*x^4-A*x
^2*c^2+B*x^2*b*c-A*b*c)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2}\left(Bx^2 + A\right)}{x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2}\left(Bx^2 + A\right)}{x^{\frac{7}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(7/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2\left(b + cx^2\right)}\left(A + Bx^2\right)}{x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**(7/2),x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**(7/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2}\left(Bx^2 + A\right)}{x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(7/2), x)
```

3.227 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{9/2}}\,dx$

Optimal. Leaf size=328

$$\frac{2\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(Ac+5bB)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5b^{3/4}\sqrt{bx^2+cx^4}}-\frac{4\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(Ac+5bB)E\left(2\right.}{5b^{3/4}\sqrt{bx^2+cx^4}}$$

```
[Out] (4*Sqrt[c]*(5*b*B + A*c)*x^(3/2)*(b + c*x^2))/(5*b*(Sqrt[b] + Sqrt[c]*x)*Sq
rt[b*x^2 + c*x^4]) - (2*(5*b*B + A*c)*Sqrt[b*x^2 + c*x^4])/(5*b*x^(3/2)) -
(2*A*(b*x^2 + c*x^4)^(3/2))/(5*b*x^(11/2)) - (4*c^(1/4)*(5*b*B + A*c)*x*(Sq
rt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*Ar
cTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*b^(3/4)*Sqrt[b*x^2 + c*x^4]) + (2
*c^(1/4)*(5*b*B + A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] +
Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*b^(3/
4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.374005, antiderivative size = 328, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2038, 2020, 2032, 329, 305, 220, 1196}

$$\frac{2\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(Ac+5bB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{bx^2+cx^4}}-\frac{4\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(Ac+5bB)E\left(2\tan^{-1}\left(\right.\right.}{5b^{3/4}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(9/2),x]
```



```
[Out] (4*Sqrt[c]*(5*b*B + A*c)*x^(3/2)*(b + c*x^2))/(5*b*(Sqrt[b] + Sqrt[c]*x)*Sq
rt[b*x^2 + c*x^4]) - (2*(5*b*B + A*c)*Sqrt[b*x^2 + c*x^4])/(5*b*x^(3/2)) -
(2*A*(b*x^2 + c*x^4)^(3/2))/(5*b*x^(11/2)) - (4*c^(1/4)*(5*b*B + A*c)*x*(Sq
rt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*Ar
cTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*b^(3/4)*Sqrt[b*x^2 + c*x^4]) + (2
*c^(1/4)*(5*b*B + A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] +
Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*b^(3/
4)*Sqrt[b*x^2 + c*x^4])
```


Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{9/2}}\,dx = -\frac{2A(bx^2+cx^4)^{3/2}}{5bx^{11/2}} - \frac{\left(2\left(-\frac{5bB}{2}-\frac{Ac}{2}\right)\right)\int \frac{\sqrt{bx^2+cx^4}}{x^{5/2}}\,dx}{5b}$$

$$= -\frac{2(5bB+Ac)\sqrt{bx^2+cx^4}}{5bx^{3/2}} - \frac{2A(bx^2+cx^4)^{3/2}}{5bx^{11/2}} + \frac{(2c(5bB+Ac))\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{5b}$$

$$= -\frac{2(5bB+Ac)\sqrt{bx^2+cx^4}}{5bx^{3/2}} - \frac{2A(bx^2+cx^4)^{3/2}}{5bx^{11/2}} + \frac{\left(2c(5bB+Ac)x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{5b\sqrt{bx^2+cx^4}}$$

$$= -\frac{2(5bB+Ac)\sqrt{bx^2+cx^4}}{5bx^{3/2}} - \frac{2A(bx^2+cx^4)^{3/2}}{5bx^{11/2}} + \frac{\left(4c(5bB+Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{b+\cdots}}\right.}{5b\sqrt{bx^2+cx^4}}$$

$$= -\frac{2(5bB+Ac)\sqrt{bx^2+cx^4}}{5bx^{3/2}} - \frac{2A(bx^2+cx^4)^{3/2}}{5bx^{11/2}} + \frac{\left(4\sqrt{c}(5bB+Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \cdots\right.}{5\sqrt{b}\sqrt{bx^2+cx^4}}$$

$$= \frac{4\sqrt{c}(5bB+Ac)x^{3/2}(b+cx^2)}{5b(\sqrt{b}+\sqrt{c}x)\sqrt{bx^2+cx^4}} - \frac{2(5bB+Ac)\sqrt{bx^2+cx^4}}{5bx^{3/2}} - \frac{2A(bx^2+cx^4)^{3/2}}{5bx^{11/2}} - \frac{4\sqrt[4]{c}(5bB\cdots}{\cdots}$$

Mathematica [C] time = 0.041659, size = 96, normalized size = 0.29

$$-\frac{2\sqrt{x^2(b+cx^2)}\left(x^2(Ac+5bB)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};-\frac{cx^2}{b}\right)+A(b+cx^2)\sqrt{\frac{cx^2}{b}+1}\right)}{5bx^{7/2}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(9/2),x]
```

```
[Out] (-2*Sqrt[x^2*(b + c*x^2)]*(A*(b + c*x^2)*Sqrt[1 + (c*x^2)/b] + (5*b*B + A*c
)*x^2*Hypergeometric2F1[-1/2, -1/4, 3/4, -((c*x^2)/b)]))/(5*b*x^(7/2)*Sqrt[
1 + (c*x^2)/b])
```

Maple [A] time = 0.032, size = 422, normalized size = 1.3

$$\frac{2}{\left(5\,cx^2+5\,b\right)b}\sqrt{cx^4+bx^2}\left(2\,A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\,\sqrt{2}\right)x^2bc-A\sqrt{(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(9/2),x)
```

```
[Out] 2/5*(c*x^4+b*x^2)^(1/2)/x^(7/2)/(c*x^2+b)*(2*A*((c*x+(-b*c)^(1/2))/(-b*c)^(
1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(
1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*
x^2*b*c-A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/
2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1
/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b*c+10*B*((c*x+(-b*c)^(1/2))/(-b*
c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*
c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/
2))*x^2*b^2-5*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*
c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b
*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^2-2*A*c^2*x^4-5*B*x^4*b*c
-3*A*b*c*x^2-5*B*x^2*b^2-A*b^2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4+bx^2}\left(Bx^2+A\right)}{x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\left(Bx^2+A\right)}{x^{\frac{9}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(9/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4+bx^2}\left(Bx^2+A\right)}{x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(9/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(9/2), x)
```

3.228 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{11/2}}\,dx$

Optimal. Leaf size=167

$$\frac{2c^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(7bB-Ac)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{21b^{5/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}(7bB-Ac)}{21bx^{5/2}}-\frac{2A\left(bx^2+cx^4\right)^{3/2}}{7bx^{13/2}}$$

```
[Out] (-2*(7*b*B - A*c)*Sqrt[b*x^2 + c*x^4])/(21*b*x^(5/2)) - (2*A*(b*x^2 + c*x^4
)^(3/2))/(7*b*x^(13/2)) + (2*c^(3/4)*(7*b*B - A*c)*x*(Sqrt[b] + Sqrt[c]*x)*
Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[
x])/b^(1/4)], 1/2])/(21*b^(5/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.250341, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2038, 2020, 2032, 329, 220}

$$\frac{2c^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(7bB-Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21b^{5/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}(7bB-Ac)}{21bx^{5/2}}-\frac{2A\left(bx^2+cx^4\right)^{3/2}}{7bx^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(11/2),x]
```

```
[Out] (-2*(7*b*B - A*c)*Sqrt[b*x^2 + c*x^4])/(21*b*x^(5/2)) - (2*A*(b*x^2 + c*x^4
)^(3/2))/(7*b*x^(13/2)) + (2*c^(3/4)*(7*b*B - A*c)*x*(Sqrt[b] + Sqrt[c]*x)*
Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[
x])/b^(1/4)], 1/2])/(21*b^(5/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
```

```
0]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{11/2}}\,dx = -\frac{2A\left(bx^2+cx^4\right)^{3/2}}{7bx^{13/2}} - \frac{\left(2\left(-\frac{7bB}{2}+\frac{Ac}{2}\right)\right)\int \frac{\sqrt{bx^2+cx^4}}{x^{7/2}}\,dx}{7b}$$

$$= -\frac{2(7bB-Ac)\sqrt{bx^2+cx^4}}{21bx^{5/2}} - \frac{2A\left(bx^2+cx^4\right)^{3/2}}{7bx^{13/2}} + \frac{(2c(7bB-Ac))\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{21b}$$

$$= -\frac{2(7bB-Ac)\sqrt{bx^2+cx^4}}{21bx^{5/2}} - \frac{2A\left(bx^2+cx^4\right)^{3/2}}{7bx^{13/2}} + \frac{\left(2c(7bB-Ac)x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,d}{21b\sqrt{bx^2+cx^4}}$$

$$= -\frac{2(7bB-Ac)\sqrt{bx^2+cx^4}}{21bx^{5/2}} - \frac{2A\left(bx^2+cx^4\right)^{3/2}}{7bx^{13/2}} + \frac{\left(4c(7bB-Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{}{\sqrt{l}}\right.}{21b\sqrt{bx^2+cx^4}}$$

$$= -\frac{2(7bB-Ac)\sqrt{bx^2+cx^4}}{21bx^{5/2}} - \frac{2A\left(bx^2+cx^4\right)^{3/2}}{7bx^{13/2}} + \frac{2c^{3/4}(7bB-Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx}{(\sqrt{b}+\sqrt{c}}}}{21b^{5/4}\sqrt{bx^2+c}}$$

Mathematica [C] time = 0.0424836, size = 98, normalized size = 0.59

$$-\frac{2\sqrt{x^2\left(b+cx^2\right)}\left(x^2(7bB-Ac)\,{}_2F_1\left(-\frac{3}{4},-\frac{1}{2};\frac{1}{4};-\frac{cx^2}{b}\right)+3A\left(b+cx^2\right)\sqrt{\frac{cx^2}{b}+1}\right)}{21bx^{9/2}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(11/2),x]
```

```
[Out] (-2*Sqrt[x^2*(b + c*x^2)]*(3*A*(b + c*x^2)*Sqrt[1 + (c*x^2)/b] + (7*b*B - A
*c)*x^2*Hypergeometric2F1[-3/4, -1/2, 1/4, -((c*x^2)/b)]))/(21*b*x^(9/2)*Sq
rt[1 + (c*x^2)/b])
```

Maple [A] time = 0.029, size = 255, normalized size = 1.5

$$-\frac{2}{\left(21\,cx^2+21\,b\right)b}\sqrt{cx^4+bx^2}\left(A\sqrt{\left(cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}}\sqrt{2}\sqrt{\left(-cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}}\sqrt{-cx\frac{1}{\sqrt{-bc}}}\text{EllipticF}\left(\sqrt{\left(cx+\sqrt{-bc}\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(11/2),x)
```

```
[Out] -2/21*(c*x^4+b*x^2)^(1/2)/x^(9/2)/(c*x^2+b)*(A*((c*x+(-b*c)^(1/2))/(-b*c)^(
1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(
1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*
(-b*c)^(1/2)*x^3*c-7*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c
*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((
c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(-b*c)^(1/2)*x^3*b+2*A*c
^2*x^4+7*B*x^4*b*c+5*A*b*c*x^2+7*B*x^2*b^2+3*A*b^2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2}(Bx^2 + A)}{x^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(11/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(11/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2}(Bx^2 + A)}{x^{\frac{11}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(11/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(11/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2}\left(Bx^2 + A\right)}{x^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(11/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(11/2), x)
```

3.229 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{13/2}}\,dx$

Optimal. Leaf size=369

$$\frac{2c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(3bB-Ac)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{15b^{7/4}\sqrt{bx^2+cx^4}}+\frac{4c^{3/2}x^{3/2}\left(b+cx^2\right)(3bB-Ac)}{15b^2\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{4c^{5/4}x\left(\sqrt{b}+\right.}{}$$

```
[Out] (4*c^(3/2)*(3*b*B - A*c)*x^(3/2)*(b + c*x^2))/(15*b^2*(Sqrt[b] + Sqrt[c]*x)
*Sqrt[b*x^2 + c*x^4]) - (2*(3*b*B - A*c)*Sqrt[b*x^2 + c*x^4])/(15*b*x^(7/2)
) - (4*c*(3*b*B - A*c)*Sqrt[b*x^2 + c*x^4])/(15*b^2*x^(3/2)) - (2*A*(b*x^2
+ c*x^4)^(3/2))/(9*b*x^(15/2)) - (4*c^(5/4)*(3*b*B - A*c)*x*(Sqrt[b] + Sqrt
[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4
)*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(7/4)*Sqrt[b*x^2 + c*x^4]) + (2*c^(5/4)*(3
*b*B - A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^
2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(7/4)*Sqrt[b*
x^2 + c*x^4])
```

Rubi [A] time = 0.436171, antiderivative size = 369, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2038, 2020, 2025, 2032, 329, 305, 220, 1196}

$$\frac{4c^{3/2}x^{3/2}\left(b+cx^2\right)(3bB-Ac)}{15b^2\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{2c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(3bB-Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{bx^2+cx^4}}-\frac{4c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\cdots}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(13/2),x]
```

```
[Out] (4*c^(3/2)*(3*b*B - A*c)*x^(3/2)*(b + c*x^2))/(15*b^2*(Sqrt[b] + Sqrt[c]*x)
*Sqrt[b*x^2 + c*x^4]) - (2*(3*b*B - A*c)*Sqrt[b*x^2 + c*x^4])/(15*b*x^(7/2)
) - (4*c*(3*b*B - A*c)*Sqrt[b*x^2 + c*x^4])/(15*b^2*x^(3/2)) - (2*A*(b*x^2
+ c*x^4)^(3/2))/(9*b*x^(15/2)) - (4*c^(5/4)*(3*b*B - A*c)*x*(Sqrt[b] + Sqrt
[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4
)*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(7/4)*Sqrt[b*x^2 + c*x^4]) + (2*c^(5/4)*(3
*b*B - A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^
2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(7/4)*Sqrt[b*
x^2 + c*x^4])
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\left(A+Bx^2\right)\sqrt{bx^2+cx^4}}{x^{13/2}}\,dx = -\frac{2A\left(bx^2+cx^4\right)^{3/2}}{9bx^{15/2}} - \frac{\left(2\left(-\frac{9bB}{2}+\frac{3Ac}{2}\right)\right)\int \frac{\sqrt{bx^2+cx^4}}{x^{9/2}}\,dx}{9b}$$

$$= -\frac{2(3bB-Ac)\sqrt{bx^2+cx^4}}{15bx^{7/2}} - \frac{2A\left(bx^2+cx^4\right)^{3/2}}{9bx^{15/2}} + \frac{(2c(3bB-Ac))\int \frac{1}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx}{15b}$$

$$= -\frac{2(3bB-Ac)\sqrt{bx^2+cx^4}}{15bx^{7/2}} - \frac{4c(3bB-Ac)\sqrt{bx^2+cx^4}}{15b^2x^{3/2}} - \frac{2A\left(bx^2+cx^4\right)^{3/2}}{9bx^{15/2}} + \frac{\left(2c^2(3bB-\right.}{}$$

$$= -\frac{2(3bB-Ac)\sqrt{bx^2+cx^4}}{15bx^{7/2}} - \frac{4c(3bB-Ac)\sqrt{bx^2+cx^4}}{15b^2x^{3/2}} - \frac{2A\left(bx^2+cx^4\right)^{3/2}}{9bx^{15/2}} + \frac{\left(2c^2(3bB-\right.}{1}$$

$$= -\frac{2(3bB-Ac)\sqrt{bx^2+cx^4}}{15bx^{7/2}} - \frac{4c(3bB-Ac)\sqrt{bx^2+cx^4}}{15b^2x^{3/2}} - \frac{2A\left(bx^2+cx^4\right)^{3/2}}{9bx^{15/2}} + \frac{\left(4c^2(3bB-\right.}{}$$

$$= -\frac{2(3bB-Ac)\sqrt{bx^2+cx^4}}{15bx^{7/2}} - \frac{4c(3bB-Ac)\sqrt{bx^2+cx^4}}{15b^2x^{3/2}} - \frac{2A\left(bx^2+cx^4\right)^{3/2}}{9bx^{15/2}} + \frac{\left(4c^{3/2}(3bB-\right.}{}$$

$$= \frac{4c^{3/2}(3bB-Ac)x^{3/2}\left(b+cx^2\right)}{15b^2\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{2(3bB-Ac)\sqrt{bx^2+cx^4}}{15bx^{7/2}} - \frac{4c(3bB-Ac)\sqrt{bx^2+cx^4}}{15b^2x^{3/2}} - \frac{2A}{}$$

Mathematica [C] time = 0.0438959, size = 99, normalized size = 0.27

$$-\frac{2\sqrt{x^2\left(b+cx^2\right)}\left(3x^2(3bB-Ac)\,{}_2F_1\left(-\frac{5}{4},-\frac{1}{2};-\frac{1}{4};-\frac{cx^2}{b}\right)+5A\left(b+cx^2\right)\sqrt{\frac{cx^2}{b}+1}\right)}{45bx^{11/2}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(13/2),x]
```

```
[Out] (-2*Sqrt[x^2*(b + c*x^2)]*(5*A*(b + c*x^2)*Sqrt[1 + (c*x^2)/b] + 3*(3*b*B -
 A*c)*x^2*Hypergeometric2F1[-5/4, -1/2, -1/4, -((c*x^2)/b)]))/(45*b*x^(11/2
)*Sqrt[1 + (c*x^2)/b])
```

Maple [A] time = 0.032, size = 452, normalized size = 1.2

$$-\frac{2}{\left(45\,cx^2+45\,b\right)b^2}\sqrt{cx^4+bx^2}\left(6\,A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\text{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)x^4bc^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(13/2),x)
```

```
[Out] -2/45*(c*x^4+b*x^2)^(1/2)/x^(11/2)/(c*x^2+b)*(6*A*((c*x+(-b*c)^(1/2))/(-b*c
)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c
)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2
))*x^4*b*c^2-3*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b
*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-
b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^4*b*c^2-18*B*((c*x+(-b*c)^(1
/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(
-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),
1/2*2^(1/2))*x^4*b^2*c+9*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*
((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*Elliptic
F(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^4*b^2*c-6*A*c^3*x^
6+18*B*x^6*b*c^2-4*A*b*c^2*x^4+27*B*x^4*b^2*c+7*A*b^2*c*x^2+9*B*x^2*b^3+5*A
*b^3)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2}(Bx^2 + A)}{x^{\frac{13}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(13/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(13/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2}(Bx^2 + A)}{x^{\frac{13}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(13/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(13/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**(13/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2}(Bx^2 + A)}{x^{\frac{13}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(13/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(13/2), x)
```

3.230 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{15/2}}\,dx$

Optimal. Leaf size=204

$$-\frac{2c^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(11bB-5Ac)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{231b^{9/4}\sqrt{bx^2+cx^4}}-\frac{4c\sqrt{bx^2+cx^4}(11bB-5Ac)}{231b^2x^{5/2}}-\frac{2\sqrt{bx^2+cx^4}(11bB-5Ac)}{77bx^{9/2}}-\frac{2A(bx^2+cx^4)^{3/2}}{11bx^{17/2}}$$

```
[Out] (-2*(11*b*B - 5*A*c)*Sqrt[b*x^2 + c*x^4])/(77*b*x^(9/2)) - (4*c*(11*b*B - 5
*A*c)*Sqrt[b*x^2 + c*x^4])/(231*b^2*x^(5/2)) - (2*A*(b*x^2 + c*x^4)^(3/2))/
(11*b*x^(17/2)) - (2*c^(7/4)*(11*b*B - 5*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[
(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b
^(1/4)], 1/2])/(231*b^(9/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.308142, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2038, 2020, 2025, 2032, 329, 220}

$$-\frac{2c^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(11bB-5Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{231b^{9/4}\sqrt{bx^2+cx^4}}-\frac{4c\sqrt{bx^2+cx^4}(11bB-5Ac)}{231b^2x^{5/2}}-\frac{2\sqrt{bx^2+cx^4}(11bB-5Ac)}{77bx^{9/2}}-\frac{2A(bx^2+cx^4)^{3/2}}{11bx^{17/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(15/2),x]
```


```
[Out] (-2*(11*b*B - 5*A*c)*Sqrt[b*x^2 + c*x^4])/(77*b*x^(9/2)) - (4*c*(11*b*B - 5
*A*c)*Sqrt[b*x^2 + c*x^4])/(231*b^2*x^(5/2)) - (2*A*(b*x^2 + c*x^4)^(3/2))/
(11*b*x^(17/2)) - (2*c^(7/4)*(11*b*B - 5*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[
(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b
^(1/4)], 1/2])/(231*b^(9/4)*Sqrt[b*x^2 + c*x^4])
```


Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
```

```
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{15/2}}\,dx = -\frac{2A(bx^2+cx^4)^{3/2}}{11bx^{17/2}} - \frac{\left(2\left(-\frac{11bB}{2}+\frac{5Ac}{2}\right)\right)\int \frac{\sqrt{bx^2+cx^4}}{x^{11/2}}\,dx}{11b}$$

$$= -\frac{2(11bB-5Ac)\sqrt{bx^2+cx^4}}{77bx^{9/2}} - \frac{2A(bx^2+cx^4)^{3/2}}{11bx^{17/2}} + \frac{(2c(11bB-5Ac))\int \frac{1}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx}{77b}$$

$$= -\frac{2(11bB-5Ac)\sqrt{bx^2+cx^4}}{77bx^{9/2}} - \frac{4c(11bB-5Ac)\sqrt{bx^2+cx^4}}{231b^2x^{5/2}} - \frac{2A(bx^2+cx^4)^{3/2}}{11bx^{17/2}} - \frac{(2c^2(11b}{}$$

$$= -\frac{2(11bB-5Ac)\sqrt{bx^2+cx^4}}{77bx^{9/2}} - \frac{4c(11bB-5Ac)\sqrt{bx^2+cx^4}}{231b^2x^{5/2}} - \frac{2A(bx^2+cx^4)^{3/2}}{11bx^{17/2}} - \frac{(2c^2(11b}{}$$

$$= -\frac{2(11bB-5Ac)\sqrt{bx^2+cx^4}}{77bx^{9/2}} - \frac{4c(11bB-5Ac)\sqrt{bx^2+cx^4}}{231b^2x^{5/2}} - \frac{2A(bx^2+cx^4)^{3/2}}{11bx^{17/2}} - \frac{(4c^2(11b}{}$$

$$= -\frac{2(11bB-5Ac)\sqrt{bx^2+cx^4}}{77bx^{9/2}} - \frac{4c(11bB-5Ac)\sqrt{bx^2+cx^4}}{231b^2x^{5/2}} - \frac{2A(bx^2+cx^4)^{3/2}}{11bx^{17/2}} - \frac{2c^{7/4}(11}{}$$

Mathematica [C] time = 0.0450107, size = 98, normalized size = 0.48

$$-\frac{2\sqrt{x^2(b+cx^2)}\left(x^2(11bB-5Ac)\,{}_2F_1\left(-\frac{7}{4},-\frac{1}{2};-\frac{3}{4};-\frac{cx^2}{b}\right)+7A(b+cx^2)\sqrt{\frac{cx^2}{b}+1}\right)}{77bx^{13/2}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(15/2),x]
```

```
[Out] (-2*Sqrt[x^2*(b + c*x^2)]*(7*A*(b + c*x^2)*Sqrt[1 + (c*x^2)/b] + (11*b*B -
5*A*c)*x^2*Hypergeometric2F1[-7/4, -1/2, -3/4, -((c*x^2)/b)]))/(77*b*x^(13/
2)*Sqrt[1 + (c*x^2)/b])
```

Maple [A] time = 0.03, size = 283, normalized size = 1.4

$$\frac{2}{(231\,cx^2+231\,b)\,b^2}\sqrt{cx^4+bx^2}\left(5\,A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\,\sqrt{2}\right)\sqrt{-bc}x^5c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(15/2),x)
```

```
[Out] 2/231*(c*x^4+b*x^2)^(1/2)/x^(13/2)/(c*x^2+b)*(5*A*((c*x+(-b*c)^(1/2))/(-b*c
)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c
)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2
))*(-b*c)^(1/2)*x^5*c^2-11*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2
)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*Ellipt
icF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(-b*c)^(1/2)*x^5*b
*c+10*A*c^3*x^6-22*B*x^6*b*c^2+4*A*b*c^2*x^4-55*B*x^4*b^2*c-27*A*b^2*c*x^2-
33*B*x^2*b^3-21*A*b^3)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2}\left(Bx^2 + A\right)}{x^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(15/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(15/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2}\left(Bx^2 + A\right)}{x^{\frac{15}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(15/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(15/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**(15/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2}\left(Bx^2 + A\right)}{x^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(15/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(15/2), x)
```

3.231 $\int x^{7/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=486

$$\frac{44b^{21/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(3bB-5Ac)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{16575c^{19/4}\sqrt{bx^2+cx^4}}-\frac{8b^2x^{9/2}\sqrt{bx^2+cx^4}(3bB-5Ac)}{7735c^2}+\frac{88b^3x^5}{}$$

```
[Out] (88*b^5*(3*b*B - 5*A*c)*x^(3/2)*(b + c*x^2))/(16575*c^(9/2)*(Sqrt[b] + Sqrt
[c]*x)*Sqrt[b*x^2 + c*x^4]) - (88*b^4*(3*b*B - 5*A*c)*Sqrt[x]*Sqrt[b*x^2 +
c*x^4])/(49725*c^4) + (88*b^3*(3*b*B - 5*A*c)*x^(5/2)*Sqrt[b*x^2 + c*x^4])/
(69615*c^3) - (8*b^2*(3*b*B - 5*A*c)*x^(9/2)*Sqrt[b*x^2 + c*x^4])/(7735*c^2
) - (4*b*(3*b*B - 5*A*c)*x^(13/2)*Sqrt[b*x^2 + c*x^4])/(595*c) - (2*(3*b*B
- 5*A*c)*x^(9/2)*(b*x^2 + c*x^4)^(3/2))/(105*c) + (2*B*x^(5/2)*(b*x^2 + c*x
^4)^(5/2))/(25*c) - (88*b^(21/4)*(3*b*B - 5*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sq
rt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x]
)/b^(1/4)], 1/2])/(16575*c^(19/4)*Sqrt[b*x^2 + c*x^4]) + (44*b^(21/4)*(3*b*
B - 5*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2
]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(16575*c^(19/4)*Sqrt
[b*x^2 + c*x^4])
```

Rubi [A] time = 0.670511, antiderivative size = 486, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2039, 2021, 2024, 2032, 329, 305, 220, 1196}

$$-\frac{8b^2x^{9/2}\sqrt{bx^2+cx^4}(3bB-5Ac)}{7735c^2}+\frac{88b^3x^{5/2}\sqrt{bx^2+cx^4}(3bB-5Ac)}{69615c^3}+\frac{88b^5x^{3/2}\left(b+cx^2\right)(3bB-5Ac)}{16575c^{9/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{88b^4\sqrt{x}\sqrt{b}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```


```
[Out] (88*b^5*(3*b*B - 5*A*c)*x^(3/2)*(b + c*x^2))/(16575*c^(9/2)*(Sqrt[b] + Sqrt
[c]*x)*Sqrt[b*x^2 + c*x^4]) - (88*b^4*(3*b*B - 5*A*c)*Sqrt[x]*Sqrt[b*x^2 +
c*x^4])/(49725*c^4) + (88*b^3*(3*b*B - 5*A*c)*x^(5/2)*Sqrt[b*x^2 + c*x^4])/
(69615*c^3) - (8*b^2*(3*b*B - 5*A*c)*x^(9/2)*Sqrt[b*x^2 + c*x^4])/(7735*c^2
) - (4*b*(3*b*B - 5*A*c)*x^(13/2)*Sqrt[b*x^2 + c*x^4])/(595*c) - (2*(3*b*B
- 5*A*c)*x^(9/2)*(b*x^2 + c*x^4)^(3/2))/(105*c) + (2*B*x^(5/2)*(b*x^2 + c*x
```

```
^4)^(5/2))/(25*c) - (88*b^(21/4)*(3*b*B - 5*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sq
rt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x]
)/b^(1/4)], 1/2])/(16575*c^(19/4)*Sqrt[b*x^2 + c*x^4]) + (44*b^(21/4)*(3*b*
B - 5*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2
]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(16575*c^(19/4)*Sqrt
[b*x^2 + c*x^4])
```

Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
```

```
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int x^{7/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{2Bx^{5/2}\left(bx^2+cx^4\right)^{5/2}}{25c}-\frac{\left(2\left(\frac{15bB}{2}-\frac{25Ac}{2}\right)\right)\int x^{7/2}\left(bx^2+cx^4\right)^{3/2}\,dx}{25c}$$

$$=-\frac{2(3bB-5Ac)x^{9/2}\left(bx^2+cx^4\right)^{3/2}}{105c}+\frac{2Bx^{5/2}\left(bx^2+cx^4\right)^{5/2}}{25c}-\frac{(2b(3bB-5Ac))\int x^{11/}}{35c}$$

$$=-\frac{4b(3bB-5Ac)x^{13/2}\sqrt{bx^2+cx^4}}{595c}-\frac{2(3bB-5Ac)x^{9/2}\left(bx^2+cx^4\right)^{3/2}}{105c}+\frac{2Bx^{5/2}\left(bx^2\right.}{25c}$$

$$=-\frac{8b^2(3bB-5Ac)x^{9/2}\sqrt{bx^2+cx^4}}{7735c^2}-\frac{4b(3bB-5Ac)x^{13/2}\sqrt{bx^2+cx^4}}{595c}-\frac{2(3bB-5Ac)}{}$$

$$=\frac{88b^3(3bB-5Ac)x^{5/2}\sqrt{bx^2+cx^4}}{69615c^3}-\frac{8b^2(3bB-5Ac)x^{9/2}\sqrt{bx^2+cx^4}}{7735c^2}-\frac{4b(3bB-5A}{5}$$

$$=-\frac{88b^4(3bB-5Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{49725c^4}+\frac{88b^3(3bB-5Ac)x^{5/2}\sqrt{bx^2+cx^4}}{69615c^3}-\frac{8b^2(3bB-}{}$$

$$=-\frac{88b^4(3bB-5Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{49725c^4}+\frac{88b^3(3bB-5Ac)x^{5/2}\sqrt{bx^2+cx^4}}{69615c^3}-\frac{8b^2(3bB-}{}$$

$$=-\frac{88b^4(3bB-5Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{49725c^4}+\frac{88b^3(3bB-5Ac)x^{5/2}\sqrt{bx^2+cx^4}}{69615c^3}-\frac{8b^2(3bB-}{}$$

$$=-\frac{88b^4(3bB-5Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{49725c^4}+\frac{88b^3(3bB-5Ac)x^{5/2}\sqrt{bx^2+cx^4}}{69615c^3}-\frac{8b^2(3bB-}{}$$

$$=\frac{88b^5(3bB-5Ac)x^{3/2}\left(b+cx^2\right)}{16575c^{9/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{88b^4(3bB-5Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{49725c^4}+\frac{88b^3(3bB-}{}$$

Mathematica [C] time = 0.214981, size = 160, normalized size = 0.33

$$\frac{2\sqrt{x}\sqrt{x^2\left(b+cx^2\right)}\left(385b^4(3bB-5Ac)\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)-\left(b+cx^2\right)^2\sqrt{\frac{cx^2}{b}+1}\left(-55b^2c\left(35A+39Bx^2\right)+65bc^2x^2\left(55A\right.\right.\right.}{116025c^4\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[x]*Sqrt[x^2*(b + c*x^2)]*(-((b + c*x^2)^2*Sqrt[1 + (c*x^2)/b]*(1155
*b^3*B - 221*c^3*x^4*(25*A + 21*B*x^2) - 55*b^2*c*(35*A + 39*B*x^2) + 65*b*
c^2*x^2*(55*A + 51*B*x^2))) + 385*b^4*(3*b*B - 5*A*c)*Hypergeometric2F1[-3/
2, 3/4, 7/4, -((c*x^2)/b)]))/(116025*c^4*Sqrt[1 + (c*x^2)/b])
```

Maple [A] time = 0.04, size = 518, normalized size = 1.1

$$-\frac{2}{348075\left(cx^2+b\right)^2c^5}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(-13923\,Bx^{14}c^7-16575\,Ax^{12}c^7-31824\,Bx^{12}bc^6-39000\,Ax^{10}bc^6-18369\,Bx^{10}b^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -2/348075*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2/c^5*(-13923*B*x^14*c^7-16
575*A*x^12*c^7-31824*B*x^12*b*c^6-39000*A*x^10*b*c^6-18369*B*x^10*b^2*c^5-2
3325*A*x^8*b^2*c^5+72*B*x^8*b^3*c^4+200*A*x^6*b^3*c^4-120*B*x^6*b^4*c^3+462
0*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-
b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(
-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^6*c-2310*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/
2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/
2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^
6*c-2772*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1
/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(
1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^7+1386*B*((c*x+(-b*c)^(1/2))/(-b*c
)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c
)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2
))*b^7-440*A*x^4*b^4*c^3+264*B*x^4*b^5*c^2-1540*A*x^2*b^5*c^2+924*B*x^2*b^6
*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(Bx^2+A\right)x^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)*x^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bcx^9 + (Bb + Ac)x^7 + Abx^5\right)\sqrt{cx^4 + bx^2}\sqrt{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*c*x^9 + (B*b + A*c)*x^7 + A*b*x^5)*sqrt(c*x^4 + b*x^2)*sqrt(x),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(B*x**2+A)*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4 + bx^2\right)^{\frac{3}{2}}\left(Bx^2 + A\right)x^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)*x^(7/2), x)
```

3.232 $\int x^{5/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=321

$$\frac{12b^{19/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(13bB-23Ac)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{33649c^{17/4}\sqrt{bx^2+cx^4}}-\frac{8b^2x^{7/2}\sqrt{bx^2+cx^4}(13bB-23Ac)}{24035c^2}+\frac{72\ldots}{\ldots}$$

```
[Out] (-24*b^4*(13*b*B - 23*A*c)*Sqrt[b*x^2 + c*x^4])/(33649*c^4*Sqrt[x]) + (72*b
^3*(13*b*B - 23*A*c)*x^(3/2)*Sqrt[b*x^2 + c*x^4])/(168245*c^3) - (8*b^2*(13
*b*B - 23*A*c)*x^(7/2)*Sqrt[b*x^2 + c*x^4])/(24035*c^2) - (4*b*(13*b*B - 23
*A*c)*x^(11/2)*Sqrt[b*x^2 + c*x^4])/(2185*c) - (2*(13*b*B - 23*A*c)*x^(7/2)
*(b*x^2 + c*x^4)^(3/2))/(437*c) + (2*B*x^(3/2)*(b*x^2 + c*x^4)^(5/2))/(23*c
) + (12*b^(19/4)*(13*b*B - 23*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)
/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/
2])/(33649*c^(17/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.489965, antiderivative size = 321, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2039, 2021, 2024, 2032, 329, 220}

$$-\frac{8b^2x^{7/2}\sqrt{bx^2+cx^4}(13bB-23Ac)}{24035c^2}+\frac{72b^3x^{3/2}\sqrt{bx^2+cx^4}(13bB-23Ac)}{168245c^3}-\frac{24b^4\sqrt{bx^2+cx^4}(13bB-23Ac)}{33649c^4\sqrt{x}}+\frac{12b^{19/4}x\left(\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```


```
[Out] (-24*b^4*(13*b*B - 23*A*c)*Sqrt[b*x^2 + c*x^4])/(33649*c^4*Sqrt[x]) + (72*b
^3*(13*b*B - 23*A*c)*x^(3/2)*Sqrt[b*x^2 + c*x^4])/(168245*c^3) - (8*b^2*(13
*b*B - 23*A*c)*x^(7/2)*Sqrt[b*x^2 + c*x^4])/(24035*c^2) - (4*b*(13*b*B - 23
*A*c)*x^(11/2)*Sqrt[b*x^2 + c*x^4])/(2185*c) - (2*(13*b*B - 23*A*c)*x^(7/2)
*(b*x^2 + c*x^4)^(3/2))/(437*c) + (2*B*x^(3/2)*(b*x^2 + c*x^4)^(5/2))/(23*c
) + (12*b^(19/4)*(13*b*B - 23*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)
/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/
2])/(33649*c^(17/4)*Sqrt[b*x^2 + c*x^4])
```


Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
```

```
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int x^{5/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{2Bx^{3/2}\left(bx^2+cx^4\right)^{5/2}}{23c}-\frac{\left(2\left(\frac{13bB}{2}-\frac{23Ac}{2}\right)\right)\int x^{5/2}\left(bx^2+cx^4\right)^{3/2}\,dx}{23c}$$

$$=-\frac{2(13bB-23Ac)x^{7/2}\left(bx^2+cx^4\right)^{3/2}}{437c}+\frac{2Bx^{3/2}\left(bx^2+cx^4\right)^{5/2}}{23c}-\frac{(6b(13bB-23Ac)}{4}$$

$$=-\frac{4b(13bB-23Ac)x^{11/2}\sqrt{bx^2+cx^4}}{2185c}-\frac{2(13bB-23Ac)x^{7/2}\left(bx^2+cx^4\right)^{3/2}}{437c}+\frac{2Bx^{3/2}}{}$$

$$=-\frac{8b^2(13bB-23Ac)x^{7/2}\sqrt{bx^2+cx^4}}{24035c^2}-\frac{4b(13bB-23Ac)x^{11/2}\sqrt{bx^2+cx^4}}{2185c}-\frac{2(13bB}{}$$

$$=\frac{72b^3(13bB-23Ac)x^{3/2}\sqrt{bx^2+cx^4}}{168245c^3}-\frac{8b^2(13bB-23Ac)x^{7/2}\sqrt{bx^2+cx^4}}{24035c^2}-\frac{4b(13b}{}$$

$$=-\frac{24b^4(13bB-23Ac)\sqrt{bx^2+cx^4}}{33649c^4\sqrt{x}}+\frac{72b^3(13bB-23Ac)x^{3/2}\sqrt{bx^2+cx^4}}{168245c^3}-\frac{8b^2(13b}{}$$

$$=-\frac{24b^4(13bB-23Ac)\sqrt{bx^2+cx^4}}{33649c^4\sqrt{x}}+\frac{72b^3(13bB-23Ac)x^{3/2}\sqrt{bx^2+cx^4}}{168245c^3}-\frac{8b^2(13b}{}$$

$$=-\frac{24b^4(13bB-23Ac)\sqrt{bx^2+cx^4}}{33649c^4\sqrt{x}}+\frac{72b^3(13bB-23Ac)x^{3/2}\sqrt{bx^2+cx^4}}{168245c^3}-\frac{8b^2(13b}{}$$

$$=-\frac{24b^4(13bB-23Ac)\sqrt{bx^2+cx^4}}{33649c^4\sqrt{x}}+\frac{72b^3(13bB-23Ac)x^{3/2}\sqrt{bx^2+cx^4}}{168245c^3}-\frac{8b^2(13b}{}$$

Mathematica [C] time = 0.207018, size = 160, normalized size = 0.5

$$\frac{2\sqrt{x^2\left(b+cx^2\right)}\left(15b^4(13bB-23Ac)\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{cx^2}{b}\right)-\left(b+cx^2\right)^2\sqrt{\frac{cx^2}{b}+1}\left(-3b^2c\left(115A+143Bx^2\right)+11bc^2x^2\left(69A\right.\right.\right.}{24035c^4\sqrt{x}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[x^2*(b + c*x^2)]*(-((b + c*x^2)^2*Sqrt[1 + (c*x^2)/b]*(195*b^3*B -
55*c^3*x^4*(23*A + 19*B*x^2) + 11*b*c^2*x^2*(69*A + 65*B*x^2) - 3*b^2*c*(11
5*A + 143*B*x^2))) + 15*b^4*(13*b*B - 23*A*c)*Hypergeometric2F1[-3/2, 1/4,
5/4, -((c*x^2)/b)]))/(24035*c^4*Sqrt[x]*Sqrt[1 + (c*x^2)/b])
```

Maple [A] time = 0.033, size = 355, normalized size = 1.1

$$-\frac{2}{168245\left(cx^2+b\right)^2c^5}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(-7315\,Bx^{13}c^7-8855\,Ax^{11}c^7-16940\,Bx^{11}bc^6-21252\,Ax^9bc^6-9933\,Bx^9b^2c^5-13\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -2/168245*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2*(-7315*B*x^13*c^7-8855*A*
x^11*c^7-16940*B*x^11*b*c^6-21252*A*x^9*b*c^6-9933*B*x^9*b^2*c^5-13041*A*x^
7*b^2*c^5+56*B*x^7*b^3*c^4+690*A*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1
/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1
/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b
^5*c+184*A*x^5*b^3*c^4-390*B*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))
^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))
^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^6-1
04*B*x^5*b^4*c^3-552*A*x^3*b^4*c^3+312*B*x^3*b^5*c^2-1380*A*x*b^5*c^2+780*B
*x*b^6*c)/c^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(Bx^2+A\right)x^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)*x^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bcx^8 + (Bb + Ac)x^6 + Abx^4\right)\sqrt{cx^4 + bx^2}\sqrt{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*c*x^8 + (B*b + A*c)*x^6 + A*b*x^4)*sqrt(c*x^4 + b*x^2)*sqrt(x),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(B*x**2+A)*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4 + bx^2\right)^{\frac{3}{2}}\left(Bx^2 + A\right)x^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)*x^(5/2), x)
```

3.233 $\int x^{3/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=447

$$-\frac{4b^{17/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(11bB-21Ac)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3315c^{15/4}\sqrt{bx^2+cx^4}}-\frac{8b^2x^{5/2}\sqrt{bx^2+cx^4}(11bB-21Ac)}{13923c^2}-\frac{8b^4}{331}$$

```
[Out] (-8*b^4*(11*b*B - 21*A*c)*x^(3/2)*(b + c*x^2))/(3315*c^(7/2)*(Sqrt[b] + Sqr
t[c]*x)*Sqrt[b*x^2 + c*x^4]) + (8*b^3*(11*b*B - 21*A*c)*Sqrt[x]*Sqrt[b*x^2
+ c*x^4])/(9945*c^3) - (8*b^2*(11*b*B - 21*A*c)*x^(5/2)*Sqrt[b*x^2 + c*x^4]
)/(13923*c^2) - (4*b*(11*b*B - 21*A*c)*x^(9/2)*Sqrt[b*x^2 + c*x^4])/(1547*c
) - (2*(11*b*B - 21*A*c)*x^(5/2)*(b*x^2 + c*x^4)^(3/2))/(357*c) + (2*B*Sqrt
[x]*(b*x^2 + c*x^4)^(5/2))/(21*c) + (8*b^(17/4)*(11*b*B - 21*A*c)*x*(Sqrt[b
] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan
[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(3315*c^(15/4)*Sqrt[b*x^2 + c*x^4]) - (4
*b^(17/4)*(11*b*B - 21*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[
b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(33
15*c^(15/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.589558, antiderivative size = 447, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2039, 2021, 2024, 2032, 329, 305, 220, 1196}

$$-\frac{8b^2x^{5/2}\sqrt{bx^2+cx^4}(11bB-21Ac)}{13923c^2}-\frac{8b^4x^{3/2}\left(b+cx^2\right)(11bB-21Ac)}{3315c^{7/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{8b^3\sqrt{x}\sqrt{bx^2+cx^4}(11bB-21Ac)}{9945c^3}-\frac{4b^{17/4}x\left(\sqrt{b}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```


```
[Out] (-8*b^4*(11*b*B - 21*A*c)*x^(3/2)*(b + c*x^2))/(3315*c^(7/2)*(Sqrt[b] + Sqr
t[c]*x)*Sqrt[b*x^2 + c*x^4]) + (8*b^3*(11*b*B - 21*A*c)*Sqrt[x]*Sqrt[b*x^2
+ c*x^4])/(9945*c^3) - (8*b^2*(11*b*B - 21*A*c)*x^(5/2)*Sqrt[b*x^2 + c*x^4]
)/(13923*c^2) - (4*b*(11*b*B - 21*A*c)*x^(9/2)*Sqrt[b*x^2 + c*x^4])/(1547*c
) - (2*(11*b*B - 21*A*c)*x^(5/2)*(b*x^2 + c*x^4)^(3/2))/(357*c) + (2*B*Sqrt
[x]*(b*x^2 + c*x^4)^(5/2))/(21*c) + (8*b^(17/4)*(11*b*B - 21*A*c)*x*(Sqrt[b
] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan
```

```
[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(3315*c^(15/4)*Sqrt[b*x^2 + c*x^4]) - (4
*b^(17/4)*(11*b*B - 21*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[
b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(33
15*c^(15/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int x^{3/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{2B\sqrt{x}\left(bx^2+cx^4\right)^{5/2}}{21c}-\frac{\left(2\left(\frac{11bB}{2}-\frac{21Ac}{2}\right)\right)\int x^{3/2}\left(bx^2+cx^4\right)^{3/2}\,dx}{21c}$$

$$=-\frac{2(11bB-21Ac)x^{5/2}\left(bx^2+cx^4\right)^{3/2}}{357c}+\frac{2B\sqrt{x}\left(bx^2+cx^4\right)^{5/2}}{21c}-\frac{(2b(11bB-21Ac))}{11}$$

$$=-\frac{4b(11bB-21Ac)x^{9/2}\sqrt{bx^2+cx^4}}{1547c}-\frac{2(11bB-21Ac)x^{5/2}\left(bx^2+cx^4\right)^{3/2}}{357c}+\frac{2B\sqrt{x}\left(\right.}{}$$

$$=-\frac{8b^2(11bB-21Ac)x^{5/2}\sqrt{bx^2+cx^4}}{13923c^2}-\frac{4b(11bB-21Ac)x^{9/2}\sqrt{bx^2+cx^4}}{1547c}-\frac{2(11bB-}{}$$

$$=\frac{8b^3(11bB-21Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{9945c^3}-\frac{8b^2(11bB-21Ac)x^{5/2}\sqrt{bx^2+cx^4}}{13923c^2}-\frac{4b(11bB-}{}$$

$$=\frac{8b^3(11bB-21Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{9945c^3}-\frac{8b^2(11bB-21Ac)x^{5/2}\sqrt{bx^2+cx^4}}{13923c^2}-\frac{4b(11bB-}{}$$

$$=\frac{8b^3(11bB-21Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{9945c^3}-\frac{8b^2(11bB-21Ac)x^{5/2}\sqrt{bx^2+cx^4}}{13923c^2}-\frac{4b(11bB-}{}$$

$$=\frac{8b^3(11bB-21Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{9945c^3}-\frac{8b^2(11bB-21Ac)x^{5/2}\sqrt{bx^2+cx^4}}{13923c^2}-\frac{4b(11bB-}{}$$

$$=-\frac{8b^4(11bB-21Ac)x^{3/2}\left(b+cx^2\right)}{3315c^{7/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{8b^3(11bB-21Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{9945c^3}-\frac{8b^2(11b}{}$$

Mathematica [C] time = 0.175096, size = 138, normalized size = 0.31

$$\frac{2\sqrt{x}\sqrt{x^2\left(b+cx^2\right)}\left(\left(b+cx^2\right)^2\sqrt{\frac{cx^2}{b}+1}\left(-bc\left(147A+143Bx^2\right)+13c^2x^2\left(21A+17Bx^2\right)+77b^2B\right)+7b^3(21Ac-11bB)\right.}{4641c^3\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[x]*Sqrt[x^2*(b + c*x^2)]*((b + c*x^2)^2*Sqrt[1 + (c*x^2)/b]*(77*b^2
*B + 13*c^2*x^2*(21*A + 17*B*x^2) - b*c*(147*A + 143*B*x^2)) + 7*b^3*(-11*b
```

```
*B + 21*A*c)*Hypergeometric2F1[-3/2, 3/4, 7/4, -((c*x^2)/b)]))/(4641*c^3*Sq
rt[1 + (c*x^2)/b])
```

Maple [A] time = 0.014, size = 494, normalized size = 1.1

$$\frac{2}{69615\left(cx^{2}+b\right)^{2}c^{4}}\left(cx^{4}+bx^{2}\right)^{\frac{3}{2}}\left(3315\,Bx^{12}c^{6}+4095\,Ax^{10}c^{6}+7800\,Bx^{10}bc^{5}+10080\,Ax^{8}bc^{5}+4665\,Bx^{8}b^{2}c^{4}+6405\,A\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 2/69615*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2/c^4*(3315*B*x^12*c^6+4095*A
*x^10*c^6+7800*B*x^10*b*c^5+10080*A*x^8*b*c^5+4665*B*x^8*b^2*c^4+6405*A*x^6
*b^2*c^4-40*B*x^6*b^3*c^3+1764*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^
(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*El
lipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^5*c-882*A*((
c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(
1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^
(1/2))^(1/2),1/2*2^(1/2))*b^5*c-924*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/
2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^6+462*B
*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c
)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*
c)^(1/2))^(1/2),1/2*2^(1/2))*b^6-168*A*x^4*b^3*c^3+88*B*x^4*b^4*c^2-588*A*x
^2*b^4*c^2+308*B*x^2*b^5*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(cx^{4}+bx^{2}\right)^{\frac{3}{2}}\left(Bx^{2}+A\right)x^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)*x^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bcx^7 + (Bb + Ac)x^5 + Abx^3\right)\sqrt{cx^4 + bx^2}\sqrt{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*c*x^7 + (B*b + A*c)*x^5 + A*b*x^3)*sqrt(c*x^4 + b*x^2)*sqrt(x),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(B*x**2+A)*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4 + bx^2\right)^{\frac{3}{2}} \left(Bx^2 + A\right)x^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)*x^(3/2), x)
```

3.234 $\int \sqrt{x}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=282

$$-\frac{4b^{15/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(9bB-19Ac)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{4389c^{13/4}\sqrt{bx^2+cx^4}}+\frac{8b^3\sqrt{bx^2+cx^4}(9bB-19Ac)}{4389c^3\sqrt{x}}-\frac{8b^2x^{3/2}\sqrt{}}{}$$

```
[Out] (8*b^3*(9*b*B - 19*A*c)*Sqrt[b*x^2 + c*x^4])/(4389*c^3*Sqrt[x]) - (8*b^2*(9
*b*B - 19*A*c)*x^(3/2)*Sqrt[b*x^2 + c*x^4])/(7315*c^2) - (4*b*(9*b*B - 19*A
*c)*x^(7/2)*Sqrt[b*x^2 + c*x^4])/(1045*c) - (2*(9*b*B - 19*A*c)*x^(3/2)*(b*
x^2 + c*x^4)^(3/2))/(285*c) + (2*B*(b*x^2 + c*x^4)^(5/2))/(19*c*Sqrt[x]) -
(4*b^(15/4)*(9*b*B - 19*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt
[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(4
389*c^(13/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.424831, antiderivative size = 282, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2039, 2021, 2024, 2032, 329, 220}

$$\frac{8b^3\sqrt{bx^2+cx^4}(9bB-19Ac)}{4389c^3\sqrt{x}}-\frac{8b^2x^{3/2}\sqrt{bx^2+cx^4}(9bB-19Ac)}{7315c^2}-\frac{4b^{15/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(9bB-19Ac)F\left(2\tan^{-}\right.}{4389c^{13/4}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```


```
[Out] (8*b^3*(9*b*B - 19*A*c)*Sqrt[b*x^2 + c*x^4])/(4389*c^3*Sqrt[x]) - (8*b^2*(9
*b*B - 19*A*c)*x^(3/2)*Sqrt[b*x^2 + c*x^4])/(7315*c^2) - (4*b*(9*b*B - 19*A
*c)*x^(7/2)*Sqrt[b*x^2 + c*x^4])/(1045*c) - (2*(9*b*B - 19*A*c)*x^(3/2)*(b*
x^2 + c*x^4)^(3/2))/(285*c) + (2*B*(b*x^2 + c*x^4)^(5/2))/(19*c*Sqrt[x]) -
(4*b^(15/4)*(9*b*B - 19*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt
[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(4
389*c^(13/4)*Sqrt[b*x^2 + c*x^4])
```


Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
```

```
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] && !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \sqrt{x}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}dx = \frac{2B\left(bx^2+cx^4\right)^{5/2}}{19c\sqrt{x}} - \frac{\left(2\left(\frac{9bB}{2}-\frac{19Ac}{2}\right)\right)\int\sqrt{x}\left(bx^2+cx^4\right)^{3/2}dx}{19c}$$

$$= -\frac{2(9bB-19Ac)x^{3/2}\left(bx^2+cx^4\right)^{3/2}}{285c} + \frac{2B\left(bx^2+cx^4\right)^{5/2}}{19c\sqrt{x}} - \frac{(2b(9bB-19Ac))\int x^{5/2}\sqrt{}}{95c}$$

$$= -\frac{4b(9bB-19Ac)x^{7/2}\sqrt{bx^2+cx^4}}{1045c} - \frac{2(9bB-19Ac)x^{3/2}\left(bx^2+cx^4\right)^{3/2}}{285c} + \frac{2B\left(bx^2+c\right.}{19c\sqrt{x}}$$

$$= -\frac{8b^2(9bB-19Ac)x^{3/2}\sqrt{bx^2+cx^4}}{7315c^2} - \frac{4b(9bB-19Ac)x^{7/2}\sqrt{bx^2+cx^4}}{1045c} - \frac{2(9bB-19A}{}$$

$$= \frac{8b^3(9bB-19Ac)\sqrt{bx^2+cx^4}}{4389c^3\sqrt{x}} - \frac{8b^2(9bB-19Ac)x^{3/2}\sqrt{bx^2+cx^4}}{7315c^2} - \frac{4b(9bB-19Ac)x}{1045}$$

$$= \frac{8b^3(9bB-19Ac)\sqrt{bx^2+cx^4}}{4389c^3\sqrt{x}} - \frac{8b^2(9bB-19Ac)x^{3/2}\sqrt{bx^2+cx^4}}{7315c^2} - \frac{4b(9bB-19Ac)x}{1045}$$

$$= \frac{8b^3(9bB-19Ac)\sqrt{bx^2+cx^4}}{4389c^3\sqrt{x}} - \frac{8b^2(9bB-19Ac)x^{3/2}\sqrt{bx^2+cx^4}}{7315c^2} - \frac{4b(9bB-19Ac)x}{1045}$$

$$= \frac{8b^3(9bB-19Ac)\sqrt{bx^2+cx^4}}{4389c^3\sqrt{x}} - \frac{8b^2(9bB-19Ac)x^{3/2}\sqrt{bx^2+cx^4}}{7315c^2} - \frac{4b(9bB-19Ac)x}{1045}$$

Mathematica [C] time = 0.168598, size = 138, normalized size = 0.49

$$\frac{2\sqrt{x^2\left(b+cx^2\right)}\left(\left(b+cx^2\right)^2\sqrt{\frac{cx^2}{b}+1}\left(-bc\left(95A+99Bx^2\right)+11c^2x^2\left(19A+15Bx^2\right)+45b^2B\right)+5b^3(19Ac-9bB)\,{}_2F_1\left(-\frac{3}{2},\right.\right.}{3135c^3\sqrt{x}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[x^2*(b + c*x^2)]*((b + c*x^2)^2*Sqrt[1 + (c*x^2)/b]*(45*b^2*B + 11*
c^2*x^2*(19*A + 15*B*x^2) - b*c*(95*A + 99*B*x^2)) + 5*b^3*(-9*b*B + 19*A*c
)*Hypergeometric2F1[-3/2, 1/4, 5/4, -((c*x^2)/b)]))/(3135*c^3*Sqrt[x]*Sqrt[
```

```
1 + (c*x^2)/b])
```

Maple [A] time = 0.015, size = 331, normalized size = 1.2

$$\frac{2}{21945\left(cx^{2}+b\right)^{2}c^{4}}\left(cx^{4}+bx^{2}\right)^{\frac{3}{2}}\left(1155\,Bx^{11}c^{6}+1463\,Ax^{9}c^{6}+2772\,Bx^{9}bc^{5}+3724\,Ax^{7}bc^{5}+1701\,Bx^{7}b^{2}c^{4}+190\,A\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)*x^(1/2),x)
```

```
[Out] 2/21945*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2*(1155*B*x^11*c^6+1463*A*x^9
*c^6+2772*B*x^9*b*c^5+3724*A*x^7*b*c^5+1701*B*x^7*b^2*c^4+190*A*((c*x+(-b*c
)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1
/2),1/2*2^(1/2))*(-b*c)^(1/2)*b^4*c+2489*A*x^5*b^2*c^4-90*B*((c*x+(-b*c)^(1
/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(
-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),
1/2*2^(1/2))*(-b*c)^(1/2)*b^5-24*B*x^5*b^3*c^3-152*A*x^3*b^3*c^3+72*B*x^3*b
^4*c^2-380*A*x*b^4*c^2+180*B*x*b^5*c)/c^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(cx^{4}+bx^{2}\right)^{\frac{3}{2}}\left(Bx^{2}+A\right)\sqrt{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)*x^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)*sqrt(x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Bcx^{6}+(Bb+Ac)x^{4}+Abx^{2}\right)\sqrt{cx^{4}+bx^{2}}\sqrt{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)*x^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*c*x^6 + (B*b + A*c)*x^4 + A*b*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x),
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x}\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}\left(A+Bx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)*x**(1/2),x)
```

```
[Out] Integral(sqrt(x)*(x**2*(b + c*x**2))**(3/2)*(A + B*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4+bx^2\right)^{\frac{3}{2}}\left(Bx^2+A\right)\sqrt{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)*x^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)*sqrt(x), x)
```

3.235 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{\sqrt{x}}\,dx$

Optimal. Leaf size=408

$$\frac{4b^{13/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(7bB-17Ac)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{1105c^{11/4}\sqrt{bx^2+cx^4}}+\frac{8b^3x^{3/2}\left(b+cx^2\right)(7bB-17Ac)}{1105c^{5/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{8b^2\sqrt{}}{}$$

```
[Out] (8*b^3*(7*b*B - 17*A*c)*x^(3/2)*(b + c*x^2))/(1105*c^(5/2)*(Sqrt[b] + Sqrt[
c]*x)*Sqrt[b*x^2 + c*x^4]) - (8*b^2*(7*b*B - 17*A*c)*Sqrt[x]*Sqrt[b*x^2 + c
*x^4])/(3315*c^2) - (4*b*(7*b*B - 17*A*c)*x^(5/2)*Sqrt[b*x^2 + c*x^4])/(663
*c) - (2*(7*b*B - 17*A*c)*Sqrt[x]*(b*x^2 + c*x^4)^(3/2))/(221*c) + (2*B*(b*
x^2 + c*x^4)^(5/2))/(17*c*x^(3/2)) - (8*b^(13/4)*(7*b*B - 17*A*c)*x*(Sqrt[b
] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan
[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(1105*c^(11/4)*Sqrt[b*x^2 + c*x^4]) + (4
*b^(13/4)*(7*b*B - 17*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b
] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(110
5*c^(11/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.515229, antiderivative size = 408, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2039, 2021, 2024, 2032, 329, 305, 220, 1196}

$$\frac{8b^3x^{3/2}\left(b+cx^2\right)(7bB-17Ac)}{1105c^{5/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{8b^2\sqrt{x}\sqrt{bx^2+cx^4}(7bB-17Ac)}{3315c^2}+\frac{4b^{13/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(7bB-17Ac)F\left(2\right.}{1105c^{11/4}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/Sqrt[x],x]
```


```
[Out] (8*b^3*(7*b*B - 17*A*c)*x^(3/2)*(b + c*x^2))/(1105*c^(5/2)*(Sqrt[b] + Sqrt[
c]*x)*Sqrt[b*x^2 + c*x^4]) - (8*b^2*(7*b*B - 17*A*c)*Sqrt[x]*Sqrt[b*x^2 + c
*x^4])/(3315*c^2) - (4*b*(7*b*B - 17*A*c)*x^(5/2)*Sqrt[b*x^2 + c*x^4])/(663
*c) - (2*(7*b*B - 17*A*c)*Sqrt[x]*(b*x^2 + c*x^4)^(3/2))/(221*c) + (2*B*(b*
x^2 + c*x^4)^(5/2))/(17*c*x^(3/2)) - (8*b^(13/4)*(7*b*B - 17*A*c)*x*(Sqrt[b
] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan
[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(1105*c^(11/4)*Sqrt[b*x^2 + c*x^4]) + (4
*b^(13/4)*(7*b*B - 17*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b
```

```
] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(110
5*c^(11/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{\sqrt{x}}dx = \frac{2B(bx^2+cx^4)^{5/2}}{17cx^{3/2}} - \frac{\left(2\left(\frac{7bB}{2}-\frac{17Ac}{2}\right)\right)\int \frac{(bx^2+cx^4)^{3/2}}{\sqrt{x}}dx}{17c}$$

$$= -\frac{2(7bB-17Ac)\sqrt{x}(bx^2+cx^4)^{3/2}}{221c} + \frac{2B(bx^2+cx^4)^{5/2}}{17cx^{3/2}} - \frac{(6b(7bB-17Ac))\int x^{3/2}\sqrt{bx^2}}{221c}$$

$$= -\frac{4b(7bB-17Ac)x^{5/2}\sqrt{bx^2+cx^4}}{663c} - \frac{2(7bB-17Ac)\sqrt{x}(bx^2+cx^4)^{3/2}}{221c} + \frac{2B(bx^2+cx^4)^{5}}{17cx^{3/2}}$$

$$= -\frac{8b^2(7bB-17Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{3315c^2} - \frac{4b(7bB-17Ac)x^{5/2}\sqrt{bx^2+cx^4}}{663c} - \frac{2(7bB-17Ac)\sqrt{}}{22}$$

$$= -\frac{8b^2(7bB-17Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{3315c^2} - \frac{4b(7bB-17Ac)x^{5/2}\sqrt{bx^2+cx^4}}{663c} - \frac{2(7bB-17Ac)\sqrt{}}{22}$$

$$= -\frac{8b^2(7bB-17Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{3315c^2} - \frac{4b(7bB-17Ac)x^{5/2}\sqrt{bx^2+cx^4}}{663c} - \frac{2(7bB-17Ac)\sqrt{}}{22}$$

$$= -\frac{8b^2(7bB-17Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{3315c^2} - \frac{4b(7bB-17Ac)x^{5/2}\sqrt{bx^2+cx^4}}{663c} - \frac{2(7bB-17Ac)\sqrt{}}{22}$$

$$= \frac{8b^3(7bB-17Ac)x^{3/2}(b+cx^2)}{1105c^{5/2}(\sqrt{b}+\sqrt{c}x)\sqrt{bx^2+cx^4}} - \frac{8b^2(7bB-17Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{3315c^2} - \frac{4b(7bB-17Ac)x}{663}$$

Mathematica [C] time = 0.143927, size = 115, normalized size = 0.28

$$\frac{2\sqrt{x}\sqrt{x^2(b+cx^2)}\left(b^2(7bB-17Ac)\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)-(b+cx^2)^2\sqrt{\frac{cx^2}{b}+1}\left(-17Ac+7bB-13Bcx^2\right)\right)}{221c^2\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*Sqrt[x^2*(b + c*x^2)]*(-((b + c*x^2)^2*Sqrt[1 + (c*x^2)/b]*(7*b*
B - 17*A*c - 13*B*c*x^2)) + b^2*(7*b*B - 17*A*c)*Hypergeometric2F1[-3/2, 3/
4, 7/4, -((c*x^2)/b)]))/(221*c^2*Sqrt[1 + (c*x^2)/b])
```

Maple [A] time = 0.015, size = 470, normalized size = 1.2

$$-\frac{2}{3315\left(cx^2+b\right)^2c^3}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(-195\,Bx^{10}c^5-255\,Ax^8c^5-480\,Bx^8bc^4-680\,Ax^6bc^4-305\,Bx^6b^2c^3+204\,A\sqrt{\frac{cx+}{\sqrt{-}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(1/2),x)
```

```
[Out] -2/3315*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2/c^3*(-195*B*x^10*c^5-255*A*
x^8*c^5-480*B*x^8*b*c^4-680*A*x^6*b*c^4-305*B*x^6*b^2*c^3+204*A*((c*x+(-b*c
)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1
/2),1/2*2^(1/2))*b^4*c-102*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2
)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*Ellipt
icF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^4*c-84*B*((c*x+(
-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))
^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2)
)^(1/2),1/2*2^(1/2))*b^5+42*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/
2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*Ellip
ticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^5-493*A*x^4*b^2
*c^3+8*B*x^4*b^3*c^2-68*A*x^2*b^3*c^2+28*B*x^2*b^4*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(Bx^2+A\right)}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/sqrt(x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bcx^5+(Bb+Ac)x^3+Abx\right)\sqrt{cx^4+bx^2}\sqrt{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*c*x^5 + (B*b + A*c)*x^3 + A*b*x)*sqrt(c*x^4 + b*x^2)*sqrt(x), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2\right)^{\frac{3}{2}}\left(Bx^2 + A\right)}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/sqrt(x), x)
```

3.236 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{3/2}} dx$

Optimal. Leaf size=239

$$\frac{4b^{11/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(bB-3Ac)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{231c^{9/4}\sqrt{bx^2+cx^4}}-\frac{8b^2\sqrt{bx^2+cx^4}(bB-3Ac)}{231c^2\sqrt{x}}-\frac{4bx^{3/2}\sqrt{bx^2+}}{7}$$

```
[Out] (-8*b^2*(b*B - 3*A*c)*Sqrt[b*x^2 + c*x^4])/(231*c^2*Sqrt[x]) - (4*b*(b*B - 
3*A*c)*x^(3/2)*Sqrt[b*x^2 + c*x^4])/(77*c) - (2*(b*B - 3*A*c)*(b*x^2 + c*x^
4)^(3/2))/(33*c*Sqrt[x]) + (2*B*(b*x^2 + c*x^4)^(5/2))/(15*c*x^(5/2)) + (4*
b^(11/4)*(b*B - 3*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + 
Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(231*c^(
9/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.369354, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2039, 2021, 2024, 2032, 329, 220}

$$-\frac{8b^2\sqrt{bx^2+cx^4}(bB-3Ac)}{231c^2\sqrt{x}}+\frac{4b^{11/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(bB-3Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{231c^{9/4}\sqrt{bx^2+cx^4}}-\frac{4bx^{3/2}\sqrt{bx^2+cx^4}(bB}{77c}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(3/2),x]
```


```
[Out] (-8*b^2*(b*B - 3*A*c)*Sqrt[b*x^2 + c*x^4])/(231*c^2*Sqrt[x]) - (4*b*(b*B - 
3*A*c)*x^(3/2)*Sqrt[b*x^2 + c*x^4])/(77*c) - (2*(b*B - 3*A*c)*(b*x^2 + c*x^
4)^(3/2))/(33*c*Sqrt[x]) + (2*B*(b*x^2 + c*x^4)^(5/2))/(15*c*x^(5/2)) + (4*
b^(11/4)*(b*B - 3*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + 
Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(231*c^(
9/4)*Sqrt[b*x^2 + c*x^4])
```


Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) + 
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
```

```
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{3/2}}\,dx = \frac{2B(bx^2+cx^4)^{5/2}}{15cx^{5/2}} - \frac{\left(2\left(\frac{5bB}{2}-\frac{15Ac}{2}\right)\right)\int \frac{(bx^2+cx^4)^{3/2}}{x^{3/2}}\,dx}{15c}$$

$$= -\frac{2(bB-3Ac)(bx^2+cx^4)^{3/2}}{33c\sqrt{x}} + \frac{2B(bx^2+cx^4)^{5/2}}{15cx^{5/2}} - \frac{(2b(bB-3Ac))\int \sqrt{x}\sqrt{bx^2+cx^4}\,d}{11c}$$

$$= -\frac{4b(bB-3Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c} - \frac{2(bB-3Ac)(bx^2+cx^4)^{3/2}}{33c\sqrt{x}} + \frac{2B(bx^2+cx^4)^{5/2}}{15cx^{5/2}} - $$

$$= -\frac{8b^2(bB-3Ac)\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}} - \frac{4b(bB-3Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c} - \frac{2(bB-3Ac)(bx^2+cx}{33c\sqrt{x}}$$

$$= -\frac{8b^2(bB-3Ac)\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}} - \frac{4b(bB-3Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c} - \frac{2(bB-3Ac)(bx^2+cx}{33c\sqrt{x}}$$

$$= -\frac{8b^2(bB-3Ac)\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}} - \frac{4b(bB-3Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c} - \frac{2(bB-3Ac)(bx^2+cx}{33c\sqrt{x}}$$

$$= -\frac{8b^2(bB-3Ac)\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}} - \frac{4b(bB-3Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c} - \frac{2(bB-3Ac)(bx^2+cx}{33c\sqrt{x}}$$

Mathematica [C] time = 0.142065, size = 115, normalized size = 0.48

$$\frac{2\sqrt{x^2(b+cx^2)}\left(5b^2(bB-3Ac)\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{cx^2}{b}\right)-(b+cx^2)^2\sqrt{\frac{cx^2}{b}+1}\,(-15Ac+5bB-11Bcx^2)\right)}{165c^2\sqrt{x}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(3/2),x]
```

```
[Out] (2*Sqrt[x^2*(b + c*x^2)]*(-((b + c*x^2)^2*Sqrt[1 + (c*x^2)/b]*(5*b*B - 15*A
*c - 11*B*c*x^2)) + 5*b^2*(b*B - 3*A*c)*Hypergeometric2F1[-3/2, 1/4, 5/4, -
((c*x^2)/b)]))/(165*c^2*Sqrt[x]*Sqrt[1 + (c*x^2)/b])
```

Maple [A] time = 0.014, size = 307, normalized size = 1.3

$$-\frac{2}{1155\left(cx^2+b\right)^2c^3}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(-77\,Bx^9c^5-105\,Ax^7c^5-196\,Bx^7bc^4+30\,A\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(3/2),x)
```

```
[Out] -2/1155*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2*(-77*B*x^9*c^5-105*A*x^7*c^
5-196*B*x^7*b*c^4+30*A*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)
*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)
*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^3*c-300*A
*x^5*b*c^4-10*B*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2
)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*Ellipt
icF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^4-131*B*x^5*b^2*
c^3-255*A*x^3*b^2*c^3+8*B*x^3*b^3*c^2-60*A*x*b^3*c^2+20*B*x*b^4*c)/c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(Bx^2+A\right)}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bcx^4+(Bb+Ac)x^2+Ab\right)\sqrt{cx^4+bx^2}\sqrt{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*c*x^4 + (B*b + A*c)*x^2 + A*b)*sqrt(c*x^4 + b*x^2)*sqrt(x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(Bx^2+A\right)}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(3/2), x)
```

3.237 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{5/2}} dx$

Optimal. Leaf size=369

$$-\frac{4b^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(3bB-13Ac)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{195c^{7/4}\sqrt{bx^2+cx^4}}-\frac{8b^2x^{3/2}\left(b+cx^2\right)(3bB-13Ac)}{195c^{3/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{8b^{9/4}x\left(\ldots\right.}{}$$

```
[Out] (-8*b^2*(3*b*B - 13*A*c)*x^(3/2)*(b + c*x^2))/(195*c^(3/2)*(Sqrt[b] + Sqrt[
c]*x)*Sqrt[b*x^2 + c*x^4]) - (4*b*(3*b*B - 13*A*c)*Sqrt[x]*Sqrt[b*x^2 + c*x
^4])/(195*c) - (2*(3*b*B - 13*A*c)*(b*x^2 + c*x^4)^(3/2))/(117*c*x^(3/2)) +
 (2*B*(b*x^2 + c*x^4)^(5/2))/(13*c*x^(7/2)) + (8*b^(9/4)*(3*b*B - 13*A*c)*x
*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[
2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(195*c^(7/4)*Sqrt[b*x^2 + c*x^4]
) - (4*b^(9/4)*(3*b*B - 13*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(S
qrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])
/(195*c^(7/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.449977, antiderivative size = 369, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2039, 2021, 2032, 329, 305, 220, 1196}

$$-\frac{8b^2x^{3/2}\left(b+cx^2\right)(3bB-13Ac)}{195c^{3/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{4b^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(3bB-13Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{195c^{7/4}\sqrt{bx^2+cx^4}}+\frac{8b^{9/4}x\left(\sqrt{b}+\sqrt{c}\ldots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(5/2),x]
```


```
[Out] (-8*b^2*(3*b*B - 13*A*c)*x^(3/2)*(b + c*x^2))/(195*c^(3/2)*(Sqrt[b] + Sqrt[
c]*x)*Sqrt[b*x^2 + c*x^4]) - (4*b*(3*b*B - 13*A*c)*Sqrt[x]*Sqrt[b*x^2 + c*x
^4])/(195*c) - (2*(3*b*B - 13*A*c)*(b*x^2 + c*x^4)^(3/2))/(117*c*x^(3/2)) +
 (2*B*(b*x^2 + c*x^4)^(5/2))/(13*c*x^(7/2)) + (8*b^(9/4)*(3*b*B - 13*A*c)*x
*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[
2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(195*c^(7/4)*Sqrt[b*x^2 + c*x^4]
) - (4*b^(9/4)*(3*b*B - 13*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(S
qrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])
```

```
/(195*c^(7/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
```

```
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}}{x^{5/2}}\,dx = \frac{2B\left(bx^2+cx^4\right)^{5/2}}{13cx^{7/2}} - \frac{\left(2\left(\frac{3bB}{2}-\frac{13Ac}{2}\right)\right)\int\frac{\left(bx^2+cx^4\right)^{3/2}}{x^{5/2}}\,dx}{13c}$$

$$= -\frac{2(3bB-13Ac)\left(bx^2+cx^4\right)^{3/2}}{117cx^{3/2}} + \frac{2B\left(bx^2+cx^4\right)^{5/2}}{13cx^{7/2}} - \frac{(2b(3bB-13Ac))\int\frac{\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx}{39c}$$

$$= -\frac{4b(3bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{195c} - \frac{2(3bB-13Ac)\left(bx^2+cx^4\right)^{3/2}}{117cx^{3/2}} + \frac{2B\left(bx^2+cx^4\right)^{5/2}}{13cx^{7/2}} -$$

$$= -\frac{4b(3bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{195c} - \frac{2(3bB-13Ac)\left(bx^2+cx^4\right)^{3/2}}{117cx^{3/2}} + \frac{2B\left(bx^2+cx^4\right)^{5/2}}{13cx^{7/2}} -$$

$$= -\frac{4b(3bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{195c} - \frac{2(3bB-13Ac)\left(bx^2+cx^4\right)^{3/2}}{117cx^{3/2}} + \frac{2B\left(bx^2+cx^4\right)^{5/2}}{13cx^{7/2}} -$$

$$= -\frac{4b(3bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{195c} - \frac{2(3bB-13Ac)\left(bx^2+cx^4\right)^{3/2}}{117cx^{3/2}} + \frac{2B\left(bx^2+cx^4\right)^{5/2}}{13cx^{7/2}} -$$

$$= -\frac{8b^2(3bB-13Ac)x^{3/2}\left(b+cx^2\right)}{195c^{3/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{4b(3bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{195c} - \frac{2(3bB-13Ac)\left(b\right.}{117cx^{3}}$$

Mathematica [C] time = 0.103269, size = 98, normalized size = 0.27

$$\frac{2\sqrt{x}\sqrt{x^2\left(b+cx^2\right)}\left(b(13Ac-3bB)\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)+3B\sqrt{\frac{cx^2}{b}+1}\left(b+cx^2\right)^2\right)}{39c\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(5/2),x]
```

```
[Out] (2*Sqrt[x]*Sqrt[x^2*(b + c*x^2)]*(3*B*(b + c*x^2)^2*Sqrt[1 + (c*x^2)/b] + b
*(-3*b*B + 13*A*c)*Hypergeometric2F1[-3/2, 3/4, 7/4, -((c*x^2)/b)]))/(39*c*
Sqrt[1 + (c*x^2)/b])
```

Maple [A] time = 0.017, size = 446, normalized size = 1.2

$$\frac{2}{585\left(cx^2+b\right)^2c^2}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(45\,Bx^8c^4+65\,Ax^6c^4+120\,Bx^6bc^3+156\,A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\mathrm{E}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(5/2),x)
```

```
[Out] 2/585*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2/c^2*(45*B*x^8*c^4+65*A*x^6*c^
4+120*B*x^6*b*c^3+156*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-
c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE((
(c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^3*c-78*A*((c*x+(-b*c)
^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2
)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2),1/2*2^(1/2))*b^3*c-36*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*
((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*Elliptic
E(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^4+18*B*((c*x+(-b*c
)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1
/2),1/2*2^(1/2))*b^4+208*A*x^4*b*c^3+87*B*x^4*b^2*c^2+143*A*x^2*b^2*c^2+12*
B*x^2*b^3*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(Bx^2+A\right)}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bcx^4 + (Bb + Ac)x^2 + Ab\right)\sqrt{cx^4 + bx^2}}{\sqrt{x}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*c*x^4 + (B*b + A*c)*x^2 + A*b)*sqrt(c*x^4 + b*x^2)/sqrt(x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2\right)^{\frac{3}{2}}\left(Bx^2 + A\right)}{x^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(5/2), x)
```

3.238 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{7/2}}\,dx$

Optimal. Leaf size=201

$$-\frac{4b^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(bB-11Ac)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{77c^{5/4}\sqrt{bx^2+cx^4}}-\frac{2\left(bx^2+cx^4\right)^{3/2}(bB-11Ac)}{77cx^{5/2}}-\frac{4b\sqrt{bx^2+\ldots}}{7\ldots}$$

```
[Out] (-4*b*(b*B - 11*A*c)*Sqrt[b*x^2 + c*x^4])/(77*c*Sqrt[x]) - (2*(b*B - 11*A*c
)*(b*x^2 + c*x^4)^(3/2))/(77*c*x^(5/2)) + (2*B*(b*x^2 + c*x^4)^(5/2))/(11*c
*x^(9/2)) - (4*b^(7/4)*(b*B - 11*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x
^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)],
 1/2])/(77*c^(5/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.311562, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2039, 2021, 2032, 329, 220}

$$-\frac{4b^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(bB-11Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77c^{5/4}\sqrt{bx^2+cx^4}}-\frac{2\left(bx^2+cx^4\right)^{3/2}(bB-11Ac)}{77cx^{5/2}}-\frac{4b\sqrt{bx^2+cx^4}(bB-\ldots}{77c\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(7/2),x]
```

```
[Out] (-4*b*(b*B - 11*A*c)*Sqrt[b*x^2 + c*x^4])/(77*c*Sqrt[x]) - (2*(b*B - 11*A*c
)*(b*x^2 + c*x^4)^(3/2))/(77*c*x^(5/2)) + (2*B*(b*x^2 + c*x^4)^(5/2))/(11*c
*x^(9/2)) - (4*b^(7/4)*(b*B - 11*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x
^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)],
 1/2])/(77*c^(5/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
```

```
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}}{x^{7/2}}\,dx = \frac{2B\left(bx^2+cx^4\right)^{5/2}}{11cx^{9/2}} - \frac{\left(2\left(\frac{bB}{2}-\frac{11Ac}{2}\right)\right)\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{7/2}}\,dx}{11c}$$

$$= -\frac{2(bB-11Ac)\left(bx^2+cx^4\right)^{3/2}}{77cx^{5/2}} + \frac{2B\left(bx^2+cx^4\right)^{5/2}}{11cx^{9/2}} - \frac{(6b(bB-11Ac))\int \frac{\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx}{77c}$$

$$= -\frac{4b(bB-11Ac)\sqrt{bx^2+cx^4}}{77c\sqrt{x}} - \frac{2(bB-11Ac)\left(bx^2+cx^4\right)^{3/2}}{77cx^{5/2}} + \frac{2B\left(bx^2+cx^4\right)^{5/2}}{11cx^{9/2}} - \frac{(4b}{}$$

$$= -\frac{4b(bB-11Ac)\sqrt{bx^2+cx^4}}{77c\sqrt{x}} - \frac{2(bB-11Ac)\left(bx^2+cx^4\right)^{3/2}}{77cx^{5/2}} + \frac{2B\left(bx^2+cx^4\right)^{5/2}}{11cx^{9/2}} - \frac{(4b}{}$$

$$= -\frac{4b(bB-11Ac)\sqrt{bx^2+cx^4}}{77c\sqrt{x}} - \frac{2(bB-11Ac)\left(bx^2+cx^4\right)^{3/2}}{77cx^{5/2}} + \frac{2B\left(bx^2+cx^4\right)^{5/2}}{11cx^{9/2}} - \frac{(8b}{}$$

$$= -\frac{4b(bB-11Ac)\sqrt{bx^2+cx^4}}{77c\sqrt{x}} - \frac{2(bB-11Ac)\left(bx^2+cx^4\right)^{3/2}}{77cx^{5/2}} + \frac{2B\left(bx^2+cx^4\right)^{5/2}}{11cx^{9/2}} - \frac{4b^7}{}$$

Mathematica [C] time = 0.0681256, size = 97, normalized size = 0.48

$$\frac{2\sqrt{x^2\left(b+cx^2\right)}\left(b(11Ac-bB)\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{cx^2}{b}\right)+B\sqrt{\frac{cx^2}{b}+1}\left(b+cx^2\right)^2\right)}{11c\sqrt{x}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(7/2),x]
```

```
[Out] (2*Sqrt[x^2*(b + c*x^2)]*(B*(b + c*x^2)^2*Sqrt[1 + (c*x^2)/b] + b*(-(b*B) +
 11*A*c)*Hypergeometric2F1[-3/2, 1/4, 5/4, -((c*x^2)/b)]))/(11*c*Sqrt[x]*Sq
rt[1 + (c*x^2)/b])
```

Maple [A] time = 0.015, size = 283, normalized size = 1.4

$$\frac{2}{77\left(cx^2+b\right)^2c^2}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(7Bx^7c^4+22A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(7/2),x)
```

```
[Out] 2/77*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2*(7*B*x^7*c^4+22*A*((c*x+(-b*c)
^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2
)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2),1/2*2^(1/2))*(-b*c)^(1/2)*b^2*c+11*A*x^5*c^4-2*B*((c*x+(-b*c)^(1/2))/(-b
*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b
*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1
/2))*(-b*c)^(1/2)*b^3+20*B*x^5*b*c^3+44*A*x^3*b*c^3+17*B*x^3*b^2*c^2+33*A*x
*b^2*c^2+4*B*x*b^3*c)/c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2\right)^{\frac{3}{2}} \left(Bx^2 + A\right)}{x^{\frac{7}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bcx^4 + (Bb + Ac)x^2 + Ab\right)\sqrt{cx^4 + bx^2}}{x^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(7/2),x, algorithm="fricas")
```

```
[Out] integral((B*c*x^4 + (B*b + A*c)*x^2 + A*b)*sqrt(c*x^4 + b*x^2)/x^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}\left(A+Bx^2\right)}{x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**(7/2),x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**(7/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(Bx^2+A\right)}{x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(7/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(7/2), x)
```

3.239 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{9/2}}\,dx$

Optimal. Leaf size=356

$$\frac{4b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(9Ac+bB)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{15c^{3/4}\sqrt{bx^2+cx^4}}-\frac{8b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(9Ac+bB)E\left(2\right.}{15c^{3/4}\sqrt{bx^2+cx^4}}$$

```
[Out] (8*b*(b*B + 9*A*c)*x^(3/2)*(b + c*x^2))/(15*Sqrt[c]*(Sqrt[b] + Sqrt[c]*x)*S
qrt[b*x^2 + c*x^4]) + (4*(b*B + 9*A*c)*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/15 + (2
*(b*B + 9*A*c)*(b*x^2 + c*x^4)^(3/2))/(9*b*x^(3/2)) - (2*A*(b*x^2 + c*x^4)^
(5/2))/(b*x^(11/2)) - (8*b^(5/4)*(b*B + 9*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt
[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/
b^(1/4)], 1/2])/(15*c^(3/4)*Sqrt[b*x^2 + c*x^4]) + (4*b^(5/4)*(b*B + 9*A*c)
*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Elliptic
F[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*c^(3/4)*Sqrt[b*x^2 + c*x^4
])
```

Rubi [A] time = 0.445831, antiderivative size = 356, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2038, 2021, 2032, 329, 305, 220, 1196}

$$\frac{4b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(9Ac+bB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{3/4}\sqrt{bx^2+cx^4}}-\frac{8b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(9Ac+bB)E\left(2\tan^{-1}\left(\right.\right.}{15c^{3/4}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(9/2),x]
```


```
[Out] (8*b*(b*B + 9*A*c)*x^(3/2)*(b + c*x^2))/(15*Sqrt[c]*(Sqrt[b] + Sqrt[c]*x)*S
qrt[b*x^2 + c*x^4]) + (4*(b*B + 9*A*c)*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/15 + (2
*(b*B + 9*A*c)*(b*x^2 + c*x^4)^(3/2))/(9*b*x^(3/2)) - (2*A*(b*x^2 + c*x^4)^
(5/2))/(b*x^(11/2)) - (8*b^(5/4)*(b*B + 9*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt
[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/
b^(1/4)], 1/2])/(15*c^(3/4)*Sqrt[b*x^2 + c*x^4]) + (4*b^(5/4)*(b*B + 9*A*c)
*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Elliptic
F[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*c^(3/4)*Sqrt[b*x^2 + c*x^4
```

```
])
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}}{x^{9/2}}\,dx = -\frac{2A\left(bx^2+cx^4\right)^{5/2}}{bx^{11/2}} - \frac{\left(2\left(-\frac{bB}{2}-\frac{9Ac}{2}\right)\right)\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{5/2}}\,dx}{b}$$

$$= \frac{2(bB+9Ac)\left(bx^2+cx^4\right)^{3/2}}{9bx^{3/2}} - \frac{2A\left(bx^2+cx^4\right)^{5/2}}{bx^{11/2}} + \frac{1}{3}(2(bB+9Ac))\int \frac{\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx$$

$$= \frac{4}{15}(bB+9Ac)\sqrt{x}\sqrt{bx^2+cx^4} + \frac{2(bB+9Ac)\left(bx^2+cx^4\right)^{3/2}}{9bx^{3/2}} - \frac{2A\left(bx^2+cx^4\right)^{5/2}}{bx^{11/2}} + \frac{1}{15}(4$$

$$= \frac{4}{15}(bB+9Ac)\sqrt{x}\sqrt{bx^2+cx^4} + \frac{2(bB+9Ac)\left(bx^2+cx^4\right)^{3/2}}{9bx^{3/2}} - \frac{2A\left(bx^2+cx^4\right)^{5/2}}{bx^{11/2}} + \frac{(4b(}{}$$

$$= \frac{4}{15}(bB+9Ac)\sqrt{x}\sqrt{bx^2+cx^4} + \frac{2(bB+9Ac)\left(bx^2+cx^4\right)^{3/2}}{9bx^{3/2}} - \frac{2A\left(bx^2+cx^4\right)^{5/2}}{bx^{11/2}} + \frac{(8b(}{}$$

$$= \frac{4}{15}(bB+9Ac)\sqrt{x}\sqrt{bx^2+cx^4} + \frac{2(bB+9Ac)\left(bx^2+cx^4\right)^{3/2}}{9bx^{3/2}} - \frac{2A\left(bx^2+cx^4\right)^{5/2}}{bx^{11/2}} + \frac{(8b^3}{}$$

$$= \frac{8b(bB+9Ac)x^{3/2}\left(b+cx^2\right)}{15\sqrt{c}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} + \frac{4}{15}(bB+9Ac)\sqrt{x}\sqrt{bx^2+cx^4} + \frac{2(bB+9Ac)\left(bx^2+cx\right.}{9bx^{3/2}}$$

Mathematica [C] time = 0.0737532, size = 85, normalized size = 0.24

$$\frac{2\sqrt{x^2\left(b+cx^2\right)}\left(\frac{x^2(9Ac+bB)\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)}{\sqrt{\frac{cx^2}{b}+1}}-\frac{3A\left(b+cx^2\right)^2}{b}\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(9/2),x]
```

```
[Out] (2*Sqrt[x^2*(b + c*x^2)]*((-3*A*(b + c*x^2)^2)/b + ((b*B + 9*A*c)*x^2*Hyper
geometric2F1[-3/2, 3/4, 7/4, -((c*x^2)/b)])/Sqrt[1 + (c*x^2)/b]))/(3*x^(3/2
))
```

Maple [A] time = 0.02, size = 429, normalized size = 1.2

$$\frac{2}{45\left(cx^2+b\right)^2c}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(5\,Bc^3x^6+108\,A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(9/2),x)
```

```
[Out] 2/45*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2*(5*B*c^3*x^6+108*A*((c*x+(-b*c
)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1
/2),1/2*2^(1/2))*b^2*c-54*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)
*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*Ellipti
cF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c+12*B*((c*x+(-
b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^
(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))
^(1/2),1/2*2^(1/2))*b^3-6*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)
*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*Ellipti
cF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^3+9*A*x^4*c^3+16*
B*x^4*b*c^2-36*A*x^2*b*c^2+11*B*x^2*b^2*c-45*A*b^2*c)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(Bx^2+A\right)}{x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(Bcx^4+(Bb+Ac)x^2+Ab\right)\sqrt{cx^4+bx^2}}{x^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(9/2),x, algorithm="fricas")
```

```
[Out] integral((B*c*x^4 + (B*b + A*c)*x^2 + A*b)*sqrt(c*x^4 + b*x^2)/x^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(Bx^2+A\right)}{x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(9/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(9/2), x)
```

3.240 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{11/2}}\,dx$

Optimal. Leaf size=200

$$\frac{4b^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(7Ac+3bB)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{21\sqrt[4]{c}\sqrt{bx^2+cx^4}}+\frac{2\left(bx^2+cx^4\right)^{3/2}(7Ac+3bB)}{21bx^{5/2}}+\frac{4\sqrt{bx^2+cx^4}}{21}$$

```
[Out] (4*(3*b*B + 7*A*c)*Sqrt[b*x^2 + c*x^4])/(21*Sqrt[x]) + (2*(3*b*B + 7*A*c)*(
b*x^2 + c*x^4)^(3/2))/(21*b*x^(5/2)) - (2*A*(b*x^2 + c*x^4)^(5/2))/(3*b*x^(
13/2)) + (4*b^(3/4)*(3*b*B + 7*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2
)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1
/2])/(21*c^(1/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.3222, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2038, 2021, 2032, 329, 220}

$$\frac{4b^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(7Ac+3bB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21\sqrt[4]{c}\sqrt{bx^2+cx^4}}+\frac{2\left(bx^2+cx^4\right)^{3/2}(7Ac+3bB)}{21bx^{5/2}}+\frac{4\sqrt{bx^2+cx^4}(7Ac+3}{21\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(11/2),x]
```


```
[Out] (4*(3*b*B + 7*A*c)*Sqrt[b*x^2 + c*x^4])/(21*Sqrt[x]) + (2*(3*b*B + 7*A*c)*(
b*x^2 + c*x^4)^(3/2))/(21*b*x^(5/2)) - (2*A*(b*x^2 + c*x^4)^(5/2))/(3*b*x^(
13/2)) + (4*b^(3/4)*(3*b*B + 7*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2
)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1
/2])/(21*c^(1/4)*Sqrt[b*x^2 + c*x^4])
```


Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
```

```
n] && !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{11/2}}\,dx = -\frac{2A(bx^2+cx^4)^{5/2}}{3bx^{13/2}} - \frac{\left(2\left(-\frac{3bB}{2}-\frac{7Ac}{2}\right)\right)\int \frac{(bx^2+cx^4)^{3/2}}{x^{7/2}}\,dx}{3b}$$

$$= \frac{2(3bB+7Ac)(bx^2+cx^4)^{3/2}}{21bx^{5/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{3bx^{13/2}} + \frac{1}{7}(2(3bB+7Ac))\int \frac{\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx$$

$$= \frac{4(3bB+7Ac)\sqrt{bx^2+cx^4}}{21\sqrt{x}} + \frac{2(3bB+7Ac)(bx^2+cx^4)^{3/2}}{21bx^{5/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{3bx^{13/2}} + \frac{1}{21}(4b$$

$$= \frac{4(3bB+7Ac)\sqrt{bx^2+cx^4}}{21\sqrt{x}} + \frac{2(3bB+7Ac)(bx^2+cx^4)^{3/2}}{21bx^{5/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{3bx^{13/2}} + \frac{(4b(3}{}$$

$$= \frac{4(3bB+7Ac)\sqrt{bx^2+cx^4}}{21\sqrt{x}} + \frac{2(3bB+7Ac)(bx^2+cx^4)^{3/2}}{21bx^{5/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{3bx^{13/2}} + \frac{(8b(3}{}$$

$$= \frac{4(3bB+7Ac)\sqrt{bx^2+cx^4}}{21\sqrt{x}} + \frac{2(3bB+7Ac)(bx^2+cx^4)^{3/2}}{21bx^{5/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{3bx^{13/2}} + \frac{4b^{3/4}(3}{}$$

Mathematica [C] time = 0.0548848, size = 101, normalized size = 0.5

$$-\frac{2\sqrt{x^2(b+cx^2)}\left(A(b+cx^2)^2\sqrt{\frac{cx^2}{b}+1} - bx^2(7Ac+3bB)\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{cx^2}{b}\right)\right)}{3bx^{5/2}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(11/2),x]
```

```
[Out] (-2*Sqrt[x^2*(b + c*x^2)]*(A*(b + c*x^2)^2*Sqrt[1 + (c*x^2)/b] - b*(3*b*B +
 7*A*c)*x^2*Hypergeometric2F1[-3/2, 1/4, 5/4, -((c*x^2)/b)]))/(3*b*x^(5/2)*
Sqrt[1 + (c*x^2)/b])
```

Maple [A] time = 0.015, size = 260, normalized size = 1.3

$$\frac{2}{21(cx^2+b)^2c}(cx^4+bx^2)^{\frac{3}{2}}\left(14A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)\sqrt{-bc}xbc\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(11/2),x)
```

```
[Out] 2/21*(c*x^4+b*x^2)^(3/2)/x^(9/2)/(c*x^2+b)^2*(14*A*((c*x+(-b*c)^(1/2))/(-b*
c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*
c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/
2))*(-b*c)^(1/2)*x*b*c+6*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*
((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*Elliptic
F(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(-b*c)^(1/2)*x*b^2+3
*B*c^3*x^6+7*A*x^4*c^3+12*B*x^4*b*c^2+9*B*x^2*b^2*c-7*A*b^2*c)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(Bx^2+A\right)}{x^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(11/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(Bcx^4+(Bb+Ac)x^2+Ab\right)\sqrt{cx^4+bx^2}}{x^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(11/2),x, algorithm="fricas")
```

```
[Out] integral((B*c*x^4 + (B*b + A*c)*x^2 + A*b)*sqrt(c*x^4 + b*x^2)/x^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2\right)^{\frac{3}{2}}\left(Bx^2 + A\right)}{x^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(11/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(11/2), x)
```

3.241 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{13/2}}\,dx$

Optimal. Leaf size=354

$$\frac{12\sqrt[4]{b}\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(Ac+bB)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5\sqrt{bx^2+cx^4}}-\frac{2\left(bx^2+cx^4\right)^{3/2}(Ac+bB)}{bx^{7/2}}+\frac{12c\sqrt{x}\sqrt{bx^2}}{}$$

```
[Out] (24*Sqrt[c]*(b*B + A*c)*x^(3/2)*(b + c*x^2))/(5*(Sqrt[b] + Sqrt[c]*x)*Sqrt[
b*x^2 + c*x^4]) + (12*c*(b*B + A*c)*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(5*b) - (2
*(b*B + A*c)*(b*x^2 + c*x^4)^(3/2))/(b*x^(7/2)) - (2*A*(b*x^2 + c*x^4)^(5/2
))/(5*b*x^(15/2)) - (24*b^(1/4)*c^(1/4)*(b*B + A*c)*x*(Sqrt[b] + Sqrt[c]*x)
*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt
[x])/b^(1/4)], 1/2])/(5*Sqrt[b*x^2 + c*x^4]) + (12*b^(1/4)*c^(1/4)*(b*B + A
*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Ellip
ticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.435397, antiderivative size = 354, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2038, 2020, 2021, 2032, 329, 305, 220, 1196}

$$-\frac{2\left(bx^2+cx^4\right)^{3/2}(Ac+bB)}{bx^{7/2}}+\frac{12c\sqrt{x}\sqrt{bx^2+cx^4}(Ac+bB)}{5b}+\frac{24\sqrt{c}x^{3/2}\left(b+cx^2\right)(Ac+bB)}{5\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{12\sqrt[4]{b}\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(13/2),x]
```


```
[Out] (24*Sqrt[c]*(b*B + A*c)*x^(3/2)*(b + c*x^2))/(5*(Sqrt[b] + Sqrt[c]*x)*Sqrt[
b*x^2 + c*x^4]) + (12*c*(b*B + A*c)*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(5*b) - (2
*(b*B + A*c)*(b*x^2 + c*x^4)^(3/2))/(b*x^(7/2)) - (2*A*(b*x^2 + c*x^4)^(5/2
))/(5*b*x^(15/2)) - (24*b^(1/4)*c^(1/4)*(b*B + A*c)*x*(Sqrt[b] + Sqrt[c]*x)
*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt
[x])/b^(1/4)], 1/2])/(5*Sqrt[b*x^2 + c*x^4]) + (12*b^(1/4)*c^(1/4)*(b*B + A
*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Ellip
ticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*Sqrt[b*x^2 + c*x^4])
```


Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
```

```
b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}}{x^{13/2}}\,dx = -\frac{2A\left(bx^2+cx^4\right)^{5/2}}{5bx^{15/2}} + -\frac{\left(2\left(-\frac{5bB}{2}-\frac{5Ac}{2}\right)\right)\int\frac{\left(bx^2+cx^4\right)^{3/2}}{x^{9/2}}\,dx}{5b}$$

$$= -\frac{2(bB+Ac)\left(bx^2+cx^4\right)^{3/2}}{bx^{7/2}} - \frac{2A\left(bx^2+cx^4\right)^{5/2}}{5bx^{15/2}} + \frac{(6c(bB+Ac))\int\frac{\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx}{b}$$

$$= \frac{12c(bB+Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5b} - \frac{2(bB+Ac)\left(bx^2+cx^4\right)^{3/2}}{bx^{7/2}} - \frac{2A\left(bx^2+cx^4\right)^{5/2}}{5bx^{15/2}} + \frac{1}{5}(12$$

$$= \frac{12c(bB+Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5b} - \frac{2(bB+Ac)\left(bx^2+cx^4\right)^{3/2}}{bx^{7/2}} - \frac{2A\left(bx^2+cx^4\right)^{5/2}}{5bx^{15/2}} + \frac{(12c}{}$$

$$= \frac{12c(bB+Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5b} - \frac{2(bB+Ac)\left(bx^2+cx^4\right)^{3/2}}{bx^{7/2}} - \frac{2A\left(bx^2+cx^4\right)^{5/2}}{5bx^{15/2}} + \frac{(24c}{}$$

$$= \frac{12c(bB+Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5b} - \frac{2(bB+Ac)\left(bx^2+cx^4\right)^{3/2}}{bx^{7/2}} - \frac{2A\left(bx^2+cx^4\right)^{5/2}}{5bx^{15/2}} + \frac{(24\sqrt{}}{}$$

$$= \frac{24\sqrt{c}(bB+Ac)x^{3/2}\left(b+cx^2\right)}{5\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} + \frac{12c(bB+Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5b} - \frac{2(bB+Ac)\left(bx^2+cx^4\right.}{bx^{7/2}}$$

Mathematica [C] time = 0.044802, size = 99, normalized size = 0.28

$$-\frac{2\sqrt{x^2\left(b+cx^2\right)}\left(5bx^2(Ac+bB)\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{4};\frac{3}{4};-\frac{cx^2}{b}\right)+A\left(b+cx^2\right)^2\sqrt{\frac{cx^2}{b}+1}\right)}{5bx^{7/2}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(13/2),x]
```

```
[Out] (-2*Sqrt[x^2*(b + c*x^2)]*(A*(b + c*x^2)^2*Sqrt[1 + (c*x^2)/b] + 5*b*(b*B +
 A*c)*x^2*Hypergeometric2F1[-3/2, -1/4, 3/4, -((c*x^2)/b)]))/(5*b*x^(7/2)*S
qrt[1 + (c*x^2)/b])
```

Maple [A] time = 0.019, size = 427, normalized size = 1.2

$$\frac{2}{5\left(cx^2+b\right)^2}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(12\,A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)x^2bc-6\,A\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(13/2),x)
```

```
[Out] 2/5*(c*x^4+b*x^2)^(3/2)/x^(11/2)/(c*x^2+b)^2*(12*A*((c*x+(-b*c)^(1/2))/(-b*
c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*
c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/
2))*x^2*b*c-6*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*
c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b
*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b*c+12*B*((c*x+(-b*c)^(1/2)
)/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*
c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2
*2^(1/2))*x^2*b^2-6*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*
x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c
*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^2+B*c^2*x^6-7*A*c^2
*x^4-4*B*x^4*b*c-8*A*b*c*x^2-5*B*x^2*b^2-A*b^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2\right)^{\frac{3}{2}}\left(Bx^2 + A\right)}{x^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(13/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(13/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bcx^4 + (Bb + Ac)x^2 + Ab\right)\sqrt{cx^4 + bx^2}}{x^{\frac{9}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(13/2),x, algorithm="fricas")
```

```
[Out] integral((B*c*x^4 + (B*b + A*c)*x^2 + A*b)*sqrt(c*x^4 + b*x^2)/x^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**(13/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2\right)^{\frac{3}{2}}\left(Bx^2 + A\right)}{x^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(13/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(13/2), x)
```

3.242 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{15/2}}\,dx$

Optimal. Leaf size=204

$$\frac{4c^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(3Ac+7bB)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{21\sqrt[4]{b}\sqrt{bx^2+cx^4}}-\frac{2\left(bx^2+cx^4\right)^{3/2}(3Ac+7bB)}{21bx^{9/2}}+\frac{4c\sqrt{bx^2+c}}{2}$$

```
[Out] (4*c*(7*b*B + 3*A*c)*Sqrt[b*x^2 + c*x^4])/(21*b*Sqrt[x]) - (2*(7*b*B + 3*A*
c)*(b*x^2 + c*x^4)^(3/2))/(21*b*x^(9/2)) - (2*A*(b*x^2 + c*x^4)^(5/2))/(7*b
*x^(17/2)) + (4*c^(3/4)*(7*b*B + 3*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c
*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)
], 1/2])/(21*b^(1/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.319058, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2038, 2020, 2021, 2032, 329, 220}

$$\frac{4c^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(3Ac+7bB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21\sqrt[4]{b}\sqrt{bx^2+cx^4}}-\frac{2\left(bx^2+cx^4\right)^{3/2}(3Ac+7bB)}{21bx^{9/2}}+\frac{4c\sqrt{bx^2+cx^4}(3Ac}{21b\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(15/2),x]
```



```
[Out] (4*c*(7*b*B + 3*A*c)*Sqrt[b*x^2 + c*x^4])/(21*b*Sqrt[x]) - (2*(7*b*B + 3*A*
c)*(b*x^2 + c*x^4)^(3/2))/(21*b*x^(9/2)) - (2*A*(b*x^2 + c*x^4)^(5/2))/(7*b
*x^(17/2)) + (4*c^(3/4)*(7*b*B + 3*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c
*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)
], 1/2])/(21*b^(1/4)*Sqrt[b*x^2 + c*x^4])
```


Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
```

```
n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{15/2}}\,dx = -\frac{2A(bx^2+cx^4)^{5/2}}{7bx^{17/2}} - \frac{\left(2\left(-\frac{7bB}{2}-\frac{3Ac}{2}\right)\right)\int \frac{(bx^2+cx^4)^{3/2}}{x^{11/2}}\,dx}{7b}$$

$$= -\frac{2(7bB+3Ac)(bx^2+cx^4)^{3/2}}{21bx^{9/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{7bx^{17/2}} + \frac{(2c(7bB+3Ac))\int \frac{\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx}{7b}$$

$$= \frac{4c(7bB+3Ac)\sqrt{bx^2+cx^4}}{21b\sqrt{x}} - \frac{2(7bB+3Ac)(bx^2+cx^4)^{3/2}}{21bx^{9/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{7bx^{17/2}} + \frac{1}{21}($$

$$= \frac{4c(7bB+3Ac)\sqrt{bx^2+cx^4}}{21b\sqrt{x}} - \frac{2(7bB+3Ac)(bx^2+cx^4)^{3/2}}{21bx^{9/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{7bx^{17/2}} + \frac{(4c}{}$$

$$= \frac{4c(7bB+3Ac)\sqrt{bx^2+cx^4}}{21b\sqrt{x}} - \frac{2(7bB+3Ac)(bx^2+cx^4)^{3/2}}{21bx^{9/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{7bx^{17/2}} + \frac{(8c}{}$$

$$= \frac{4c(7bB+3Ac)\sqrt{bx^2+cx^4}}{21b\sqrt{x}} - \frac{2(7bB+3Ac)(bx^2+cx^4)^{3/2}}{21bx^{9/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{7bx^{17/2}} + \frac{4c^{3/}}{}$$

Mathematica [C] time = 0.0523253, size = 101, normalized size = 0.5

$$-\frac{2\sqrt{x^2(b+cx^2)}\left(bx^2(3Ac+7bB)\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{4};\frac{1}{4};-\frac{cx^2}{b}\right)+3A(b+cx^2)^2\sqrt{\frac{cx^2}{b}+1}\right)}{21bx^{9/2}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(15/2),x]
```

```
[Out] (-2*Sqrt[x^2*(b + c*x^2)]*(3*A*(b + c*x^2)^2*Sqrt[1 + (c*x^2)/b] + b*(7*b*B
 + 3*A*c)*x^2*Hypergeometric2F1[-3/2, -3/4, 1/4, -((c*x^2)/b)]))/(21*b*x^(9
/2)*Sqrt[1 + (c*x^2)/b])
```

Maple [A] time = 0.016, size = 254, normalized size = 1.3

$$\frac{2}{21(cx^2+b)^2}(cx^4+bx^2)^{\frac{3}{2}}\left(6A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)\sqrt{-bc}x^3c+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(15/2),x)
```

```
[Out] 2/21*(c*x^4+b*x^2)^(3/2)/x^(13/2)/(c*x^2+b)^2*(6*A*((c*x+(-b*c)^(1/2))/(-b*
c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*
c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/
2))*(-b*c)^(1/2)*x^3*c+14*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)
*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*Ellipti
cF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(-b*c)^(1/2)*x^3*b+
7*B*c^2*x^6-9*A*c^2*x^4-12*A*b*c*x^2-7*B*x^2*b^2-3*A*b^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(Bx^2+A\right)}{x^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(15/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(15/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bcx^4+(Bb+Ac)x^2+Ab\right)\sqrt{cx^4+bx^2}}{x^{\frac{11}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(15/2),x, algorithm="fricas")
```

```
[Out] integral((B*c*x^4 + (B*b + A*c)*x^2 + A*b)*sqrt(c*x^4 + b*x^2)/x^(11/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**(15/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^4 + bx^2)^{\frac{3}{2}}(Bx^2 + A)}{x^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(15/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(15/2), x)
```

3.243 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{17/2}}\,dx$

Optimal. Leaf size=364

$$\frac{4c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(Ac+9bB)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{15b^{3/4}\sqrt{bx^2+cx^4}}-\frac{8c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(Ac+9bB)E\left(2\right.}{15b^{3/4}\sqrt{bx^2+cx^4}}$$

```
[Out] (8*c^(3/2)*(9*b*B + A*c)*x^(3/2)*(b + c*x^2))/(15*b*(Sqrt[b] + Sqrt[c]*x)*S
qrt[b*x^2 + c*x^4]) - (4*c*(9*b*B + A*c)*Sqrt[b*x^2 + c*x^4])/(15*b*x^(3/2)
) - (2*(9*b*B + A*c)*(b*x^2 + c*x^4)^(3/2))/(45*b*x^(11/2)) - (2*A*(b*x^2 +
 c*x^4)^(5/2))/(9*b*x^(19/2)) - (8*c^(5/4)*(9*b*B + A*c)*x*(Sqrt[b] + Sqrt[
c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)
*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[b*x^2 + c*x^4]) + (4*c^(5/4)*(9*
b*B + A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2
]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[b*x
^2 + c*x^4])
```

Rubi [A] time = 0.445657, antiderivative size = 364, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2038, 2020, 2032, 329, 305, 220, 1196}

$$\frac{4c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(Ac+9bB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{bx^2+cx^4}}-\frac{8c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(Ac+9bB)E\left(2\tan^{-1}\left(\right.\right.}{15b^{3/4}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(17/2),x]
```


```
[Out] (8*c^(3/2)*(9*b*B + A*c)*x^(3/2)*(b + c*x^2))/(15*b*(Sqrt[b] + Sqrt[c]*x)*S
qrt[b*x^2 + c*x^4]) - (4*c*(9*b*B + A*c)*Sqrt[b*x^2 + c*x^4])/(15*b*x^(3/2)
) - (2*(9*b*B + A*c)*(b*x^2 + c*x^4)^(3/2))/(45*b*x^(11/2)) - (2*A*(b*x^2 +
 c*x^4)^(5/2))/(9*b*x^(19/2)) - (8*c^(5/4)*(9*b*B + A*c)*x*(Sqrt[b] + Sqrt[
c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)
*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[b*x^2 + c*x^4]) + (4*c^(5/4)*(9*
b*B + A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2
]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[b*x
```

```
^2 + c*x^4])
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}}{x^{17/2}}\,dx = -\frac{2A\left(bx^2+cx^4\right)^{5/2}}{9bx^{19/2}} - \frac{\left(2\left(-\frac{9bB}{2}-\frac{Ac}{2}\right)\right)\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{13/2}}\,dx}{9b}$$

$$= -\frac{2(9bB+Ac)\left(bx^2+cx^4\right)^{3/2}}{45bx^{11/2}} - \frac{2A\left(bx^2+cx^4\right)^{5/2}}{9bx^{19/2}} + \frac{(2c(9bB+Ac))\int \frac{\sqrt{bx^2+cx^4}}{x^{5/2}}\,dx}{15b}$$

$$= -\frac{4c(9bB+Ac)\sqrt{bx^2+cx^4}}{15bx^{3/2}} - \frac{2(9bB+Ac)\left(bx^2+cx^4\right)^{3/2}}{45bx^{11/2}} - \frac{2A\left(bx^2+cx^4\right)^{5/2}}{9bx^{19/2}} + \frac{\left(4c^2(9\right.}{}$$

$$= -\frac{4c(9bB+Ac)\sqrt{bx^2+cx^4}}{15bx^{3/2}} - \frac{2(9bB+Ac)\left(bx^2+cx^4\right)^{3/2}}{45bx^{11/2}} - \frac{2A\left(bx^2+cx^4\right)^{5/2}}{9bx^{19/2}} + \frac{\left(4c^2(9\right.}{}$$

$$= -\frac{4c(9bB+Ac)\sqrt{bx^2+cx^4}}{15bx^{3/2}} - \frac{2(9bB+Ac)\left(bx^2+cx^4\right)^{3/2}}{45bx^{11/2}} - \frac{2A\left(bx^2+cx^4\right)^{5/2}}{9bx^{19/2}} + \frac{\left(8c^2(9\right.}{}$$

$$= -\frac{4c(9bB+Ac)\sqrt{bx^2+cx^4}}{15bx^{3/2}} - \frac{2(9bB+Ac)\left(bx^2+cx^4\right)^{3/2}}{45bx^{11/2}} - \frac{2A\left(bx^2+cx^4\right)^{5/2}}{9bx^{19/2}} + \frac{\left(8c^{3/2}(\right.}{}$$

$$= \frac{8c^{3/2}(9bB+Ac)x^{3/2}\left(b+cx^2\right)}{15b\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{4c(9bB+Ac)\sqrt{bx^2+cx^4}}{15bx^{3/2}} - \frac{2(9bB+Ac)\left(bx^2+cx^4\right)^{3/2}}{45bx^{11/2}}$$

Mathematica [C] time = 0.0549277, size = 100, normalized size = 0.27

$$-\frac{2\sqrt{x^2\left(b+cx^2\right)}\left(bx^2(Ac+9bB)\,{}_2F_1\left(-\frac{3}{2},-\frac{5}{4};-\frac{1}{4};-\frac{cx^2}{b}\right)+5A\left(b+cx^2\right)^2\sqrt{\frac{cx^2}{b}+1}\right)}{45bx^{11/2}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(17/2),x]
```

```
[Out] (-2*Sqrt[x^2*(b + c*x^2)]*(5*A*(b + c*x^2)^2*Sqrt[1 + (c*x^2)/b] + b*(9*b*B
 + A*c)*x^2*Hypergeometric2F1[-3/2, -5/4, -1/4, -((c*x^2)/b)]))/(45*b*x^(11
/2)*Sqrt[1 + (c*x^2)/b])
```

Maple [A] time = 0.02, size = 452, normalized size = 1.2

$$\frac{2}{45\left(cx^2+b\right)^2b}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(12\,A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\text{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)x^4bc^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(17/2),x)
```

```
[Out] 2/45*(c*x^4+b*x^2)^(3/2)/x^(15/2)/(c*x^2+b)^2*(12*A*((c*x+(-b*c)^(1/2))/(-b
*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b
*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1
/2))*x^4*b*c^2-6*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(
-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+
(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^4*b*c^2+108*B*((c*x+(-b*c)
^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2
)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2),1/2*2^(1/2))*x^4*b^2*c-54*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1
/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*Elli
pticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^4*b^2*c-12*A*c
^3*x^6-63*B*x^6*b*c^2-23*A*b*c^2*x^4-72*B*x^4*b^2*c-16*A*b^2*c*x^2-9*B*x^2*
b^3-5*A*b^3)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(Bx^2+A\right)}{x^{\frac{17}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(17/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(17/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bcx^4+(Bb+Ac)x^2+Ab\right)\sqrt{cx^4+bx^2}}{x^{\frac{13}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(17/2),x, algorithm="fricas")
```

```
[Out] integral((B*c*x^4 + (B*b + A*c)*x^2 + A*b)*sqrt(c*x^4 + b*x^2)/x^(13/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**(17/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(Bx^2+A\right)}{x^{\frac{17}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(17/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(17/2), x)
```

3.244 $\int \frac{x^{13/2}\left(A+Bx^2\right)}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=243

$$\frac{b^{11/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(13bB-15Ac)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{77c^{17/4}\sqrt{bx^2+cx^4}}-\frac{2b^2\sqrt{bx^2+cx^4}(13bB-15Ac)}{77c^4\sqrt{x}}-\frac{2x^{7/2}\sqrt{bx^2}}{}$$

```
[Out] (-2*b^2*(13*b*B - 15*A*c)*Sqrt[b*x^2 + c*x^4])/(77*c^4*Sqrt[x]) + (6*b*(13*
b*B - 15*A*c)*x^(3/2)*Sqrt[b*x^2 + c*x^4])/(385*c^3) - (2*(13*b*B - 15*A*c)
*x^(7/2)*Sqrt[b*x^2 + c*x^4])/(165*c^2) + (2*B*x^(11/2)*Sqrt[b*x^2 + c*x^4]
)/(15*c) + (b^(11/4)*(13*b*B - 15*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*
x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)]
, 1/2])/(77*c^(17/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.367642, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2039, 2024, 2032, 329, 220}

$$-\frac{2b^2\sqrt{bx^2+cx^4}(13bB-15Ac)}{77c^4\sqrt{x}}+\frac{b^{11/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(13bB-15Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77c^{17/4}\sqrt{bx^2+cx^4}}-\frac{2x^{7/2}\sqrt{bx^2+cx^4}(}{165c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(13/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```


```
[Out] (-2*b^2*(13*b*B - 15*A*c)*Sqrt[b*x^2 + c*x^4])/(77*c^4*Sqrt[x]) + (6*b*(13*
b*B - 15*A*c)*x^(3/2)*Sqrt[b*x^2 + c*x^4])/(385*c^3) - (2*(13*b*B - 15*A*c)
*x^(7/2)*Sqrt[b*x^2 + c*x^4])/(165*c^2) + (2*B*x^(11/2)*Sqrt[b*x^2 + c*x^4]
)/(15*c) + (b^(11/4)*(13*b*B - 15*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*
x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)]
, 1/2])/(77*c^(17/4)*Sqrt[b*x^2 + c*x^4])
```


Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
```

```
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{x^{13/2}\left(A+Bx^2\right)}{\sqrt{bx^2+cx^4}}dx = \frac{2Bx^{11/2}\sqrt{bx^2+cx^4}}{15c} - \frac{\left(2\left(\frac{13bB}{2}-\frac{15Ac}{2}\right)\right)\int \frac{x^{13/2}}{\sqrt{bx^2+cx^4}}dx}{15c}$$

$$= -\frac{2(13bB-15Ac)x^{7/2}\sqrt{bx^2+cx^4}}{165c^2} + \frac{2Bx^{11/2}\sqrt{bx^2+cx^4}}{15c} + \frac{(3b(13bB-15Ac))\int \frac{x^{9/2}}{\sqrt{bx^2+cx^4}}dx}{55c^2}$$

$$= \frac{6b(13bB-15Ac)x^{3/2}\sqrt{bx^2+cx^4}}{385c^3} - \frac{2(13bB-15Ac)x^{7/2}\sqrt{bx^2+cx^4}}{165c^2} + \frac{2Bx^{11/2}\sqrt{bx^2+cx^4}}{15c} - \frac{(3b^2}{}$$

$$= -\frac{2b^2(13bB-15Ac)\sqrt{bx^2+cx^4}}{77c^4\sqrt{x}} + \frac{6b(13bB-15Ac)x^{3/2}\sqrt{bx^2+cx^4}}{385c^3} - \frac{2(13bB-15Ac)x^{7/2}\sqrt{bx^2+}}{165c^2}$$

$$= -\frac{2b^2(13bB-15Ac)\sqrt{bx^2+cx^4}}{77c^4\sqrt{x}} + \frac{6b(13bB-15Ac)x^{3/2}\sqrt{bx^2+cx^4}}{385c^3} - \frac{2(13bB-15Ac)x^{7/2}\sqrt{bx^2+}}{165c^2}$$

$$= -\frac{2b^2(13bB-15Ac)\sqrt{bx^2+cx^4}}{77c^4\sqrt{x}} + \frac{6b(13bB-15Ac)x^{3/2}\sqrt{bx^2+cx^4}}{385c^3} - \frac{2(13bB-15Ac)x^{7/2}\sqrt{bx^2+}}{165c^2}$$

$$= -\frac{2b^2(13bB-15Ac)\sqrt{bx^2+cx^4}}{77c^4\sqrt{x}} + \frac{6b(13bB-15Ac)x^{3/2}\sqrt{bx^2+cx^4}}{385c^3} - \frac{2(13bB-15Ac)x^{7/2}\sqrt{bx^2+}}{165c^2}$$

Mathematica [C] time = 0.158961, size = 143, normalized size = 0.59

$$\frac{2x^{3/2}\left(15b^3\sqrt{\frac{cx^2}{b}+1}(13bB-15Ac)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^2}{b}\right)-\left(b+cx^2\right)\left(-9b^2c\left(25A+13Bx^2\right)+bc^2x^2\left(135A+91Bx^2\right)-7c^3x^4\right.\right.}{1155c^4\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(13/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*x^(3/2)*(-((b + c*x^2)*(195*b^3*B - 7*c^3*x^4*(15*A + 11*B*x^2) - 9*b^2*
c*(25*A + 13*B*x^2) + b*c^2*x^2*(135*A + 91*B*x^2))) + 15*b^3*(13*b*B - 15*
A*c)*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^2)/b)]))/(
1155*c^4*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.025, size = 298, normalized size = 1.2

$$-\frac{1}{1155\,c^5}\sqrt{x}\left(-154\,Bx^9c^5-210\,Ax^7c^5+28\,Bx^7bc^4+225\,A\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\mathrm{EllipticF}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/1155/(c*x^4+b*x^2)^(1/2)*x^(1/2)*(-154*B*x^9*c^5-210*A*x^7*c^5+28*B*x^7*
b*c^4+225*A*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((
-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(
((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^3*c+60*A*x^5*b*c^4-1
95*B*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-
b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(
-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^4-52*B*x^5*b^2*c^3-180*A*x^
3*b^2*c^3+156*B*x^3*b^3*c^2-450*A*x*b^3*c^2+390*B*x*b^4*c)/c^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2+A\right)x^{\frac{13}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^(13/2)/sqrt(c*x^4 + b*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(Bx^6+Ax^4\right)\sqrt{cx^4+bx^2}\sqrt{x}}{cx^2+b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^6 + A*x^4)*sqrt(c*x^4 + b*x^2)*sqrt(x)/(c*x^2 + b), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(13/2)*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)x^{\frac{13}{2}}}{\sqrt{cx^4 + bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^(13/2)/sqrt(c*x^4 + b*x^2), x)
```

3.245 $\int \frac{x^{11/2}\left(A+Bx^2\right)}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=369

$$-\frac{7b^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(11bB-13Ac)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{195c^{15/4}\sqrt{bx^2+cx^4}}-\frac{14b^2x^{3/2}\left(b+cx^2\right)(11bB-13Ac)}{195c^{7/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{14b}{}$$

```
[Out] (-14*b^2*(11*b*B - 13*A*c)*x^(3/2)*(b + c*x^2))/(195*c^(7/2)*(Sqrt[b] + Sqr
t[c]*x)*Sqrt[b*x^2 + c*x^4]) + (14*b*(11*b*B - 13*A*c)*Sqrt[x]*Sqrt[b*x^2 +
 c*x^4])/(585*c^3) - (2*(11*b*B - 13*A*c)*x^(5/2)*Sqrt[b*x^2 + c*x^4])/(117
*c^2) + (2*B*x^(9/2)*Sqrt[b*x^2 + c*x^4])/(13*c) + (14*b^(9/4)*(11*b*B - 13
*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Ell
ipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(195*c^(15/4)*Sqrt[b*x^2
+ c*x^4]) - (7*b^(9/4)*(11*b*B - 13*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b +
c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4
)], 1/2])/(195*c^(15/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.444791, antiderivative size = 369, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2039, 2024, 2032, 329, 305, 220, 1196}

$$-\frac{14b^2x^{3/2}\left(b+cx^2\right)(11bB-13Ac)}{195c^{7/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{7b^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(11bB-13Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{195c^{15/4}\sqrt{bx^2+cx^4}}+\frac{14b^{9/4}x\left(\sqrt{b}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(11/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```


```
[Out] (-14*b^2*(11*b*B - 13*A*c)*x^(3/2)*(b + c*x^2))/(195*c^(7/2)*(Sqrt[b] + Sqr
t[c]*x)*Sqrt[b*x^2 + c*x^4]) + (14*b*(11*b*B - 13*A*c)*Sqrt[x]*Sqrt[b*x^2 +
 c*x^4])/(585*c^3) - (2*(11*b*B - 13*A*c)*x^(5/2)*Sqrt[b*x^2 + c*x^4])/(117
*c^2) + (2*B*x^(9/2)*Sqrt[b*x^2 + c*x^4])/(13*c) + (14*b^(9/4)*(11*b*B - 13
*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Ell
ipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(195*c^(15/4)*Sqrt[b*x^2
+ c*x^4]) - (7*b^(9/4)*(11*b*B - 13*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b +
c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4
```

```
)], 1/2])/(195*c^(15/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^{11/2}\left(A+Bx^2\right)}{\sqrt{bx^2+cx^4}}\,dx = \frac{2Bx^{9/2}\sqrt{bx^2+cx^4}}{13c} - \frac{\left(2\left(\frac{11bB}{2}-\frac{13Ac}{2}\right)\right)\int \frac{x^{11/2}}{\sqrt{bx^2+cx^4}}\,dx}{13c}$$

$$= -\frac{2(11bB-13Ac)x^{5/2}\sqrt{bx^2+cx^4}}{117c^2} + \frac{2Bx^{9/2}\sqrt{bx^2+cx^4}}{13c} + \frac{(7b(11bB-13Ac))\int \frac{x^{7/2}}{\sqrt{bx^2+cx^4}}\,dx}{117c^2}$$

$$= \frac{14b(11bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{585c^3} - \frac{2(11bB-13Ac)x^{5/2}\sqrt{bx^2+cx^4}}{117c^2} + \frac{2Bx^{9/2}\sqrt{bx^2+cx^4}}{13c} - \frac{(7b^{}}{}$$

$$= \frac{14b(11bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{585c^3} - \frac{2(11bB-13Ac)x^{5/2}\sqrt{bx^2+cx^4}}{117c^2} + \frac{2Bx^{9/2}\sqrt{bx^2+cx^4}}{13c} - \frac{\left(7b\right.}{}$$

$$= \frac{14b(11bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{585c^3} - \frac{2(11bB-13Ac)x^{5/2}\sqrt{bx^2+cx^4}}{117c^2} + \frac{2Bx^{9/2}\sqrt{bx^2+cx^4}}{13c} - \frac{\left(14\right.}{}$$

$$= \frac{14b(11bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{585c^3} - \frac{2(11bB-13Ac)x^{5/2}\sqrt{bx^2+cx^4}}{117c^2} + \frac{2Bx^{9/2}\sqrt{bx^2+cx^4}}{13c} - \frac{\left(14\right.}{}$$

$$= -\frac{14b^2(11bB-13Ac)x^{3/2}\left(b+cx^2\right)}{195c^{7/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} + \frac{14b(11bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{585c^3} - \frac{2(11bB-13Ac)x^{5/2}\sqrt{}}{117c^2}$$

Mathematica [C] time = 0.138221, size = 122, normalized size = 0.33

$$\frac{2x^{5/2}\left(\left(b+cx^2\right)\left(-bc\left(91A+55Bx^2\right)+5c^2x^2\left(13A+9Bx^2\right)+77b^2B\right)+7b^2\sqrt{\frac{cx^2}{b}+1}(13Ac-11bB)\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)\right)}{585c^3\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(11/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*x^(5/2)*((b + c*x^2)*(77*b^2*B + 5*c^2*x^2*(13*A + 9*B*x^2) - b*c*(91*A
+ 55*B*x^2)) + 7*b^2*(-11*b*B + 13*A*c)*Sqrt[1 + (c*x^2)/b]*Hypergeometric2
F1[1/2, 3/4, 7/4, -((c*x^2)/b)]))/(585*c^3*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.025, size = 437, normalized size = 1.2

$$\frac{1}{585\,c^4}\sqrt{x}\left(90\,Bx^8c^4+130\,Ax^6c^4-20\,Bx^6bc^3+546\,A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/585/(c*x^4+b*x^2)^(1/2)*x^(1/2)/c^4*(90*B*x^8*c^4+130*A*x^6*c^4-20*B*x^6*
b*c^3+546*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(
1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^
(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^3*c-273*A*((c*x+(-b*c)^(1/2))/(-b
*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b
*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1
/2))*b^3*c-462*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b
*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-
b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^4+231*B*((c*x+(-b*c)^(1/2))/
(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/
(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2
^(1/2))*b^4-52*A*x^4*b*c^3+44*B*x^4*b^2*c^2-182*A*x^2*b^2*c^2+154*B*x^2*b^3
*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)x^{\frac{11}{2}}}{\sqrt{cx^4 + bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^(11/2)/sqrt(c*x^4 + b*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(Bx^5 + Ax^3)\sqrt{cx^4 + bx^2}\sqrt{x}}{cx^2 + b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^5 + A*x^3)*sqrt(c*x^4 + b*x^2)*sqrt(x)/(c*x^2 + b), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(11/2)*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)x^{\frac{11}{2}}}{\sqrt{cx^4 + bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^(11/2)/sqrt(c*x^4 + b*x^2), x)
```

3.246 $\int \frac{x^{9/2}\left(A+Bx^2\right)}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=204

$$-\frac{5b^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(9bB-11Ac)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{231c^{13/4}\sqrt{bx^2+cx^4}}-\frac{2x^{3/2}\sqrt{bx^2+cx^4}(9bB-11Ac)}{77c^2}+\frac{10b\sqrt{bx}}{}$$

```
[Out] (10*b*(9*b*B - 11*A*c)*Sqrt[b*x^2 + c*x^4])/(231*c^3*Sqrt[x]) - (2*(9*b*B -
 11*A*c)*x^(3/2)*Sqrt[b*x^2 + c*x^4])/(77*c^2) + (2*B*x^(7/2)*Sqrt[b*x^2 +
c*x^4])/(11*c) - (5*b^(7/4)*(9*b*B - 11*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(
b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^
(1/4)], 1/2])/(231*c^(13/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.313829, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2039, 2024, 2032, 329, 220}

$$-\frac{5b^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(9bB-11Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{231c^{13/4}\sqrt{bx^2+cx^4}}-\frac{2x^{3/2}\sqrt{bx^2+cx^4}(9bB-11Ac)}{77c^2}+\frac{10b\sqrt{bx^2+cx^4}(9}{231c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(9/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```


```
[Out] (10*b*(9*b*B - 11*A*c)*Sqrt[b*x^2 + c*x^4])/(231*c^3*Sqrt[x]) - (2*(9*b*B -
 11*A*c)*x^(3/2)*Sqrt[b*x^2 + c*x^4])/(77*c^2) + (2*B*x^(7/2)*Sqrt[b*x^2 +
c*x^4])/(11*c) - (5*b^(7/4)*(9*b*B - 11*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(
b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^
(1/4)], 1/2])/(231*c^(13/4)*Sqrt[b*x^2 + c*x^4])
```


Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
```

```
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{x^{9/2}\left(A+Bx^2\right)}{\sqrt{bx^2+cx^4}}\,dx = \frac{2Bx^{7/2}\sqrt{bx^2+cx^4}}{11c} - \frac{\left(2\left(\frac{9bB}{2}-\frac{11Ac}{2}\right)\right)\int \frac{x^{9/2}}{\sqrt{bx^2+cx^4}}\,dx}{11c}$$

$$= -\frac{2(9bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c^2} + \frac{2Bx^{7/2}\sqrt{bx^2+cx^4}}{11c} + \frac{(5b(9bB-11Ac))\int \frac{x^{5/2}}{\sqrt{bx^2+cx^4}}\,dx}{77c^2}$$

$$= \frac{10b(9bB-11Ac)\sqrt{bx^2+cx^4}}{231c^3\sqrt{x}} - \frac{2(9bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c^2} + \frac{2Bx^{7/2}\sqrt{bx^2+cx^4}}{11c} - \frac{\left(5b^2(9bB\right.}{}$$

$$= \frac{10b(9bB-11Ac)\sqrt{bx^2+cx^4}}{231c^3\sqrt{x}} - \frac{2(9bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c^2} + \frac{2Bx^{7/2}\sqrt{bx^2+cx^4}}{11c} - \frac{\left(5b^2(9bB\right.}{}$$

$$= \frac{10b(9bB-11Ac)\sqrt{bx^2+cx^4}}{231c^3\sqrt{x}} - \frac{2(9bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c^2} + \frac{2Bx^{7/2}\sqrt{bx^2+cx^4}}{11c} - \frac{\left(10b^2(9bB\right.}{}$$

$$= \frac{10b(9bB-11Ac)\sqrt{bx^2+cx^4}}{231c^3\sqrt{x}} - \frac{2(9bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c^2} + \frac{2Bx^{7/2}\sqrt{bx^2+cx^4}}{11c} - \frac{5b^{7/4}(9bB}{}$$

Mathematica [C] time = 0.131712, size = 122, normalized size = 0.6

$$\frac{2x^{3/2}\left(\left(b+cx^2\right)\left(-bc\left(55A+27Bx^2\right)+3c^2x^2\left(11A+7Bx^2\right)+45b^2B\right)+5b^2\sqrt{\frac{cx^2}{b}+1}(11Ac-9bB)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^2}{b}\right)\right)}{231c^3\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(9/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*x^(3/2)*((b + c*x^2)*(45*b^2*B + 3*c^2*x^2*(11*A + 7*B*x^2) - b*c*(55*A
+ 27*B*x^2)) + 5*b^2*(-9*b*B + 11*A*c)*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F
1[1/4, 1/2, 5/4, -((c*x^2)/b)]))/(231*c^3*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.016, size = 274, normalized size = 1.3

$$\frac{1}{231\,c^4}\sqrt{x}\left(42\,Bx^7c^4+55\,A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)\sqrt{-bc}b^2c+66\,A\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/231/(c*x^4+b*x^2)^(1/2)*x^(1/2)*(42*B*x^7*c^4+55*A*((c*x+(-b*c)^(1/2))/(-
b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-
b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(
1/2))*(-b*c)^(1/2)*b^2*c+66*A*x^5*c^4-45*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2)
)^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2)
)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(-b*
c)^(1/2)*b^3-12*B*x^5*b*c^3-44*A*x^3*b*c^3+36*B*x^3*b^2*c^2-110*A*x*b^2*c^2
+90*B*x*b^3*c)/c^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)x^{\frac{9}{2}}}{\sqrt{cx^4 + bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^(9/2)/sqrt(c*x^4 + b*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(Bx^4 + Ax^2)\sqrt{cx^4 + bx^2}\sqrt{x}}{cx^2 + b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^4 + A*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x)/(c*x^2 + b), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2+A)x^{\frac{9}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^(9/2)/sqrt(c*x^4 + b*x^2), x)
```

3.247 $\int \frac{x^{7/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=330

$$\frac{b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(7bB-9Ac)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{15c^{11/4}\sqrt{bx^2+cx^4}}-\frac{2b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(7bB-9Ac)E\left(\right.}{15c^{11/4}\sqrt{bx^2+cx^4}}$$

```
[Out] (2*b*(7*b*B - 9*A*c)*x^(3/2)*(b + c*x^2))/(15*c^(5/2)*(Sqrt[b] + Sqrt[c]*x)
*Sqrt[b*x^2 + c*x^4]) - (2*(7*b*B - 9*A*c)*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(45
*c^2) + (2*B*x^(5/2)*Sqrt[b*x^2 + c*x^4])/(9*c) - (2*b^(5/4)*(7*b*B - 9*A*c
)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Ellipti
cE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*c^(11/4)*Sqrt[b*x^2 + c*x
^4]) + (b^(5/4)*(7*b*B - 9*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(S
qrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])
/(15*c^(11/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.376646, antiderivative size = 330, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2039, 2024, 2032, 329, 305, 220, 1196}

$$\frac{b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(7bB-9Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{11/4}\sqrt{bx^2+cx^4}}-\frac{2b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(7bB-9Ac)E\left(2\tan^{-1}\right.}{15c^{11/4}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(7/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```



```
[Out] (2*b*(7*b*B - 9*A*c)*x^(3/2)*(b + c*x^2))/(15*c^(5/2)*(Sqrt[b] + Sqrt[c]*x)
*Sqrt[b*x^2 + c*x^4]) - (2*(7*b*B - 9*A*c)*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(45
*c^2) + (2*B*x^(5/2)*Sqrt[b*x^2 + c*x^4])/(9*c) - (2*b^(5/4)*(7*b*B - 9*A*c
)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Ellipti
cE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*c^(11/4)*Sqrt[b*x^2 + c*x
^4]) + (b^(5/4)*(7*b*B - 9*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(S
qrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])
/(15*c^(11/4)*Sqrt[b*x^2 + c*x^4])
```


Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^{7/2}\left(A+Bx^2\right)}{\sqrt{bx^2+cx^4}}\,dx = \frac{2Bx^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{\left(2\left(\frac{7bB}{2}-\frac{9Ac}{2}\right)\right)\int \frac{x^{7/2}}{\sqrt{bx^2+cx^4}}\,dx}{9c}$$

$$= -\frac{2(7bB-9Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{45c^2}+\frac{2Bx^{5/2}\sqrt{bx^2+cx^4}}{9c}+\frac{(b(7bB-9Ac))\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{15c^2}$$

$$= -\frac{2(7bB-9Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{45c^2}+\frac{2Bx^{5/2}\sqrt{bx^2+cx^4}}{9c}+\frac{\left(b(7bB-9Ac)x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{15c^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{2(7bB-9Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{45c^2}+\frac{2Bx^{5/2}\sqrt{bx^2+cx^4}}{9c}+\frac{\left(2b(7bB-9Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx}}\right.}{15c^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{2(7bB-9Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{45c^2}+\frac{2Bx^{5/2}\sqrt{bx^2+cx^4}}{9c}+\frac{\left(2b^{3/2}(7bB-9Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{}{\sqrt{b}}\right.}{15c^{5/2}\sqrt{bx^2+cx^4}}$$

$$= \frac{2b(7bB-9Ac)x^{3/2}\left(b+cx^2\right)}{15c^{5/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{2(7bB-9Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{45c^2}+\frac{2Bx^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{2b^{5/4}(7bB-}{}$$

Mathematica [C] time = 0.117432, size = 97, normalized size = 0.29

$$\frac{2x^{5/2}\left(b\sqrt{\frac{cx^2}{b}+1}(7bB-9Ac)\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)-\left(b+cx^2\right)\left(-9Ac+7bB-5Bcx^2\right)\right)}{45c^2\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(7/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*x^(5/2)*(-((b + c*x^2)*(7*b*B - 9*A*c - 5*B*c*x^2)) + b*(7*b*B - 9*A*c)*
Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[1/2, 3/4, 7/4, -((c*x^2)/b)]))/(45*c^
2*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.017, size = 413, normalized size = 1.3

$$-\frac{1}{45\,c^3}\sqrt{x}\left(-10\,Bc^3x^6+54\,A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\text{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)b^2c-27\,A\sqrt{\frac{c}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/45/(c*x^4+b*x^2)^(1/2)*x^(1/2)/c^3*(-10*B*c^3*x^6+54*A*((c*x+(-b*c)^(1/2
))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x
*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/
2*2^(1/2))*b^2*c-27*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*
x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c
*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c-42*B*((c*x+(-b*c)^(
1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*
(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)
,1/2*2^(1/2))*b^3+21*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c
*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((
c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^3-18*A*x^4*c^3+4*B*x^4
*b*c^2-18*A*x^2*b*c^2+14*B*x^2*b^2*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2+A)x^{\frac{7}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^(7/2)/sqrt(c*x^4 + b*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\left(Bx^3+Ax\right)\sqrt{x}}{cx^2+b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(B*x^3 + A*x)*sqrt(x)/(c*x^2 + b), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2+A\right)x^{\frac{7}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^(7/2)/sqrt(c*x^4 + b*x^2), x)
```

3.248 $\int \frac{x^{5/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=167

$$\frac{b^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(5bB-7Ac)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{21c^{9/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}(5bB-7Ac)}{21c^2\sqrt{x}}+\frac{2Bx^{3/2}\sqrt{bx^2+c}}{7c}$$

```
[Out] (-2*(5*b*B - 7*A*c)*Sqrt[b*x^2 + c*x^4])/(21*c^2*Sqrt[x]) + (2*B*x^(3/2)*Sq
rt[b*x^2 + c*x^4])/(7*c) + (b^(3/4)*(5*b*B - 7*A*c)*x*(Sqrt[b] + Sqrt[c]*x)
*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt
[x])/b^(1/4)], 1/2])/(21*c^(9/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.255178, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2039, 2024, 2032, 329, 220}

$$\frac{b^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(5bB-7Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21c^{9/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}(5bB-7Ac)}{21c^2\sqrt{x}}+\frac{2Bx^{3/2}\sqrt{bx^2+cx^4}}{7c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(5/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```



```
[Out] (-2*(5*b*B - 7*A*c)*Sqrt[b*x^2 + c*x^4])/(21*c^2*Sqrt[x]) + (2*B*x^(3/2)*Sq
rt[b*x^2 + c*x^4])/(7*c) + (b^(3/4)*(5*b*B - 7*A*c)*x*(Sqrt[b] + Sqrt[c]*x)
*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt
[x])/b^(1/4)], 1/2])/(21*c^(9/4)*Sqrt[b*x^2 + c*x^4])
```


Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] && !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{x^{5/2}\left(A+Bx^2\right)}{\sqrt{bx^2+cx^4}}\,dx = \frac{2Bx^{3/2}\sqrt{bx^2+cx^4}}{7c} - \frac{\left(2\left(\frac{5bB}{2}-\frac{7Ac}{2}\right)\right)\int \frac{x^{5/2}}{\sqrt{bx^2+cx^4}}\,dx}{7c}$$

$$= -\frac{2(5bB-7Ac)\sqrt{bx^2+cx^4}}{21c^2\sqrt{x}} + \frac{2Bx^{3/2}\sqrt{bx^2+cx^4}}{7c} + \frac{(b(5bB-7Ac))\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{21c^2}$$

$$= -\frac{2(5bB-7Ac)\sqrt{bx^2+cx^4}}{21c^2\sqrt{x}} + \frac{2Bx^{3/2}\sqrt{bx^2+cx^4}}{7c} + \frac{\left(b(5bB-7Ac)x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{21c^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{2(5bB-7Ac)\sqrt{bx^2+cx^4}}{21c^2\sqrt{x}} + \frac{2Bx^{3/2}\sqrt{bx^2+cx^4}}{7c} + \frac{\left(2b(5bB-7Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\right.}{21c^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{2(5bB-7Ac)\sqrt{bx^2+cx^4}}{21c^2\sqrt{x}} + \frac{2Bx^{3/2}\sqrt{bx^2+cx^4}}{7c} + \frac{b^{3/4}(5bB-7Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(\right.}{21c^{9/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] time = 0.111283, size = 97, normalized size = 0.58

$$\frac{2x^{3/2}\left(b\sqrt{\frac{cx^2}{b}+1}(5bB-7Ac)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^2}{b}\right)-\left(b+cx^2\right)\left(-7Ac+5bB-3Bcx^2\right)\right)}{21c^2\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(5/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*x^(3/2)*(-((b + c*x^2)*(5*b*B - 7*A*c - 3*B*c*x^2)) + b*(5*b*B - 7*A*c)*
Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^2)/b)]))/(21*c^
2*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.015, size = 248, normalized size = 1.5

$$-\frac{1}{21\,c^3}\sqrt{x}\left(7\,A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\text{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\,\sqrt{2}\right)\sqrt{-bc}bc-5\,B\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/21/(c*x^4+b*x^2)^(1/2)*x^(1/2)*(7*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1
/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1
/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(-b*c)^(
1/2)*b*c-5*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^
(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)
^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(-b*c)^(1/2)*b^2-6*B*c^3*x^5-14*A*
x^3*c^3+4*B*x^3*b*c^2-14*A*b*c^2*x+10*B*b^2*c*x)/c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)x^{\frac{5}{2}}}{\sqrt{cx^4 + bx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^(5/2)/sqrt(c*x^4 + b*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{cx^4 + bx^2}(Bx^2 + A)\sqrt{x}}{cx^2 + b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*sqrt(x)/(c*x^2 + b), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)x^{\frac{5}{2}}}{\sqrt{cx^4 + bx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^(5/2)/sqrt(c*x^4 + b*x^2), x)
```

3.249 $\int \frac{x^{3/2}\left(A+Bx^2\right)}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=293

$$-\frac{\sqrt[4]{b}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(3bB-5Ac)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5c^{7/4}\sqrt{bx^2+cx^4}}-\frac{2x^{3/2}\left(b+cx^2\right)(3bB-5Ac)}{5c^{3/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{2\sqrt[4]{b}x\left(\sqrt{b}+\right.}{}$$

```
[Out] (-2*(3*b*B - 5*A*c)*x^(3/2)*(b + c*x^2))/(5*c^(3/2)*(Sqrt[b] + Sqrt[c]*x)*S
qrt[b*x^2 + c*x^4]) + (2*B*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(5*c) + (2*b^(1/4)*
(3*b*B - 5*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]
*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*c^(7/4)*Sqrt
[b*x^2 + c*x^4]) - (b^(1/4)*(3*b*B - 5*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b
 + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(
1/4)], 1/2])/(5*c^(7/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.31129, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2039, 2032, 329, 305, 220, 1196}

$$-\frac{2x^{3/2}\left(b+cx^2\right)(3bB-5Ac)}{5c^{3/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{\sqrt[4]{b}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(3bB-5Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{7/4}\sqrt{bx^2+cx^4}}+\frac{2\sqrt[4]{b}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\ }}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(3/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```


```
[Out] (-2*(3*b*B - 5*A*c)*x^(3/2)*(b + c*x^2))/(5*c^(3/2)*(Sqrt[b] + Sqrt[c]*x)*S
qrt[b*x^2 + c*x^4]) + (2*B*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(5*c) + (2*b^(1/4)*
(3*b*B - 5*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]
*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*c^(7/4)*Sqrt
[b*x^2 + c*x^4]) - (b^(1/4)*(3*b*B - 5*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b
 + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(
1/4)], 1/2])/(5*c^(7/4)*Sqrt[b*x^2 + c*x^4])
```


Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
```

```
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] && !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^{3/2}\left(A+Bx^2\right)}{\sqrt{bx^2+cx^4}}\,dx = \frac{2B\sqrt{x}\sqrt{bx^2+cx^4}}{5c} - \frac{\left(2\left(\frac{3bB}{2}-\frac{5Ac}{2}\right)\right)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{5c}$$

$$= \frac{2B\sqrt{x}\sqrt{bx^2+cx^4}}{5c} - \frac{\left(2\left(\frac{3bB}{2}-\frac{5Ac}{2}\right)x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{5c\sqrt{bx^2+cx^4}}$$

$$= \frac{2B\sqrt{x}\sqrt{bx^2+cx^4}}{5c} - \frac{\left(4\left(\frac{3bB}{2}-\frac{5Ac}{2}\right)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{5c\sqrt{bx^2+cx^4}}$$

$$= \frac{2B\sqrt{x}\sqrt{bx^2+cx^4}}{5c} - \frac{\left(4\sqrt{b}\left(\frac{3bB}{2}-\frac{5Ac}{2}\right)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{5c^{3/2}\sqrt{bx^2+cx^4}} + \frac{\left(4\sqrt{b}\left(\frac{3bB}{2}-\frac{5Ac}{2}\right.\right.}{}$$

$$= -\frac{2(3bB-5Ac)x^{3/2}\left(b+cx^2\right)}{5c^{3/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} + \frac{2B\sqrt{x}\sqrt{bx^2+cx^4}}{5c} + \frac{2\sqrt[4]{b}(3bB-5Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}}{5c^{7/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] time = 0.0878932, size = 81, normalized size = 0.28

$$\frac{2x^{5/2}\left(\sqrt{\frac{cx^2}{b}+1}(5Ac-3bB)\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)+3B\left(b+cx^2\right)\right)}{15c\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(3/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*x^(5/2)*(3*B*(b + c*x^2) + (-3*b*B + 5*A*c)*Sqrt[1 + (c*x^2)/b]*Hypergeo
metric2F1[1/2, 3/4, 7/4, -((c*x^2)/b)]))/(15*c*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.014, size = 378, normalized size = 1.3

$$\frac{1}{5\,c^2}\sqrt{x}\left(10\,A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\text{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)bc-5\,A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/5/(c*x^4+b*x^2)^(1/2)*x^(1/2)/c^2*(10*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))
^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))
^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*c-5
*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b
*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-
b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*c-6*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1
/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1
/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^2+3*B*
((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)
^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c
)^(1/2))^(1/2),1/2*2^(1/2))*b^2+2*B*c^2*x^4+2*B*x^2*b*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)x^{\frac{3}{2}}}{\sqrt{cx^4 + bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^(3/2)/sqrt(c*x^4 + b*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2}(Bx^2 + A)\sqrt{x}}{cx^3 + bx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*sqrt(x)/(c*x^3 + b*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}\left(A+Bx^{2}\right)}{\sqrt{x^{2}\left(b+cx^{2}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**(3/2)*(A + B*x**2)/sqrt(x**2*(b + c*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^{2}+A\right)x^{\frac{3}{2}}}{\sqrt{cx^{4}+bx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^(3/2)/sqrt(c*x^4 + b*x^2), x)
```

3.250 $\int \frac{\sqrt{x}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=130

$$\frac{2B\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(bB-3Ac)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}c^{5/4}\sqrt{bx^2+cx^4}}$$

```
[Out] (2*B*Sqrt[b*x^2 + c*x^4])/(3*c*Sqrt[x]) - ((b*B - 3*A*c)*x*(Sqrt[b] + Sqrt[
c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)
*Sqrt[x])/b^(1/4)], 1/2])/(3*b^(1/4)*c^(5/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.198333, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2039, 2032, 329, 220}

$$\frac{2B\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(bB-3Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}c^{5/4}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*B*Sqrt[b*x^2 + c*x^4])/(3*c*Sqrt[x]) - ((b*B - 3*A*c)*x*(Sqrt[b] + Sqrt[
c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)
*Sqrt[x])/b^(1/4)], 1/2])/(3*b^(1/4)*c^(5/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}\left(A+Bx^2\right)}{\sqrt{bx^2+cx^4}}\,dx &= \frac{2B\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{\left(2\left(\frac{bB}{2}-\frac{3Ac}{2}\right)\right)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{3c}\\
&= \frac{2B\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{\left(2\left(\frac{bB}{2}-\frac{3Ac}{2}\right)x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{3c\sqrt{bx^2+cx^4}}\\
&= \frac{2B\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{\left(4\left(\frac{bB}{2}-\frac{3Ac}{2}\right)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{3c\sqrt{bx^2+cx^4}}\\
&= \frac{2B\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{(bB-3Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}c^{5/4}\sqrt{bx^2+cx^4}}
\end{aligned}$$

Mathematica [C] time = 0.0621478, size = 80, normalized size = 0.62

$$\frac{2x^{3/2}\left(\sqrt{\frac{cx^2}{b}+1}(3Ac-bB)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^2}{b}\right)+B\left(b+cx^2\right)\right)}{3c\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*x^(3/2)*(B*(b + c*x^2) + (-(b*B) + 3*A*c)*Sqrt[1 + (c*x^2)/b]*Hypergeome
tric2F1[1/4, 1/2, 5/4, -((c*x^2)/b)]))/(3*c*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.015, size = 216, normalized size = 1.7

$$\frac{1}{3c^2}\sqrt{x}\left(3A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\text{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)\sqrt{-bc}c-B\sqrt{\left(cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/3/(c*x^4+b*x^2)^(1/2)*x^(1/2)*(3*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2
)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2
)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(-b*c)^(1/
2)*c-B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))
/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2)
)/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(-b*c)^(1/2)*b+2*B*c^2*x^3+2*B*b*c*x)/c^
2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2+A\right)\sqrt{x}}{\sqrt{cx^4+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*sqrt(x)/sqrt(c*x^4 + b*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bx^2+A\right)\sqrt{x}}{\sqrt{cx^4+bx^2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)*sqrt(x)/sqrt(c*x^4 + b*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}\left(A+Bx^2\right)}{\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*x**(1/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(x)*(A + B*x**2)/sqrt(x**2*(b + c*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2+A\right)\sqrt{x}}{\sqrt{cx^4+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*sqrt(x)/sqrt(c*x^4 + b*x^2), x)
```

3.251 $\int \frac{A+Bx^2}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=281

$$\frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(Ac+bB)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{b^{3/4}c^{3/4}\sqrt{bx^2+cx^4}}-\frac{2x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(Ac+bB)E\left(2\tan^{-1}\left(\right.\right.}{b^{3/4}c^{3/4}\sqrt{bx^2+cx^4}}$$

```
[Out] (2*(b*B + A*c)*x^(3/2)*(b + c*x^2))/(b*Sqrt[c]*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b
*x^2 + c*x^4]) - (2*A*Sqrt[b*x^2 + c*x^4])/(b*x^(3/2)) - (2*(b*B + A*c)*x*(
Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*
ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(b^(3/4)*c^(3/4)*Sqrt[b*x^2 + c*x^
4]) + ((b*B + A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt
[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(b^(3/4)*c^(
3/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.309211, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2038, 2032, 329, 305, 220, 1196}

$$\frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(Ac+bB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}c^{3/4}\sqrt{bx^2+cx^4}}-\frac{2x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(Ac+bB)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}c^{3/4}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(Sqrt[x]*Sqrt[b*x^2 + c*x^4]),x]
```


```
[Out] (2*(b*B + A*c)*x^(3/2)*(b + c*x^2))/(b*Sqrt[c]*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b
*x^2 + c*x^4]) - (2*A*Sqrt[b*x^2 + c*x^4])/(b*x^(3/2)) - (2*(b*B + A*c)*x*(
Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*
ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(b^(3/4)*c^(3/4)*Sqrt[b*x^2 + c*x^
4]) + ((b*B + A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt
[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(b^(3/4)*c^(
3/4)*Sqrt[b*x^2 + c*x^4])
```


Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
```

```
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{A+Bx^2}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx = -\frac{2A\sqrt{bx^2+cx^4}}{bx^{3/2}} + \frac{(bB+Ac)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{b}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{bx^{3/2}} + \frac{\left((bB+Ac)x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{b\sqrt{bx^2+cx^4}}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{bx^{3/2}} + \frac{\left(2(bB+Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{b\sqrt{bx^2+cx^4}}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{bx^{3/2}} + \frac{\left(2(bB+Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{b}\sqrt{c}\sqrt{bx^2+cx^4}} - \frac{\left(2(bB+Ac)x\sqrt{b+cx\right.}{\sqrt{b}\sqrt{}}$$

$$= \frac{2(bB+Ac)x^{3/2}\left(b+cx^2\right)}{b\sqrt{c}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{2A\sqrt{bx^2+cx^4}}{bx^{3/2}} - \frac{2(bB+Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-}\right.}{b^{3/4}c^{3/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] time = 0.0488705, size = 82, normalized size = 0.29

$$\frac{2\sqrt{x}\left(x^2\sqrt{\frac{cx^2}{b}+1}(Ac+bB)\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)-3A\left(b+cx^2\right)\right)}{3b\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(Sqrt[x]*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (2*Sqrt[x]*(-3*A*(b + c*x^2) + (b*B + A*c)*x^2*Sqrt[1 + (c*x^2)/b]*Hypergeo
metric2F1[1/2, 3/4, 7/4, -((c*x^2)/b)]))/(3*b*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.018, size = 377, normalized size = 1.3

$$\frac{1}{bc}\sqrt{x}\left(2A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\text{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)bc-A\sqrt{\left(cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}}\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(1/2)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/(c*x^4+b*x^2)^(1/2)*x^(1/2)*(2*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*
2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*
EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*c-A*((c*x+
(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2)
)^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2
))^(1/2),1/2*2^(1/2))*b*c+2*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/
2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*Ellip
ticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^2-B*((c*x+(-b*c
)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1
/2),1/2*2^(1/2))*b^2-2*A*x^2*c^2-2*A*b*c)/c/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\sqrt{cx^4 + bx^2}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*sqrt(x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{cx^4 + bx^2}\left(Bx^2 + A\right)\sqrt{x}}{cx^5 + bx^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*sqrt(x)/(c*x^5 + b*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A+Bx^2}{\sqrt{x}\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(1/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(sqrt(x)*sqrt(x**2*(b + c*x**2))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2+A}{\sqrt{cx^4+bx^2}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*sqrt(x)), x)
```

3.252 $\int \frac{A+Bx^2}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=131

$$\frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(3bB-Ac)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt[4]{c}\sqrt{bx^2+cx^4}}-\frac{2A\sqrt{bx^2+cx^4}}{3bx^{5/2}}$$

```
[Out] (-2*A*Sqrt[b*x^2 + c*x^4])/(3*b*x^(5/2)) + ((3*b*B - A*c)*x*(Sqrt[b] + Sqrt
[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4
)*Sqrt[x])/b^(1/4)], 1/2])/(3*b^(5/4)*c^(1/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.203731, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2038, 2032, 329, 220}

$$\frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(3bB-Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3b^{5/4}\sqrt[4]{c}\sqrt{bx^2+cx^4}}-\frac{2A\sqrt{bx^2+cx^4}}{3bx^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(3/2)*Sqrt[b*x^2 + c*x^4]),x]
```


```
[Out] (-2*A*Sqrt[b*x^2 + c*x^4])/(3*b*x^(5/2)) + ((3*b*B - A*c)*x*(Sqrt[b] + Sqrt
[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4
)*Sqrt[x])/b^(1/4)], 1/2])/(3*b^(5/4)*c^(1/4)*Sqrt[b*x^2 + c*x^4])
```


Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^2}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx &= -\frac{2A\sqrt{bx^2+cx^4}}{3bx^{5/2}}-\frac{\left(2\left(-\frac{3bB}{2}+\frac{Ac}{2}\right)\right)\int\frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{3b}\\
&= -\frac{2A\sqrt{bx^2+cx^4}}{3bx^{5/2}}-\frac{\left(2\left(-\frac{3bB}{2}+\frac{Ac}{2}\right)x\sqrt{b+cx^2}\right)\int\frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{3b\sqrt{bx^2+cx^4}}\\
&= -\frac{2A\sqrt{bx^2+cx^4}}{3bx^{5/2}}-\frac{\left(4\left(-\frac{3bB}{2}+\frac{Ac}{2}\right)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{3b\sqrt{bx^2+cx^4}}\\
&= -\frac{2A\sqrt{bx^2+cx^4}}{3bx^{5/2}}+\frac{(3bB-Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3b^{5/4}\sqrt[4]{c}\sqrt{bx^2+cx^4}}
\end{aligned}$$

Mathematica [C] time = 0.0468396, size = 82, normalized size = 0.63

$$-\frac{2\left(x^2\sqrt{\frac{cx^2}{b}+1}(Ac-3bB)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^2}{b}\right)+A\left(b+cx^2\right)\right)}{3b\sqrt{x}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(3/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*(A*(b + c*x^2) + (-3*b*B + A*c)*x^2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2
F1[1/4, 1/2, 5/4, -((c*x^2)/b)]))/(3*b*Sqrt[x]*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.015, size = 219, normalized size = 1.7

$$-\frac{1}{3\,bc}\left(A\sqrt{\left(cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}}\sqrt{2}\sqrt{\left(-cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}}\sqrt{-cx\frac{1}{\sqrt{-bc}}}\mathrm{EllipticF}\left(\sqrt{\left(cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{-bc}xc-3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/3/(c*x^4+b*x^2)^(1/2)/x^(1/2)*(A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)
*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)
*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(-b*c)^(1/2
)*x*c-3*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/
2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1
/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(-b*c)^(1/2)*x*b+2*A*x^2*c^2+2*A*b*c)
/c/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\sqrt{cx^4 + bx^2}x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\left(Bx^2+A\right)\sqrt{x}}{cx^6+bx^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*sqrt(x)/(c*x^6 + b*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A+Bx^2}{x^{\frac{3}{2}}\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(3/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**(3/2)*sqrt(x**2*(b + c*x**2))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2+A}{\sqrt{cx^4+bx^2}x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^(3/2)), x)
```

3.253 $\int \frac{A+Bx^2}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=332

$$\frac{\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(5bB-3Ac)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5b^{7/4}\sqrt{bx^2+cx^4}}+\frac{2\sqrt{c}x^{3/2}\left(b+cx^2\right)(5bB-3Ac)}{5b^2\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{5b^2}$$

```
[Out] (2*Sqrt[c]*(5*b*B - 3*A*c)*x^(3/2)*(b + c*x^2))/(5*b^2*(Sqrt[b] + Sqrt[c]*x
)*Sqrt[b*x^2 + c*x^4]) - (2*A*Sqrt[b*x^2 + c*x^4])/(5*b*x^(7/2)) - (2*(5*b*
B - 3*A*c)*Sqrt[b*x^2 + c*x^4])/(5*b^2*x^(3/2)) - (2*c^(1/4)*(5*b*B - 3*A*c
)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Ellipti
cE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*b^(7/4)*Sqrt[b*x^2 + c*x^4
]) + (c^(1/4)*(5*b*B - 3*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqr
t[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(
5*b^(7/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.378017, antiderivative size = 332, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2038, 2025, 2032, 329, 305, 220, 1196}

$$\frac{2\sqrt{c}x^{3/2}\left(b+cx^2\right)(5bB-3Ac)}{5b^2\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}(5bB-3Ac)}{5b^2x^{3/2}}+\frac{\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(5bB-3Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\right.}{5b^{7/4}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(5/2)*Sqrt[b*x^2 + c*x^4]),x]
```


```
[Out] (2*Sqrt[c]*(5*b*B - 3*A*c)*x^(3/2)*(b + c*x^2))/(5*b^2*(Sqrt[b] + Sqrt[c]*x
)*Sqrt[b*x^2 + c*x^4]) - (2*A*Sqrt[b*x^2 + c*x^4])/(5*b*x^(7/2)) - (2*(5*b*
B - 3*A*c)*Sqrt[b*x^2 + c*x^4])/(5*b^2*x^(3/2)) - (2*c^(1/4)*(5*b*B - 3*A*c
)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Ellipti
cE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*b^(7/4)*Sqrt[b*x^2 + c*x^4
]) + (c^(1/4)*(5*b*B - 3*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqr
t[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(
5*b^(7/4)*Sqrt[b*x^2 + c*x^4])
```


Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
```

```
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx = -\frac{2A\sqrt{bx^2+cx^4}}{5bx^{7/2}} - \frac{\left(2\left(-\frac{5bB}{2}+\frac{3Ac}{2}\right)\right)\int \frac{1}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx}{5b}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{5bx^{7/2}} - \frac{2(5bB-3Ac)\sqrt{bx^2+cx^4}}{5b^2x^{3/2}} + \frac{(c(5bB-3Ac))\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{5b^2}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{5bx^{7/2}} - \frac{2(5bB-3Ac)\sqrt{bx^2+cx^4}}{5b^2x^{3/2}} + \frac{\left(c(5bB-3Ac)x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{5b^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{5bx^{7/2}} - \frac{2(5bB-3Ac)\sqrt{bx^2+cx^4}}{5b^2x^{3/2}} + \frac{\left(2c(5bB-3Ac)x\sqrt{b+cx^2}\right)\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x\right.}{5b^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{5bx^{7/2}} - \frac{2(5bB-3Ac)\sqrt{bx^2+cx^4}}{5b^2x^{3/2}} + \frac{\left(2\sqrt{c}(5bB-3Ac)x\sqrt{b+cx^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx\right.}{5b^{3/2}\sqrt{bx^2+cx^4}}$$

$$= \frac{2\sqrt{c}(5bB-3Ac)x^{3/2}\left(b+cx^2\right)}{5b^2\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{2A\sqrt{bx^2+cx^4}}{5bx^{7/2}} - \frac{2(5bB-3Ac)\sqrt{bx^2+cx^4}}{5b^2x^{3/2}} - \frac{2\sqrt[4]{c}(5bB-3Ac)x}{}$$

Mathematica [C] time = 0.0446758, size = 83, normalized size = 0.25

$$-\frac{2\left(x^2\sqrt{\frac{cx^2}{b}+1}(5bB-3Ac)\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{cx^2}{b}\right)+A\left(b+cx^2\right)\right)}{5bx^{3/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(5/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*(A*(b + c*x^2) + (5*b*B - 3*A*c)*x^2*Sqrt[1 + (c*x^2)/b]*Hypergeometric
2F1[-1/4, 1/2, 3/4, -((c*x^2)/b)]))/(5*b*x^(3/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.018, size = 413, normalized size = 1.2

$$-\frac{1}{5\,b^2}\left(6\,A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\text{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)x^2bc-3\,A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-c}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/5/(c*x^4+b*x^2)^(1/2)/x^(3/2)*(6*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/
2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b*c-3
*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b
*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-
b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b*c-10*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2
))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2
))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^2
*b^2+5*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2
))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/
2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^2-6*A*c^2*x^4+10*B*x^4*b*c-4*A*b
*c*x^2+10*B*x^2*b^2+2*A*b^2)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\sqrt{cx^4 + bx^2}x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\left(Bx^2+A\right)\sqrt{x}}{cx^7+bx^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*sqrt(x)/(c*x^7 + b*x^5), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A+Bx^2}{x^{\frac{5}{2}}\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(5/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**(5/2)*sqrt(x**2*(b + c*x**2))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2+A}{\sqrt{cx^4+bx^2}x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^(5/2)), x)
```

3.254 $\int \frac{A+Bx^2}{x^{7/2}\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=167

$$-\frac{c^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(7bB-5Ac)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{21b^{9/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}(7bB-5Ac)}{21b^2x^{5/2}}-\frac{2A\sqrt{bx^2+cx^4}}{7bx^{9/2}}$$

```
[Out] (-2*A*Sqrt[b*x^2 + c*x^4])/(7*b*x^(9/2)) - (2*(7*b*B - 5*A*c)*Sqrt[b*x^2 +
c*x^4])/(21*b^2*x^(5/2)) - (c^(3/4)*(7*b*B - 5*A*c)*x*(Sqrt[b] + Sqrt[c]*x)
*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt
[x])/b^(1/4)], 1/2])/(21*b^(9/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.253239, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2038, 2025, 2032, 329, 220}

$$-\frac{c^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(7bB-5Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21b^{9/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}(7bB-5Ac)}{21b^2x^{5/2}}-\frac{2A\sqrt{bx^2+cx^4}}{7bx^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(7/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*A*Sqrt[b*x^2 + c*x^4])/(7*b*x^(9/2)) - (2*(7*b*B - 5*A*c)*Sqrt[b*x^2 +
c*x^4])/(21*b^2*x^(5/2)) - (c^(3/4)*(7*b*B - 5*A*c)*x*(Sqrt[b] + Sqrt[c]*x)
*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt
[x])/b^(1/4)], 1/2])/(21*b^(9/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
```

```
0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^{7/2}\sqrt{bx^2+cx^4}}\,dx = -\frac{2A\sqrt{bx^2+cx^4}}{7bx^{9/2}} - \frac{\left(2\left(-\frac{7bB}{2}+\frac{5Ac}{2}\right)\right)\int \frac{1}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx}{7b}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{7bx^{9/2}} - \frac{2(7bB-5Ac)\sqrt{bx^2+cx^4}}{21b^2x^{5/2}} - \frac{(c(7bB-5Ac))\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{21b^2}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{7bx^{9/2}} - \frac{2(7bB-5Ac)\sqrt{bx^2+cx^4}}{21b^2x^{5/2}} - \frac{\left(c(7bB-5Ac)x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{21b^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{7bx^{9/2}} - \frac{2(7bB-5Ac)\sqrt{bx^2+cx^4}}{21b^2x^{5/2}} - \frac{\left(2c(7bB-5Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx,\right.}{21b^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{7bx^{9/2}} - \frac{2(7bB-5Ac)\sqrt{bx^2+cx^4}}{21b^2x^{5/2}} - \frac{c^{3/4}(7bB-5Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\,\text{ta}\right.}{21b^{9/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] time = 0.0450328, size = 85, normalized size = 0.51

$$\frac{2x^2\sqrt{\frac{cx^2}{b}+1}(5Ac-7bB)\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};-\frac{cx^2}{b}\right)-6A\left(b+cx^2\right)}{21bx^{5/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(7/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-6*A*(b + c*x^2) + 2*(-7*b*B + 5*A*c)*x^2*Sqrt[1 + (c*x^2)/b]*Hypergeometr
ic2F1[-3/4, 1/2, 1/4, -((c*x^2)/b)])/(21*b*x^(5/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.016, size = 247, normalized size = 1.5

$$\frac{1}{21\,b^2}\left(5\,A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\text{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)\sqrt{-bc}x^3c-7\,B\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(7/2)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/21/(c*x^4+b*x^2)^(1/2)/x^(5/2)*(5*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/
2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(-b*c)^(1
/2)*x^3*c-7*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)
^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c
)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(-b*c)^(1/2)*x^3*b+10*A*c^2*x^4-1
4*B*x^4*b*c+4*A*b*c*x^2-14*B*x^2*b^2-6*A*b^2)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\sqrt{cx^4 + bx^2}x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(7/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^(7/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{cx^4 + bx^2}\left(Bx^2 + A\right)\sqrt{x}}{cx^8 + bx^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(7/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*sqrt(x)/(c*x^8 + b*x^6), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + Bx^2}{x^{\frac{7}{2}}\sqrt{x^2\left(b + cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(7/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**(7/2)*sqrt(x**2*(b + c*x**2))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\sqrt{cx^4 + bx^2}x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(7/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^(7/2)), x)
```

3.255 $\int \frac{A+Bx^2}{x^{9/2}\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=369

$$-\frac{c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(9bB-7Ac)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{15b^{11/4}\sqrt{bx^2+cx^4}}-\frac{2c^{3/2}x^{3/2}\left(b+cx^2\right)(9bB-7Ac)}{15b^3\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{2c^{5/4}x\left(\sqrt{b}\right.}{}$$

```
[Out] (-2*c^(3/2)*(9*b*B - 7*A*c)*x^(3/2)*(b + c*x^2))/(15*b^3*(Sqrt[b] + Sqrt[c]
*x)*Sqrt[b*x^2 + c*x^4]) - (2*A*Sqrt[b*x^2 + c*x^4])/(9*b*x^(11/2)) - (2*(9
*b*B - 7*A*c)*Sqrt[b*x^2 + c*x^4])/(45*b^2*x^(7/2)) + (2*c*(9*b*B - 7*A*c)*
Sqrt[b*x^2 + c*x^4])/(15*b^3*x^(3/2)) + (2*c^(5/4)*(9*b*B - 7*A*c)*x*(Sqrt[
b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTa
n[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(11/4)*Sqrt[b*x^2 + c*x^4]) - (c^
(5/4)*(9*b*B - 7*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + S
qrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(11
/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.440989, antiderivative size = 369, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2038, 2025, 2032, 329, 305, 220, 1196}

$$-\frac{2c^{3/2}x^{3/2}\left(b+cx^2\right)(9bB-7Ac)}{15b^3\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(9bB-7Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{11/4}\sqrt{bx^2+cx^4}}+\frac{2c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(9/2)*Sqrt[b*x^2 + c*x^4]),x]
```



```
[Out] (-2*c^(3/2)*(9*b*B - 7*A*c)*x^(3/2)*(b + c*x^2))/(15*b^3*(Sqrt[b] + Sqrt[c]
*x)*Sqrt[b*x^2 + c*x^4]) - (2*A*Sqrt[b*x^2 + c*x^4])/(9*b*x^(11/2)) - (2*(9
*b*B - 7*A*c)*Sqrt[b*x^2 + c*x^4])/(45*b^2*x^(7/2)) + (2*c*(9*b*B - 7*A*c)*
Sqrt[b*x^2 + c*x^4])/(15*b^3*x^(3/2)) + (2*c^(5/4)*(9*b*B - 7*A*c)*x*(Sqrt[
b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTa
n[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(11/4)*Sqrt[b*x^2 + c*x^4]) - (c^
(5/4)*(9*b*B - 7*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + S
qrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(11
/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
```

```
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^{9/2}\sqrt{bx^2+cx^4}}\,dx = -\frac{2A\sqrt{bx^2+cx^4}}{9bx^{11/2}} - \frac{\left(2\left(-\frac{9bB}{2}+\frac{7Ac}{2}\right)\right)\int \frac{1}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx}{9b}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{9bx^{11/2}} - \frac{2(9bB-7Ac)\sqrt{bx^2+cx^4}}{45b^2x^{7/2}} - \frac{(c(9bB-7Ac))\int \frac{1}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx}{15b^2}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{9bx^{11/2}} - \frac{2(9bB-7Ac)\sqrt{bx^2+cx^4}}{45b^2x^{7/2}} + \frac{2c(9bB-7Ac)\sqrt{bx^2+cx^4}}{15b^3x^{3/2}} - \frac{\left(c^2(9bB-7Ac)\right)\int \cdots}{15b^3}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{9bx^{11/2}} - \frac{2(9bB-7Ac)\sqrt{bx^2+cx^4}}{45b^2x^{7/2}} + \frac{2c(9bB-7Ac)\sqrt{bx^2+cx^4}}{15b^3x^{3/2}} - \frac{\left(c^2(9bB-7Ac)x\sqrt{b\cdots}\right.}{15b^3\sqrt{bx\cdots}}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{9bx^{11/2}} - \frac{2(9bB-7Ac)\sqrt{bx^2+cx^4}}{45b^2x^{7/2}} + \frac{2c(9bB-7Ac)\sqrt{bx^2+cx^4}}{15b^3x^{3/2}} - \frac{\left(2c^2(9bB-7Ac)x\sqrt{\cdots}\right.}{\cdots}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{9bx^{11/2}} - \frac{2(9bB-7Ac)\sqrt{bx^2+cx^4}}{45b^2x^{7/2}} + \frac{2c(9bB-7Ac)\sqrt{bx^2+cx^4}}{15b^3x^{3/2}} - \frac{\left(2c^{3/2}(9bB-7Ac)x\cdots\right.}{\cdots}$$

$$= -\frac{2c^{3/2}(9bB-7Ac)x^{3/2}\left(b+cx^2\right)}{15b^3\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{2A\sqrt{bx^2+cx^4}}{9bx^{11/2}} - \frac{2(9bB-7Ac)\sqrt{bx^2+cx^4}}{45b^2x^{7/2}} + \frac{2c(9bB-7Ac)\cdots}{15b^3x^3\cdots}$$

Mathematica [C] time = 0.044351, size = 84, normalized size = 0.23

$$-\frac{2\left(x^2\sqrt{\frac{cx^2}{b}+1}(9bB-7Ac)\,{}_2F_1\left(-\frac{5}{4},\frac{1}{2};-\frac{1}{4};-\frac{cx^2}{b}\right)+5A\left(b+cx^2\right)\right)}{45bx^{7/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(9/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*(5*A*(b + c*x^2) + (9*b*B - 7*A*c)*x^2*Sqrt[1 + (c*x^2)/b]*Hypergeometr
ic2F1[-5/4, 1/2, -1/4, -((c*x^2)/b)]))/(45*b*x^(7/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.02, size = 443, normalized size = 1.2

$$\frac{1}{45\,b^3}\left(42\,A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\,\sqrt{2}\right)x^4bc^2-21\,A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(9/2)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/45/(c*x^4+b*x^2)^(1/2)/x^(7/2)*(42*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1
/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1
/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^4*b*c^
2-21*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))
/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2)
)/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^4*b*c^2-54*B*((c*x+(-b*c)^(1/2))/(-b*c
)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c
)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2
))*x^4*b^2*c+27*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-
b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(
-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^4*b^2*c-42*A*c^3*x^6+54*B*x
^6*b*c^2-28*A*b*c^2*x^4+36*B*x^4*b^2*c+4*A*b^2*c*x^2-18*B*x^2*b^3-10*A*b^3)
/b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\sqrt{cx^4 + bx^2}x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(9/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^(9/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\left(Bx^2+A\right)\sqrt{x}}{cx^9+bx^7},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(9/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*sqrt(x)/(c*x^9 + b*x^7), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(9/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2+A}{\sqrt{cx^4+bx^2}x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(9/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^(9/2)), x)
```

3.256 $\int \frac{A+Bx^2}{x^{11/2}\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=204

$$\frac{5c^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(11bB-9Ac)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{231b^{13/4}\sqrt{bx^2+cx^4}}+\frac{10c\sqrt{bx^2+cx^4}(11bB-9Ac)}{231b^3x^{5/2}}-\frac{2\sqrt{bx^2+c}}{77}$$

```
[Out] (-2*A*Sqrt[b*x^2 + c*x^4])/(11*b*x^(13/2)) - (2*(11*b*B - 9*A*c)*Sqrt[b*x^2
 + c*x^4])/(77*b^2*x^(9/2)) + (10*c*(11*b*B - 9*A*c)*Sqrt[b*x^2 + c*x^4])/(
231*b^3*x^(5/2)) + (5*c^(7/4)*(11*b*B - 9*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt
[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/
b^(1/4)], 1/2])/(231*b^(13/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.311279, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2038, 2025, 2032, 329, 220}

$$\frac{5c^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(11bB-9Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{231b^{13/4}\sqrt{bx^2+cx^4}}+\frac{10c\sqrt{bx^2+cx^4}(11bB-9Ac)}{231b^3x^{5/2}}-\frac{2\sqrt{bx^2+cx^4}(11bB}{77b^2x^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(11/2)*Sqrt[b*x^2 + c*x^4]),x]
```



```
[Out] (-2*A*Sqrt[b*x^2 + c*x^4])/(11*b*x^(13/2)) - (2*(11*b*B - 9*A*c)*Sqrt[b*x^2
 + c*x^4])/(77*b^2*x^(9/2)) + (10*c*(11*b*B - 9*A*c)*Sqrt[b*x^2 + c*x^4])/(
231*b^3*x^(5/2)) + (5*c^(7/4)*(11*b*B - 9*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt
[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/
b^(1/4)], 1/2])/(231*b^(13/4)*Sqrt[b*x^2 + c*x^4])
```


Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] && !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
```

```
|| (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^{11/2}\sqrt{bx^2+cx^4}}\,dx = -\frac{2A\sqrt{bx^2+cx^4}}{11bx^{13/2}} - \frac{\left(2\left(-\frac{11bB}{2}+\frac{9Ac}{2}\right)\right)\int \frac{1}{x^{7/2}\sqrt{bx^2+cx^4}}\,dx}{11b}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{11bx^{13/2}} - \frac{2(11bB-9Ac)\sqrt{bx^2+cx^4}}{77b^2x^{9/2}} - \frac{(5c(11bB-9Ac))\int \frac{1}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx}{77b^2}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{11bx^{13/2}} - \frac{2(11bB-9Ac)\sqrt{bx^2+cx^4}}{77b^2x^{9/2}} + \frac{10c(11bB-9Ac)\sqrt{bx^2+cx^4}}{231b^3x^{5/2}} + \frac{\left(5c^2(11bB-9\right.}{2}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{11bx^{13/2}} - \frac{2(11bB-9Ac)\sqrt{bx^2+cx^4}}{77b^2x^{9/2}} + \frac{10c(11bB-9Ac)\sqrt{bx^2+cx^4}}{231b^3x^{5/2}} + \frac{\left(5c^2(11bB-9\right.}{2}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{11bx^{13/2}} - \frac{2(11bB-9Ac)\sqrt{bx^2+cx^4}}{77b^2x^{9/2}} + \frac{10c(11bB-9Ac)\sqrt{bx^2+cx^4}}{231b^3x^{5/2}} + \frac{\left(10c^2(11bB-\right.}{}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{11bx^{13/2}} - \frac{2(11bB-9Ac)\sqrt{bx^2+cx^4}}{77b^2x^{9/2}} + \frac{10c(11bB-9Ac)\sqrt{bx^2+cx^4}}{231b^3x^{5/2}} + \frac{5c^{7/4}(11bB-9}{}$$

Mathematica [C] time = 0.0435078, size = 84, normalized size = 0.41

$$-\frac{2\left(x^2\sqrt{\frac{cx^2}{b}+1}(11bB-9Ac)\,{}_2F_1\left(-\frac{7}{4},\frac{1}{2};-\frac{3}{4};-\frac{cx^2}{b}\right)+7A\left(b+cx^2\right)\right)}{77bx^{9/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(11/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*(7*A*(b + c*x^2) + (11*b*B - 9*A*c)*x^2*Sqrt[1 + (c*x^2)/b]*Hypergeomet
ric2F1[-7/4, 1/2, -3/4, -((c*x^2)/b)]))/(77*b*x^(9/2)*Sqrt[x^2*(b + c*x^2)]
)
```

Maple [A] time = 0.019, size = 274, normalized size = 1.3

$$-\frac{1}{231\,b^3}\left(45\,A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)\sqrt{-bc}x^5c^2-55\,B\sqrt{\frac{cx+\sqrt{-b}}{\sqrt{-bc}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(11/2)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/231/(c*x^4+b*x^2)^(1/2)/x^(9/2)*(45*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^
(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^
(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(-b*c)
^(1/2)*x^5*c^2-55*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+
(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x
+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(-b*c)^(1/2)*x^5*b*c+90*A*c
^3*x^6-110*B*x^6*b*c^2+36*A*b*c^2*x^4-44*B*x^4*b^2*c-12*A*b^2*c*x^2+66*B*x^
2*b^3+42*A*b^3)/b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\sqrt{cx^4 + bx^2}x^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(11/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^(11/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2}\left(Bx^2 + A\right)\sqrt{x}}{cx^{10} + bx^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(11/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*sqrt(x)/(c*x^10 + b*x^8), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(11/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\sqrt{cx^4 + bx^2}x^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(11/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^(11/2)), x)
```

3.257 $\int \frac{x^{17/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=251

$$-\frac{15b^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(13bB-11Ac)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{154c^{17/4}\sqrt{bx^2+cx^4}}+\frac{x^{7/2}\sqrt{bx^2+cx^4}(13bB-11Ac)}{11bc^2}-\frac{9x^{3/2}\sqrt{}}{}$$

```
[Out] -(((b*B - A*c)*x^(15/2))/(b*c*Sqrt[b*x^2 + c*x^4])) + (15*b*(13*b*B - 11*A*
c)*Sqrt[b*x^2 + c*x^4])/(77*c^4*Sqrt[x]) - (9*(13*b*B - 11*A*c)*x^(3/2)*Sqr
t[b*x^2 + c*x^4])/(77*c^3) + ((13*b*B - 11*A*c)*x^(7/2)*Sqrt[b*x^2 + c*x^4]
)/(11*b*c^2) - (15*b^(7/4)*(13*b*B - 11*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(
b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^
(1/4)], 1/2])/(154*c^(17/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.389588, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2037, 2024, 2032, 329, 220}

$$-\frac{15b^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(13bB-11Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{154c^{17/4}\sqrt{bx^2+cx^4}}+\frac{x^{7/2}\sqrt{bx^2+cx^4}(13bB-11Ac)}{11bc^2}-\frac{9x^{3/2}\sqrt{bx^2+cx}}{77}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(17/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```


```
[Out] -(((b*B - A*c)*x^(15/2))/(b*c*Sqrt[b*x^2 + c*x^4])) + (15*b*(13*b*B - 11*A*
c)*Sqrt[b*x^2 + c*x^4])/(77*c^4*Sqrt[x]) - (9*(13*b*B - 11*A*c)*x^(3/2)*Sqr
t[b*x^2 + c*x^4])/(77*c^3) + ((13*b*B - 11*A*c)*x^(7/2)*Sqrt[b*x^2 + c*x^4]
)/(11*b*c^2) - (15*b^(7/4)*(13*b*B - 11*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(
b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^
(1/4)], 1/2])/(154*c^(17/4)*Sqrt[b*x^2 + c*x^4])
```


Rule 2037

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> -Simp[(e^(j - 1)*(b*c - a*d)*(e*x)^(m - j +
 1)*(a*x^j + b*x^(j + n))^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(e^j*(a*d*(m
```

```
+ j*p + 1) - b*c*(m + n + p*(j + n) + 1)))/(a*b*n*(p + 1)), Int[(e*x)^(m -
j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m, n},
 x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &
& GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{x^{17/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{(bB-Ac)x^{15/2}}{bc\sqrt{bx^2+cx^4}} + \frac{\left(\frac{13bB}{2}-\frac{11Ac}{2}\right)\int \frac{x^{13/2}}{\sqrt{bx^2+cx^4}}\,dx}{bc}$$

$$= -\frac{(bB-Ac)x^{15/2}}{bc\sqrt{bx^2+cx^4}} + \frac{(13bB-11Ac)x^{7/2}\sqrt{bx^2+cx^4}}{11bc^2} - \frac{(9(13bB-11Ac))\int \frac{x^{9/2}}{\sqrt{bx^2+cx^4}}\,dx}{22c^2}$$

$$= -\frac{(bB-Ac)x^{15/2}}{bc\sqrt{bx^2+cx^4}} - \frac{9(13bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c^3} + \frac{(13bB-11Ac)x^{7/2}\sqrt{bx^2+cx^4}}{11bc^2} + \frac{(45b(13bB}{}$$

$$= -\frac{(bB-Ac)x^{15/2}}{bc\sqrt{bx^2+cx^4}} + \frac{15b(13bB-11Ac)\sqrt{bx^2+cx^4}}{77c^4\sqrt{x}} - \frac{9(13bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c^3} + \frac{(13bB-11}{}$$

$$= -\frac{(bB-Ac)x^{15/2}}{bc\sqrt{bx^2+cx^4}} + \frac{15b(13bB-11Ac)\sqrt{bx^2+cx^4}}{77c^4\sqrt{x}} - \frac{9(13bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c^3} + \frac{(13bB-11}{}$$

$$= -\frac{(bB-Ac)x^{15/2}}{bc\sqrt{bx^2+cx^4}} + \frac{15b(13bB-11Ac)\sqrt{bx^2+cx^4}}{77c^4\sqrt{x}} - \frac{9(13bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c^3} + \frac{(13bB-11}{}$$

$$= -\frac{(bB-Ac)x^{15/2}}{bc\sqrt{bx^2+cx^4}} + \frac{15b(13bB-11Ac)\sqrt{bx^2+cx^4}}{77c^4\sqrt{x}} - \frac{9(13bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c^3} + \frac{(13bB-11}{}$$

Mathematica [C] time = 0.165654, size = 134, normalized size = 0.53

$$\frac{x^{3/2}\left(15b^2\sqrt{\frac{cx^2}{b}+1}(11Ac-13bB)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^2}{b}\right)+b^2\left(78Bcx^2-165Ac\right)-2bc^2x^2\left(33A+13Bx^2\right)+2c^3x^4\left(11A+7Bx\right.\right.}{77c^4\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(17/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x^(3/2)*(195*b^3*B + 2*c^3*x^4*(11*A + 7*B*x^2) - 2*b*c^2*x^2*(33*A + 13*B
*x^2) + b^2*(-165*A*c + 78*B*c*x^2) + 15*b^2*(-13*b*B + 11*A*c)*Sqrt[1 + (c
*x^2)/b]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^2)/b)]))/(77*c^4*Sqrt[x^2*
(b + c*x^2)])
```

Maple [A] time = 0.038, size = 281, normalized size = 1.1

$$\frac{cx^2+b}{154\,c^5}x^{\frac{5}{2}}\left(28\,Bx^7c^4+165\,A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)\sqrt{-bc}b^2c+44\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/154/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(28*B*x^7*c^4+165*A*((c*x+(-b*c
)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1
/2),1/2*2^(1/2))*(-b*c)^(1/2)*b^2*c+44*A*x^5*c^4-195*B*((c*x+(-b*c)^(1/2))/
(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/
(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2
^(1/2))*(-b*c)^(1/2)*b^3-52*B*x^5*b*c^3-132*A*x^3*b*c^3+156*B*x^3*b^2*c^2-3
30*A*x*b^2*c^2+390*B*x*b^3*c)/c^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(Bx^2+A\right)x^{\frac{17}{2}}}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^(17/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(Bx^6+Ax^4\right)\sqrt{cx^4+bx^2}\sqrt{x}}{c^2x^4+2\,bcx^2+b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^6 + A*x^4)*sqrt(c*x^4 + b*x^2)*sqrt(x)/(c^2*x^4 + 2*b*c*x^2 +
 b^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(17/2)*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2 + A\right)x^{\frac{17}{2}}}{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^(17/2)/(c*x^4 + b*x^2)^(3/2), x)
```

3.258 $\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=377

$$\frac{7b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(11bB-9Ac)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{30c^{15/4}\sqrt{bx^2+cx^4}}-\frac{7b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(11bB-9A}{15c^{15/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -(((b*B - A*c)*x^(13/2))/(b*c*Sqrt[b*x^2 + c*x^4])) + (7*b*(11*b*B - 9*A*c)
*x^(3/2)*(b + c*x^2))/(15*c^(7/2)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]
) - (7*(11*b*B - 9*A*c)*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(45*c^3) + ((11*b*B -
9*A*c)*x^(5/2)*Sqrt[b*x^2 + c*x^4])/(9*b*c^2) - (7*b^(5/4)*(11*b*B - 9*A*c)
*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Elliptic
E[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*c^(15/4)*Sqrt[b*x^2 + c*x^
4]) + (7*b^(5/4)*(11*b*B - 9*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/
(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2
])/(30*c^(15/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.460616, antiderivative size = 377, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2037, 2024, 2032, 329, 305, 220, 1196}

$$\frac{7b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(11bB-9Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{30c^{15/4}\sqrt{bx^2+cx^4}}-\frac{7b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(11bB-9Ac)E\left(2\,\text{ta}\right.}{15c^{15/4}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(15/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```


```
[Out] -(((b*B - A*c)*x^(13/2))/(b*c*Sqrt[b*x^2 + c*x^4])) + (7*b*(11*b*B - 9*A*c)
*x^(3/2)*(b + c*x^2))/(15*c^(7/2)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]
) - (7*(11*b*B - 9*A*c)*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(45*c^3) + ((11*b*B -
9*A*c)*x^(5/2)*Sqrt[b*x^2 + c*x^4])/(9*b*c^2) - (7*b^(5/4)*(11*b*B - 9*A*c)
*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Elliptic
E[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*c^(15/4)*Sqrt[b*x^2 + c*x^
4]) + (7*b^(5/4)*(11*b*B - 9*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/
(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2
```

```
])/(30*c^(15/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 2037

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> -Simp[(e^(j - 1)*(b*c - a*d)*(e*x)^(m - j +
 1)*(a*x^j + b*x^(j + n))^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(e^j*(a*d*(m
+ j*p + 1) - b*c*(m + n + p*(j + n) + 1)))/(a*b*n*(p + 1)), Int[(e*x)^(m -
j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m, n},
 x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &
& GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^{15/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{(bB-Ac)x^{13/2}}{bc\sqrt{bx^2+cx^4}}+\frac{\left(\frac{11bB}{2}-\frac{9Ac}{2}\right)\int \frac{x^{11/2}}{\sqrt{bx^2+cx^4}}\,dx}{bc}$$

$$= -\frac{(bB-Ac)x^{13/2}}{bc\sqrt{bx^2+cx^4}}+\frac{(11bB-9Ac)x^{5/2}\sqrt{bx^2+cx^4}}{9bc^2}-\frac{(7(11bB-9Ac))\int \frac{x^{7/2}}{\sqrt{bx^2+cx^4}}\,dx}{18c^2}$$

$$= -\frac{(bB-Ac)x^{13/2}}{bc\sqrt{bx^2+cx^4}}-\frac{7(11bB-9Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{45c^3}+\frac{(11bB-9Ac)x^{5/2}\sqrt{bx^2+cx^4}}{9bc^2}+\frac{(7b(11bB-}{}$$

$$= -\frac{(bB-Ac)x^{13/2}}{bc\sqrt{bx^2+cx^4}}-\frac{7(11bB-9Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{45c^3}+\frac{(11bB-9Ac)x^{5/2}\sqrt{bx^2+cx^4}}{9bc^2}+\frac{\left(7b(11bB-\right.}{}$$

$$= -\frac{(bB-Ac)x^{13/2}}{bc\sqrt{bx^2+cx^4}}-\frac{7(11bB-9Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{45c^3}+\frac{(11bB-9Ac)x^{5/2}\sqrt{bx^2+cx^4}}{9bc^2}+\frac{\left(7b(11bB-\right.}{}$$

$$= -\frac{(bB-Ac)x^{13/2}}{bc\sqrt{bx^2+cx^4}}-\frac{7(11bB-9Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{45c^3}+\frac{(11bB-9Ac)x^{5/2}\sqrt{bx^2+cx^4}}{9bc^2}+\frac{\left(7b^{3/2}(11bB\right.}{}$$

$$= -\frac{(bB-Ac)x^{13/2}}{bc\sqrt{bx^2+cx^4}}+\frac{7b(11bB-9Ac)x^{3/2}\left(b+cx^2\right)}{15c^{7/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{7(11bB-9Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{45c^3}+\frac{(11bB-9}{}$$

Mathematica [C] time = 0.124633, size = 110, normalized size = 0.29

$$\frac{2x^{5/2}\left(7b\sqrt{\frac{cx^2}{b}+1}(9Ac-11bB)\,_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{cx^2}{b}\right)-bc\left(63A+11Bx^2\right)+c^2x^2\left(9A+5Bx^2\right)+77b^2B\right)}{45c^3\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(15/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*x^(5/2)*(77*b^2*B + c^2*x^2*(9*A + 5*B*x^2) - b*c*(63*A + 11*B*x^2) + 7*
b*(-11*b*B + 9*A*c)*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[3/4, 3/2, 7/4, -(
(c*x^2)/b)]))/(45*c^3*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.034, size = 420, normalized size = 1.1

$$-\frac{cx^2+b}{90\,c^4}x^{\frac{5}{2}}\left(-20\,Bc^3x^6+378\,A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)b^2c-189\,A\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/90/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(-20*B*c^3*x^6+378*A*((c*x+(-b*
c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1
/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(
1/2),1/2*2^(1/2))*b^2*c-189*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/
2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*Ellip
ticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c-462*B*((c*x
+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2
))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/
2))^(1/2),1/2*2^(1/2))*b^3+231*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^
(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*El
lipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^3-36*A*x^4*c
^3+44*B*x^4*b*c^2-126*A*x^2*b*c^2+154*B*x^2*b^2*c)/c^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2 + A\right)x^{\frac{15}{2}}}{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^(15/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bx^5 + Ax^3\right)\sqrt{cx^4 + bx^2}\sqrt{x}}{c^2x^4 + 2\,bcx^2 + b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^5 + A*x^3)*sqrt(c*x^4 + b*x^2)*sqrt(x)/(c^2*x^4 + 2*b*c*x^2 +
 b^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(15/2)*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)x^{\frac{15}{2}}}{(cx^4 + bx^2)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^(15/2)/(c*x^4 + b*x^2)^(3/2), x)
```

3.259 $\int \frac{x^{13/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=214

$$\frac{5b^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(9bB-7Ac)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{42c^{13/4}\sqrt{bx^2+cx^4}}+\frac{x^{3/2}\sqrt{bx^2+cx^4}(9bB-7Ac)}{7bc^2}-\frac{5\sqrt{bx^2+cx^4}}{21c}$$

```
[Out] -(((b*B - A*c)*x^(11/2))/(b*c*Sqrt[b*x^2 + c*x^4])) - (5*(9*b*B - 7*A*c)*Sq
rt[b*x^2 + c*x^4])/(21*c^3*Sqrt[x]) + ((9*b*B - 7*A*c)*x^(3/2)*Sqrt[b*x^2 +
 c*x^4])/(7*b*c^2) + (5*b^(3/4)*(9*b*B - 7*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqr
t[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])
/b^(1/4)], 1/2])/(42*c^(13/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.326987, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2037, 2024, 2032, 329, 220}

$$\frac{5b^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(9bB-7Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{42c^{13/4}\sqrt{bx^2+cx^4}}+\frac{x^{3/2}\sqrt{bx^2+cx^4}(9bB-7Ac)}{7bc^2}-\frac{5\sqrt{bx^2+cx^4}(9bB-7}{21c^3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```


```
[Out] -(((b*B - A*c)*x^(11/2))/(b*c*Sqrt[b*x^2 + c*x^4])) - (5*(9*b*B - 7*A*c)*Sq
rt[b*x^2 + c*x^4])/(21*c^3*Sqrt[x]) + ((9*b*B - 7*A*c)*x^(3/2)*Sqrt[b*x^2 +
 c*x^4])/(7*b*c^2) + (5*b^(3/4)*(9*b*B - 7*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqr
t[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])
/b^(1/4)], 1/2])/(42*c^(13/4)*Sqrt[b*x^2 + c*x^4])
```


Rule 2037

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> -Simp[(e^(j - 1)*(b*c - a*d)*(e*x)^(m - j +
 1)*(a*x^j + b*x^(j + n))^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(e^j*(a*d*(m
+ j*p + 1) - b*c*(m + n + p*(j + n) + 1)))/(a*b*n*(p + 1)), Int[(e*x)^(m -
j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m, n},
```

```
 x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &
& GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{x^{13/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{(bB-Ac)x^{11/2}}{bc\sqrt{bx^2+cx^4}} + \frac{\left(\frac{9bB}{2}-\frac{7Ac}{2}\right)\int \frac{x^{9/2}}{\sqrt{bx^2+cx^4}}\,dx}{bc}$$

$$= -\frac{(bB-Ac)x^{11/2}}{bc\sqrt{bx^2+cx^4}} + \frac{(9bB-7Ac)x^{3/2}\sqrt{bx^2+cx^4}}{7bc^2} - \frac{(5(9bB-7Ac))\int \frac{x^{5/2}}{\sqrt{bx^2+cx^4}}\,dx}{14c^2}$$

$$= -\frac{(bB-Ac)x^{11/2}}{bc\sqrt{bx^2+cx^4}} - \frac{5(9bB-7Ac)\sqrt{bx^2+cx^4}}{21c^3\sqrt{x}} + \frac{(9bB-7Ac)x^{3/2}\sqrt{bx^2+cx^4}}{7bc^2} + \frac{(5b(9bB-7Ac))}{42c}$$

$$= -\frac{(bB-Ac)x^{11/2}}{bc\sqrt{bx^2+cx^4}} - \frac{5(9bB-7Ac)\sqrt{bx^2+cx^4}}{21c^3\sqrt{x}} + \frac{(9bB-7Ac)x^{3/2}\sqrt{bx^2+cx^4}}{7bc^2} + \frac{\left(5b(9bB-7Ac)\right.}{42c}$$

$$= -\frac{(bB-Ac)x^{11/2}}{bc\sqrt{bx^2+cx^4}} - \frac{5(9bB-7Ac)\sqrt{bx^2+cx^4}}{21c^3\sqrt{x}} + \frac{(9bB-7Ac)x^{3/2}\sqrt{bx^2+cx^4}}{7bc^2} + \frac{\left(5b(9bB-7Ac)\right.}{}$$

$$= -\frac{(bB-Ac)x^{11/2}}{bc\sqrt{bx^2+cx^4}} - \frac{5(9bB-7Ac)\sqrt{bx^2+cx^4}}{21c^3\sqrt{x}} + \frac{(9bB-7Ac)x^{3/2}\sqrt{bx^2+cx^4}}{7bc^2} + \frac{5b^{3/4}(9bB-7Ac}{}$$

Mathematica [C] time = 0.133174, size = 110, normalized size = 0.51

$$\frac{x^{3/2}\left(5b\sqrt{\frac{cx^2}{b}+1}(9bB-7Ac)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^2}{b}\right)+bc\left(35A-18Bx^2\right)+2c^2x^2\left(7A+3Bx^2\right)-45b^2B\right)}{21c^3\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x^(3/2)*(-45*b^2*B + b*c*(35*A - 18*B*x^2) + 2*c^2*x^2*(7*A + 3*B*x^2) + 5
*b*(9*b*B - 7*A*c)*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[1/4, 1/2, 5/4, -((
c*x^2)/b)]))/(21*c^3*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.019, size = 255, normalized size = 1.2

$$-\frac{cx^2+b}{42\,c^4}x^{\frac{5}{2}}\left(35\,A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\text{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)\sqrt{-bc}bc-45\,B\sqrt{\frac{cx+\sqrt{-}}{\sqrt{-bc}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/42/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(35*A*((c*x+(-b*c)^(1/2))/(-b*c
)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c
)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2
))*(-b*c)^(1/2)*b*c-45*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((
-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(
((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(-b*c)^(1/2)*b^2-12*B*
c^3*x^5-28*A*x^3*c^3+36*B*x^3*b*c^2-70*A*b*c^2*x+90*B*b^2*c*x)/c^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2 + A\right)x^{\frac{13}{2}}}{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^(13/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bx^4 + Ax^2\right)\sqrt{cx^4 + bx^2}\sqrt{x}}{c^2x^4 + 2\,bcx^2 + b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^4 + A*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x)/(c^2*x^4 + 2*b*c*x^2 +
 b^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(13/2)*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)x^{\frac{13}{2}}}{(cx^4 + bx^2)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^(13/2)/(c*x^4 + b*x^2)^(3/2), x)
```

3.260 $\int \frac{x^{11/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=340

$$-\frac{3\sqrt[4]{b}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(7bB-5Ac)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{10c^{11/4}\sqrt{bx^2+cx^4}}-\frac{3x^{3/2}\left(b+cx^2\right)(7bB-5Ac)}{5c^{5/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{\sqrt{x}\sqrt{bx^2+}}{}$$

```
[Out] -(((b*B - A*c)*x^(9/2))/(b*c*Sqrt[b*x^2 + c*x^4])) - (3*(7*b*B - 5*A*c)*x^(
3/2)*(b + c*x^2))/(5*c^(5/2)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) + (
(7*b*B - 5*A*c)*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(5*b*c^2) + (3*b^(1/4)*(7*b*B
- 5*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*
EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*c^(11/4)*Sqrt[b*x^2
 + c*x^4]) - (3*b^(1/4)*(7*b*B - 5*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c
*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)
], 1/2])/(10*c^(11/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.396641, antiderivative size = 340, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2037, 2024, 2032, 329, 305, 220, 1196}

$$-\frac{3x^{3/2}\left(b+cx^2\right)(7bB-5Ac)}{5c^{5/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{\sqrt{x}\sqrt{bx^2+cx^4}(7bB-5Ac)}{5bc^2}-\frac{3\sqrt[4]{b}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(7bB-5Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}}{\sqrt[4]{}}\right.\right.}{10c^{11/4}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```



```
[Out] -(((b*B - A*c)*x^(9/2))/(b*c*Sqrt[b*x^2 + c*x^4])) - (3*(7*b*B - 5*A*c)*x^(
3/2)*(b + c*x^2))/(5*c^(5/2)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) + (
(7*b*B - 5*A*c)*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(5*b*c^2) + (3*b^(1/4)*(7*b*B
- 5*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*
EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*c^(11/4)*Sqrt[b*x^2
 + c*x^4]) - (3*b^(1/4)*(7*b*B - 5*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c
*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)
], 1/2])/(10*c^(11/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 2037

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> -Simp[(e^(j - 1)*(b*c - a*d)*(e*x)^(m - j +
 1)*(a*x^j + b*x^(j + n))^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(e^j*(a*d*(m
+ j*p + 1) - b*c*(m + n + p*(j + n) + 1)))/(a*b*n*(p + 1)), Int[(e*x)^(m -
j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m, n},
 x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &
& GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
```

```
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^{11/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{(bB-Ac)x^{9/2}}{bc\sqrt{bx^2+cx^4}}+\frac{\left(\frac{7bB}{2}-\frac{5Ac}{2}\right)\int \frac{x^{7/2}}{\sqrt{bx^2+cx^4}}\,dx}{bc}$$

$$= -\frac{(bB-Ac)x^{9/2}}{bc\sqrt{bx^2+cx^4}}+\frac{(7bB-5Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5bc^2}-\frac{(3(7bB-5Ac))\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{10c^2}$$

$$= -\frac{(bB-Ac)x^{9/2}}{bc\sqrt{bx^2+cx^4}}+\frac{(7bB-5Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5bc^2}-\frac{\left(3(7bB-5Ac)x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{10c^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{(bB-Ac)x^{9/2}}{bc\sqrt{bx^2+cx^4}}+\frac{(7bB-5Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5bc^2}-\frac{\left(3(7bB-5Ac)x\sqrt{b+cx^2}\right)\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x\right.}{5c^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{(bB-Ac)x^{9/2}}{bc\sqrt{bx^2+cx^4}}+\frac{(7bB-5Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5bc^2}-\frac{\left(3\sqrt{b}(7bB-5Ac)x\sqrt{b+cx^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,d\right.}{5c^{5/2}\sqrt{bx^2+cx^4}}$$

$$= -\frac{(bB-Ac)x^{9/2}}{bc\sqrt{bx^2+cx^4}}-\frac{3(7bB-5Ac)x^{3/2}\left(b+cx^2\right)}{5c^{5/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{(7bB-5Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5bc^2}+\frac{3\sqrt[4]{b}(7bB-5Ac}{}$$

Mathematica [C] time = 0.102897, size = 85, normalized size = 0.25

$$\frac{2x^{5/2}\left(\sqrt{\frac{cx^2}{b}+1}(7bB-5Ac)\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{cx^2}{b}\right)+5Ac-7bB+Bcx^2\right)}{5c^2\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*x^(5/2)*(-7*b*B + 5*A*c + B*c*x^2 + (7*b*B - 5*A*c)*Sqrt[1 + (c*x^2)/b]*
Hypergeometric2F1[3/4, 3/2, 7/4, -((c*x^2)/b)]))/(5*c^2*Sqrt[x^2*(b + c*x^2
)])
```

Maple [A] time = 0.018, size = 394, normalized size = 1.2

$$\frac{cx^2+b}{10\,c^3}x^{\frac{5}{2}}\left(30\,A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\,\sqrt{2}\right)bc-15\,A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/10/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(30*A*((c*x+(-b*c)^(1/2))/(-b*c)
^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)
^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2)
)*b*c-15*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1
/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(
1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*c-42*B*((c*x+(-b*c)^(1/2))/(-b*c)^
(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^
(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))
*b^2+21*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/
2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1
/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^2+4*B*c^2*x^4-10*A*x^2*c^2+14*B*x^2
*b*c)/c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2+A\right)x^{\frac{11}{2}}}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^(11/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\left(Bx^3+Ax\right)\sqrt{x}}{c^2x^4+2\,bcx^2+b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(B*x^3 + A*x)*sqrt(x)/(c^2*x^4 + 2*b*c*x^2 + b
^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(11/2)*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2+A\right)x^{\frac{11}{2}}}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^(11/2)/(c*x^4 + b*x^2)^(3/2), x)
```

3.261 $\int \frac{x^{9/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=178

$$-\frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(5bB-3Ac)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{6\sqrt[4]{b}c^{9/4}\sqrt{bx^2+cx^4}}+\frac{\sqrt{bx^2+cx^4}(5bB-3Ac)}{3bc^2\sqrt{x}}-\frac{x^{7/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

```
[Out] -(((b*B - A*c)*x^(7/2))/(b*c*Sqrt[b*x^2 + c*x^4])) + ((5*b*B - 3*A*c)*Sqrt[
b*x^2 + c*x^4])/(3*b*c^2*Sqrt[x]) - ((5*b*B - 3*A*c)*x*(Sqrt[b] + Sqrt[c]*x
)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqr
t[x])/b^(1/4)], 1/2])/(6*b^(1/4)*c^(9/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.272466, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2037, 2024, 2032, 329, 220}

$$\frac{\sqrt{bx^2+cx^4}(5bB-3Ac)}{3bc^2\sqrt{x}}-\frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(5bB-3Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{6\sqrt[4]{b}c^{9/4}\sqrt{bx^2+cx^4}}-\frac{x^{7/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```


```
[Out] -(((b*B - A*c)*x^(7/2))/(b*c*Sqrt[b*x^2 + c*x^4])) + ((5*b*B - 3*A*c)*Sqrt[
b*x^2 + c*x^4])/(3*b*c^2*Sqrt[x]) - ((5*b*B - 3*A*c)*x*(Sqrt[b] + Sqrt[c]*x
)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqr
t[x])/b^(1/4)], 1/2])/(6*b^(1/4)*c^(9/4)*Sqrt[b*x^2 + c*x^4])
```


Rule 2037

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> -Simp[(e^(j - 1)*(b*c - a*d)*(e*x)^(m - j +
 1)*(a*x^j + b*x^(j + n))^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(e^j*(a*d*(m
+ j*p + 1) - b*c*(m + n + p*(j + n) + 1)))/(a*b*n*(p + 1)), Int[(e*x)^(m -
j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m, n},
 x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &
```

```
& GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{x^{9/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{(bB-Ac)x^{7/2}}{bc\sqrt{bx^2+cx^4}} + \frac{\left(\frac{5bB}{2}-\frac{3Ac}{2}\right)\int \frac{x^{5/2}}{\sqrt{bx^2+cx^4}}\,dx}{bc}$$

$$= -\frac{(bB-Ac)x^{7/2}}{bc\sqrt{bx^2+cx^4}} + \frac{(5bB-3Ac)\sqrt{bx^2+cx^4}}{3bc^2\sqrt{x}} - \frac{(5bB-3Ac)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{6c^2}$$

$$= -\frac{(bB-Ac)x^{7/2}}{bc\sqrt{bx^2+cx^4}} + \frac{(5bB-3Ac)\sqrt{bx^2+cx^4}}{3bc^2\sqrt{x}} - \frac{\left((5bB-3Ac)x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{6c^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{(bB-Ac)x^{7/2}}{bc\sqrt{bx^2+cx^4}} + \frac{(5bB-3Ac)\sqrt{bx^2+cx^4}}{3bc^2\sqrt{x}} - \frac{\left((5bB-3Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{\right.}{3c^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{(bB-Ac)x^{7/2}}{bc\sqrt{bx^2+cx^4}} + \frac{(5bB-3Ac)\sqrt{bx^2+cx^4}}{3bc^2\sqrt{x}} - \frac{(5bB-3Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\right.\right.}{6\sqrt[4]{b}c^{9/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] time = 0.107913, size = 86, normalized size = 0.48

$$\frac{x^{3/2}\left(\sqrt{\frac{cx^2}{b}+1}(3Ac-5bB)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^2}{b}\right)-3Ac+5bB+2Bcx^2\right)}{3c^2\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x^(3/2)*(5*b*B - 3*A*c + 2*B*c*x^2 + (-5*b*B + 3*A*c)*Sqrt[1 + (c*x^2)/b]*
Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^2)/b)]))/(3*c^2*Sqrt[x^2*(b + c*x^2
)])
```

Maple [A] time = 0.017, size = 230, normalized size = 1.3

$$\frac{cx^2+b}{6\,c^3}x^{\frac{5}{2}}\left(3\,A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\text{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)\sqrt{-bc}c-5\,B\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/6/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(3*A*((c*x+(-b*c)^(1/2))/(-b*c)^(
1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(
1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*
(-b*c)^(1/2)*c-5*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(
-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+
(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(-b*c)^(1/2)*b+4*B*c^2*x^3-6
*A*c^2*x+10*B*b*c*x)/c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2 + A\right)x^{\frac{9}{2}}}{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^(9/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2}\left(Bx^2 + A\right)\sqrt{x}}{c^2x^4 + 2\,bcx^2 + b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*sqrt(x)/(c^2*x^4 + 2*b*c*x^2 + b^2
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)x^{\frac{9}{2}}}{(cx^4 + bx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^(9/2)/(c*x^4 + b*x^2)^(3/2), x)
```

3.262 $\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=299

$$\frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(3bB-Ac)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2b^{3/4}c^{7/4}\sqrt{bx^2+cx^4}}-\frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(3bB-Ac)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\ldots\right.}{b^{3/4}c^{7/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -(((b*B - A*c)*x^(5/2))/(b*c*Sqrt[b*x^2 + c*x^4])) + ((3*b*B - A*c)*x^(3/2)
*(b + c*x^2))/(b*c^(3/2)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) - ((3*b
*B - A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]
*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(b^(3/4)*c^(7/4)*Sqrt
[b*x^2 + c*x^4]) + ((3*b*B - A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/
(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2
])/(2*b^(3/4)*c^(7/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.327944, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2037, 2032, 329, 305, 220, 1196}

$$\frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(3bB-Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2b^{3/4}c^{7/4}\sqrt{bx^2+cx^4}}-\frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(3bB-Ac)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}c^{7/4}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```


```
[Out] -(((b*B - A*c)*x^(5/2))/(b*c*Sqrt[b*x^2 + c*x^4])) + ((3*b*B - A*c)*x^(3/2)
*(b + c*x^2))/(b*c^(3/2)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) - ((3*b
*B - A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]
*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(b^(3/4)*c^(7/4)*Sqrt
[b*x^2 + c*x^4]) + ((3*b*B - A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/
(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2
])/(2*b^(3/4)*c^(7/4)*Sqrt[b*x^2 + c*x^4])
```


Rule 2037

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> -Simp[(e^(j - 1)*(b*c - a*d)*(e*x)^(m - j +
 1)*(a*x^j + b*x^(j + n))^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(e^j*(a*d*(m
+ j*p + 1) - b*c*(m + n + p*(j + n) + 1)))/(a*b*n*(p + 1)), Int[(e*x)^(m -
j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m, n},
 x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &
& GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^{7/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{(bB-Ac)x^{5/2}}{bc\sqrt{bx^2+cx^4}}+\frac{\left(\frac{3bB}{2}-\frac{Ac}{2}\right)\int\frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{bc}$$

$$= -\frac{(bB-Ac)x^{5/2}}{bc\sqrt{bx^2+cx^4}}+\frac{\left(\left(\frac{3bB}{2}-\frac{Ac}{2}\right)x\sqrt{b+cx^2}\right)\int\frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{bc\sqrt{bx^2+cx^4}}$$

$$= -\frac{(bB-Ac)x^{5/2}}{bc\sqrt{bx^2+cx^4}}+\frac{\left(2\left(\frac{3bB}{2}-\frac{Ac}{2}\right)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int\frac{x^2}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{bc\sqrt{bx^2+cx^4}}$$

$$= -\frac{(bB-Ac)x^{5/2}}{bc\sqrt{bx^2+cx^4}}+\frac{\left(2\left(\frac{3bB}{2}-\frac{Ac}{2}\right)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{\sqrt{b}c^{3/2}\sqrt{bx^2+cx^4}}-\frac{\left(2\left(\frac{3bB}{2}-\frac{Ac}{2}\right)x\sqrt{b+c\right.}{\sqrt{b}c^{3}}$$

$$= -\frac{(bB-Ac)x^{5/2}}{bc\sqrt{bx^2+cx^4}}+\frac{(3bB-Ac)x^{3/2}\left(b+cx^2\right)}{bc^{3/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{(3bB-Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-}\right.}{b^{3/4}c^{7/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] time = 0.086856, size = 78, normalized size = 0.26

$$-\frac{2x^{5/2}\left(\sqrt{\frac{cx^2}{b}+1}(3bB-Ac)\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{cx^2}{b}\right)-3bB\right)}{3bc\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-2*x^(5/2)*(-3*b*B + (3*b*B - A*c)*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[3
/4, 3/2, 7/4, -((c*x^2)/b)]))/(3*b*c*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.019, size = 388, normalized size = 1.3

$$-\frac{cx^2+b}{2\,bc^2}x^{\frac{5}{2}}\left(2\,A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\text{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)bc-A\sqrt{\left(cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/2/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(2*A*((c*x+(-b*c)^(1/2))/(-b*c)^
(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^
(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))
*b*c-A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))
/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2)
)/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*c-6*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2)
)^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2)
)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^2+
3*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-
b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(
-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^2-2*A*x^2*c^2+2*B*x^2*b*c)/c^2/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)x^{\frac{7}{2}}}{(cx^4 + bx^2)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^(7/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2}(Bx^2 + A)\sqrt{x}}{c^2x^5 + 2\,bcx^3 + b^2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*sqrt(x)/(c^2*x^5 + 2*b*c*x^3 + b^2
*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2+A\right)x^{\frac{7}{2}}}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^(7/2)/(c*x^4 + b*x^2)^(3/2), x)
```

3.263 $\int \frac{x^{5/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=137

$$\frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(Ac+bB)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2b^{5/4}c^{5/4}\sqrt{bx^2+cx^4}}-\frac{x^{3/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

```
[Out] -(((b*B - A*c)*x^(3/2))/(b*c*Sqrt[b*x^2 + c*x^4])) + ((b*B + A*c)*x*(Sqrt[b
] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan
[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(2*b^(5/4)*c^(5/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.220644, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2037, 2032, 329, 220}

$$\frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(Ac+bB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2b^{5/4}c^{5/4}\sqrt{bx^2+cx^4}}-\frac{x^{3/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```


```
[Out] -(((b*B - A*c)*x^(3/2))/(b*c*Sqrt[b*x^2 + c*x^4])) + ((b*B + A*c)*x*(Sqrt[b
] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan
[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(2*b^(5/4)*c^(5/4)*Sqrt[b*x^2 + c*x^4])
```


Rule 2037

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> -Simp[(e^(j - 1)*(b*c - a*d)*(e*x)^(m - j +
 1)*(a*x^j + b*x^(j + n))^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(e^j*(a*d*(m
+ j*p + 1) - b*c*(m + n + p*(j + n) + 1)))/(a*b*n*(p + 1)), Int[(e*x)^(m -
j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m, n},
 x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &
& GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{5/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^{3/2}}\,dx &= -\frac{(bB-Ac)x^{3/2}}{bc\sqrt{bx^2+cx^4}}+\frac{(bB+Ac)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{2bc}\\
&= -\frac{(bB-Ac)x^{3/2}}{bc\sqrt{bx^2+cx^4}}+\frac{\left((bB+Ac)x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{2bc\sqrt{bx^2+cx^4}}\\
&= -\frac{(bB-Ac)x^{3/2}}{bc\sqrt{bx^2+cx^4}}+\frac{\left((bB+Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{bc\sqrt{bx^2+cx^4}}\\
&= -\frac{(bB-Ac)x^{3/2}}{bc\sqrt{bx^2+cx^4}}+\frac{(bB+Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2b^{5/4}c^{5/4}\sqrt{bx^2+cx^4}}
\end{aligned}$$

Mathematica [C] time = 0.0596482, size = 76, normalized size = 0.55

$$\frac{x^{3/2}\left(\sqrt{\frac{cx^2}{b}+1}(Ac+bB)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^2}{b}\right)+Ac-bB\right)}{bc\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x^(3/2)*(-(b*B) + A*c + (b*B + A*c)*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[
1/4, 1/2, 5/4, -((c*x^2)/b)]))/(b*c*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.019, size = 222, normalized size = 1.6

$$\frac{cx^2+b}{2\,bc^2}x^{\frac{5}{2}}\left(A\sqrt{\left(cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}}\sqrt{2}\sqrt{\left(-cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}}\sqrt{-cx\frac{1}{\sqrt{-bc}}}\text{EllipticF}\left(\sqrt{\left(cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{-bcc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/2/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/
2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/
2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(-
b*c)^(1/2)*c+B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c
)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*
c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(-b*c)^(1/2)*b+2*A*c^2*x-2*B*b*c
*x)/b/c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2+A\right)x^{\frac{5}{2}}}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^(5/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\left(Bx^2+A\right)\sqrt{x}}{c^2x^6+2\,bcx^4+b^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*sqrt(x)/(c^2*x^6 + 2*b*c*x^4 + b^2
*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2+A\right)x^{\frac{5}{2}}}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^(5/2)/(c*x^4 + b*x^2)^(3/2), x)
```

3.264 $\int \frac{x^{3/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=318

$$-\frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(bB-3Ac)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2b^{7/4}c^{3/4}\sqrt{bx^2+cx^4}}+\frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(bB-3Ac)E\left(2\tan^{-1}\right.}{b^{7/4}c^{3/4}\sqrt{bx^2+cx^4}}$$

```
[Out] (-2*A*Sqrt[x])/(b*Sqrt[b*x^2 + c*x^4]) + ((b*B - 3*A*c)*x^(5/2))/(b^2*Sqrt[
b*x^2 + c*x^4]) - ((b*B - 3*A*c)*x^(3/2)*(b + c*x^2))/(b^2*Sqrt[c]*(Sqrt[b]
 + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) + ((b*B - 3*A*c)*x*(Sqrt[b] + Sqrt[c]*x)
*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt
[x])/b^(1/4)], 1/2])/(b^(7/4)*c^(3/4)*Sqrt[b*x^2 + c*x^4]) - ((b*B - 3*A*c)
*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Elliptic
F[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(2*b^(7/4)*c^(3/4)*Sqrt[b*x^2
+ c*x^4])
```

Rubi [A] time = 0.389588, antiderivative size = 318, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2038, 2023, 2032, 329, 305, 220, 1196}

$$-\frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(bB-3Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2b^{7/4}c^{3/4}\sqrt{bx^2+cx^4}}+\frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(bB-3Ac)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\right.}{b^{7/4}c^{3/4}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```


```
[Out] (-2*A*Sqrt[x])/(b*Sqrt[b*x^2 + c*x^4]) + ((b*B - 3*A*c)*x^(5/2))/(b^2*Sqrt[
b*x^2 + c*x^4]) - ((b*B - 3*A*c)*x^(3/2)*(b + c*x^2))/(b^2*Sqrt[c]*(Sqrt[b]
 + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) + ((b*B - 3*A*c)*x*(Sqrt[b] + Sqrt[c]*x)
*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt
[x])/b^(1/4)], 1/2])/(b^(7/4)*c^(3/4)*Sqrt[b*x^2 + c*x^4]) - ((b*B - 3*A*c)
*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Elliptic
F[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(2*b^(7/4)*c^(3/4)*Sqrt[b*x^2
+ c*x^4])
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
```

```
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^{3/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{2A\sqrt{x}}{b\sqrt{bx^2+cx^4}} - \frac{\left(2\left(-\frac{bB}{2}+\frac{3Ac}{2}\right)\right)\int \frac{x^{7/2}}{\left(bx^2+cx^4\right)^{3/2}}\,dx}{b}$$

$$= -\frac{2A\sqrt{x}}{b\sqrt{bx^2+cx^4}} + \frac{(bB-3Ac)x^{5/2}}{b^2\sqrt{bx^2+cx^4}} - \frac{(bB-3Ac)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{2b^2}$$

$$= -\frac{2A\sqrt{x}}{b\sqrt{bx^2+cx^4}} + \frac{(bB-3Ac)x^{5/2}}{b^2\sqrt{bx^2+cx^4}} - \frac{\left((bB-3Ac)x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{2b^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{2A\sqrt{x}}{b\sqrt{bx^2+cx^4}} + \frac{(bB-3Ac)x^{5/2}}{b^2\sqrt{bx^2+cx^4}} - \frac{\left((bB-3Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{b^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{2A\sqrt{x}}{b\sqrt{bx^2+cx^4}} + \frac{(bB-3Ac)x^{5/2}}{b^2\sqrt{bx^2+cx^4}} - \frac{\left((bB-3Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{b^{3/2}\sqrt{c}\sqrt{bx^2+cx^4}} + \frac{\left((bB\ \cdots\right.}{\cdots}$$

$$= -\frac{2A\sqrt{x}}{b\sqrt{bx^2+cx^4}} + \frac{(bB-3Ac)x^{5/2}}{b^2\sqrt{bx^2+cx^4}} - \frac{(bB-3Ac)x^{3/2}\left(b+cx^2\right)}{b^2\sqrt{c}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} + \frac{(bB-3Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\cdots}}{b^{7/4}c^{3/4}\cdots}$$

Mathematica [C] time = 0.0469074, size = 77, normalized size = 0.24

$$\frac{2\sqrt{x}\left(x^2\sqrt{\frac{cx^2}{b}+1}(bB-3Ac)\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{cx^2}{b}\right)-3Ab\right)}{3b^2\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[x]*(-3*A*b + (b*B - 3*A*c)*x^2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F
1[3/4, 3/2, 7/4, -((c*x^2)/b)]))/(3*b^2*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.02, size = 392, normalized size = 1.2

$$\frac{cx^2+b}{2\,b^2c}x^{\frac{5}{2}}\left(6\,A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)bc-3\,A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/2/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(6*A*((c*x+(-b*c)^(1/2))/(-b*c)^(
1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(
1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*
b*c-3*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2)
)/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2
))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*c-2*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2
))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2
))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^2
+B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b
*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-
b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^2-6*A*x^2*c^2+2*B*x^2*b*c-4*A*b*c)/c/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2+A\right)x^{\frac{3}{2}}}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^(3/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\left(Bx^2+A\right)\sqrt{x}}{c^2x^7+2\,bcx^5+b^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*sqrt(x)/(c^2*x^7 + 2*b*c*x^5 + b^2
*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(Bx^2+A\right)x^{\frac{3}{2}}}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^(3/2)/(c*x^4 + b*x^2)^(3/2), x)
```

3.265 $\int \frac{\sqrt{x}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=167

$$\frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(3bB-5Ac)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{6b^{9/4}\sqrt[4]{c}\sqrt{bx^2+cx^4}}+\frac{x^{3/2}(3bB-5Ac)}{3b^2\sqrt{bx^2+cx^4}}-\frac{2A}{3b\sqrt{x}\sqrt{bx^2+cx^4}}$$

```
[Out] (-2*A)/(3*b*Sqrt[x]*Sqrt[b*x^2 + c*x^4]) + ((3*b*B - 5*A*c)*x^(3/2))/(3*b^2
*Sqrt[b*x^2 + c*x^4]) + ((3*b*B - 5*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b +
c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4
)], 1/2])/(6*b^(9/4)*c^(1/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.272043, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2038, 2023, 2032, 329, 220}

$$\frac{x^{3/2}(3bB-5Ac)}{3b^2\sqrt{bx^2+cx^4}}+\frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(3bB-5Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{6b^{9/4}\sqrt[4]{c}\sqrt{bx^2+cx^4}}-\frac{2A}{3b\sqrt{x}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```


```
[Out] (-2*A)/(3*b*Sqrt[x]*Sqrt[b*x^2 + c*x^4]) + ((3*b*B - 5*A*c)*x^(3/2))/(3*b^2
*Sqrt[b*x^2 + c*x^4]) + ((3*b*B - 5*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b +
c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4
)], 1/2])/(6*b^(9/4)*c^(1/4)*Sqrt[b*x^2 + c*x^4])
```


Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
```

```
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{\sqrt{x}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{2A}{3b\sqrt{x}\sqrt{bx^2+cx^4}} - \frac{\left(2\left(-\frac{3bB}{2}+\frac{5Ac}{2}\right)\right)\int \frac{x^{5/2}}{\left(bx^2+cx^4\right)^{3/2}}\,dx}{3b}$$

$$= -\frac{2A}{3b\sqrt{x}\sqrt{bx^2+cx^4}} + \frac{(3bB-5Ac)x^{3/2}}{3b^2\sqrt{bx^2+cx^4}} + \frac{(3bB-5Ac)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{6b^2}$$

$$= -\frac{2A}{3b\sqrt{x}\sqrt{bx^2+cx^4}} + \frac{(3bB-5Ac)x^{3/2}}{3b^2\sqrt{bx^2+cx^4}} + \frac{\left((3bB-5Ac)x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{6b^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{2A}{3b\sqrt{x}\sqrt{bx^2+cx^4}} + \frac{(3bB-5Ac)x^{3/2}}{3b^2\sqrt{bx^2+cx^4}} + \frac{\left((3bB-5Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{3b^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{2A}{3b\sqrt{x}\sqrt{bx^2+cx^4}} + \frac{(3bB-5Ac)x^{3/2}}{3b^2\sqrt{bx^2+cx^4}} + \frac{(3bB-5Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\right.}{6b^{9/4}\sqrt[4]{c}\sqrt{bx^2+cx^4}}$$

Mathematica [C] time = 0.0491973, size = 92, normalized size = 0.55

$$\frac{x^2\sqrt{\frac{cx^2}{b}+1}(3bB-5Ac)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^2}{b}\right)-2Ab-5Acx^2+3bBx^2}{3b^2\sqrt{x}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-2*A*b + 3*b*B*x^2 - 5*A*c*x^2 + (3*b*B - 5*A*c)*x^2*Sqrt[1 + (c*x^2)/b]*H
ypergeometric2F1[1/4, 1/2, 5/4, -((c*x^2)/b)])/(3*b^2*Sqrt[x]*Sqrt[x^2*(b +
 c*x^2)])
```

Maple [A] time = 0.02, size = 235, normalized size = 1.4

$$-\frac{cx^2+b}{6\,b^2c}x^{\frac{3}{2}}\left(5\,A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\text{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)\sqrt{-bc}xc-3\,B\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/6/(c*x^4+b*x^2)^(3/2)*x^(3/2)*(c*x^2+b)*(5*A*((c*x+(-b*c)^(1/2))/(-b*c)^
(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^
(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))
*(-b*c)^(1/2)*x*c-3*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*
x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c
*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(-b*c)^(1/2)*x*b+10*A*x^2
*c^2-6*B*x^2*b*c+4*A*b*c)/c/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2+A)\sqrt{x}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*sqrt(x)/(c*x^4 + b*x^2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}(Bx^2+A)\sqrt{x}}{c^2x^8+2\,bcx^6+b^2x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*sqrt(x)/(c^2*x^8 + 2*b*c*x^6 + b^2
*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}\left(A+Bx^{2}\right)}{\left(x^{2}\left(b+cx^{2}\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*x**(1/2)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(sqrt(x)*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^{2}+A\right)\sqrt{x}}{\left(cx^{4}+bx^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*sqrt(x)/(c*x^4 + b*x^2)^(3/2), x)
```

3.266 $\int \frac{A+Bx^2}{\sqrt{x}\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=368

$$\frac{3\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(5bB-7Ac)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{10b^{11/4}\sqrt{bx^2+cx^4}}+\frac{3\sqrt{c}x^{3/2}\left(b+cx^2\right)(5bB-7Ac)}{5b^3\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{\sqrt{x}(5bB-}{5b^2\sqrt{bx^2}}$$

```
[Out] (-2*A)/(5*b*x^(3/2)*Sqrt[b*x^2 + c*x^4]) + ((5*b*B - 7*A*c)*Sqrt[x])/(5*b^2
*Sqrt[b*x^2 + c*x^4]) + (3*Sqrt[c]*(5*b*B - 7*A*c)*x^(3/2)*(b + c*x^2))/(5*
b^3*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) - (3*(5*b*B - 7*A*c)*Sqrt[b*
x^2 + c*x^4])/(5*b^3*x^(3/2)) - (3*c^(1/4)*(5*b*B - 7*A*c)*x*(Sqrt[b] + Sqr
t[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/
4)*Sqrt[x])/b^(1/4)], 1/2])/(5*b^(11/4)*Sqrt[b*x^2 + c*x^4]) + (3*c^(1/4)*(
5*b*B - 7*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*
x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(10*b^(11/4)*Sqr
t[b*x^2 + c*x^4])
```

Rubi [A] time = 0.450657, antiderivative size = 368, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2038, 2023, 2025, 2032, 329, 305, 220, 1196}

$$\frac{3\sqrt{c}x^{3/2}\left(b+cx^2\right)(5bB-7Ac)}{5b^3\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{\sqrt{x}(5bB-7Ac)}{5b^2\sqrt{bx^2+cx^4}}-\frac{3\sqrt{bx^2+cx^4}(5bB-7Ac)}{5b^3x^{3/2}}+\frac{3\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(5bB-7}{10b^{11/4}\sqrt{bx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(Sqrt[x]*(b*x^2 + c*x^4)^(3/2)),x]
```


```
[Out] (-2*A)/(5*b*x^(3/2)*Sqrt[b*x^2 + c*x^4]) + ((5*b*B - 7*A*c)*Sqrt[x])/(5*b^2
*Sqrt[b*x^2 + c*x^4]) + (3*Sqrt[c]*(5*b*B - 7*A*c)*x^(3/2)*(b + c*x^2))/(5*
b^3*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) - (3*(5*b*B - 7*A*c)*Sqrt[b*
x^2 + c*x^4])/(5*b^3*x^(3/2)) - (3*c^(1/4)*(5*b*B - 7*A*c)*x*(Sqrt[b] + Sqr
t[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/
4)*Sqrt[x])/b^(1/4)], 1/2])/(5*b^(11/4)*Sqrt[b*x^2 + c*x^4]) + (3*c^(1/4)*(
5*b*B - 7*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*
x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(10*b^(11/4)*Sqr
```

```
t[b*x^2 + c*x^4])
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{A+Bx^2}{\sqrt{x}\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{2A}{5bx^{3/2}\sqrt{bx^2+cx^4}} - \frac{\left(2\left(-\frac{5bB}{2}+\frac{7Ac}{2}\right)\right)\int \frac{x^{3/2}}{\left(bx^2+cx^4\right)^{3/2}}\,dx}{5b}$$

$$= -\frac{2A}{5bx^{3/2}\sqrt{bx^2+cx^4}} + \frac{(5bB-7Ac)\sqrt{x}}{5b^2\sqrt{bx^2+cx^4}} + \frac{(3(5bB-7Ac))\int \frac{1}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx}{10b^2}$$

$$= -\frac{2A}{5bx^{3/2}\sqrt{bx^2+cx^4}} + \frac{(5bB-7Ac)\sqrt{x}}{5b^2\sqrt{bx^2+cx^4}} - \frac{3(5bB-7Ac)\sqrt{bx^2+cx^4}}{5b^3x^{3/2}} + \frac{(3c(5bB-7Ac))\int \frac{x^{3/2}}{\sqrt{bx^2+c}}}{10b^3}$$

$$= -\frac{2A}{5bx^{3/2}\sqrt{bx^2+cx^4}} + \frac{(5bB-7Ac)\sqrt{x}}{5b^2\sqrt{bx^2+cx^4}} - \frac{3(5bB-7Ac)\sqrt{bx^2+cx^4}}{5b^3x^{3/2}} + \frac{\left(3c(5bB-7Ac)x\sqrt{b+cx}\right.}{10b^3\sqrt{bx^2+c}}$$

$$= -\frac{2A}{5bx^{3/2}\sqrt{bx^2+cx^4}} + \frac{(5bB-7Ac)\sqrt{x}}{5b^2\sqrt{bx^2+cx^4}} - \frac{3(5bB-7Ac)\sqrt{bx^2+cx^4}}{5b^3x^{3/2}} + \frac{\left(3c(5bB-7Ac)x\sqrt{b+cx}\right.}{5b^3}$$

$$= -\frac{2A}{5bx^{3/2}\sqrt{bx^2+cx^4}} + \frac{(5bB-7Ac)\sqrt{x}}{5b^2\sqrt{bx^2+cx^4}} - \frac{3(5bB-7Ac)\sqrt{bx^2+cx^4}}{5b^3x^{3/2}} + \frac{\left(3\sqrt{c}(5bB-7Ac)x\sqrt{b+}\right.}{5b^{5/2}}$$

$$= -\frac{2A}{5bx^{3/2}\sqrt{bx^2+cx^4}} + \frac{(5bB-7Ac)\sqrt{x}}{5b^2\sqrt{bx^2+cx^4}} + \frac{3\sqrt{c}(5bB-7Ac)x^{3/2}\left(b+cx^2\right)}{5b^3\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{3(5bB-7Ac)\sqrt{bx^2}}{5b^3x^{3/2}}$$

<u>Mathematica</u> [C] time = 0.0431953, size = 79, normalized size = 0.21

$$\frac{2x^2\sqrt{\frac{cx^2}{b}+1}(7Ac-5bB)\,{}_2F_1\left(-\frac{1}{4},\frac{3}{2};\frac{3}{4};-\frac{cx^2}{b}\right)-2Ab}{5b^2x^{3/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(Sqrt[x]*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-2*A*b + 2*(-5*b*B + 7*A*c)*x^2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-1/4
, 3/2, 3/4, -((c*x^2)/b)])/(5*b^2*x^(3/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.021, size = 420, normalized size = 1.1

$$-\frac{cx^2+b}{10\,b^3}\sqrt{x}\left(42\,A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\text{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)x^2bc-21\,A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(c*x^4+b*x^2)^(3/2)/x^(1/2),x)
```

```
[Out] -1/10/(c*x^4+b*x^2)^(3/2)*x^(1/2)*(c*x^2+b)*(42*A*((c*x+(-b*c)^(1/2))/(-b*c
)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c
)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2
))*x^2*b*c-21*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*
c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b
*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b*c-30*B*((c*x+(-b*c)^(1/2)
)/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*
c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2
*2^(1/2))*x^2*b^2+15*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c
*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((
c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^2-42*A*c^2*x^4+30*
B*x^4*b*c-28*A*b*c*x^2+20*B*x^2*b^2+4*A*b^2)/b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{Bx^2+A}{\left(cx^4+bx^2\right)^{\frac{3}{2}}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^(3/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/((c*x^4 + b*x^2)^(3/2)*sqrt(x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\left(Bx^2+A\right)\sqrt{x}}{c^2x^9+2\,bcx^7+b^2x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^(3/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*sqrt(x)/(c^2*x^9 + 2*b*c*x^7 + b^2
*x^5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2)**(3/2)/x**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\left(cx^4 + bx^2\right)^{\frac{3}{2}}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^(3/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/((c*x^4 + b*x^2)^(3/2)*sqrt(x)), x)
```

3.267 $\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=203

$$-\frac{5c^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(7bB-9Ac)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{42b^{13/4}\sqrt{bx^2+cx^4}}-\frac{5\sqrt{bx^2+cx^4}(7bB-9Ac)}{21b^3x^{5/2}}+\frac{7bB-9A}{7b^2\sqrt{x}\sqrt{bx^2}}$$

```
[Out] (-2*A)/(7*b*x^(5/2)*Sqrt[b*x^2 + c*x^4]) + (7*b*B - 9*A*c)/(7*b^2*Sqrt[x]*S
qrt[b*x^2 + c*x^4]) - (5*(7*b*B - 9*A*c)*Sqrt[b*x^2 + c*x^4])/(21*b^3*x^(5/
2)) - (5*c^(3/4)*(7*b*B - 9*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(
Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2]
)/(42*b^(13/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.327939, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2038, 2023, 2025, 2032, 329, 220}

$$-\frac{5c^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(7bB-9Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{42b^{13/4}\sqrt{bx^2+cx^4}}-\frac{5\sqrt{bx^2+cx^4}(7bB-9Ac)}{21b^3x^{5/2}}+\frac{7bB-9Ac}{7b^2\sqrt{x}\sqrt{bx^2+cx^4}}-$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(3/2)*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-2*A)/(7*b*x^(5/2)*Sqrt[b*x^2 + c*x^4]) + (7*b*B - 9*A*c)/(7*b^2*Sqrt[x]*S
qrt[b*x^2 + c*x^4]) - (5*(7*b*B - 9*A*c)*Sqrt[b*x^2 + c*x^4])/(21*b^3*x^(5/
2)) - (5*c^(3/4)*(7*b*B - 9*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(
Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2]
)/(42*b^(13/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
```

```
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{2A}{7bx^{5/2}\sqrt{bx^2+cx^4}} - \frac{\left(2\left(-\frac{7bB}{2}+\frac{9Ac}{2}\right)\right)\int \frac{\sqrt{x}}{\left(bx^2+cx^4\right)^{3/2}}\,dx}{7b}$$

$$= -\frac{2A}{7bx^{5/2}\sqrt{bx^2+cx^4}} + \frac{7bB-9Ac}{7b^2\sqrt{x}\sqrt{bx^2+cx^4}} + \frac{(5(7bB-9Ac))\int \frac{1}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx}{14b^2}$$

$$= -\frac{2A}{7bx^{5/2}\sqrt{bx^2+cx^4}} + \frac{7bB-9Ac}{7b^2\sqrt{x}\sqrt{bx^2+cx^4}} - \frac{5(7bB-9Ac)\sqrt{bx^2+cx^4}}{21b^3x^{5/2}} - \frac{(5c(7bB-9Ac))\int \ldots}{42b^3}$$

$$= -\frac{2A}{7bx^{5/2}\sqrt{bx^2+cx^4}} + \frac{7bB-9Ac}{7b^2\sqrt{x}\sqrt{bx^2+cx^4}} - \frac{5(7bB-9Ac)\sqrt{bx^2+cx^4}}{21b^3x^{5/2}} - \frac{\left(5c(7bB-9Ac)x\sqrt{b\ldots}\right.}{42b^3\sqrt{b\ldots}}$$

$$= -\frac{2A}{7bx^{5/2}\sqrt{bx^2+cx^4}} + \frac{7bB-9Ac}{7b^2\sqrt{x}\sqrt{bx^2+cx^4}} - \frac{5(7bB-9Ac)\sqrt{bx^2+cx^4}}{21b^3x^{5/2}} - \frac{\left(5c(7bB-9Ac)x\sqrt{b\ldots}\right.}{2\ldots}$$

$$= -\frac{2A}{7bx^{5/2}\sqrt{bx^2+cx^4}} + \frac{7bB-9Ac}{7b^2\sqrt{x}\sqrt{bx^2+cx^4}} - \frac{5(7bB-9Ac)\sqrt{bx^2+cx^4}}{21b^3x^{5/2}} - \frac{5c^{3/4}(7bB-9Ac)x\left(\ldots\right.}{\ldots}$$

Mathematica [C] time = 0.0457401, size = 79, normalized size = 0.39

$$\frac{2x^2\sqrt{\frac{cx^2}{b}+1}(9Ac-7bB)\,{}_2F_1\left(-\frac{3}{4},\frac{3}{2};\frac{1}{4};-\frac{cx^2}{b}\right)-6Ab}{21b^2x^{5/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(3/2)*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-6*A*b + 2*(-7*b*B + 9*A*c)*x^2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-3/4
, 3/2, 1/4, -((c*x^2)/b)])/(21*b^2*x^(5/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.02, size = 254, normalized size = 1.3

$$\frac{cx^2+b}{42\,b^3}\left(45\,A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)\sqrt{-bc}x^3c-35\,B\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/42/(c*x^4+b*x^2)^(3/2)/x^(1/2)*(c*x^2+b)*(45*A*((c*x+(-b*c)^(1/2))/(-b*c)
^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)
^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2)
)*(-b*c)^(1/2)*x^3*c-35*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*(
(-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF
(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(-b*c)^(1/2)*x^3*b+90
*A*c^2*x^4-70*B*x^4*b*c+36*A*b*c*x^2-28*B*x^2*b^2-12*A*b^2)/b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\left(cx^4 + bx^2\right)^{\frac{3}{2}} x^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/((c*x^4 + b*x^2)^(3/2)*x^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2}\left(Bx^2 + A\right)\sqrt{x}}{c^2x^{10} + 2\,bcx^8 + b^2x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*sqrt(x)/(c^2*x^10 + 2*b*c*x^8 + b^
2*x^6), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(3/2)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\left(cx^4 + bx^2\right)^{\frac{3}{2}} x^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/((c*x^4 + b*x^2)^(3/2)*x^(3/2)), x)
```

3.268 $\int \frac{A+Bx^2}{x^{5/2}\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=405

$$-\frac{7c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(9bB-11Ac)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{30b^{15/4}\sqrt{bx^2+cx^4}}-\frac{7c^{3/2}x^{3/2}\left(b+cx^2\right)(9bB-11Ac)}{15b^4\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{7c^{5/4}x\left(\right.}{}$$

```
[Out] (-2*A)/(9*b*x^(7/2)*Sqrt[b*x^2 + c*x^4]) + (9*b*B - 11*A*c)/(9*b^2*x^(3/2)*
Sqrt[b*x^2 + c*x^4]) - (7*c^(3/2)*(9*b*B - 11*A*c)*x^(3/2)*(b + c*x^2))/(15
*b^4*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) - (7*(9*b*B - 11*A*c)*Sqrt[
b*x^2 + c*x^4])/(45*b^3*x^(7/2)) + (7*c*(9*b*B - 11*A*c)*Sqrt[b*x^2 + c*x^4
])/(15*b^4*x^(3/2)) + (7*c^(5/4)*(9*b*B - 11*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*S
qrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x
])/b^(1/4)], 1/2])/(15*b^(15/4)*Sqrt[b*x^2 + c*x^4]) - (7*c^(5/4)*(9*b*B -
11*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*E
llipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(30*b^(15/4)*Sqrt[b*x^2
 + c*x^4])
```

Rubi [A] time = 0.522016, antiderivative size = 405, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2038, 2023, 2025, 2032, 329, 305, 220, 1196}

$$-\frac{7c^{3/2}x^{3/2}\left(b+cx^2\right)(9bB-11Ac)}{15b^4\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{7c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(9bB-11Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{30b^{15/4}\sqrt{bx^2+cx^4}}+\frac{7c^{5/4}x\left(\sqrt{b}+\sqrt{c}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(5/2)*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-2*A)/(9*b*x^(7/2)*Sqrt[b*x^2 + c*x^4]) + (9*b*B - 11*A*c)/(9*b^2*x^(3/2)*
Sqrt[b*x^2 + c*x^4]) - (7*c^(3/2)*(9*b*B - 11*A*c)*x^(3/2)*(b + c*x^2))/(15
*b^4*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) - (7*(9*b*B - 11*A*c)*Sqrt[
b*x^2 + c*x^4])/(45*b^3*x^(7/2)) + (7*c*(9*b*B - 11*A*c)*Sqrt[b*x^2 + c*x^4
])/(15*b^4*x^(3/2)) + (7*c^(5/4)*(9*b*B - 11*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*S
qrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x
])/b^(1/4)], 1/2])/(15*b^(15/4)*Sqrt[b*x^2 + c*x^4]) - (7*c^(5/4)*(9*b*B -
11*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*E
```

```
llipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(30*b^(15/4)*Sqrt[b*x^2
 + c*x^4])
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^{5/2}\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{2A}{9bx^{7/2}\sqrt{bx^2+cx^4}} - \frac{\left(2\left(-\frac{9bB}{2}+\frac{11Ac}{2}\right)\right)\int \frac{1}{\sqrt{x}\left(bx^2+cx^4\right)^{3/2}}\,dx}{9b}$$

$$= -\frac{2A}{9bx^{7/2}\sqrt{bx^2+cx^4}} + \frac{9bB-11Ac}{9b^2x^{3/2}\sqrt{bx^2+cx^4}} + \frac{(7(9bB-11Ac))\int \frac{1}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx}{18b^2}$$

$$= -\frac{2A}{9bx^{7/2}\sqrt{bx^2+cx^4}} + \frac{9bB-11Ac}{9b^2x^{3/2}\sqrt{bx^2+cx^4}} - \frac{7(9bB-11Ac)\sqrt{bx^2+cx^4}}{45b^3x^{7/2}} - \frac{(7c(9bB-11Ac))\int}{30b^3}$$

$$= -\frac{2A}{9bx^{7/2}\sqrt{bx^2+cx^4}} + \frac{9bB-11Ac}{9b^2x^{3/2}\sqrt{bx^2+cx^4}} - \frac{7(9bB-11Ac)\sqrt{bx^2+cx^4}}{45b^3x^{7/2}} + \frac{7c(9bB-11Ac)\sqrt{b}}{15b^4x^{3/2}}$$

$$= -\frac{2A}{9bx^{7/2}\sqrt{bx^2+cx^4}} + \frac{9bB-11Ac}{9b^2x^{3/2}\sqrt{bx^2+cx^4}} - \frac{7(9bB-11Ac)\sqrt{bx^2+cx^4}}{45b^3x^{7/2}} + \frac{7c(9bB-11Ac)\sqrt{b}}{15b^4x^{3/2}}$$

$$= -\frac{2A}{9bx^{7/2}\sqrt{bx^2+cx^4}} + \frac{9bB-11Ac}{9b^2x^{3/2}\sqrt{bx^2+cx^4}} - \frac{7(9bB-11Ac)\sqrt{bx^2+cx^4}}{45b^3x^{7/2}} + \frac{7c(9bB-11Ac)\sqrt{b}}{15b^4x^{3/2}}$$

$$= -\frac{2A}{9bx^{7/2}\sqrt{bx^2+cx^4}} + \frac{9bB-11Ac}{9b^2x^{3/2}\sqrt{bx^2+cx^4}} - \frac{7(9bB-11Ac)\sqrt{bx^2+cx^4}}{45b^3x^{7/2}} + \frac{7c(9bB-11Ac)\sqrt{b}}{15b^4x^{3/2}}$$

$$= -\frac{2A}{9bx^{7/2}\sqrt{bx^2+cx^4}} + \frac{9bB-11Ac}{9b^2x^{3/2}\sqrt{bx^2+cx^4}} - \frac{7c^{3/2}(9bB-11Ac)x^{3/2}\left(b+cx^2\right)}{15b^4\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{7(9bB-11Ac}{45b^3}$$

Mathematica [C] time = 0.0447571, size = 79, normalized size = 0.2

$$\frac{2x^2\sqrt{\frac{cx^2}{b}+1}(11Ac-9bB)\,{}_2F_1\left(-\frac{5}{4},\frac{3}{2};-\frac{1}{4};-\frac{cx^2}{b}\right)-10Ab}{45b^2x^{7/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(5/2)*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-10*A*b + 2*(-9*b*B + 11*A*c)*x^2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-5
/4, 3/2, -1/4, -((c*x^2)/b)])/(45*b^2*x^(7/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.022, size = 450, normalized size = 1.1

$$\frac{cx^2+b}{90\,b^4}\left(462\,A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\,\sqrt{2}\right)x^4bc^2-231\,A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/90/(c*x^4+b*x^2)^(3/2)/x^(3/2)*(c*x^2+b)*(462*A*((c*x+(-b*c)^(1/2))/(-b*c
)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c
)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2
))*x^4*b*c^2-231*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(
-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+
(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^4*b*c^2-378*B*((c*x+(-b*c)
^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2
)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2),1/2*2^(1/2))*x^4*b^2*c+189*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(
1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*Ell
ipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^4*b^2*c-462*A
*c^3*x^6+378*B*x^6*b*c^2-308*A*b*c^2*x^4+252*B*x^4*b^2*c+44*A*b^2*c*x^2-36*
B*x^2*b^3-20*A*b^3)/b^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2+A}{\left(cx^4+bx^2\right)^{\frac{3}{2}}x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/((c*x^4 + b*x^2)^(3/2)*x^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{cx^4+bx^2}\left(Bx^2+A\right)\sqrt{x}}{c^2x^{11}+2\,bcx^9+b^2x^7},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*sqrt(x)/(c^2*x^11 + 2*b*c*x^9 + b^
2*x^7), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(5/2)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\left(cx^4 + bx^2\right)^{\frac{3}{2}} x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/((c*x^4 + b*x^2)^(3/2)*x^(5/2)), x)
```

3.269 $\int x^m \left(A+Bx^2\right) \left(bx^2+cx^4\right)^3 \, dx$

Optimal. Leaf size=96

$$\frac{b^2x^{m+9}(3Ac+bB)}{m+9}+\frac{Ab^3x^{m+7}}{m+7}+\frac{c^2x^{m+13}(Ac+3bB)}{m+13}+\frac{3bcx^{m+11}(Ac+bB)}{m+11}+\frac{Bc^3x^{m+15}}{m+15}$$

```
[Out] (A*b^3*x^(7 + m))/(7 + m) + (b^2*(b*B + 3*A*c)*x^(9 + m))/(9 + m) + (3*b*c*
(b*B + A*c)*x^(11 + m))/(11 + m) + (c^2*(3*b*B + A*c)*x^(13 + m))/(13 + m)
+ (B*c^3*x^(15 + m))/(15 + m)
```

Rubi [A] time = 0.0699623, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1584, 448}

$$\frac{b^2x^{m+9}(3Ac+bB)}{m+9}+\frac{Ab^3x^{m+7}}{m+7}+\frac{c^2x^{m+13}(Ac+3bB)}{m+13}+\frac{3bcx^{m+11}(Ac+bB)}{m+11}+\frac{Bc^3x^{m+15}}{m+15}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(A + B*x^2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (A*b^3*x^(7 + m))/(7 + m) + (b^2*(b*B + 3*A*c)*x^(9 + m))/(9 + m) + (3*b*c*
(b*B + A*c)*x^(11 + m))/(11 + m) + (c^2*(3*b*B + A*c)*x^(13 + m))/(13 + m)
+ (B*c^3*x^(15 + m))/(15 + m)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int x^m \left(A+Bx^2\right)\left(bx^2+cx^4\right)^3\,dx = \int x^{6+m}\left(A+Bx^2\right)\left(b+cx^2\right)^3\,dx$$
$$= \int \left(Ab^3x^{6+m} + b^2(bB+3Ac)x^{8+m} + 3bc(bB+Ac)x^{10+m} + c^2(3bB+Ac)x^{12+m} + Bc^3\right.$$
$$= \frac{Ab^3x^{7+m}}{7+m} + \frac{b^2(bB+3Ac)x^{9+m}}{9+m} + \frac{3bc(bB+Ac)x^{11+m}}{11+m} + \frac{c^2(3bB+Ac)x^{13+m}}{13+m} + \frac{Bc^3x}{15}$$

Mathematica [A] time = 0.091216, size = 89, normalized size = 0.93

$$x^{m+7}\left(\frac{b^2x^2(3Ac+bB)}{m+9} + \frac{Ab^3}{m+7} + \frac{c^2x^6(Ac+3bB)}{m+13} + \frac{3bcx^4(Ac+bB)}{m+11} + \frac{Bc^3x^8}{m+15}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(A + B*x^2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] x^(7 + m)*((A*b^3)/(7 + m) + (b^2*(b*B + 3*A*c)*x^2)/(9 + m) + (3*b*c*(b*B
+ A*c)*x^4)/(11 + m) + (c^2*(3*b*B + A*c)*x^6)/(13 + m) + (B*c^3*x^8)/(15 +
 m))
```

Maple [B] time = 0.051, size = 474, normalized size = 4.9

$$\frac{x^{7+m}\left(Bc^3m^4x^8 + 40\,Bc^3m^3x^8 + Ac^3m^4x^6 + 3\,Bbc^2m^4x^6 + 590\,Bc^3m^2x^8 + 42\,Ac^3m^3x^6 + 126\,Bbc^2m^3x^6 + 3800\,Bc^3mx^8\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(B*x^2+A)*(c*x^4+b*x^2)^3,x)
```

```
[Out] x^(7+m)*(B*c^3*m^4*x^8+40*B*c^3*m^3*x^8+A*c^3*m^4*x^6+3*B*b*c^2*m^4*x^6+590
*B*c^3*m^2*x^8+42*A*c^3*m^3*x^6+126*B*b*c^2*m^3*x^6+3800*B*c^3*m*x^8+3*A*b*
c^2*m^4*x^4+644*A*c^3*m^2*x^6+3*B*b^2*c*m^4*x^4+1932*B*b*c^2*m^2*x^6+9009*B
*c^3*x^8+132*A*b*c^2*m^3*x^4+4278*A*c^3*m*x^6+132*B*b^2*c*m^3*x^4+12834*B*b
*c^2*m*x^6+3*A*b^2*c*m^4*x^2+2118*A*b*c^2*m^2*x^4+10395*A*c^3*x^6+B*b^3*m^4
*x^2+2118*B*b^2*c*m^2*x^4+31185*B*b*c^2*x^6+138*A*b^2*c*m^3*x^2+14652*A*b*c
^2*m*x^4+46*B*b^3*m^3*x^2+14652*B*b^2*c*m*x^4+A*b^3*m^4+2328*A*b^2*c*m^2*x^
2+36855*A*b*c^2*x^4+776*B*b^3*m^2*x^2+36855*B*b^2*c*x^4+48*A*b^3*m^3+16998*
A*b^2*c*m*x^2+5666*B*b^3*m*x^2+854*A*b^3*m^2+45045*A*b^2*c*x^2+15015*B*b^3*
```

```
x^2+6672*A*b^3*m+19305*A*b^3)/(15+m)/(13+m)/(11+m)/(9+m)/(7+m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 2.29343, size = 910, normalized size = 9.48

$$\frac{\left(\left(Bc^3m^4 + 40\,Bc^3m^3 + 590\,Bc^3m^2 + 3800\,Bc^3m + 9009\,Bc^3\right)x^{15} + \left(\left(3\,Bbc^2 + Ac^3\right)m^4 + 31185\,Bbc^2 + 10395\,Ac^3 + 42\,(3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="fricas")

[Out] ((B*c^3*m^4 + 40*B*c^3*m^3 + 590*B*c^3*m^2 + 3800*B*c^3*m + 9009*B*c^3)*x^1
5 + ((3*B*b*c^2 + A*c^3)*m^4 + 31185*B*b*c^2 + 10395*A*c^3 + 42*(3*B*b*c^2
+ A*c^3)*m^3 + 644*(3*B*b*c^2 + A*c^3)*m^2 + 4278*(3*B*b*c^2 + A*c^3)*m)*x^
13 + 3*((B*b^2*c + A*b*c^2)*m^4 + 12285*B*b^2*c + 12285*A*b*c^2 + 44*(B*b^2
*c + A*b*c^2)*m^3 + 706*(B*b^2*c + A*b*c^2)*m^2 + 4884*(B*b^2*c + A*b*c^2)*
m)*x^11 + ((B*b^3 + 3*A*b^2*c)*m^4 + 15015*B*b^3 + 45045*A*b^2*c + 46*(B*b^
3 + 3*A*b^2*c)*m^3 + 776*(B*b^3 + 3*A*b^2*c)*m^2 + 5666*(B*b^3 + 3*A*b^2*c)
*m)*x^9 + (A*b^3*m^4 + 48*A*b^3*m^3 + 854*A*b^3*m^2 + 6672*A*b^3*m + 19305*
A*b^3)*x^7)*x^m/(m^5 + 55*m^4 + 1190*m^3 + 12650*m^2 + 66009*m + 135135)
```

Sympy [A] time = 11.2351, size = 2077, normalized size = 21.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(B*x**2+A)*(c*x**4+b*x**2)**3,x)
```

```
[Out] Piecewise((-A*b**3/(8*x**8) - A*b**2*c/(2*x**6) - 3*A*b*c**2/(4*x**4) - A*c
**3/(2*x**2) - B*b**3/(6*x**6) - 3*B*b**2*c/(4*x**4) - 3*B*b*c**2/(2*x**2)
+ B*c**3*log(x), Eq(m, -15)), (-A*b**3/(6*x**6) - 3*A*b**2*c/(4*x**4) - 3*A
*b*c**2/(2*x**2) + A*c**3*log(x) - B*b**3/(4*x**4) - 3*B*b**2*c/(2*x**2) +
3*B*b*c**2*log(x) + B*c**3*x**2/2, Eq(m, -13)), (-A*b**3/(4*x**4) - 3*A*b**
2*c/(2*x**2) + 3*A*b*c**2*log(x) + A*c**3*x**2/2 - B*b**3/(2*x**2) + 3*B*b*
*2*c*log(x) + 3*B*b*c**2*x**2/2 + B*c**3*x**4/4, Eq(m, -11)), (-A*b**3/(2*x
**2) + 3*A*b**2*c*log(x) + 3*A*b*c**2*x**2/2 + A*c**3*x**4/4 + B*b**3*log(x
) + 3*B*b**2*c*x**2/2 + 3*B*b*c**2*x**4/4 + B*c**3*x**6/6, Eq(m, -9)), (A*b
**3*log(x) + 3*A*b**2*c*x**2/2 + 3*A*b*c**2*x**4/4 + A*c**3*x**6/6 + B*b**3
*x**2/2 + 3*B*b**2*c*x**4/4 + B*b*c**2*x**6/2 + B*c**3*x**8/8, Eq(m, -7)),
(A*b**3*m**4*x**7*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m +
 135135) + 48*A*b**3*m**3*x**7*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**
2 + 66009*m + 135135) + 854*A*b**3*m**2*x**7*x**m/(m**5 + 55*m**4 + 1190*m*
*3 + 12650*m**2 + 66009*m + 135135) + 6672*A*b**3*m*x**7*x**m/(m**5 + 55*m*
*4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + 19305*A*b**3*x**7*x**m/(m
**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + 3*A*b**2*c*m**
4*x**9*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) +
138*A*b**2*c*m**3*x**9*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 6600
9*m + 135135) + 2328*A*b**2*c*m**2*x**9*x**m/(m**5 + 55*m**4 + 1190*m**3 +
12650*m**2 + 66009*m + 135135) + 16998*A*b**2*c*m*x**9*x**m/(m**5 + 55*m**4
 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + 45045*A*b**2*c*x**9*x**m/(m
**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + 3*A*b*c**2*m**
4*x**11*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) +
 132*A*b*c**2*m**3*x**11*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66
009*m + 135135) + 2118*A*b*c**2*m**2*x**11*x**m/(m**5 + 55*m**4 + 1190*m**3
 + 12650*m**2 + 66009*m + 135135) + 14652*A*b*c**2*m*x**11*x**m/(m**5 + 55*
m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + 36855*A*b*c**2*x**11*x*
*m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + A*c**3*m*
*4*x**13*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135)
+ 42*A*c**3*m**3*x**13*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 6600
9*m + 135135) + 644*A*c**3*m**2*x**13*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12
650*m**2 + 66009*m + 135135) + 4278*A*c**3*m*x**13*x**m/(m**5 + 55*m**4 + 1
190*m**3 + 12650*m**2 + 66009*m + 135135) + 10395*A*c**3*x**13*x**m/(m**5 +
 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + B*b**3*m**4*x**9*x*
*m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + 46*B*b**3
*m**3*x**9*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135
) + 776*B*b**3*m**2*x**9*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66
009*m + 135135) + 5666*B*b**3*m*x**9*x**m/(m**5 + 55*m**4 + 1190*m**3 + 126
50*m**2 + 66009*m + 135135) + 15015*B*b**3*x**9*x**m/(m**5 + 55*m**4 + 1190
*m**3 + 12650*m**2 + 66009*m + 135135) + 3*B*b**2*c*m**4*x**11*x**m/(m**5 +
 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + 132*B*b**2*c*m**3*x
```

```
**11*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + 21
18*B*b**2*c*m**2*x**11*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 6600
9*m + 135135) + 14652*B*b**2*c*m*x**11*x**m/(m**5 + 55*m**4 + 1190*m**3 + 1
2650*m**2 + 66009*m + 135135) + 36855*B*b**2*c*x**11*x**m/(m**5 + 55*m**4 +
 1190*m**3 + 12650*m**2 + 66009*m + 135135) + 3*B*b*c**2*m**4*x**13*x**m/(m
**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + 126*B*b*c**2*m
**3*x**13*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135)
 + 1932*B*b*c**2*m**2*x**13*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 +
 66009*m + 135135) + 12834*B*b*c**2*m*x**13*x**m/(m**5 + 55*m**4 + 1190*m**
3 + 12650*m**2 + 66009*m + 135135) + 31185*B*b*c**2*x**13*x**m/(m**5 + 55*m
**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + B*c**3*m**4*x**15*x**m/(
m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + 40*B*c**3*m**
3*x**15*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) +
 590*B*c**3*m**2*x**15*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 6600
9*m + 135135) + 3800*B*c**3*m*x**15*x**m/(m**5 + 55*m**4 + 1190*m**3 + 1265
0*m**2 + 66009*m + 135135) + 9009*B*c**3*x**15*x**m/(m**5 + 55*m**4 + 1190*
m**3 + 12650*m**2 + 66009*m + 135135), True))
```

Giac [B] time = 1.29304, size = 814, normalized size = 8.48

$$\frac{Bc^3m^4x^{15}x^m + 40\,Bc^3m^3x^{15}x^m + 3\,Bbc^2m^4x^{13}x^m + Ac^3m^4x^{13}x^m + 590\,Bc^3m^2x^{15}x^m + 126\,Bbc^2m^3x^{13}x^m + 42\,Ac^3m^3x^{13}x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] (B*c^3*m^4*x^15*x^m + 40*B*c^3*m^3*x^15*x^m + 3*B*b*c^2*m^4*x^13*x^m + A*c^
3*m^4*x^13*x^m + 590*B*c^3*m^2*x^15*x^m + 126*B*b*c^2*m^3*x^13*x^m + 42*A*c
^3*m^3*x^13*x^m + 3800*B*c^3*m*x^15*x^m + 3*B*b^2*c*m^4*x^11*x^m + 3*A*b*c^
2*m^4*x^11*x^m + 1932*B*b*c^2*m^2*x^13*x^m + 644*A*c^3*m^2*x^13*x^m + 9009*
B*c^3*x^15*x^m + 132*B*b^2*c*m^3*x^11*x^m + 132*A*b*c^2*m^3*x^11*x^m + 1283
4*B*b*c^2*m*x^13*x^m + 4278*A*c^3*m*x^13*x^m + B*b^3*m^4*x^9*x^m + 3*A*b^2*
c*m^4*x^9*x^m + 2118*B*b^2*c*m^2*x^11*x^m + 2118*A*b*c^2*m^2*x^11*x^m + 311
85*B*b*c^2*x^13*x^m + 10395*A*c^3*x^13*x^m + 46*B*b^3*m^3*x^9*x^m + 138*A*b
^2*c*m^3*x^9*x^m + 14652*B*b^2*c*m*x^11*x^m + 14652*A*b*c^2*m*x^11*x^m + A*
b^3*m^4*x^7*x^m + 776*B*b^3*m^2*x^9*x^m + 2328*A*b^2*c*m^2*x^9*x^m + 36855*
B*b^2*c*x^11*x^m + 36855*A*b*c^2*x^11*x^m + 48*A*b^3*m^3*x^7*x^m + 5666*B*b
^3*m*x^9*x^m + 16998*A*b^2*c*m*x^9*x^m + 854*A*b^3*m^2*x^7*x^m + 15015*B*b^
3*x^9*x^m + 45045*A*b^2*c*x^9*x^m + 6672*A*b^3*m*x^7*x^m + 19305*A*b^3*x^7*
x^m)/(m^5 + 55*m^4 + 1190*m^3 + 12650*m^2 + 66009*m + 135135)
```

3.270 $\int x^m \left(A+Bx^2\right)\left(bx^2+cx^4\right)^2\,dx$

Optimal. Leaf size=71

$$\frac{Ab^2x^{m+5}}{m+5}+\frac{bx^{m+7}(2Ac+bB)}{m+7}+\frac{cx^{m+9}(Ac+2bB)}{m+9}+\frac{Bc^2x^{m+11}}{m+11}$$

```
[Out] (A*b^2*x^(5 + m))/(5 + m) + (b*(b*B + 2*A*c)*x^(7 + m))/(7 + m) + (c*(2*b*B
 + A*c)*x^(9 + m))/(9 + m) + (B*c^2*x^(11 + m))/(11 + m)
```

Rubi [A] time = 0.0519871, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1584, 448}

$$\frac{Ab^2x^{m+5}}{m+5}+\frac{bx^{m+7}(2Ac+bB)}{m+7}+\frac{cx^{m+9}(Ac+2bB)}{m+9}+\frac{Bc^2x^{m+11}}{m+11}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (A*b^2*x^(5 + m))/(5 + m) + (b*(b*B + 2*A*c)*x^(7 + m))/(7 + m) + (c*(2*b*B
 + A*c)*x^(9 + m))/(9 + m) + (B*c^2*x^(11 + m))/(11 + m)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int x^m \left(A+Bx^2\right)\left(bx^2+cx^4\right)^2 dx = \int x^{4+m}\left(A+Bx^2\right)\left(b+cx^2\right)^2 dx$$
$$= \int \left(Ab^2x^{4+m}+b(bB+2Ac)x^{6+m}+c(2bB+Ac)x^{8+m}+Bc^2x^{10+m}\right) dx$$
$$= \frac{Ab^2x^{5+m}}{5+m}+\frac{b(bB+2Ac)x^{7+m}}{7+m}+\frac{c(2bB+Ac)x^{9+m}}{9+m}+\frac{Bc^2x^{11+m}}{11+m}$$

Mathematica [A] time = 0.0571561, size = 66, normalized size = 0.93

$$x^{m+5}\left(\frac{Ab^2}{m+5}+\frac{cx^4(Ac+2bB)}{m+9}+\frac{bx^2(2Ac+bB)}{m+7}+\frac{Bc^2x^6}{m+11}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] x^(5 + m)*((A*b^2)/(5 + m) + (b*(b*B + 2*A*c)*x^2)/(7 + m) + (c*(2*b*B + A*
c)*x^4)/(9 + m) + (B*c^2*x^6)/(11 + m))
```

Maple [B] time = 0.006, size = 262, normalized size = 3.7

$$x^{5+m}\left(Bc^2m^3x^6+21\,Bc^2m^2x^6+Ac^2m^3x^4+2\,Bbcm^3x^4+143\,Bc^2mx^6+23\,Ac^2m^2x^4+46\,Bbcm^2x^4+315\,Bc^2x^6+2\,Abcm\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(B*x^2+A)*(c*x^4+b*x^2)^2,x)
```

```
[Out] x^(5+m)*(B*c^2*m^3*x^6+21*B*c^2*m^2*x^6+A*c^2*m^3*x^4+2*B*b*c*m^3*x^4+143*B
*c^2*m*x^6+23*A*c^2*m^2*x^4+46*B*b*c*m^2*x^4+315*B*c^2*x^6+2*A*b*c*m^3*x^2+
167*A*c^2*m*x^4+B*b^2*m^3*x^2+334*B*b*c*m*x^4+50*A*b*c*m^2*x^2+385*A*c^2*x^
4+25*B*b^2*m^2*x^2+770*B*b*c*x^4+A*b^2*m^3+398*A*b*c*m*x^2+199*B*b^2*m*x^2+
27*A*b^2*m^2+990*A*b*c*x^2+495*B*b^2*x^2+239*A*b^2*m+693*A*b^2)/(11+m)/(9+m
)/(7+m)/(5+m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.30448, size = 512, normalized size = 7.21

$$\frac{\left(\left(Bc^2m^3 + 21\,Bc^2m^2 + 143\,Bc^2m + 315\,Bc^2\right)x^{11} + \left(\left(2\,Bbc + Ac^2\right)m^3 + 770\,Bbc + 385\,Ac^2 + 23\,\left(2\,Bbc + Ac^2\right)m^2 + 167\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] ((B*c^2*m^3 + 21*B*c^2*m^2 + 143*B*c^2*m + 315*B*c^2)*x^11 + ((2*B*b*c + A*
c^2)*m^3 + 770*B*b*c + 385*A*c^2 + 23*(2*B*b*c + A*c^2)*m^2 + 167*(2*B*b*c
+ A*c^2)*m)*x^9 + ((B*b^2 + 2*A*b*c)*m^3 + 495*B*b^2 + 990*A*b*c + 25*(B*b^
2 + 2*A*b*c)*m^2 + 199*(B*b^2 + 2*A*b*c)*m)*x^7 + (A*b^2*m^3 + 27*A*b^2*m^2
 + 239*A*b^2*m + 693*A*b^2)*x^5)*x^m/(m^4 + 32*m^3 + 374*m^2 + 1888*m + 346
5)
```

Sympy [A] time = 5.04163, size = 1051, normalized size = 14.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(B*x**2+A)*(c*x**4+b*x**2)**2,x)
```

```
[Out] Piecewise((-A*b**2/(6*x**6) - A*b*c/(2*x**4) - A*c**2/(2*x**2) - B*b**2/(4*
x**4) - B*b*c/x**2 + B*c**2*log(x), Eq(m, -11)), (-A*b**2/(4*x**4) - A*b*c/
x**2 + A*c**2*log(x) - B*b**2/(2*x**2) + 2*B*b*c*log(x) + B*c**2*x**2/2, Eq
(m, -9)), (-A*b**2/(2*x**2) + 2*A*b*c*log(x) + A*c**2*x**2/2 + B*b**2*log(x
) + B*b*c*x**2 + B*c**2*x**4/4, Eq(m, -7)), (A*b**2*log(x) + A*b*c*x**2 + A
*c**2*x**4/4 + B*b**2*x**2/2 + B*b*c*x**4/2 + B*c**2*x**6/6, Eq(m, -5)), (A
*b**2*m**3*x**5*x**m/(m**4 + 32*m**3 + 374*m**2 + 1888*m + 3465) + 27*A*b**
2*m**2*x**5*x**m/(m**4 + 32*m**3 + 374*m**2 + 1888*m + 3465) + 239*A*b**2*m
*x**5*x**m/(m**4 + 32*m**3 + 374*m**2 + 1888*m + 3465) + 693*A*b**2*x**5*x*
```

```
*m/(m**4 + 32*m**3 + 374*m**2 + 1888*m + 3465) + 2*A*b*c*m**3*x**7*x**m/(m*
*4 + 32*m**3 + 374*m**2 + 1888*m + 3465) + 50*A*b*c*m**2*x**7*x**m/(m**4 +
32*m**3 + 374*m**2 + 1888*m + 3465) + 398*A*b*c*m*x**7*x**m/(m**4 + 32*m**3
 + 374*m**2 + 1888*m + 3465) + 990*A*b*c*x**7*x**m/(m**4 + 32*m**3 + 374*m*
*2 + 1888*m + 3465) + A*c**2*m**3*x**9*x**m/(m**4 + 32*m**3 + 374*m**2 + 18
88*m + 3465) + 23*A*c**2*m**2*x**9*x**m/(m**4 + 32*m**3 + 374*m**2 + 1888*m
 + 3465) + 167*A*c**2*m*x**9*x**m/(m**4 + 32*m**3 + 374*m**2 + 1888*m + 346
5) + 385*A*c**2*x**9*x**m/(m**4 + 32*m**3 + 374*m**2 + 1888*m + 3465) + B*b
**2*m**3*x**7*x**m/(m**4 + 32*m**3 + 374*m**2 + 1888*m + 3465) + 25*B*b**2*
m**2*x**7*x**m/(m**4 + 32*m**3 + 374*m**2 + 1888*m + 3465) + 199*B*b**2*m*x
**7*x**m/(m**4 + 32*m**3 + 374*m**2 + 1888*m + 3465) + 495*B*b**2*x**7*x**m
/(m**4 + 32*m**3 + 374*m**2 + 1888*m + 3465) + 2*B*b*c*m**3*x**9*x**m/(m**4
 + 32*m**3 + 374*m**2 + 1888*m + 3465) + 46*B*b*c*m**2*x**9*x**m/(m**4 + 32
*m**3 + 374*m**2 + 1888*m + 3465) + 334*B*b*c*m*x**9*x**m/(m**4 + 32*m**3 +
 374*m**2 + 1888*m + 3465) + 770*B*b*c*x**9*x**m/(m**4 + 32*m**3 + 374*m**2
 + 1888*m + 3465) + B*c**2*m**3*x**11*x**m/(m**4 + 32*m**3 + 374*m**2 + 188
8*m + 3465) + 21*B*c**2*m**2*x**11*x**m/(m**4 + 32*m**3 + 374*m**2 + 1888*m
 + 3465) + 143*B*c**2*m*x**11*x**m/(m**4 + 32*m**3 + 374*m**2 + 1888*m + 34
65) + 315*B*c**2*x**11*x**m/(m**4 + 32*m**3 + 374*m**2 + 1888*m + 3465), Tr
ue))
```

Giac [B] time = 1.34679, size = 459, normalized size = 6.46

$$\frac{Bc^2m^3x^{11}x^m + 21\,Bc^2m^2x^{11}x^m + 2\,Bbcm^3x^9x^m + Ac^2m^3x^9x^m + 143\,Bc^2mx^{11}x^m + 46\,Bbcm^2x^9x^m + 23\,Ac^2m^2x^9x^m + 315}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] (B*c^2*m^3*x^11*x^m + 21*B*c^2*m^2*x^11*x^m + 2*B*b*c*m^3*x^9*x^m + A*c^2*m
^3*x^9*x^m + 143*B*c^2*m*x^11*x^m + 46*B*b*c*m^2*x^9*x^m + 23*A*c^2*m^2*x^9
*x^m + 315*B*c^2*x^11*x^m + B*b^2*m^3*x^7*x^m + 2*A*b*c*m^3*x^7*x^m + 334*B
*b*c*m*x^9*x^m + 167*A*c^2*m*x^9*x^m + 25*B*b^2*m^2*x^7*x^m + 50*A*b*c*m^2*
x^7*x^m + 770*B*b*c*x^9*x^m + 385*A*c^2*x^9*x^m + A*b^2*m^3*x^5*x^m + 199*B
*b^2*m*x^7*x^m + 398*A*b*c*m*x^7*x^m + 27*A*b^2*m^2*x^5*x^m + 495*B*b^2*x^7
*x^m + 990*A*b*c*x^7*x^m + 239*A*b^2*m*x^5*x^m + 693*A*b^2*x^5*x^m)/(m^4 +
32*m^3 + 374*m^2 + 1888*m + 3465)
```

3.271 $\int x^m \left(A+Bx^2\right) \left(bx^2+cx^4\right) dx$

Optimal. Leaf size=45

$$\frac{x^{m+5}(Ac+bB)}{m+5}+\frac{Abx^{m+3}}{m+3}+\frac{Bcx^{m+7}}{m+7}$$

```
[Out] (A*b*x^(3 + m))/(3 + m) + ((b*B + A*c)*x^(5 + m))/(5 + m) + (B*c*x^(7 + m))
/(7 + m)
```

Rubi [A] time = 0.0298884, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1584, 448}

$$\frac{x^{m+5}(Ac+bB)}{m+5}+\frac{Abx^{m+3}}{m+3}+\frac{Bcx^{m+7}}{m+7}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(A + B*x^2)*(b*x^2 + c*x^4),x]
```

```
[Out] (A*b*x^(3 + m))/(3 + m) + ((b*B + A*c)*x^(5 + m))/(5 + m) + (B*c*x^(7 + m))
/(7 + m)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int x^m \left(A+Bx^2\right)\left(bx^2+cx^4\right)dx = \int x^{2+m}\left(A+Bx^2\right)\left(b+cx^2\right)dx$$
$$= \int \left(Abx^{2+m}+(bB+Ac)x^{4+m}+Bcx^{6+m}\right)dx$$
$$= \frac{Abx^{3+m}}{3+m}+\frac{(bB+Ac)x^{5+m}}{5+m}+\frac{Bcx^{7+m}}{7+m}$$

Mathematica [A] time = 0.0399325, size = 42, normalized size = 0.93

$$x^{m+3}\left(\frac{x^2(Ac+bB)}{m+5}+\frac{Ab}{m+3}+\frac{Bcx^4}{m+7}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(A + B*x^2)*(b*x^2 + c*x^4),x]
```

```
[Out] x^(3 + m)*((A*b)/(3 + m) + ((b*B + A*c)*x^2)/(5 + m) + (B*c*x^4)/(7 + m))
```

Maple [B] time = 0.003, size = 110, normalized size = 2.4

$$\frac{x^{3+m}\left(Bcm^2x^4+8\,Bcmx^4+Acm^2x^2+Bbm^2x^2+15\,Bcx^4+10\,Acmx^2+10\,Bbmx^2+Abm^2+21\,Ax^2c+21\,Bx^2b+12\,Ab\right.}{(7+m)\,(5+m)\,(3+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(B*x^2+A)*(c*x^4+b*x^2),x)
```

```
[Out] x^(3+m)*(B*c*m^2*x^4+8*B*c*m*x^4+A*c*m^2*x^2+B*b*m^2*x^2+15*B*c*x^4+10*A*c*
m*x^2+10*B*b*m*x^2+A*b*m^2+21*A*c*x^2+21*B*b*x^2+12*A*b*m+35*A*b)/(7+m)/(5+
m)/(3+m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.31435, size = 223, normalized size = 4.96

$$\frac{\left(\left(Bcm^2+8\,Bcm+15\,Bc\right)x^7+\left((Bb+Ac)m^2+21\,Bb+21\,Ac+10\,(Bb+Ac)m\right)x^5+\left(Abm^2+12\,Abm+35\,Ab\right)x^3\right)x^m}{m^3+15\,m^2+71\,m+105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] ((B*c*m^2 + 8*B*c*m + 15*B*c)*x^7 + ((B*b + A*c)*m^2 + 21*B*b + 21*A*c + 10
*(B*b + A*c)*m)*x^5 + (A*b*m^2 + 12*A*b*m + 35*A*b)*x^3)*x^m/(m^3 + 15*m^2
+ 71*m + 105)
```

Sympy [A] time = 1.8105, size = 415, normalized size = 9.22

$$\begin{cases} -\frac{Ab}{4x^4}-\frac{Ac}{2x^2}-\frac{Bb}{2x^2}+Bc\log{(x)} \\ -\frac{Ab}{2x^2}+Ac\log{(x)}+Bb\log{(x)}+\frac{Bcx^2}{2} \\ Ab\log{(x)}+\frac{Acx^2}{2}+\frac{Bbx^2}{2}+\frac{Bcx^4}{4} \\ \frac{Abm^2x^3x^m}{m^3+15m^2+71m+105}+\frac{12Abmx^3x^m}{m^3+15m^2+71m+105}+\frac{35Abx^3x^m}{m^3+15m^2+71m+105}+\frac{Acm^2x^5x^m}{m^3+15m^2+71m+105}+\frac{10Acmx^5x^m}{m^3+15m^2+71m+105}+\frac{21Acx^5x^m}{m^3+15m^2+71m+105}+\frac{}{m^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(B*x**2+A)*(c*x**4+b*x**2),x)
```

```
[Out] Piecewise((-A*b/(4*x**4) - A*c/(2*x**2) - B*b/(2*x**2) + B*c*log(x), Eq(m,
-7)), (-A*b/(2*x**2) + A*c*log(x) + B*b*log(x) + B*c*x**2/2, Eq(m, -5)), (A
*b*log(x) + A*c*x**2/2 + B*b*x**2/2 + B*c*x**4/4, Eq(m, -3)), (A*b*m**2*x**
3*x**m/(m**3 + 15*m**2 + 71*m + 105) + 12*A*b*m*x**3*x**m/(m**3 + 15*m**2 +
 71*m + 105) + 35*A*b*x**3*x**m/(m**3 + 15*m**2 + 71*m + 105) + A*c*m**2*x*
*5*x**m/(m**3 + 15*m**2 + 71*m + 105) + 10*A*c*m*x**5*x**m/(m**3 + 15*m**2
+ 71*m + 105) + 21*A*c*x**5*x**m/(m**3 + 15*m**2 + 71*m + 105) + B*b*m**2*x
**5*x**m/(m**3 + 15*m**2 + 71*m + 105) + 10*B*b*m*x**5*x**m/(m**3 + 15*m**2
 + 71*m + 105) + 21*B*b*x**5*x**m/(m**3 + 15*m**2 + 71*m + 105) + B*c*m**2*
```

```
x**7*x**m/(m**3 + 15*m**2 + 71*m + 105) + 8*B*c*m*x**7*x**m/(m**3 + 15*m**2
 + 71*m + 105) + 15*B*c*x**7*x**m/(m**3 + 15*m**2 + 71*m + 105), True))
```

Giac [B] time = 1.40694, size = 201, normalized size = 4.47

$$\frac{Bcm^2x^7x^m + 8\,Bcmx^7x^m + Bbm^2x^5x^m + Acm^2x^5x^m + 15\,Bcx^7x^m + 10\,Bbmx^5x^m + 10\,Acmx^5x^m + Abm^2x^3x^m + 21\,Bbx^5x}{m^3 + 15\,m^2 + 71\,m + 105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] (B*c*m^2*x^7*x^m + 8*B*c*m*x^7*x^m + B*b*m^2*x^5*x^m + A*c*m^2*x^5*x^m + 15
*B*c*x^7*x^m + 10*B*b*m*x^5*x^m + 10*A*c*m*x^5*x^m + A*b*m^2*x^3*x^m + 21*B
*b*x^5*x^m + 21*A*c*x^5*x^m + 12*A*b*m*x^3*x^m + 35*A*b*x^3*x^m)/(m^3 + 15*
m^2 + 71*m + 105)
```

3.272 $\int \frac{x^m(A+Bx^2)}{bx^2+cx^4}\,dx$

Optimal. Leaf size=71

$$\frac{x^{m-1}(bB-Ac)\,{}_2F_1\left(1,\frac{m-1}{2};\frac{m+1}{2};-\frac{cx^2}{b}\right)}{bc(1-m)}-\frac{Bx^{m-1}}{c(1-m)}$$

```
[Out] -((B*x^(-1 + m))/(c*(1 - m))) + ((b*B - A*c)*x^(-1 + m)*Hypergeometric2F1[1
, (-1 + m)/2, (1 + m)/2, -((c*x^2)/b)])/(b*c*(1 - m))
```

Rubi [A] time = 0.0470955, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 459, 364}

$$\frac{x^{m-1}(bB-Ac)\,{}_2F_1\left(1,\frac{m-1}{2};\frac{m+1}{2};-\frac{cx^2}{b}\right)}{bc(1-m)}-\frac{Bx^{m-1}}{c(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] -((B*x^(-1 + m))/(c*(1 - m))) + ((b*B - A*c)*x^(-1 + m)*Hypergeometric2F1[1
, (-1 + m)/2, (1 + m)/2, -((c*x^2)/b)])/(b*c*(1 - m))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^m\left(A+Bx^2\right)}{bx^2+cx^4}\,dx &= \int \frac{x^{-2+m}\left(A+Bx^2\right)}{b+cx^2}\,dx\\ &= -\frac{Bx^{-1+m}}{c(1-m)}-\frac{(bB(-1+m)-Ac(-1+m))\int \frac{x^{-2+m}}{b+cx^2}\,dx}{c(-1+m)}\\ &= -\frac{Bx^{-1+m}}{c(1-m)}+\frac{(bB-Ac)x^{-1+m}\,{}_2F_1\left(1,\frac{1}{2}(-1+m);\frac{1+m}{2};-\frac{cx^2}{b}\right)}{bc(1-m)}\end{aligned}$$

Mathematica [A] time = 0.0615062, size = 55, normalized size = 0.77

$$\frac{x^{m-1}\left((Ac-bB)\,{}_2F_1\left(1,\frac{m-1}{2};\frac{m+1}{2};-\frac{cx^2}{b}\right)+bB\right)}{bc(m-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (x^(-1 + m)*(b*B + (-(b*B) + A*c)*Hypergeometric2F1[1, (-1 + m)/2, (1 + m)/
2, -((c*x^2)/b)]))/(b*c*(-1 + m))
```

Maple [F] time = 0.234, size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2+A\right)x^m}{cx^4+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(B*x^2+A)/(c*x^4+b*x^2),x)
```

```
[Out] int(x^m*(B*x^2+A)/(c*x^4+b*x^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)x^m}{cx^4 + bx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^m/(c*x^4 + b*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(Bx^2 + A)x^m}{cx^4 + bx^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)*x^m/(c*x^4 + b*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m\left(A + Bx^2\right)}{x^2\left(b + cx^2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] Integral(x**m*(A + B*x**2)/(x**2*(b + c*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)x^m}{cx^4 + bx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^m/(c*x^4 + b*x^2), x)
```

3.273 $\int \frac{x^m (A+Bx^2)}{(bx^2+cx^4)^2} dx$

Optimal. Leaf size=98

$$\frac{x^{m-3}(bB(3-m)-Ac(5-m))\,{}_2F_1\left(1,\frac{m-3}{2};\frac{m-1}{2};-\frac{cx^2}{b}\right)}{2b^2c(3-m)}-\frac{x^{m-3}(bB-Ac)}{2bc\left(b+cx^2\right)}$$

```
[Out] -((b*B - A*c)*x^(-3 + m))/(2*b*c*(b + c*x^2)) + ((b*B*(3 - m) - A*c*(5 - m)
)*x^(-3 + m)*Hypergeometric2F1[1, (-3 + m)/2, (-1 + m)/2, -((c*x^2)/b)])/(2
*b^2*c*(3 - m))
```

Rubi [A] time = 0.0595799, antiderivative size = 92, normalized size of antiderivative = 0.94, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 457, 364}

$$\frac{x^{m-3}\left(\frac{bB}{c}-\frac{A(5-m)}{3-m}\right)\,{}_2F_1\left(1,\frac{m-3}{2};\frac{m-1}{2};-\frac{cx^2}{b}\right)}{2b^2}-\frac{x^{m-3}(bB-Ac)}{2bc\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -((b*B - A*c)*x^(-3 + m))/(2*b*c*(b + c*x^2)) + (((b*B)/c - (A*(5 - m))/(3
- m))*x^(-3 + m)*Hypergeometric2F1[1, (-3 + m)/2, (-1 + m)/2, -((c*x^2)/b)]
)/(2*b^2)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
```

```
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^m\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx &= \int \frac{x^{-4+m}\left(A+Bx^2\right)}{\left(b+cx^2\right)^2}\,dx \\
&= -\frac{(bB-Ac)x^{-3+m}}{2bc\left(b+cx^2\right)}+\frac{(-Ac(-5+m)+bB(-3+m))\int \frac{x^{-4+m}}{b+cx^2}\,dx}{2bc} \\
&= -\frac{(bB-Ac)x^{-3+m}}{2bc\left(b+cx^2\right)}+\frac{\left(\frac{bB}{c}-\frac{A(5-m)}{3-m}\right)x^{-3+m}\,{}_2F_1\left(1,\frac{1}{2}(-3+m);\frac{1}{2}(-1+m);-\frac{cx^2}{b}\right)}{2b^2}
\end{aligned}$$

Mathematica [A] time = 0.0749948, size = 80, normalized size = 0.82

$$\frac{x^{m-3}\left((Ac-bB)\,{}_2F_1\left(2,\frac{m-3}{2};\frac{m-1}{2};-\frac{cx^2}{b}\right)+bB\,{}_2F_1\left(1,\frac{m-3}{2};\frac{m-1}{2};-\frac{cx^2}{b}\right)\right)}{b^2c(m-3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (x^(-3 + m)*(b*B*Hypergeometric2F1[1, (-3 + m)/2, (-1 + m)/2, -((c*x^2)/b)]
 + (-(b*B) + A*c)*Hypergeometric2F1[2, (-3 + m)/2, (-1 + m)/2, -((c*x^2)/b)
]))/(b^2*c*(-3 + m))
```

Maple [F] time = 0.235, size = 0, normalized size = 0.

$$\int \frac{\left(B x^{2}+A\right) x^{m}}{\left(c x^{4}+b x^{2}\right)^{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(B*x^2+A)/(c*x^4+b*x^2)^2,x)
```

```
[Out] int(x^m*(B*x^2+A)/(c*x^4+b*x^2)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B x^{2}+A\right)x^{m}}{\left(c x^{4}+b x^{2}\right)^{2}}\,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^m/(c*x^4 + b*x^2)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(B x^{2}+A\right)x^{m}}{c^{2} x^{8}+2 \, b c x^{6}+b^{2} x^{4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)*x^m/(c^2*x^8 + 2*b*c*x^6 + b^2*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m\left(A+Bx^2\right)}{x^4\left(b+cx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Integral(x**m*(A + B*x**2)/(x**4*(b + c*x**2)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2+A\right)x^m}{\left(cx^4+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^m/(c*x^4 + b*x^2)^2, x)
```

3.274 $\int x^m \left(A+Bx^2\right)\left(bx^2+cx^4\right)^p\,dx$

Optimal. Leaf size=140

$$\frac{Bx^{m-1}\left(bx^2+cx^4\right)^{p+1}}{c(m+4p+3)}-\frac{x^{m+1}\left(\frac{cx^2}{b}+1\right)^{-p}\left(bx^2+cx^4\right)^p(bB(m+2p+1)-Ac(m+4p+3))\,{}_2F_1\left(-p,\frac{1}{2}(m+2p+1);\frac{1}{2}(m\right.}{c(m+2p+1)(m+4p+3)}$$

```
[Out] (B*x^(-1 + m)*(b*x^2 + c*x^4)^(1 + p))/(c*(3 + m + 4*p)) - ((b*B*(1 + m + 2
*p) - A*c*(3 + m + 4*p))*x^(1 + m)*(b*x^2 + c*x^4)^p*Hypergeometric2F1[-p,
(1 + m + 2*p)/2, (3 + m + 2*p)/2, -((c*x^2)/b)])/(c*(1 + m + 2*p)*(3 + m +
4*p)*(1 + (c*x^2)/b)^p)
```

Rubi [A] time = 0.136641, antiderivative size = 126, normalized size of antiderivative = 0.9, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2039, 2032, 365, 364}

$$x^{m+1}\left(\frac{cx^2}{b}+1\right)^{-p}\left(bx^2+cx^4\right)^p\left(\frac{A}{m+2p+1}-\frac{bB}{c(m+4p+3)}\right){}_2F_1\left(-p,\frac{1}{2}(m+2p+1);\frac{1}{2}(m+2p+3);-\frac{cx^2}{b}\right)+\frac{Bx^{m-1}\left(\right.}{c(m}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(A + B*x^2)*(b*x^2 + c*x^4)^p,x]
```

```
[Out] (B*x^(-1 + m)*(b*x^2 + c*x^4)^(1 + p))/(c*(3 + m + 4*p)) + ((A/(1 + m + 2*p
) - (b*B)/(c*(3 + m + 4*p)))*x^(1 + m)*(b*x^2 + c*x^4)^p*Hypergeometric2F1[
-p, (1 + m + 2*p)/2, (3 + m + 2*p)/2, -((c*x^2)/b)])/(1 + (c*x^2)/b)^p
```

Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
```

```
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^m\left(A+Bx^2\right)\left(bx^2+cx^4\right)^p\,dx &= \frac{Bx^{-1+m}\left(bx^2+cx^4\right)^{1+p}}{c(3+m+4p)}-\left(-A+\frac{bB(1+m+2p)}{c(3+m+4p)}\right)\int x^m\left(bx^2+cx^4\right)^p\,dx\\ &= \frac{Bx^{-1+m}\left(bx^2+cx^4\right)^{1+p}}{c(3+m+4p)}-\left(\left(-A+\frac{bB(1+m+2p)}{c(3+m+4p)}\right)x^{-2p}\left(b+cx^2\right)^{-p}\left(bx^2+cx^4\right)^p\right)\int x\\ &= \frac{Bx^{-1+m}\left(bx^2+cx^4\right)^{1+p}}{c(3+m+4p)}-\left(\left(-A+\frac{bB(1+m+2p)}{c(3+m+4p)}\right)x^{-2p}\left(1+\frac{cx^2}{b}\right)^{-p}\left(bx^2+cx^4\right)^p\right)\int\\ &= \frac{Bx^{-1+m}\left(bx^2+cx^4\right)^{1+p}}{c(3+m+4p)}+\left(\frac{A}{1+m+2p}-\frac{bB}{c(3+m+4p)}\right)x^{1+m}\left(1+\frac{cx^2}{b}\right)^{-p}\left(bx^2+cx^4\right.\end{aligned}$$

Mathematica [A] time = 0.0978766, size = 135, normalized size = 0.96

$$\frac{x^{m+1}\left(x^2\left(b+cx^2\right)\right)^p\left(\frac{cx^2}{b}+1\right)^{-p}\left(A(m+2p+3)\,{}_2F_1\left(-p,\frac{1}{2}(m+2p+1);\frac{1}{2}(m+2p+3);-\frac{cx^2}{b}\right)+Bx^2(m+2p+1)\,{}_2F_1\left(-p\right.\right.}{(m+2p+1)(m+2p+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(A + B*x^2)*(b*x^2 + c*x^4)^p,x]
```

```
[Out] (x^(1 + m)*(x^2*(b + c*x^2))^p*(A*(3 + m + 2*p)*Hypergeometric2F1[-p, (1 +
m + 2*p)/2, (3 + m + 2*p)/2, -((c*x^2)/b)] + B*(1 + m + 2*p)*x^2*Hypergeome
tric2F1[-p, (3 + m + 2*p)/2, (5 + m + 2*p)/2, -((c*x^2)/b)]))/((1 + m + 2*p
)*(3 + m + 2*p)*(1 + (c*x^2)/b)^p)
```

Maple [F] time = 0.313, size = 0, normalized size = 0.

$$\int x^{m} \left(B x^{2}+A \right) \left(c \,x^{4}+b \,x^{2}\right)^{p}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(B*x^2+A)*(c*x^4+b*x^2)^p,x)
```

```
[Out] int(x^m*(B*x^2+A)*(c*x^4+b*x^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Bx^2 + A\right)\left(cx^4 + bx^2\right)^p x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*(c*x^4 + b*x^2)^p*x^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Bx^2 + A\right)\left(cx^4 + bx^2\right)^p x^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2)^p,x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)*(c*x^4 + b*x^2)^p*x^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(B*x**2+A)*(c*x**4+b*x**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Bx^2+A\right)\left(cx^4+bx^2\right)^p x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2)^p,x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*(c*x^4 + b*x^2)^p*x^m, x)
```

3.275 $\int x^{-1+n-jp}(c+dx^n)\left(ax^j+bx^{j+n}\right)^p\,dx$

Optimal. Leaf size=95

$$\frac{dx^{n-j(p+1)}\left(ax^j+bx^{j+n}\right)^{p+1}}{bn(p+2)}-\frac{x^{-j(p+1)}(ad-bc(p+2))\left(ax^j+bx^{j+n}\right)^{p+1}}{b^2n(p+1)(p+2)}$$

```
[Out] -(((a*d - b*c*(2 + p))*(a*x^j + b*x^(j + n))^(1 + p))/(b^2*n*(1 + p)*(2 + p
)*x^(j*(1 + p)))) + (d*x^(n - j*(1 + p))*(a*x^j + b*x^(j + n))^(1 + p))/(b*
n*(2 + p))
```

Rubi [A] time = 0.156137, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2039, 2014}

$$\frac{dx^{n-j(p+1)}\left(ax^j+bx^{j+n}\right)^{p+1}}{bn(p+2)}-\frac{x^{-j(p+1)}(ad-bc(p+2))\left(ax^j+bx^{j+n}\right)^{p+1}}{b^2n(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n - j*p)*(c + d*x^n)*(a*x^j + b*x^(j + n))^p,x]
```

```
[Out] -(((a*d - b*c*(2 + p))*(a*x^j + b*x^(j + n))^(1 + p))/(b^2*n*(1 + p)*(2 + p
)*x^(j*(1 + p)))) + (d*x^(n - j*(1 + p))*(a*x^j + b*x^(j + n))^(1 + p))/(b*
n*(2 + p))
```

Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
```

```
j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int x^{-1+n-jp}(c+dx^n)\left(ax^j+bx^{j+n}\right)^p\,dx=\frac{dx^{n-j(1+p)}\left(ax^j+bx^{j+n}\right)^{1+p}}{bn(2+p)}-\left(-c+\frac{ad}{b(2+p)}\right)\int x^{-1+n-jp}\left(ax^j+bx^{j+n}\right)^p\,dx$$
$$=\frac{\left(c-\frac{ad}{b(2+p)}\right)x^{-j(1+p)}\left(ax^j+bx^{j+n}\right)^{1+p}}{bn(1+p)}+\frac{dx^{n-j(1+p)}\left(ax^j+bx^{j+n}\right)^{1+p}}{bn(2+p)}$$

Mathematica [A] time = 0.0685855, size = 63, normalized size = 0.66

$$\frac{x^{-jp}(a+bx^n)\left(x^j(a+bx^n)\right)^p\left(-ad+bc(p+2)+bd(p+1)x^n\right)}{b^2n(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n - j*p)*(c + d*x^n)*(a*x^j + b*x^(j + n))^p,x]
```

```
[Out] ((a + b*x^n)*(x^j*(a + b*x^n))^p*(-(a*d) + b*c*(2 + p) + b*d*(1 + p)*x^n))/
(b^2*n*(1 + p)*(2 + p)*x^(j*p))
```

Maple [F] time = 0.547, size = 0, normalized size = 0.

$$\int x^{-jp+n-1}(c+dx^n)\left(ax^j+bx^{j+n}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-j*p+n-1)*(c+d*x^n)*(a*x^j+b*x^(j+n))^p,x)
```

```
[Out] int(x^(-j*p+n-1)*(c+d*x^n)*(a*x^j+b*x^(j+n))^p,x)
```

Maxima [A] time = 1.47314, size = 151, normalized size = 1.59

$$\frac{(bx^n+a)ce^{\left(-jp\log(x)+p\log(bx^n+a)+p\log\left(x^j\right)\right)}}{bn\left(p+1\right)}+\frac{\left(b^2\left(p+1\right)x^{2n}+abpx^n-a^2\right)de^{\left(-jp\log(x)+p\log(bx^n+a)+p\log\left(x^j\right)\right)}}{\left(p^2+3p+2\right)b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-j*p+n-1)*(c+d*x^n)*(a*x^j+b*x^(j+n))^p,x, algorithm="maxima")
```

```
[Out] (b*x^n + a)*c*e^(-j*p*log(x) + p*log(b*x^n + a) + p*log(x^j))/(b*n*(p + 1))
 + (b^2*(p + 1)*x^(2*n) + a*b*p*x^n - a^2)*d*e^(-j*p*log(x) + p*log(b*x^n +
 a) + p*log(x^j))/((p^2 + 3*p + 2)*b^2*n)
```

Fricas [A] time = 2.4282, size = 300, normalized size = 3.16

$$\frac{\left(\left(b^2dp+b^2d\right)xx^{-jp+n-1}x^{2\,n}+\left(2\,b^2c+\left(b^2c+abd\right)p\right)xx^{-jp+n-1}x^n+\left(abcp+2\,abc-a^2d\right)xx^{-jp+n-1}\right)\left(\frac{(bx^n+a)x^{j+n}}{x^n}\right)^p}{\left(b^2np^2+3\,b^2np+2\,b^2n\right)x^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-j*p+n-1)*(c+d*x^n)*(a*x^j+b*x^(j+n))^p,x, algorithm="fricas")
```

```
[Out] ((b^2*d*p + b^2*d)*x*x^(-j*p + n - 1)*x^(2*n) + (2*b^2*c + (b^2*c + a*b*d)*
p)*x*x^(-j*p + n - 1)*x^n + (a*b*c*p + 2*a*b*c - a^2*d)*x*x^(-j*p + n - 1))
*((b*x^n + a)*x^(j + n)/x^n)^p/((b^2*n*p^2 + 3*b^2*n*p + 2*b^2*n)*x^n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-j*p+n-1)*(c+d*x**n)*(a*x**j+b*x**(j+n))**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx^n+c)\left(bx^{j+n}+ax^j\right)^px^{-jp+n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-j*p+n-1)*(c+d*x^n)*(a*x^j+b*x^(j+n))^p,x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)*(b*x^(j + n) + a*x^j)^p*x^(-j*p + n - 1), x)
```

3.276 $\int (ex)^m (c+dx^n)^q \left(ax^j+bx^{j+n}\right)^p \, dx$

Optimal. Leaf size=113

$$\frac{x(ex)^m \left(\frac{bx^n}{a}+1\right)^{-p} (c+dx^n)^q \left(\frac{dx^n}{c}+1\right)^{-q} \left(ax^j+bx^{j+n}\right)^p F_1\left(\frac{m+jp+1}{n};-p,-q;\frac{m+n+jp+1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{jp+m+1}$$

```
[Out] (x*(e*x)^m*(c + d*x^n)^q*(a*x^j + b*x^(j + n))^p*AppellF1[(1 + m + j*p)/n,
-p, -q, (1 + m + n + j*p)/n, -((b*x^n)/a), -((d*x^n)/c)])/((1 + m + j*p)*(1
 + (b*x^n)/a)^p*(1 + (d*x^n)/c)^q)
```

Rubi [A] time = 0.223829, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2042, 511, 510}

$$\frac{x(ex)^m \left(\frac{bx^n}{a}+1\right)^{-p} (c+dx^n)^q \left(\frac{dx^n}{c}+1\right)^{-q} \left(ax^j+bx^{j+n}\right)^p F_1\left(\frac{m+jp+1}{n};-p,-q;\frac{m+n+jp+1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{jp+m+1}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*(c + d*x^n)^q*(a*x^j + b*x^(j + n))^p,x]
```

```
[Out] (x*(e*x)^m*(c + d*x^n)^q*(a*x^j + b*x^(j + n))^p*AppellF1[(1 + m + j*p)/n,
-p, -q, (1 + m + n + j*p)/n, -((b*x^n)/a), -((d*x^n)/c)])/((1 + m + j*p)*(1
 + (b*x^n)/a)^p*(1 + (d*x^n)/c)^q)
```

Rule 2042

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[(e^IntPart[m]*(e*x)^FracPart[m]*
(a*x^j + b*x^(j + n))^FracPart[p])/(x^(FracPart[m] + j*FracPart[p])*(a + b*
x^n)^FracPart[p]), Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; F
reeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
&& NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
```

```
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int (ex)^m (c+dx^n)^q \left(ax^j+bx^{j+n}\right)^p\,dx = \left(x^{-m-jp}(ex)^m (a+bx^n)^{-p}\left(ax^j+bx^{j+n}\right)^p\right)\int x^{m+jp}(a+bx^n)^p (c+dx^n)^q\,dx$$

$$= \left(x^{-m-jp}(ex)^m\left(1+\frac{bx^n}{a}\right)^{-p}\left(ax^j+bx^{j+n}\right)^p\right)\int x^{m+jp}\left(1+\frac{bx^n}{a}\right)^p (c+dx^n)^q\,dx$$

$$= \left(x^{-m-jp}(ex)^m\left(1+\frac{bx^n}{a}\right)^{-p}(c+dx^n)^q\left(1+\frac{dx^n}{c}\right)^{-q}\left(ax^j+bx^{j+n}\right)^p\right)\int x^{m+jp}\left(1+\frac{bx^n}{a}\right.$$

$$= \frac{x(ex)^m\left(1+\frac{bx^n}{a}\right)^{-p}(c+dx^n)^q\left(1+\frac{dx^n}{c}\right)^{-q}\left(ax^j+bx^{j+n}\right)^p F_1\left(\frac{1+m+jp}{n};-p,-q;\frac{1+m+n+}{n}\right.}{1+m+jp}$$

Mathematica [A] time = 0.16384, size = 111, normalized size = 0.98

$$\frac{x(ex)^m\left(\frac{bx^n}{a}+1\right)^{-p}(c+dx^n)^q\left(\frac{dx^n}{c}+1\right)^{-q}\left(x^j(a+bx^n)\right)^p F_1\left(\frac{m+jp+1}{n};-p,-q;\frac{m+n+jp+1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{jp+m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*(c + d*x^n)^q*(a*x^j + b*x^(j + n))^p,x]
```

```
[Out] (x*(e*x)^m*(x^j*(a + b*x^n))^p*(c + d*x^n)^q*AppellF1[(1 + m + j*p)/n, -p,
-q, (1 + m + n + j*p)/n, -((b*x^n)/a), -((d*x^n)/c)])/((1 + m + j*p)*(1 + (
b*x^n)/a)^p*(1 + (d*x^n)/c)^q)
```

Maple [F] time = 2.108, size = 0, normalized size = 0.

$$\int (ex)^m (c+dx^n)^q \left(ax^j+bx^{j+n}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(c+d*x^n)^q*(a*x^j+b*x^(j+n))^p,x)
```

```
[Out] int((e*x)^m*(c+d*x^n)^q*(a*x^j+b*x^(j+n))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^{j+n}+ax^{j}\right)^{p}(dx^{n}+c)^{q}\,(ex)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(c+d*x^n)^q*(a*x^j+b*x^(j+n))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^(j + n) + a*x^j)^p*(d*x^n + c)^q*(e*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(bx^{j+n}+ax^{j}\right)^{p}(dx^{n}+c)^{q}\,(ex)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(c+d*x^n)^q*(a*x^j+b*x^(j+n))^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^(j + n) + a*x^j)^p*(d*x^n + c)^q*(e*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(c+d*x**n)**q*(a*x**j+b*x**(j+n))**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^{j+n} + ax^{j}\right)^{p} (dx^{n} + c)^{q} (ex)^{m} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(c+d*x^n)^q*(a*x^j+b*x^(j+n))^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^(j + n) + a*x^j)^p*(d*x^n + c)^q*(e*x)^m, x)
```

3.277 $\int (ex)^{7/4} (c + dx^n)^q \left(ax^j + bx^{j+n}\right)^{5/3} dx$

Optimal. Leaf size=129

$$\frac{12ae(ex)^{3/4}x^{j+2}\left(ax^j + bx^{j+n}\right)^{2/3}(c + dx^n)^q\left(\frac{dx^n}{c}+1\right)^{-q}F_1\left(\frac{20j+33}{12n};-\frac{5}{3},-q;\frac{20j+12n+33}{12n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{(20j+33)\left(\frac{bx^n}{a}+1\right)^{2/3}}$$

```
[Out] (12*a*e*x^(2 + j)*(e*x)^(3/4)*(c + d*x^n)^q*(a*x^j + b*x^(j + n))^(2/3)*App
ellF1[(33 + 20*j)/(12*n), -5/3, -q, (33 + 20*j + 12*n)/(12*n), -((b*x^n)/a)
, -((d*x^n)/c)])/((33 + 20*j)*(1 + (b*x^n)/a)^(2/3)*(1 + (d*x^n)/c)^q)
```

Rubi [A] time = 0.356796, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {2042, 511, 510}

$$\frac{12ae(ex)^{3/4}x^{j+2}\left(ax^j + bx^{j+n}\right)^{2/3}(c + dx^n)^q\left(\frac{dx^n}{c}+1\right)^{-q}F_1\left(\frac{20j+33}{12n};-\frac{5}{3},-q;\frac{20j+12n+33}{12n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{(20j+33)\left(\frac{bx^n}{a}+1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(7/4)*(c + d*x^n)^q*(a*x^j + b*x^(j + n))^(5/3),x]
```



```
[Out] (12*a*e*x^(2 + j)*(e*x)^(3/4)*(c + d*x^n)^q*(a*x^j + b*x^(j + n))^(2/3)*App
ellF1[(33 + 20*j)/(12*n), -5/3, -q, (33 + 20*j + 12*n)/(12*n), -((b*x^n)/a)
, -((d*x^n)/c)])/((33 + 20*j)*(1 + (b*x^n)/a)^(2/3)*(1 + (d*x^n)/c)^q)
```


Rule 2042

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[(e^IntPart[m]*(e*x)^FracPart[m]*
(a*x^j + b*x^(j + n))^FracPart[p])/(x^(FracPart[m] + j*FracPart[p])*(a + b*
x^n)^FracPart[p]), Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; F
reeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
&& NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
```

```
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int (ex)^{7/4}(c+dx^n)^q\left(ax^j+bx^{j+n}\right)^{5/3}\,dx = \frac{\left(ex^{-\frac{3}{4}-\frac{2j}{3}}(ex)^{3/4}\left(ax^j+bx^{j+n}\right)^{2/3}\right)\int x^{\frac{7}{4}+\frac{5j}{3}}(a+bx^n)^{5/3}(c+dx^n)^q\,dx}{(a+bx^n)^{2/3}}$$

$$= \frac{\left(aex^{-\frac{3}{4}-\frac{2j}{3}}(ex)^{3/4}\left(ax^j+bx^{j+n}\right)^{2/3}\right)\int x^{\frac{7}{4}+\frac{5j}{3}}\left(1+\frac{bx^n}{a}\right)^{5/3}(c+dx^n)^q\,dx}{\left(1+\frac{bx^n}{a}\right)^{2/3}}$$

$$= \frac{\left(aex^{-\frac{3}{4}-\frac{2j}{3}}(ex)^{3/4}(c+dx^n)^q\left(1+\frac{dx^n}{c}\right)^{-q}\left(ax^j+bx^{j+n}\right)^{2/3}\right)\int x^{\frac{7}{4}+\frac{5j}{3}}\left(1+\frac{bx^n}{a}\right)^{5/3}\left(1 \cdots\right.}{\left(1+\frac{bx^n}{a}\right)^{2/3}}$$

$$= \frac{12aex^{2+j}(ex)^{3/4}(c+dx^n)^q\left(1+\frac{dx^n}{c}\right)^{-q}\left(ax^j+bx^{j+n}\right)^{2/3}F_1\left(\frac{33+20j}{12n};-\frac{5}{3},-q;\frac{33+20j+1\cdots}{12n}\right.}{(33+20j)\left(1+\frac{bx^n}{a}\right)^{2/3}}$$

Mathematica [A] time = 0.211557, size = 210, normalized size = 1.63

$$\frac{12(ex)^{7/4}x^{j+1}\left(x^j(a+bx^n)\right)^{2/3}(c+dx^n)^q\left(\frac{dx^n}{c}+1\right)^{-q}\left(b(20j+33)x^nF_1\left(\frac{20j+12n+33}{12n};-\frac{2}{3},-q;\frac{20j+24n+33}{12n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)+a(20j+\cdots\right.}{(20j+33)(20j+12n+33)\left(\frac{bx^n}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*x)^(7/4)*(c + d*x^n)^q*(a*x^j + b*x^(j + n))^(5/3),x]
```

```
[Out] (12*x^(1 + j)*(e*x)^(7/4)*(x^j*(a + b*x^n))^(2/3)*(c + d*x^n)^q*(a*(33 + 20
*j + 12*n)*AppellF1[(33 + 20*j)/(12*n), -2/3, -q, (11/4 + (5*j)/3 + n)/n, -
((b*x^n)/a), -((d*x^n)/c)] + b*(33 + 20*j)*x^n*AppellF1[(33 + 20*j + 12*n)/
(12*n), -2/3, -q, (33 + 20*j + 24*n)/(12*n), -((b*x^n)/a), -((d*x^n)/c)]))/
((33 + 20*j)*(33 + 20*j + 12*n)*(1 + (b*x^n)/a)^(2/3)*(1 + (d*x^n)/c)^q)
```

Maple [F] time = 0.823, size = 0, normalized size = 0.

$$\int (ex)^{\frac{7}{4}} (c+dx^n)^q \left(ax^j+bx^{j+n}\right)^{\frac{5}{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/4)*(c+d*x^n)^q*(a*x^j+b*x^(j+n))^(5/3),x)
```

```
[Out] int((e*x)^(7/4)*(c+d*x^n)^q*(a*x^j+b*x^(j+n))^(5/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^{j+n}+ax^j\right)^{\frac{5}{3}} (ex)^{\frac{7}{4}} (dx^n+c)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/4)*(c+d*x^n)^q*(a*x^j+b*x^(j+n))^(5/3),x, algorithm="max
ima")
```

```
[Out] integrate((b*x^(j + n) + a*x^j)^(5/3)*(e*x)^(7/4)*(d*x^n + c)^q, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bexx^{j+n}+aexx^j\right)\left(bx^{j+n}+ax^j\right)^{\frac{2}{3}} (ex)^{\frac{3}{4}} (dx^n+c)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/4)*(c+d*x^n)^q*(a*x^j+b*x^(j+n))^(5/3),x, algorithm="fri
cas")
```

```
[Out] integral((b*e*x*x^(j + n) + a*e*x*x^j)*(b*x^(j + n) + a*x^j)^(2/3)*(e*x)^(3
/4)*(d*x^n + c)^q, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/4)*(c+d*x**n)**q*(a*x**j+b*x**(j+n))**(5/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^{j+n} + ax^j\right)^{\frac{5}{3}} (ex)^{\frac{7}{4}} (dx^n + c)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/4)*(c+d*x^n)^q*(a*x^j+b*x^(j+n))^(5/3),x, algorithm="gia
c")
```

```
[Out] integrate((b*x^(j + n) + a*x^j)^(5/3)*(e*x)^(7/4)*(d*x^n + c)^q, x)
```

3.278 $\int \frac{4+3x^4}{5x+2x^5} \, dx$

Optimal. Leaf size=19

$$\frac{7}{40}\log\left(2x^4+5\right)+\frac{4\log(x)}{5}$$

```
[Out] (4*Log[x])/5 + (7*Log[5 + 2*x^4])/40
```

Rubi [A] time = 0.0257862, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1593, 446, 72}

$$\frac{7}{40}\log\left(2x^4+5\right)+\frac{4\log(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 3*x^4)/(5*x + 2*x^5),x]
```

```
[Out] (4*Log[x])/5 + (7*Log[5 + 2*x^4])/40
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{4+3x^4}{5x+2x^5}\,dx &= \int \frac{4+3x^4}{x\left(5+2x^4\right)}\,dx\\ &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{4+3x}{x(5+2x)}\,dx, x, x^4\right)\\ &= \frac{1}{4}\operatorname{Subst}\left(\int \left(\frac{4}{5x}+\frac{7}{5(5+2x)}\right)dx, x, x^4\right)\\ &= \frac{4\log(x)}{5}+\frac{7}{40}\log\left(5+2x^4\right)\end{aligned}$$

Mathematica [A] time = 0.0052056, size = 19, normalized size = 1.

$$\frac{7}{40}\log\left(2x^4+5\right)+\frac{4\log(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 3*x^4)/(5*x + 2*x^5),x]
```

```
[Out] (4*Log[x])/5 + (7*Log[5 + 2*x^4])/40
```

Maple [A] time = 0.049, size = 16, normalized size = 0.8

$$\frac{4\ln(x)}{5}+\frac{7\ln\left(2x^4+5\right)}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^4+4)/(2*x^5+5*x),x)
```

```
[Out] 4/5*ln(x)+7/40*ln(2*x^4+5)
```

Maxima [A] time = 1.69763, size = 20, normalized size = 1.05

$$\frac{7}{40}\log\left(2x^4+5\right)+\frac{4}{5}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4+4)/(2*x^5+5*x),x, algorithm="maxima")
```

```
[Out] 7/40*log(2*x^4 + 5) + 4/5*log(x)
```

Fricas [A] time = 2.01541, size = 46, normalized size = 2.42

$$\frac{7}{40}\log\left(2x^4+5\right)+\frac{4}{5}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4+4)/(2*x^5+5*x),x, algorithm="fricas")
```

```
[Out] 7/40*log(2*x^4 + 5) + 4/5*log(x)
```

Sympy [A] time = 0.099904, size = 17, normalized size = 0.89

$$\frac{4\log\left(x\right)}{5}+\frac{7\log\left(2x^4+5\right)}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**4+4)/(2*x**5+5*x),x)
```

```
[Out] 4*log(x)/5 + 7*log(2*x**4 + 5)/40
```

Giac [A] time = 1.20917, size = 23, normalized size = 1.21

$$\frac{7}{40}\log\left(2x^4+5\right)+\frac{1}{5}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4+4)/(2*x^5+5*x),x, algorithm="giac")
```

```
[Out] 7/40*log(2*x^4 + 5) + 1/5*log(x^4)
```

3.279 $\int \frac{1+x^6}{x-x^7}\,dx$

Optimal. Leaf size=15

$$\log(x)-\frac{1}{3}\log\left(1-x^6\right)$$

```
[Out] Log[x] - Log[1 - x^6]/3
```

Rubi [A] time = 0.0219739, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1593, 446, 72}

$$\log(x)-\frac{1}{3}\log\left(1-x^6\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^6)/(x - x^7),x]
```

```
[Out] Log[x] - Log[1 - x^6]/3
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+x^6}{x-x^7}\,dx &= \int \frac{1+x^6}{x\left(1-x^6\right)}\,dx \\
&= \frac{1}{6}\,\text{Subst}\left(\int \frac{1+x}{(1-x)x}\,dx, x, x^6\right) \\
&= \frac{1}{6}\,\text{Subst}\left(\int \left(-\frac{2}{-1+x}+\frac{1}{x}\right)dx, x, x^6\right) \\
&= \log(x)-\frac{1}{3}\log\left(1-x^6\right)
\end{aligned}
$$

Mathematica [A] time = 0.0050982, size = 15, normalized size = 1.

$$\log(x)-\frac{1}{3}\log\left(1-x^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^6)/(x - x^7),x]
```

```
[Out] Log[x] - Log[1 - x^6]/3
```

Maple [B] time = 0.055, size = 36, normalized size = 2.4

$$-\frac{\ln\left(x^2-x+1\right)}{3}+\ln(x)-\frac{\ln(1+x)}{3}-\frac{\ln(-1+x)}{3}-\frac{\ln\left(x^2+x+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+1)/(-x^7+x),x)
```

```
[Out] -1/3*ln(x^2-x+1)+ln(x)-1/3*ln(1+x)-1/3*ln(-1+x)-1/3*ln(x^2+x+1)
```

Maxima [B] time = 1.76386, size = 47, normalized size = 3.13

$$-\frac{1}{3}\log\left(x^2+x+1\right)-\frac{1}{3}\log\left(x^2-x+1\right)-\frac{1}{3}\log(x+1)-\frac{1}{3}\log(x-1)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/(-x^7+x),x, algorithm="maxima")
```

```
[Out] -1/3*log(x^2 + x + 1) - 1/3*log(x^2 - x + 1) - 1/3*log(x + 1) - 1/3*log(x -
 1) + log(x)
```

Fricas [A] time = 2.11852, size = 38, normalized size = 2.53

$$-\frac{1}{3}\log\left(x^6-1\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/(-x^7+x),x, algorithm="fricas")
```

```
[Out] -1/3*log(x^6 - 1) + log(x)
```

Sympy [A] time = 0.133748, size = 10, normalized size = 0.67

$$\log\left(x\right)-\frac{\log\left(x^6-1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+1)/(-x**7+x),x)
```

```
[Out] log(x) - log(x**6 - 1)/3
```

Giac [A] time = 1.24728, size = 22, normalized size = 1.47

$$\frac{1}{6}\log\left(x^6\right)-\frac{1}{3}\log\left(\left|x^6-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/(-x^7+x),x, algorithm="giac")
```

```
[Out] 1/6*log(x^6) - 1/3*log(abs(x^6 - 1))
```

3.280 $\int \frac{8+5x^{10}}{2x-x^{11}}\,dx$

Optimal. Leaf size=17

$$4\log(x)-\frac{9}{10}\log\left(2-x^{10}\right)$$

```
[Out] 4*Log[x] - (9*Log[2 - x^10])/10
```

Rubi [A] time = 0.025, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1593, 446, 72}

$$4\log(x)-\frac{9}{10}\log\left(2-x^{10}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(8 + 5*x^10)/(2*x - x^11),x]
```

```
[Out] 4*Log[x] - (9*Log[2 - x^10])/10
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{8+5x^{10}}{2x-x^{11}}\,dx &= \int \frac{8+5x^{10}}{x\left(2-x^{10}\right)}\,dx\\ &= \frac{1}{10}\,\text{Subst}\left(\int \frac{8+5x}{(2-x)x}\,dx, x, x^{10}\right)\\ &= \frac{1}{10}\,\text{Subst}\left(\int \left(-\frac{9}{-2+x}+\frac{4}{x}\right)dx, x, x^{10}\right)\\ &= 4\log(x)-\frac{9}{10}\log\left(2-x^{10}\right)\end{aligned}$$

Mathematica [A] time = 0.0051205, size = 17, normalized size = 1.

$$4\log(x)-\frac{9}{10}\log\left(2-x^{10}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(8 + 5*x^10)/(2*x - x^11),x]
```

```
[Out] 4*Log[x] - (9*Log[2 - x^10])/10
```

Maple [A] time = 0.052, size = 14, normalized size = 0.8

$$4\,\ln(x)-\frac{9\,\ln\left(x^{10}-2\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^10+8)/(-x^11+2*x),x)
```

```
[Out] 4*ln(x)-9/10*ln(x^10-2)
```

Maxima [A] time = 1.75488, size = 18, normalized size = 1.06

$$-\frac{9}{10}\,\log\left(x^{10}-2\right)+4\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^10+8)/(-x^11+2*x),x, algorithm="maxima")
```

```
[Out] -9/10*log(x^10 - 2) + 4*log(x)
```

Fricas [A] time = 2.06155, size = 43, normalized size = 2.53

$$-\frac{9}{10}\,\log\left(x^{10}-2\right)+4\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^10+8)/(-x^11+2*x),x, algorithm="fricas")
```

```
[Out] -9/10*log(x^10 - 2) + 4*log(x)
```

Sympy [A] time = 0.16126, size = 14, normalized size = 0.82

$$4\log\left(x\right)-\frac{9\log\left(x^{10}-2\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**10+8)/(-x**11+2*x),x)
```

```
[Out] 4*log(x) - 9*log(x**10 - 2)/10
```

Giac [A] time = 1.22903, size = 22, normalized size = 1.29

$$\frac{2}{5}\,\log\left(x^{10}\right)-\frac{9}{10}\,\log\left(\left|x^{10}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^10+8)/(-x^11+2*x),x, algorithm="giac")
```

```
[Out] 2/5*log(x^10) - 9/10*log(abs(x^10 - 2))
```

3.281 $\int \frac{-3+2x}{-x^2+x^3}\,dx$

Optimal. Leaf size=16

$$-\frac{3}{x}-\log(1-x)+\log(x)$$

```
[Out] -3/x - Log[1 - x] + Log[x]
```

Rubi [A] time = 0.0136957, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1593, 77}

$$-\frac{3}{x}-\log(1-x)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + 2*x)/(-x^2 + x^3),x]
```

```
[Out] -3/x - Log[1 - x] + Log[x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \frac{-3+2x}{-x^2+x^3}\,dx = \int \frac{-3+2x}{(-1+x)x^2}\,dx$$
$$= \int \left(\frac{1}{1-x}+\frac{3}{x^2}+\frac{1}{x}\right)dx$$
$$= -\frac{3}{x}-\log(1-x)+\log(x)$$

Mathematica [A] time = 0.0032564, size = 16, normalized size = 1.

$$-\frac{3}{x}-\log(1-x)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + 2*x)/(-x^2 + x^3),x]
```

```
[Out] -3/x - Log[1 - x] + Log[x]
```

Maple [A] time = 0.051, size = 15, normalized size = 0.9

$$\ln(x)-3x^{-1}-\ln(-1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3+2*x)/(x^3-x^2),x)
```

```
[Out] ln(x)-3/x-ln(-1+x)
```

Maxima [A] time = 1.12229, size = 19, normalized size = 1.19

$$-\frac{3}{x}-\log(x-1)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*x)/(x^3-x^2),x, algorithm="maxima")
```

```
[Out] -3/x - log(x - 1) + log(x)
```

Fricas [A] time = 2.06159, size = 46, normalized size = 2.88

$$-\frac{x\log(x-1)-x\log(x)+3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*x)/(x^3-x^2),x, algorithm="fricas")
```

```
[Out] -(x*log(x - 1) - x*log(x) + 3)/x
```

Sympy [A] time = 0.147027, size = 10, normalized size = 0.62

$$\log(x)-\log(x-1)-\frac{3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*x)/(x**3-x**2),x)
```

```
[Out] log(x) - log(x - 1) - 3/x
```

Giac [A] time = 1.24333, size = 22, normalized size = 1.38

$$-\frac{3}{x}-\log(|x-1|)+\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*x)/(x^3-x^2),x, algorithm="giac")
```

```
[Out] -3/x - log(abs(x - 1)) + log(abs(x))
```

3.282 $\int \frac{ax^m+bx^n}{cx^m+dx^n}\,dx$

Optimal. Leaf size=54

$$\frac{x(bc-ad)\,{}_2F_1\left(1,\frac{1}{m-n};1+\frac{1}{m-n};-\frac{cx^{m-n}}{d}\right)}{cd}+\frac{ax}{c}$$

```
[Out] (a*x)/c + ((b*c - a*d)*x*Hypergeometric2F1[1, (m - n)^(-1), 1 + (m - n)^(-1
), -((c*x^(m - n))/d)])/(c*d)
```

Rubi [A] time = 0.0497989, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {1593, 1584, 388, 245}

$$\frac{x(bc-ad)\,{}_2F_1\left(1,\frac{1}{m-n};1+\frac{1}{m-n};-\frac{cx^{m-n}}{d}\right)}{cd}+\frac{ax}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^m + b*x^n)/(c*x^m + d*x^n),x]
```

```
[Out] (a*x)/c + ((b*c - a*d)*x*Hypergeometric2F1[1, (m - n)^(-1), 1 + (m - n)^(-1
), -((c*x^(m - n))/d)])/(c*d)
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
```

```
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{ax^m+bx^n}{cx^m+dx^n}\,dx &= \int \frac{x^n\left(b+ax^{m-n}\right)}{cx^m+dx^n}\,dx\\ &= \int \frac{b+ax^{m-n}}{d+cx^{m-n}}\,dx\\ &= \frac{ax}{c}-\frac{(-bc+ad)\int \frac{1}{d+cx^{m-n}}\,dx}{c}\\ &= \frac{ax}{c}+\frac{(bc-ad)x\,{}_2F_1\left(1,\frac{1}{m-n};1+\frac{1}{m-n};-\frac{cx^{m-n}}{d}\right)}{cd}\end{aligned}$$

Mathematica [A] time = 0.0192134, size = 52, normalized size = 0.96

$$\frac{x\left((bc-ad)\,{}_2F_1\left(1,\frac{1}{m-n};1+\frac{1}{m-n};-\frac{cx^{m-n}}{d}\right)+ad\right)}{cd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^m + b*x^n)/(c*x^m + d*x^n),x]
```

```
[Out] (x*(a*d + (b*c - a*d)*Hypergeometric2F1[1, (m - n)^(-1), 1 + (m - n)^(-1),
-((c*x^(m - n))/d)]))/(c*d)
```

Maple [F] time = 0.767, size = 0, normalized size = 0.

$$\int \frac{ax^m+bx^n}{cx^m+dx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^m+b*x^n)/(c*x^m+d*x^n),x)
```

```
[Out] int((a*x^m+b*x^n)/(c*x^m+d*x^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-(bc - ad)\int \frac{x^m}{cdx^m + d^2x^n}\,dx + \frac{bx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m+b*x^n)/(c*x^m+d*x^n),x, algorithm="maxima")
```

```
[Out] -(b*c - a*d)*integrate(x^m/(c*d*x^m + d^2*x^n), x) + b*x/d
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ax^m + bx^n}{cx^m + dx^n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m+b*x^n)/(c*x^m+d*x^n),x, algorithm="fricas")
```

```
[Out] integral((a*x^m + b*x^n)/(c*x^m + d*x^n), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax^m + bx^n}{cx^m + dx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**m+b*x**n)/(c*x**m+d*x**n),x)
```

```
[Out] Integral((a*x**m + b*x**n)/(c*x**m + d*x**n), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax^m + bx^n}{cx^m + dx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m+b*x^n)/(c*x^m+d*x^n),x, algorithm="giac")
```

```
[Out] integrate((a*x^m + b*x^n)/(c*x^m + d*x^n), x)
```

3.283 $\int x^m (a+bx^n)^p \left(a(1+m+q)x^q + b(1+m+n(1+p)+q)\right.$

Optimal. Leaf size=18

$$x^{m+q+1}(a+bx^n)^{p+1}$$

```
[Out] x^(1 + m + q)*(a + b*x^n)^(1 + p)
```

Rubi [A] time = 0.0375639, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {1584, 449}

$$x^{m+q+1}(a+bx^n)^{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^n)^p*(a*(1 + m + q)*x^q + b*(1 + m + n*(1 + p) + q)*x^(n +
 q)),x]
```

```
[Out] x^(1 + m + q)*(a + b*x^n)^(1 + p)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int x^m (a+bx^n)^p \left(a(1+m+q)x^q + b(1+m+n(1+p)+q)x^{n+q}\right) dx = \int x^{m+q} (a+bx^n)^p \left(a(1+m+q) + b(1+m+n(1\right.$$
$$= x^{1+m+q}(a+bx^n)^{1+p}$$

Mathematica [C] time = 0.170488, size = 116, normalized size = 6.44

$$x^{m+q+1}(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(\frac{bx^n(m+np+n+q+1)\,{}_2F_1\left(-p,\frac{m+n+q+1}{n};\frac{m+2n+q+1}{n};-\frac{bx^n}{a}\right)}{m+n+q+1}+a\,{}_2F_1\left(-p,\frac{m+q+1}{n};\frac{m+}{}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^n)^p*(a*(1 + m + q)*x^q + b*(1 + m + n*(1 + p) + q)*
x^(n + q)),x]
```

```
[Out] (x^(1 + m + q)*(a + b*x^n)^p*(a*Hypergeometric2F1[-p, (1 + m + q)/n, (1 + m
 + n + q)/n, -((b*x^n)/a)] + (b*(1 + m + n + n*p + q)*x^n*Hypergeometric2F1
[-p, (1 + m + n + q)/n, (1 + m + 2*n + q)/n, -((b*x^n)/a)])/(1 + m + n + q)
))/(1 + (b*x^n)/a)^p
```

Maple [F] time = 0.745, size = 0, normalized size = 0.

$$\int x^m(a+bx^n)^p\left(a\left(1+m+q\right)x^q+b\left(1+m+n\left(1+p\right)+q\right)x^{n+q}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a+b*x^n)^p*(a*(1+m+q)*x^q+b*(1+m+n*(1+p)+q)*x^(n+q)),x)
```

```
[Out] int(x^m*(a+b*x^n)^p*(a*(1+m+q)*x^q+b*(1+m+n*(1+p)+q)*x^(n+q)),x)
```

Maxima [B] time = 1.43783, size = 50, normalized size = 2.78

$$\left(axx^m+bxe^{(m\log(x)+n\log(x))}\right)e^{\left(p\log(bx^n+a)+q\log(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^n)^p*(a*(1+m+q)*x^q+b*(1+m+n*(1+p)+q)*x^(n+q)),x, algo
rithm="maxima")
```

```
[Out] (a*x*x^m + b*x*e^(m*log(x) + n*log(x)))*e^(p*log(b*x^n + a) + q*log(x))
```

Fricas [B] time = 2.54495, size = 88, normalized size = 4.89

$$(bxx^m x^{n+q} + axx^m x^q)\left(\frac{bx^{n+q} + ax^q}{x^q}\right)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^n)^p*(a*(1+m+q)*x^q+b*(1+m+n*(1+p)+q)*x^(n+q)),x, algo
rithm="fricas")
```

```
[Out] (b*x*x^m*x^(n + q) + a*x*x^m*x^q)*((b*x^(n + q) + a*x^q)/x^q)^p
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*x**n)**p*(a*(1+m+q)*x**q+b*(1+m+n*(1+p)+q)*x**(n+q)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.38679, size = 65, normalized size = 3.61

$$(bx^n + a)^p bxx^n e^{(m\log(x)+q\log(x))} + (bx^n + a)^p axe^{(m\log(x)+q\log(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^n)^p*(a*(1+m+q)*x^q+b*(1+m+n*(1+p)+q)*x^(n+q)),x, algo
rithm="giac")
```

```
[Out] (b*x^n + a)^p*b*x*x^n*e^(m*log(x) + q*log(x)) + (b*x^n + a)^p*a*x*e^(m*log(
x) + q*log(x))
```

3.284 $\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{c+dx}\,dx$

Optimal. Leaf size=64

$$\frac{x^m\left(a+\frac{b}{x}\right)^n\left(\frac{b}{ax}+1\right)^{-n}F_1\left(-m;-n,1;1-m;-\frac{b}{ax},-\frac{c}{dx}\right)}{dm}$$

```
[Out] ((a + b/x)^n*x^m*AppellF1[-m, -n, 1, 1 - m, -(b/(a*x)), -(c/(d*x))])/(d*m*(
1 + b/(a*x))^n)
```

Rubi [A] time = 0.0643364, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {514, 497, 135, 133}

$$\frac{x^m\left(a+\frac{b}{x}\right)^n\left(\frac{b}{ax}+1\right)^{-n}F_1\left(-m;-n,1;1-m;-\frac{b}{ax},-\frac{c}{dx}\right)}{dm}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x)^n*x^m)/(c + d*x),x]
```

```
[Out] ((a + b/x)^n*x^m*AppellF1[-m, -n, 1, 1 - m, -(b/(a*x)), -(c/(d*x))])/(d*m*(
1 + b/(a*x))^n)
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 497

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> -Dist[(e*x)^m*(x^(-1))^m, Subst[Int[((a + b/x^n)^p*(c
+ d/x^n)^q)/x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e, m, p, q}, x
] && NeQ[b*c - a*d, 0] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 135

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart
[n], Int[(b*x)^m*(1 + (d*x)/c)^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{c+dx}\,dx &= \int \frac{\left(a+\frac{b}{x}\right)^n x^{-1+m}}{d+\frac{c}{x}}\,dx \\
&= -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-1-m}(a+bx)^n}{d+cx}\,dx, x, \frac{1}{x}\right)\right) \\
&= -\left(\left(\left(a+\frac{b}{x}\right)^n \left(1+\frac{b}{ax}\right)^{-n} \left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-1-m}\left(1+\frac{bx}{a}\right)^n}{d+cx}\,dx, x, \frac{1}{x}\right)\right) \\
&= \frac{\left(a+\frac{b}{x}\right)^n \left(1+\frac{b}{ax}\right)^{-n} x^m F_1\left(-m;-n,1;1-m;-\frac{b}{ax},-\frac{c}{dx}\right)}{dm}
\end{aligned}$$

Mathematica [F] time = 0.0699305, size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((a + b/x)^n*x^m)/(c + d*x),x]
```

```
[Out] Integrate[((a + b/x)^n*x^m)/(c + d*x), x]
```

Maple [F] time = 0.525, size = 0, normalized size = 0.

$$\int \frac{x^{m}}{dx+c}\left(a+\frac{b}{x}\right)^{n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^n*x^m/(d*x+c),x)
```

```
[Out] int((a+b/x)^n*x^m/(d*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x}\right)^{n}x^{m}}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n*x^m/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^n*x^m/(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^{m}\left(\frac{ax+b}{x}\right)^{n}}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n*x^m/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(x^m*((a*x + b)/x)^n/(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \left(a + \frac{b}{x}\right)^n}{c + dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**n*x**m/(d*x+c),x)
```

```
[Out] Integral(x**m*(a + b/x)**n/(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + \frac{b}{x}\right)^n x^m}{dx + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n*x^m/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((a + b/x)^n*x^m/(d*x + c), x)
```

3.285 $\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{c+dx} \, dx$

Optimal. Leaf size=195

$$\frac{\left(a+\frac{b}{x}\right)^{n+1}\left(2a^2c^2-2abcdn-b^2d^2(1-n)n\right){}_2F_1\left(1,n+1;n+2;\frac{b}{ax}+1\right)}{2a^3d^3(n+1)}-\frac{x\left(a+\frac{b}{x}\right)^{n+1}(2ac+bd(1-n))}{2a^2d^2}-\frac{c^3\left(a+\frac{b}{x}\right)^{n+1}}{d}$$

```
[Out] -((2*a*c + b*d*(1 - n))*(a + b/x)^(1 + n)*x)/(2*a^2*d^2) + ((a + b/x)^(1 +
n)*x^2)/(2*a*d) - (c^3*(a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n,
 (c*(a + b/x))/(a*c - b*d)])/(d^3*(a*c - b*d)*(1 + n)) + ((2*a^2*c^2 - 2*a*
b*c*d*n - b^2*d^2*(1 - n)*n)*(a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n,
2 + n, 1 + b/(a*x)])/(2*a^3*d^3*(1 + n))
```

Rubi [A] time = 0.215586, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {514, 446, 103, 151, 156, 65, 68}

$$\frac{\left(a+\frac{b}{x}\right)^{n+1}\left(2a^2c^2-2abcdn-b^2d^2(1-n)n\right){}_2F_1\left(1,n+1;n+2;\frac{b}{ax}+1\right)}{2a^3d^3(n+1)}-\frac{x\left(a+\frac{b}{x}\right)^{n+1}(2ac+bd(1-n))}{2a^2d^2}-\frac{c^3\left(a+\frac{b}{x}\right)^{n+1}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x)^n*x^2)/(c + d*x),x]
```

```
[Out] -((2*a*c + b*d*(1 - n))*(a + b/x)^(1 + n)*x)/(2*a^2*d^2) + ((a + b/x)^(1 +
n)*x^2)/(2*a*d) - (c^3*(a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n,
 (c*(a + b/x))/(a*c - b*d)])/(d^3*(a*c - b*d)*(1 + n)) + ((2*a^2*c^2 - 2*a*
b*c*d*n - b^2*d^2*(1 - n)*n)*(a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n,
2 + n, 1 + b/(a*x)])/(2*a^3*d^3*(1 + n))
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
```

```
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{c+dx}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n x}{d+\frac{c}{x}}\,dx$$

$$= -\operatorname{Subst}\left(\int \frac{(a+bx)^n}{x^3(d+cx)}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\left(a+\frac{b}{x}\right)^{1+n} x^2}{2ad} + \frac{\operatorname{Subst}\left(\int \frac{(a+bx)^n(2ac+bd(1-n)+bc(1-n)x)}{x^2(d+cx)}\,dx, x, \frac{1}{x}\right)}{2ad}$$

$$= -\frac{(2ac+bd(1-n))\left(a+\frac{b}{x}\right)^{1+n} x}{2a^2d^2} + \frac{\left(a+\frac{b}{x}\right)^{1+n} x^2}{2ad} - \frac{\operatorname{Subst}\left(\int \frac{(a+bx)^n\left(2a^2c^2-2abcdn-b^2d^2(1-n)n-bc(2ac+bd(1-\right.}{x(d+cx)}\right.}{2a^2d^2}$$

$$= -\frac{(2ac+bd(1-n))\left(a+\frac{b}{x}\right)^{1+n} x}{2a^2d^2} + \frac{\left(a+\frac{b}{x}\right)^{1+n} x^2}{2ad} + \frac{c^3\operatorname{Subst}\left(\int \frac{(a+bx)^n}{d+cx}\,dx, x, \frac{1}{x}\right)}{d^3} - \frac{\left(2a^2c^2-2abcdn\right.}{}$$

$$= -\frac{(2ac+bd(1-n))\left(a+\frac{b}{x}\right)^{1+n} x}{2a^2d^2} + \frac{\left(a+\frac{b}{x}\right)^{1+n} x^2}{2ad} - \frac{c^3\left(a+\frac{b}{x}\right)^{1+n} {}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d^3(ac-bd)(1+n)} + \frac{\left(2a^2\right.}{}$$

Mathematica [A] time = 0.125946, size = 157, normalized size = 0.81

$$\frac{(ax+b)\left(a+\frac{b}{x}\right)^n\left((ac-bd)\left(\left(2a^2c^2-2abcdn+b^2d^2(n-1)n\right){}_2F_1\left(1,n+1;n+2;\frac{b}{ax}+1\right)+ad(n+1)x(a(dx-2c)+bd(\right.\right.}{2a^3d^3(n+1)x(ac-bd)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x)^n*x^2)/(c + d*x),x]
```

```
[Out] ((a + b/x)^n*(b + a*x)*(-2*a^3*c^3*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a
 + b/x))/(a*c - b*d)] + (a*c - b*d)*(a*d*(1 + n)*x*(b*d*(-1 + n) + a*(-2*c
+ d*x)) + (2*a^2*c^2 - 2*a*b*c*d*n + b^2*d^2*(-1 + n)*n)*Hypergeometric2F1[
1, 1 + n, 2 + n, 1 + b/(a*x)])))/(2*a^3*d^3*(a*c - b*d)*(1 + n)*x)
```

Maple [F] time = 0.497, size = 0, normalized size = 0.

$$\int \frac{x^2}{dx+c}\left(a+\frac{b}{x}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^n*x^2/(d*x+c),x)
```

```
[Out] int((a+b/x)^n*x^2/(d*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{dx+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n*x^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^n*x^2/(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^2\left(\frac{ax+b}{x}\right)^n}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n*x^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(x^2*((a*x + b)/x)^n/(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**n*x**2/(d*x+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n*x^2/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((a + b/x)^n*x^2/(d*x + c), x)
```

3.286 $\int \frac{\left(a+\frac{b}{x}\right)^n x}{c+dx} dx$

Optimal. Leaf size=131

$$-\frac{\left(a+\frac{b}{x}\right)^{n+1}(ac-bdn)\,{}_2F_1\left(1,n+1;n+2;\frac{b}{ax}+1\right)}{a^2d^2(n+1)}+\frac{c^2\left(a+\frac{b}{x}\right)^{n+1}{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d^2(n+1)(ac-bd)}+\frac{x\left(a+\frac{b}{x}\right)^{n+1}}{ad}$$

```
[Out] ((a + b/x)^(1 + n)*x)/(a*d) + (c^2*(a + b/x)^(1 + n)*Hypergeometric2F1[1, 1
 + n, 2 + n, (c*(a + b/x))/(a*c - b*d)])/(d^2*(a*c - b*d)*(1 + n)) - ((a*c
- b*d*n)*(a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + b/(a*x)])
/(a^2*d^2*(1 + n))
```

Rubi [A] time = 0.0971801, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {514, 375, 103, 156, 65, 68}

$$-\frac{\left(a+\frac{b}{x}\right)^{n+1}(ac-bdn)\,{}_2F_1\left(1,n+1;n+2;\frac{b}{ax}+1\right)}{a^2d^2(n+1)}+\frac{c^2\left(a+\frac{b}{x}\right)^{n+1}{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d^2(n+1)(ac-bd)}+\frac{x\left(a+\frac{b}{x}\right)^{n+1}}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x)^n*x)/(c + d*x),x]
```

```
[Out] ((a + b/x)^(1 + n)*x)/(a*d) + (c^2*(a + b/x)^(1 + n)*Hypergeometric2F1[1, 1
 + n, 2 + n, (c*(a + b/x))/(a*c - b*d)])/(d^2*(a*c - b*d)*(1 + n)) - ((a*c
- b*d*n)*(a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + b/(a*x)])
/(a^2*d^2*(1 + n))
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x}\right)^n x}{c+dx}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{d+\frac{c}{x}}\,dx$$
$$= -\operatorname{Subst}\left(\int \frac{(a+bx)^n}{x^2(d+cx)}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{\left(a+\frac{b}{x}\right)^{1+n} x}{ad} + \frac{\operatorname{Subst}\left(\int \frac{(a+bx)^n(ac-bdn-bcnx)}{x(d+cx)}\,dx, x, \frac{1}{x}\right)}{ad}$$
$$= \frac{\left(a+\frac{b}{x}\right)^{1+n} x}{ad} - \frac{c^2 \operatorname{Subst}\left(\int \frac{(a+bx)^n}{d+cx}\,dx, x, \frac{1}{x}\right)}{d^2} + \frac{(ac-bdn)\operatorname{Subst}\left(\int \frac{(a+bx)^n}{x}\,dx, x, \frac{1}{x}\right)}{ad^2}$$
$$= \frac{\left(a+\frac{b}{x}\right)^{1+n} x}{ad} + \frac{c^2\left(a+\frac{b}{x}\right)^{1+n} {}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d^2(ac-bd)(1+n)} - \frac{(ac-bdn)\left(a+\frac{b}{x}\right)^{1+n} {}_2F_1\left(1,1+n;2+\right.}{a^2d^2(1+n)}$$

Mathematica [A] time = 0.0632969, size = 119, normalized size = 0.91

$$\frac{(ax+b)\left(a+\frac{b}{x}\right)^n\left(a^2c^2\,{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)+(ac-bd)\left((bdn-ac)\,{}_2F_1\left(1,n+1;n+2;\frac{b}{ax}+1\right)+ad(n+1)x\right)\right)}{a^2d^2(n+1)x(ac-bd)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x)^n*x)/(c + d*x),x]
```

```
[Out] ((a + b/x)^n*(b + a*x)*(a^2*c^2*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a +
b/x))/(a*c - b*d)] + (a*c - b*d)*(a*d*(1 + n)*x + (-(a*c) + b*d*n)*Hypergeo
metric2F1[1, 1 + n, 2 + n, 1 + b/(a*x)])))/(a^2*d^2*(a*c - b*d)*(1 + n)*x)
```

Maple [F] time = 0.495, size = 0, normalized size = 0.

$$\int \frac{x}{dx+c}\left(a+\frac{b}{x}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^n*x/(d*x+c),x)
```

```
[Out] int((a+b/x)^n*x/(d*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x}\right)^n x}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n*x/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^n*x/(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x\left(\frac{ax+b}{x}\right)^n}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n*x/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(x*((a*x + b)/x)^n/(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**n*x/(d*x+c),x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x}\right)^{n} x}{dx+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n*x/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((a + b/x)^n*x/(d*x + c), x)
```

3.287 $\int \frac{\left(a+\frac{b}{x}\right)^n}{c+dx}\,dx$

Optimal. Leaf size=101

$$\frac{\left(a+\frac{b}{x}\right)^{n+1}{}_2F_1\left(1,n+1;n+2;\frac{b}{ax}+1\right)}{ad(n+1)}-\frac{c\left(a+\frac{b}{x}\right)^{n+1}{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d(n+1)(ac-bd)}$$

```
[Out] -((c*(a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*
c - b*d)])/(d*(a*c - b*d)*(1 + n))) + ((a + b/x)^(1 + n)*Hypergeometric2F1[
1, 1 + n, 2 + n, 1 + b/(a*x)])/(a*d*(1 + n))
```

Rubi [A] time = 0.0555461, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {434, 446, 86, 65, 68}

$$\frac{\left(a+\frac{b}{x}\right)^{n+1}{}_2F_1\left(1,n+1;n+2;\frac{b}{ax}+1\right)}{ad(n+1)}-\frac{c\left(a+\frac{b}{x}\right)^{n+1}{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d(n+1)(ac-bd)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^n/(c + d*x),x]
```

```
[Out] -((c*(a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*
c - b*d)])/(d*(a*c - b*d)*(1 + n))) + ((a + b/x)^(1 + n)*Hypergeometric2F1[
1, 1 + n, 2 + n, 1 + b/(a*x)])/(a*d*(1 + n))
```

Rule 434

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[((a + b*x^n)^p*(d + c*x^n)^q)/x^(n*q), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+\frac{b}{x}\right)^n}{c+dx}\,dx &= \int \frac{\left(a+\frac{b}{x}\right)^n}{\left(d+\frac{c}{x}\right)x}\,dx\\
&= -\operatorname{Subst}\left(\int \frac{(a+bx)^n}{x(d+cx)}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{\operatorname{Subst}\left(\int \frac{(a+bx)^n}{x}\,dx, x, \frac{1}{x}\right)}{d}+\frac{c\operatorname{Subst}\left(\int \frac{(a+bx)^n}{d+cx}\,dx, x, \frac{1}{x}\right)}{d}\\
&= -\frac{c\left(a+\frac{b}{x}\right)^{1+n}{}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d(ac-bd)(1+n)}+\frac{\left(a+\frac{b}{x}\right)^{1+n}{}_2F_1\left(1,1+n;2+n;1+\frac{b}{ax}\right)}{ad(1+n)}
\end{aligned}$$

Mathematica [A] time = 0.0301838, size = 97, normalized size = 0.96

$$\frac{(ax+b)\left(a+\frac{b}{x}\right)^n\left(ac\,{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)+(bd-ac)\,{}_2F_1\left(1,n+1;n+2;\frac{b}{ax}+1\right)\right)}{ad(n+1)x(bd-ac)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^n/(c + d*x),x]
```

```
[Out] ((a + b/x)^n*(b + a*x)*(a*c*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x)
)/(a*c - b*d)] + (-(a*c) + b*d)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + b/(a
*x)]))/(a*d*(-(a*c) + b*d)*(1 + n)*x)
```

Maple [F] time = 0.498, size = 0, normalized size = 0.

$$\int \frac{1}{dx+c}\left(a+\frac{b}{x}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^n/(d*x+c),x)
```

```
[Out] int((a+b/x)^n/(d*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^n/(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\frac{ax+b}{x}\right)^{n}}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(((a*x + b)/x)^n/(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x}\right)^{n}}{c+dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**n/(d*x+c),x)
```

```
[Out] Integral((a + b/x)**n/(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x}\right)^{n}}{dx+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((a + b/x)^n/(d*x + c), x)
```

3.288 $\int \frac{\left(a+\frac{b}{x}\right)^n}{x(c+dx)}\,dx$

Optimal. Leaf size=54

$$\frac{\left(a+\frac{b}{x}\right)^{n+1} {}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(n+1)(ac-bd)}$$

```
[Out] ((a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c -
b*d)])/((a*c - b*d)*(1 + n))
```

Rubi [A] time = 0.039041, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {514, 444, 68}

$$\frac{\left(a+\frac{b}{x}\right)^{n+1} {}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(n+1)(ac-bd)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^n/(x*(c + d*x)),x]
```

```
[Out] ((a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c -
b*d)])/((a*c - b*d)*(1 + n))
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{\left(d+\frac{c}{x}\right)x^2}\,dx$$
$$= -\operatorname{Subst}\left(\int \frac{(a+bx)^n}{d+cx}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{\left(a+\frac{b}{x}\right)^{1+n} {}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(ac-bd)(1+n)}$$

Mathematica [A] time = 0.0154163, size = 54, normalized size = 1.

$$\frac{\left(a+\frac{b}{x}\right)^{n+1} {}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(n+1)(ac-bd)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^n/(x*(c + d*x)),x]
```

```
[Out] ((a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c -
b*d)])/((a*c - b*d)*(1 + n))
```

Maple [F] time = 0.531, size = 0, normalized size = 0.

$$\int \frac{1}{x\,(dx+c)}\left(a+\frac{b}{x}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^n/x/(d*x+c),x)
```

```
[Out] int((a+b/x)^n/x/(d*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{(dx+c)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^n/((d*x + c)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\frac{ax+b}{x}\right)^n}{dx^2+cx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(((a*x + b)/x)^n/(d*x^2 + c*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**n/x/(d*x+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x}\right)^{n}}{(dx+c)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((a + b/x)^n/((d*x + c)*x), x)
```

3.289 $\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2(c+dx)}\,dx$

Optimal. Leaf size=84

$$-\frac{d\left(a+\frac{b}{x}\right)^{n+1} {}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c(n+1)(ac-bd)}-\frac{\left(a+\frac{b}{x}\right)^{n+1}}{bc(n+1)}$$

```
[Out] -((a + b/x)^(1 + n)/(b*c*(1 + n))) - (d*(a + b/x)^(1 + n)*Hypergeometric2F1
[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c - b*d)])/(c*(a*c - b*d)*(1 + n))
```

Rubi [A] time = 0.0555754, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {514, 446, 80, 68}

$$-\frac{d\left(a+\frac{b}{x}\right)^{n+1} {}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c(n+1)(ac-bd)}-\frac{\left(a+\frac{b}{x}\right)^{n+1}}{bc(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^n/(x^2*(c + d*x)),x]
```



```
[Out] -((a + b/x)^(1 + n)/(b*c*(1 + n))) - (d*(a + b/x)^(1 + n)*Hypergeometric2F1
[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c - b*d)])/(c*(a*c - b*d)*(1 + n))
```


Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2(c+dx)}\,dx &= \int \frac{\left(a+\frac{b}{x}\right)^n}{\left(d+\frac{c}{x}\right)x^3}\,dx \\
&= -\operatorname{Subst}\left(\int \frac{x(a+bx)^n}{d+cx}\,dx, x, \frac{1}{x}\right) \\
&= -\frac{\left(a+\frac{b}{x}\right)^{1+n}}{bc(1+n)} + \frac{d\operatorname{Subst}\left(\int \frac{(a+bx)^n}{d+cx}\,dx, x, \frac{1}{x}\right)}{c} \\
&= -\frac{\left(a+\frac{b}{x}\right)^{1+n}}{bc(1+n)} - \frac{d\left(a+\frac{b}{x}\right)^{1+n}{}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c(ac-bd)(1+n)}
\end{aligned}
$$

Mathematica [A] time = 0.0234315, size = 77, normalized size = 0.92

$$
\frac{(ax+b)\left(a+\frac{b}{x}\right)^n\left(bd\,{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)+ac-bd\right)}{bc(n+1)x(bd-ac)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^n/(x^2*(c + d*x)),x]
```

```
[Out] ((a + b/x)^n*(b + a*x)*(a*c - b*d + b*d*Hypergeometric2F1[1, 1 + n, 2 + n,
(c*(a + b/x))/(a*c - b*d)]))/(b*c*(-(a*c) + b*d)*(1 + n)*x)
```

Maple [F] time = 0.511, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 (dx + c)} \left(a + \frac{b}{x}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^n/x^2/(d*x+c),x)
```

```
[Out] int((a+b/x)^n/x^2/(d*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + \frac{b}{x}\right)^n}{(dx + c)x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^n/((d*x + c)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\frac{ax+b}{x}\right)^n}{dx^3 + cx^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(((a*x + b)/x)^n/(d*x^3 + c*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**n/x**2/(d*x+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + \frac{b}{x}\right)^n}{(dx + c)x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x^2/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((a + b/x)^n/((d*x + c)*x^2), x)
```

3.290 $\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3(c+dx)}\,dx$

Optimal. Leaf size=115

$$\frac{(ac+bd)\left(a+\frac{b}{x}\right)^{n+1}}{b^2c^2(n+1)}-\frac{\left(a+\frac{b}{x}\right)^{n+2}}{b^2c(n+2)}+\frac{d^2\left(a+\frac{b}{x}\right)^{n+1}{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c^2(n+1)(ac-bd)}$$

```
[Out] ((a*c + b*d)*(a + b/x)^(1 + n))/(b^2*c^2*(1 + n)) - (a + b/x)^(2 + n)/(b^2*
c*(2 + n)) + (d^2*(a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(
a + b/x))/(a*c - b*d)])/(c^2*(a*c - b*d)*(1 + n))
```

Rubi [A] time = 0.0962083, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {514, 446, 88, 68}

$$\frac{(ac+bd)\left(a+\frac{b}{x}\right)^{n+1}}{b^2c^2(n+1)}-\frac{\left(a+\frac{b}{x}\right)^{n+2}}{b^2c(n+2)}+\frac{d^2\left(a+\frac{b}{x}\right)^{n+1}{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c^2(n+1)(ac-bd)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^n/(x^3*(c + d*x)),x]
```

```
[Out] ((a*c + b*d)*(a + b/x)^(1 + n))/(b^2*c^2*(1 + n)) - (a + b/x)^(2 + n)/(b^2*
c*(2 + n)) + (d^2*(a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(
a + b/x))/(a*c - b*d)])/(c^2*(a*c - b*d)*(1 + n))
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3(c+dx)}\,dx &= \int \frac{\left(a+\frac{b}{x}\right)^n}{\left(d+\frac{c}{x}\right)x^4}\,dx\\
&= -\text{Subst}\left(\int \frac{x^2(a+bx)^n}{d+cx}\,dx, x, \frac{1}{x}\right)\\
&= -\text{Subst}\left(\int \left(\frac{(-ac-bd)(a+bx)^n}{bc^2}+\frac{(a+bx)^{1+n}}{bc}+\frac{d^2(a+bx)^n}{c^2(d+cx)}\right)dx, x, \frac{1}{x}\right)\\
&= \frac{(ac+bd)\left(a+\frac{b}{x}\right)^{1+n}}{b^2c^2(1+n)}-\frac{\left(a+\frac{b}{x}\right)^{2+n}}{b^2c(2+n)}-\frac{d^2\,\text{Subst}\left(\int \frac{(a+bx)^n}{d+cx}\,dx, x, \frac{1}{x}\right)}{c^2}\\
&= \frac{(ac+bd)\left(a+\frac{b}{x}\right)^{1+n}}{b^2c^2(1+n)}-\frac{\left(a+\frac{b}{x}\right)^{2+n}}{b^2c(2+n)}+\frac{d^2\left(a+\frac{b}{x}\right)^{1+n}{}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c^2(ac-bd)(1+n)}
\end{aligned}$$

Mathematica [A] time = 0.0939186, size = 112, normalized size = 0.97

$$-\frac{(ax+b)\left(a+\frac{b}{x}\right)^n\left(b^2d^2(n+2)x\,{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)+(ac-bd)(acx-bc(n+1)+bd(n+2)x)\right)}{b^2c^2(n+1)(n+2)x^2(bd-ac)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^n/(x^3*(c + d*x)),x]
```

```
[Out] -(((a + b/x)^n*(b + a*x)*((a*c - b*d)*(-(b*c*(1 + n)) + a*c*x + b*d*(2 + n)
*x) + b^2*d^2*(2 + n)*x*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a
*c - b*d)]))/(b^2*c^2*(-(a*c) + b*d)*(1 + n)*(2 + n)*x^2))
```

Maple [F] time = 0.52, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(dx+c\right)}\left(a+\frac{b}{x}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^n/x^3/(d*x+c),x)
```

```
[Out] int((a+b/x)^n/x^3/(d*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{(dx+c)x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^n/((d*x + c)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\frac{ax+b}{x}\right)^n}{dx^4+cx^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(((a*x + b)/x)^n/(d*x^4 + c*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**n/x**3/(d*x+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + \frac{b}{x}\right)^n}{(dx + c)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x^3/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((a + b/x)^n/((d*x + c)*x^3), x)
```

3.291 $\int \frac{\left(a+\frac{b}{x}\right)^n}{x^5(c+dx)}\,dx$

Optimal. Leaf size=207

$$\frac{(ac+bd)\left(a^2c^2+b^2d^2\right)\left(a+\frac{b}{x}\right)^{n+1}}{b^4c^4(n+1)}-\frac{\left(3a^2c^2+2abcd+b^2d^2\right)\left(a+\frac{b}{x}\right)^{n+2}}{b^4c^3(n+2)}+\frac{(3ac+bd)\left(a+\frac{b}{x}\right)^{n+3}}{b^4c^2(n+3)}-\frac{\left(a+\frac{b}{x}\right)^{n+4}}{b^4c(n+4)}+\frac{d^4\left(a+\right.}{}$$

```
[Out] ((a*c + b*d)*(a^2*c^2 + b^2*d^2)*(a + b/x)^(1 + n))/(b^4*c^4*(1 + n)) - ((3
*a^2*c^2 + 2*a*b*c*d + b^2*d^2)*(a + b/x)^(2 + n))/(b^4*c^3*(2 + n)) + ((3*
a*c + b*d)*(a + b/x)^(3 + n))/(b^4*c^2*(3 + n)) - (a + b/x)^(4 + n)/(b^4*c*
(4 + n)) + (d^4*(a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a
+ b/x))/(a*c - b*d)])/(c^4*(a*c - b*d)*(1 + n))
```

Rubi [A] time = 0.157829, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {514, 446, 88, 68}

$$\frac{(ac+bd)\left(a^2c^2+b^2d^2\right)\left(a+\frac{b}{x}\right)^{n+1}}{b^4c^4(n+1)}-\frac{\left(3a^2c^2+2abcd+b^2d^2\right)\left(a+\frac{b}{x}\right)^{n+2}}{b^4c^3(n+2)}+\frac{(3ac+bd)\left(a+\frac{b}{x}\right)^{n+3}}{b^4c^2(n+3)}-\frac{\left(a+\frac{b}{x}\right)^{n+4}}{b^4c(n+4)}+\frac{d^4\left(a+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^n/(x^5*(c + d*x)),x]
```

```
[Out] ((a*c + b*d)*(a^2*c^2 + b^2*d^2)*(a + b/x)^(1 + n))/(b^4*c^4*(1 + n)) - ((3
*a^2*c^2 + 2*a*b*c*d + b^2*d^2)*(a + b/x)^(2 + n))/(b^4*c^3*(2 + n)) + ((3*
a*c + b*d)*(a + b/x)^(3 + n))/(b^4*c^2*(3 + n)) - (a + b/x)^(4 + n)/(b^4*c*
(4 + n)) + (d^4*(a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a
+ b/x))/(a*c - b*d)])/(c^4*(a*c - b*d)*(1 + n))
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^5(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{\left(d+\frac{c}{x}\right)x^6}\,dx$$

$$= -\operatorname{Subst}\left(\int \frac{x^4(a+bx)^n}{d+cx}\,dx, x, \frac{1}{x}\right)$$

$$= -\operatorname{Subst}\left(\int\left(\frac{(ac+bd)\left(-a^2c^2-b^2d^2\right)(a+bx)^n}{b^3c^4} + \frac{\left(3a^2c^2+2abcd+b^2d^2\right)(a+bx)^{1+n}}{b^3c^3} + \frac{(-3ac-bc}{b}\right.\right.$$

$$= \frac{(ac+bd)\left(a^2c^2+b^2d^2\right)\left(a+\frac{b}{x}\right)^{1+n}}{b^4c^4(1+n)} - \frac{\left(3a^2c^2+2abcd+b^2d^2\right)\left(a+\frac{b}{x}\right)^{2+n}}{b^4c^3(2+n)} + \frac{(3ac+bd)\left(a+\frac{b}{x}\right)^{3+n}}{b^4c^2(3+n)} -$$

$$= \frac{(ac+bd)\left(a^2c^2+b^2d^2\right)\left(a+\frac{b}{x}\right)^{1+n}}{b^4c^4(1+n)} - \frac{\left(3a^2c^2+2abcd+b^2d^2\right)\left(a+\frac{b}{x}\right)^{2+n}}{b^4c^3(2+n)} + \frac{(3ac+bd)\left(a+\frac{b}{x}\right)^{3+n}}{b^4c^2(3+n)} -$$

Mathematica [A] time = 0.162881, size = 184, normalized size = 0.89

$$\frac{\left(a+\frac{b}{x}\right)^{n+1}\left(-\frac{c\left(a+\frac{b}{x}\right)\left(3a^2c^2+2abcd+b^2d^2\right)}{b^4(n+2)}+\frac{(ac+bd)\left(a^2c^2+b^2d^2\right)}{b^4(n+1)}+\frac{c^2\left(a+\frac{b}{x}\right)^2(3ac+bd)}{b^4(n+3)}-\frac{c^3\left(a+\frac{b}{x}\right)^3}{b^4(n+4)}+\frac{d^4\,{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(n+1)(ac-bd)}\right)}{c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^n/(x^5*(c + d*x)),x]
```

```
[Out] ((a + b/x)^(1 + n)*(((a*c + b*d)*(a^2*c^2 + b^2*d^2))/(b^4*(1 + n)) - (c*(3
*a^2*c^2 + 2*a*b*c*d + b^2*d^2)*(a + b/x))/(b^4*(2 + n)) + (c^2*(3*a*c + b*
d)*(a + b/x)^2)/(b^4*(3 + n)) - (c^3*(a + b/x)^3)/(b^4*(4 + n)) + (d^4*Hype
rgeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c - b*d)])/((a*c - b*d)*(1
+ n))))/c^4
```

Maple [F] time = 0.511, size = 0, normalized size = 0.

$$\int \frac{1}{x^5\,(dx+c)}\left(a+\frac{b}{x}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^n/x^5/(d*x+c),x)
```

```
[Out] int((a+b/x)^n/x^5/(d*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{(dx+c)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x^5/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^n/((d*x + c)*x^5), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\frac{ax+b}{x}\right)^n}{dx^6+cx^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x^5/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(((a*x + b)/x)^n/(d*x^6 + c*x^5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**n/x**5/(d*x+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{(dx+c)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x^5/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((a + b/x)^n/((d*x + c)*x^5), x)
```

3.292 $\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{(c+dx)^2}\,dx$

Optimal. Leaf size=73

$$-\frac{x^{m-1}\left(a+\frac{b}{x}\right)^n\left(\frac{b}{ax}+1\right)^{-n}F_1\left(1-m;-n,2;2-m;-\frac{b}{ax},-\frac{c}{dx}\right)}{d^2(1-m)}$$

```
[Out] -(((a + b/x)^n*x^(-1 + m)*AppellF1[1 - m, -n, 2, 2 - m, -(b/(a*x)), -(c/(d*
x))])/(d^2*(1 - m)*(1 + b/(a*x))^n))
```

Rubi [A] time = 0.0708469, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {514, 497, 135, 133}

$$-\frac{x^{m-1}\left(a+\frac{b}{x}\right)^n\left(\frac{b}{ax}+1\right)^{-n}F_1\left(1-m;-n,2;2-m;-\frac{b}{ax},-\frac{c}{dx}\right)}{d^2(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x)^n*x^m)/(c + d*x)^2,x]
```

```
[Out] -(((a + b/x)^n*x^(-1 + m)*AppellF1[1 - m, -n, 2, 2 - m, -(b/(a*x)), -(c/(d*
x))])/(d^2*(1 - m)*(1 + b/(a*x))^n))
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 497

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> -Dist[(e*x)^m*(x^(-1))^m, Subst[Int[((a + b/x^n)^p*(c
+ d/x^n)^q)/x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e, m, p, q}, x
] && NeQ[b*c - a*d, 0] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 135

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart
[n], Int[(b*x)^m*(1 + (d*x)/c)^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{(c+dx)^2}\,dx &= \int \frac{\left(a+\frac{b}{x}\right)^n x^{-2+m}}{\left(d+\frac{c}{x}\right)^2}\,dx \\
&= -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-m}(a+bx)^n}{(d+cx)^2}\,dx, x, \frac{1}{x}\right)\right) \\
&= -\left(\left(\left(a+\frac{b}{x}\right)^n\left(1+\frac{b}{ax}\right)^{-n}\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-m}\left(1+\frac{bx}{a}\right)^n}{(d+cx)^2}\,dx, x, \frac{1}{x}\right)\right) \\
&= -\frac{\left(a+\frac{b}{x}\right)^n\left(1+\frac{b}{ax}\right)^{-n} x^{-1+m} F_1\left(1-m;-n,2;2-m;-\frac{b}{ax},-\frac{c}{dx}\right)}{d^2(1-m)}
\end{aligned}$$

Mathematica [F] time = 0.0712447, size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((a + b/x)^n*x^m)/(c + d*x)^2,x]
```

```
[Out] Integrate[((a + b/x)^n*x^m)/(c + d*x)^2, x]
```

Maple [F] time = 0.561, size = 0, normalized size = 0.

$$\int \frac{x^m}{(dx+c)^2}\left(a+\frac{b}{x}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^n*x^m/(d*x+c)^2,x)

[Out] int((a+b/x)^n*x^m/(d*x+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{(dx+c)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n*x^m/(d*x+c)^2,x, algorithm="maxima")

[Out] integrate((a + b/x)^n*x^m/(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^m \left(\frac{ax+b}{x}\right)^n}{d^2x^2+2\,cdx+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n*x^m/(d*x+c)^2,x, algorithm="fricas")

[Out] integral(x^m*((a*x + b)/x)^n/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \left(a+\frac{b}{x}\right)^n}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**n*x**m/(d*x+c)**2,x)
```

```
[Out] Integral(x**m*(a + b/x)**n/(c + d*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n*x^m/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((a + b/x)^n*x^m/(d*x + c)^2, x)
```

3.293 $\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{(c+dx)^2}\,dx$

Optimal. Leaf size=202

$$-\frac{\left(a+\frac{b}{x}\right)^{n+1}(2ac-bdn)\,{}_2F_1\left(1,n+1;n+2;\frac{b}{ax}+1\right)}{a^2d^3(n+1)}+\frac{c^2\left(a+\frac{b}{x}\right)^{n+1}(2ac-bd(2-n))\,{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d^3(n+1)(ac-bd)^2}+\frac{c(2a}{ad^2}$$

```
[Out] (c*(2*a*c - b*d)*(a + b/x)^(1 + n))/(a*d^2*(a*c - b*d)*(d + c/x)) + ((a + b
/x)^(1 + n)*x)/(a*d*(d + c/x)) + (c^2*(2*a*c - b*d*(2 - n))*(a + b/x)^(1 +
n)*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c - b*d)])/(d^3*(a*c
 - b*d)^2*(1 + n)) - ((2*a*c - b*d*n)*(a + b/x)^(1 + n)*Hypergeometric2F1[1
, 1 + n, 2 + n, 1 + b/(a*x)])/(a^2*d^3*(1 + n))
```

Rubi [A] time = 0.238604, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {514, 375, 103, 151, 156, 65, 68}

$$-\frac{\left(a+\frac{b}{x}\right)^{n+1}(2ac-bdn)\,{}_2F_1\left(1,n+1;n+2;\frac{b}{ax}+1\right)}{a^2d^3(n+1)}+\frac{c^2\left(a+\frac{b}{x}\right)^{n+1}(2ac-bd(2-n))\,{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d^3(n+1)(ac-bd)^2}+\frac{c(2a}{ad^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x)^n*x^2)/(c + d*x)^2,x]
```


```
[Out] (c*(2*a*c - b*d)*(a + b/x)^(1 + n))/(a*d^2*(a*c - b*d)*(d + c/x)) + ((a + b
/x)^(1 + n)*x)/(a*d*(d + c/x)) + (c^2*(2*a*c - b*d*(2 - n))*(a + b/x)^(1 +
n)*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c - b*d)])/(d^3*(a*c
 - b*d)^2*(1 + n)) - ((2*a*c - b*d*n)*(a + b/x)^(1 + n)*Hypergeometric2F1[1
, 1 + n, 2 + n, 1 + b/(a*x)])/(a^2*d^3*(1 + n))
```


Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
```

```
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{(c+dx)^2}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{\left(d+\frac{c}{x}\right)^2}\,dx$$

$$= -\operatorname{Subst}\left(\int \frac{(a+bx)^n}{x^2(d+cx)^2}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\left(a+\frac{b}{x}\right)^{1+n} x}{ad\left(d+\frac{c}{x}\right)} + \frac{\operatorname{Subst}\left(\int \frac{(a+bx)^n(2ac-bdn+bc(1-n)x)}{x(d+cx)^2}\,dx, x, \frac{1}{x}\right)}{ad}$$

$$= \frac{c(2ac-bd)\left(a+\frac{b}{x}\right)^{1+n}}{ad^2(ac-bd)\left(d+\frac{c}{x}\right)} + \frac{\left(a+\frac{b}{x}\right)^{1+n} x}{ad\left(d+\frac{c}{x}\right)} + \frac{\operatorname{Subst}\left(\int \frac{(a+bx)^n((ac-bd)(2ac-bdn)-bc(2ac-bd)nx)}{x(d+cx)}\,dx, x, \frac{1}{x}\right)}{ad^2(ac-bd)}$$

$$= \frac{c(2ac-bd)\left(a+\frac{b}{x}\right)^{1+n}}{ad^2(ac-bd)\left(d+\frac{c}{x}\right)} + \frac{\left(a+\frac{b}{x}\right)^{1+n} x}{ad\left(d+\frac{c}{x}\right)} - \frac{\left(c^2(2ac-bd(2-n))\right)\operatorname{Subst}\left(\int \frac{(a+bx)^n}{d+cx}\,dx, x, \frac{1}{x}\right)}{d^3(ac-bd)} + \frac{(2ac-bd}{}$$

$$= \frac{c(2ac-bd)\left(a+\frac{b}{x}\right)^{1+n}}{ad^2(ac-bd)\left(d+\frac{c}{x}\right)} + \frac{\left(a+\frac{b}{x}\right)^{1+n} x}{ad\left(d+\frac{c}{x}\right)} + \frac{c^2(2ac-bd(2-n))\left(a+\frac{b}{x}\right)^{1+n} {}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d^3(ac-bd)^2(1+n)}$$

Mathematica [A] time = 0.167212, size = 179, normalized size = 0.89

$$\frac{\left(a+\frac{b}{x}\right)^{n+1}\left((c+dx)\left(a^2c^2(2ac+bd(n-2))\,{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)-(ac-bd)^2(2ac-bdn)\,{}_2F_1\left(1,n+1;n+2;\frac{b}{ax}+1\right.\right.\right.}{a^2d^3(n+1)(c+dx)(ac-bd)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x)^n*x^2)/(c + d*x)^2,x]
```

```
[Out] ((a + b/x)^(1 + n)*(a*c*d*(a*c - b*d)*(2*a*c - b*d)*(1 + n)*x + a*d^2*(a*c
- b*d)^2*(1 + n)*x^2 + (c + d*x)*(a^2*c^2*(2*a*c + b*d*(-2 + n))*Hypergeome
tric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c - b*d)] - (a*c - b*d)^2*(2*a*c
- b*d*n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + b/(a*x)])))/(a^2*d^3*(a*c -
```

```
b*d)^2*(1 + n)*(c + d*x))
```

Maple [F] time = 0.53, size = 0, normalized size = 0.

$$\int \frac{x^2}{(dx+c)^2}\left(a+\frac{b}{x}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^n*x^2/(d*x+c)^2,x)
```

```
[Out] int((a+b/x)^n*x^2/(d*x+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{(dx+c)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n*x^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^n*x^2/(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^2\left(\frac{ax+b}{x}\right)^n}{d^2x^2+2\,cdx+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n*x^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(x^2*((a*x + b)/x)^n/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**n*x**2/(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + \frac{b}{x}\right)^{n} x^{2}}{(dx + c)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n*x^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((a + b/x)^n*x^2/(d*x + c)^2, x)
```

3.294 $\int \frac{\left(a+\frac{b}{x}\right)^n x}{(c+dx)^2} dx$

Optimal. Leaf size=150

$$-\frac{c\left(a+\frac{b}{x}\right)^{n+1}(ac-bd(1-n))\,{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d^2(n+1)(ac-bd)^2}-\frac{c\left(a+\frac{b}{x}\right)^{n+1}}{d\left(\frac{c}{x}+d\right)(ac-bd)}+\frac{\left(a+\frac{b}{x}\right)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{b}{ax}+1\right)}{ad^2(n+1)}$$

```
[Out] -((c*(a + b/x)^(1 + n))/(d*(a*c - b*d)*(d + c/x))) - (c*(a*c - b*d*(1 - n))
*(a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c -
b*d)])/(d^2*(a*c - b*d)^2*(1 + n)) + ((a + b/x)^(1 + n)*Hypergeometric2F1[1
, 1 + n, 2 + n, 1 + b/(a*x)])/(a*d^2*(1 + n))
```

Rubi [A] time = 0.11531, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {514, 446, 103, 156, 65, 68}

$$-\frac{c\left(a+\frac{b}{x}\right)^{n+1}(ac-bd(1-n))\,{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d^2(n+1)(ac-bd)^2}-\frac{c\left(a+\frac{b}{x}\right)^{n+1}}{d\left(\frac{c}{x}+d\right)(ac-bd)}+\frac{\left(a+\frac{b}{x}\right)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{b}{ax}+1\right)}{ad^2(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x)^n*x)/(c + d*x)^2,x]
```


```
[Out] -((c*(a + b/x)^(1 + n))/(d*(a*c - b*d)*(d + c/x))) - (c*(a*c - b*d*(1 - n))
*(a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c -
b*d)])/(d^2*(a*c - b*d)^2*(1 + n)) + ((a + b/x)^(1 + n)*Hypergeometric2F1[1
, 1 + n, 2 + n, 1 + b/(a*x)])/(a*d^2*(1 + n))
```


Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x}\right)^n x}{(c+dx)^2}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{\left(d+\frac{c}{x}\right)^2 x}\,dx$$

$$= -\operatorname{Subst}\left(\int \frac{(a+bx)^n}{x(d+cx)^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{c\left(a+\frac{b}{x}\right)^{1+n}}{d(ac-bd)\left(d+\frac{c}{x}\right)} - \frac{\operatorname{Subst}\left(\int \frac{(a+bx)^n(ac-bd-bcnx)}{x(d+cx)}\,dx, x, \frac{1}{x}\right)}{d(ac-bd)}$$

$$= -\frac{c\left(a+\frac{b}{x}\right)^{1+n}}{d(ac-bd)\left(d+\frac{c}{x}\right)} - \frac{\operatorname{Subst}\left(\int \frac{(a+bx)^n}{x}\,dx, x, \frac{1}{x}\right)}{d^2} + \frac{(c(ac-bd(1-n)))\operatorname{Subst}\left(\int \frac{(a+bx)^n}{d+cx}\,dx, x, \frac{1}{x}\right)}{d^2(ac-bd)}$$

$$= -\frac{c\left(a+\frac{b}{x}\right)^{1+n}}{d(ac-bd)\left(d+\frac{c}{x}\right)} - \frac{c(ac-bd(1-n))\left(a+\frac{b}{x}\right)^{1+n}{}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d^2(ac-bd)^2(1+n)} + \frac{\left(a+\frac{b}{x}\right)^{1+n}{}_2F_1\left(1\right.}{ad^2}$$

Mathematica [A] time = 0.109819, size = 120, normalized size = 0.8

$$\frac{\left(a+\frac{b}{x}\right)^{n+1}\left(-\frac{c(ac+bd(n-1))\,{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(n+1)(ac-bd)^2} - \frac{cdx}{(c+dx)(ac-bd)} + \frac{{}_2F_1\left(1,n+1;n+2;\frac{b}{ax}+1\right)}{a(n+1)}\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x)^n*x)/(c + d*x)^2,x]
```

```
[Out] ((a + b/x)^(1 + n)*(-((c*d*x)/((a*c - b*d)*(c + d*x))) - (c*(a*c + b*d*(-1
+ n))*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c - b*d)])/((a*c
- b*d)^2*(1 + n)) + Hypergeometric2F1[1, 1 + n, 2 + n, 1 + b/(a*x)]/(a*(1 +
 n))))/d^2
```

Maple [F] time = 0.507, size = 0, normalized size = 0.

$$\int \frac{x}{(dx+c)^2}\left(a+\frac{b}{x}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^n*x/(d*x+c)^2,x)
```

```
[Out] int((a+b/x)^n*x/(d*x+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x}\right)^{n}x}{(dx+c)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n*x/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^n*x/(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x\left(\frac{ax+b}{x}\right)^{n}}{d^{2}x^{2}+2\,cdx+c^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n*x/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(x*((a*x + b)/x)^n/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**n*x/(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x}\right)^n x}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n*x/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((a + b/x)^n*x/(d*x + c)^2, x)
```

3.295 $\int \frac{\left(a+\frac{b}{x}\right)^n}{(c+dx)^2}\,dx$

Optimal. Leaf size=56

$$-\frac{b\left(a+\frac{b}{x}\right)^{n+1}{}_2F_1\left(2,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(n+1)(ac-bd)^2}$$

```
[Out] -((b*(a + b/x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, (c*(a + b/x))/(a*
c - b*d)])/((a*c - b*d)^2*(1 + n)))
```

Rubi [A] time = 0.0326302, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {434, 444, 68}

$$-\frac{b\left(a+\frac{b}{x}\right)^{n+1}{}_2F_1\left(2,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(n+1)(ac-bd)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^n/(c + d*x)^2,x]
```

```
[Out] -((b*(a + b/x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, (c*(a + b/x))/(a*
c - b*d)])/((a*c - b*d)^2*(1 + n)))
```

Rule 434

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[((a + b*x^n)^p*(d + c*x^n)^q)/x^(n*q), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x}\right)^n}{(c+dx)^2}\,dx &= \int \frac{\left(a+\frac{b}{x}\right)^n}{\left(d+\frac{c}{x}\right)^2 x^2}\,dx\\ &= -\mathrm{Subst}\left(\int \frac{(a+bx)^n}{(d+cx)^2}\,dx, x, \frac{1}{x}\right)\\ &= -\frac{b\left(a+\frac{b}{x}\right)^{1+n}\,{}_2F_1\left(2,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(ac-bd)^2(1+n)}\end{aligned}$$

Mathematica [A] time = 0.0216635, size = 57, normalized size = 1.02

$$-\frac{b\left(a+\frac{b}{x}\right)^{n+1}\,{}_2F_1\left(2,n+1;n+2;-\frac{c\left(a+\frac{b}{x}\right)}{bd-ac}\right)}{(n+1)(bd-ac)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^n/(c + d*x)^2,x]
```

```
[Out] -((b*(a + b/x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, -((c*(a + b/x))/(
-(a*c) + b*d))])/((-(a*c) + b*d)^2*(1 + n)))
```

Maple [F] time = 0.497, size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)^2}\left(a+\frac{b}{x}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^n/(d*x+c)^2,x)
```

```
[Out] int((a+b/x)^n/(d*x+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x}\right)^{n}}{(dx+c)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^n/(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(\frac{ax+b}{x}\right)^{n}}{d^{2}x^{2}+2\,cdx+c^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(((a*x + b)/x)^n/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**n/(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + \frac{b}{x}\right)^n}{(dx + c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((a + b/x)^n/(d*x + c)^2, x)
```

3.296 $\int \frac{\left(a+\frac{b}{x}\right)^n}{x(c+dx)^2}\,dx$

Optimal. Leaf size=105

$$\frac{\left(a+\frac{b}{x}\right)^{n+1}(ac-bd(n+1))\,{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c(n+1)(ac-bd)^2}-\frac{d\left(a+\frac{b}{x}\right)^{n+1}}{c\left(\frac{c}{x}+d\right)(ac-bd)}$$

```
[Out] -((d*(a + b/x)^(1 + n))/(c*(a*c - b*d)*(d + c/x))) + ((a*c - b*d*(1 + n))*(
a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c - b*
d)])/(c*(a*c - b*d)^2*(1 + n))
```

Rubi [A] time = 0.0679815, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {514, 446, 78, 68}

$$\frac{\left(a+\frac{b}{x}\right)^{n+1}(ac-bd(n+1))\,{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c(n+1)(ac-bd)^2}-\frac{d\left(a+\frac{b}{x}\right)^{n+1}}{c\left(\frac{c}{x}+d\right)(ac-bd)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^n/(x*(c + d*x)^2),x]
```


```
[Out] -((d*(a + b/x)^(1 + n))/(c*(a*c - b*d)*(d + c/x))) + ((a*c - b*d*(1 + n))*(
a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c - b*
d)])/(c*(a*c - b*d)^2*(1 + n))
```


Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+\frac{b}{x}\right)^n}{x(c+dx)^2}\,dx &= \int \frac{\left(a+\frac{b}{x}\right)^n}{\left(d+\frac{c}{x}\right)^2 x^3}\,dx\\
&= -\operatorname{Subst}\left(\int \frac{x(a+bx)^n}{(d+cx)^2}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{d\left(a+\frac{b}{x}\right)^{1+n}}{c(ac-bd)\left(d+\frac{c}{x}\right)} - \frac{(ac-bd(1+n))\operatorname{Subst}\left(\int \frac{(a+bx)^n}{d+cx}\,dx, x, \frac{1}{x}\right)}{c(ac-bd)}\\
&= -\frac{d\left(a+\frac{b}{x}\right)^{1+n}}{c(ac-bd)\left(d+\frac{c}{x}\right)} + \frac{(ac-bd(1+n))\left(a+\frac{b}{x}\right)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c(ac-bd)^2(1+n)}
\end{aligned}
$$

Mathematica [A] time = 0.0520035, size = 88, normalized size = 0.84

$$
\frac{\left(a+\frac{b}{x}\right)^{n+1}\left(\frac{(ac-bd(n+1))\,{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{n+1}+\frac{dx(bd-ac)}{c+dx}\right)}{c(ac-bd)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^n/(x*(c + d*x)^2),x]
```

```
[Out] ((a + b/x)^(1 + n)*((d*(-(a*c) + b*d)*x)/(c + d*x) + ((a*c - b*d*(1 + n))*H
ypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c - b*d)])/(1 + n)))/(c*
(a*c - b*d)^2)
```

Maple [F] time = 0.52, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(dx+c\right)^{2}}\left(a+\frac{b}{x}\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^n/x/(d*x+c)^2,x)
```

```
[Out] int((a+b/x)^n/x/(d*x+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x}\right)^{n}}{\left(dx+c\right)^{2}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^n/((d*x + c)^2*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(\frac{ax+b}{x}\right)^{n}}{d^{2}x^{3}+2\,cdx^{2}+c^{2}x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(((a*x + b)/x)^n/(d^2*x^3 + 2*c*d*x^2 + c^2*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**n/x/(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + \frac{b}{x}\right)^n}{(dx + c)^2 x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((a + b/x)^n/((d*x + c)^2*x), x)
```

3.297 $\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2(c+dx)^2}\,dx$

Optimal. Leaf size=133

$$\frac{d^2\left(a+\frac{b}{x}\right)^{n+1}}{c^2\left(\frac{c}{x}+d\right)(ac-bd)}-\frac{d\left(a+\frac{b}{x}\right)^{n+1}(2ac-bd(n+2))\,{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c^2(n+1)(ac-bd)^2}-\frac{\left(a+\frac{b}{x}\right)^{n+1}}{bc^2(n+1)}$$

```
[Out] -((a + b/x)^(1 + n)/(b*c^2*(1 + n))) + (d^2*(a + b/x)^(1 + n))/(c^2*(a*c -
b*d)*(d + c/x)) - (d*(2*a*c - b*d*(2 + n))*(a + b/x)^(1 + n)*Hypergeometric
2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c - b*d)])/(c^2*(a*c - b*d)^2*(1 + n)
)
```

Rubi [A] time = 0.130979, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {514, 446, 89, 80, 68}

$$\frac{d^2\left(a+\frac{b}{x}\right)^{n+1}}{c^2\left(\frac{c}{x}+d\right)(ac-bd)}-\frac{d\left(a+\frac{b}{x}\right)^{n+1}(2ac-bd(n+2))\,{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c^2(n+1)(ac-bd)^2}-\frac{\left(a+\frac{b}{x}\right)^{n+1}}{bc^2(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^n/(x^2*(c + d*x)^2),x]
```


```
[Out] -((a + b/x)^(1 + n)/(b*c^2*(1 + n))) + (d^2*(a + b/x)^(1 + n))/(c^2*(a*c -
b*d)*(d + c/x)) - (d*(2*a*c - b*d*(2 + n))*(a + b/x)^(1 + n)*Hypergeometric
2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c - b*d)])/(c^2*(a*c - b*d)^2*(1 + n)
)
```


Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2(c+dx)^2}\,dx &= \int \frac{\left(a+\frac{b}{x}\right)^n}{\left(d+\frac{c}{x}\right)^2 x^4}\,dx \\
&= -\operatorname{Subst}\left(\int \frac{x^2(a+bx)^n}{(d+cx)^2}\,dx, x, \frac{1}{x}\right) \\
&= \frac{d^2\left(a+\frac{b}{x}\right)^{1+n}}{c^2(ac-bd)\left(d+\frac{c}{x}\right)} - \frac{\operatorname{Subst}\left(\int \frac{(a+bx)^n(-d(ac-bd(1+n))+c(ac-bd)x)}{d+cx}\,dx, x, \frac{1}{x}\right)}{c^2(ac-bd)} \\
&= -\frac{\left(a+\frac{b}{x}\right)^{1+n}}{bc^2(1+n)} + \frac{d^2\left(a+\frac{b}{x}\right)^{1+n}}{c^2(ac-bd)\left(d+\frac{c}{x}\right)} + \frac{(d(2ac-bd(2+n)))\operatorname{Subst}\left(\int \frac{(a+bx)^n}{d+cx}\,dx, x, \frac{1}{x}\right)}{c^2(ac-bd)} \\
&= -\frac{\left(a+\frac{b}{x}\right)^{1+n}}{bc^2(1+n)} + \frac{d^2\left(a+\frac{b}{x}\right)^{1+n}}{c^2(ac-bd)\left(d+\frac{c}{x}\right)} - \frac{d(2ac-bd(2+n))\left(a+\frac{b}{x}\right)^{1+n} {}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c^2(ac-bd)^2(1+n)}
\end{aligned}
$$

Mathematica [A] time = 0.0716438, size = 125, normalized size = 0.94

$$
-\frac{(ax+b)\left(a+\frac{b}{x}\right)^n\left(bd(c+dx)(2ac-bd(n+2))\,{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)+(ac-bd)(ac(c+dx)-bd(c+d(n+2)x))\right)}{bc^2(n+1)x(c+dx)(ac-bd)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^n/(x^2*(c + d*x)^2),x]
```

```
[Out] -(((a + b/x)^n*(b + a*x)*((a*c - b*d)*(a*c*(c + d*x) - b*d*(c + d*(2 + n)*x
)) + b*d*(2*a*c - b*d*(2 + n))*(c + d*x)*Hypergeometric2F1[1, 1 + n, 2 + n,
 (c*(a + b/x))/(a*c - b*d)]))/(b*c^2*(a*c - b*d)^2*(1 + n)*x*(c + d*x)))
```

Maple [F] time = 0.535, size = 0, normalized size = 0.

$$
\int \frac{1}{x^2\,(dx+c)^2}\left(a+\frac{b}{x}\right)^n dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^n/x^2/(d*x+c)^2,x)
```

```
[Out] int((a+b/x)^n/x^2/(d*x+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x}\right)^{n}}{(dx+c)^{2}x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^n/((d*x + c)^2*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\frac{ax+b}{x}\right)^{n}}{d^{2}x^{4}+2\,cdx^{3}+c^{2}x^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(((a*x + b)/x)^n/(d^2*x^4 + 2*c*d*x^3 + c^2*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**n/x**2/(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + \frac{b}{x}\right)^n}{\left(dx + c\right)^2 x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((a + b/x)^n/((d*x + c)^2*x^2), x)
```

3.298 $\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3(c+dx)^2}\,dx$

Optimal. Leaf size=217

$$-\frac{\left(a+\frac{b}{x}\right)^{n+1}\left(d(bd(n+2)(ac+bd(n+3))-ac(ac+bd(3n+5)))-\frac{c(ac-bd)(ac+bd(n+3))}{x}\right)}{b^2c^3(n+1)(n+2)\left(\frac{c}{x}+d\right)(ac-bd)}+\frac{d^2\left(a+\frac{b}{x}\right)^{n+1}(3ac-bd(n+3)}{c^3(n+1)(a}$$

```
[Out] -(((a + b/x)^(1 + n)*(d*(b*d*(2 + n)*(a*c + b*d*(3 + n)) - a*c*(a*c + b*d*(
5 + 3*n))) - (c*(a*c - b*d)*(a*c + b*d*(3 + n)))/x))/(b^2*c^3*(a*c - b*d)*(
1 + n)*(2 + n)*(d + c/x))) - (a + b/x)^(1 + n)/(b*c*(2 + n)*(d + c/x)*x^2)
+ (d^2*(3*a*c - b*d*(3 + n))*(a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n,
2 + n, (c*(a + b/x))/(a*c - b*d)])/(c^3*(a*c - b*d)^2*(1 + n))
```

Rubi [A] time = 0.263849, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {514, 446, 100, 146, 68}

$$-\frac{\left(a+\frac{b}{x}\right)^{n+1}\left(d(bd(n+2)(ac+bd(n+3))-ac(ac+bd(3n+5)))-\frac{c(ac-bd)(ac+bd(n+3))}{x}\right)}{b^2c^3(n+1)(n+2)\left(\frac{c}{x}+d\right)(ac-bd)}+\frac{d^2\left(a+\frac{b}{x}\right)^{n+1}(3ac-bd(n+3)}{c^3(n+1)(a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^n/(x^3*(c + d*x)^2),x]
```

```
[Out] -(((a + b/x)^(1 + n)*(d*(b*d*(2 + n)*(a*c + b*d*(3 + n)) - a*c*(a*c + b*d*(
5 + 3*n))) - (c*(a*c - b*d)*(a*c + b*d*(3 + n)))/x))/(b^2*c^3*(a*c - b*d)*(
1 + n)*(2 + n)*(d + c/x))) - (a + b/x)^(1 + n)/(b*c*(2 + n)*(d + c/x)*x^2)
+ (d^2*(3*a*c - b*d*(3 + n))*(a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n,
2 + n, (c*(a + b/x))/(a*c - b*d)])/(c^3*(a*c - b*d)^2*(1 + n))
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 146

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m
 + 1)*(m + n + 3)), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3(c+dx)^2}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{\left(d+\frac{c}{x}\right)^2 x^5}\,dx$$

$$= -\text{Subst}\left(\int \frac{x^3(a+bx)^n}{(d+cx)^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\left(a+\frac{b}{x}\right)^{1+n}}{bc(2+n)\left(d+\frac{c}{x}\right)x^2} - \frac{\text{Subst}\left(\int \frac{x(a+bx)^n(-2ad+(-ac-bd(3+n))x)}{(d+cx)^2}\,dx, x, \frac{1}{x}\right)}{bc(2+n)}$$

$$= -\frac{\left(a+\frac{b}{x}\right)^{1+n}\left(d(bd(2+n)(ac+bd(3+n))-ac(ac+bd(5+3n)))-\frac{c(ac-bd)(ac+bd(3+n))}{x}\right)}{b^2c^3(ac-bd)(1+n)(2+n)\left(d+\frac{c}{x}\right)} - \frac{\left(a+\frac{b}{x}\right.}{bc(2+n)\left(\right.}$$

$$= -\frac{\left(a+\frac{b}{x}\right)^{1+n}\left(d(bd(2+n)(ac+bd(3+n))-ac(ac+bd(5+3n)))-\frac{c(ac-bd)(ac+bd(3+n))}{x}\right)}{b^2c^3(ac-bd)(1+n)(2+n)\left(d+\frac{c}{x}\right)} - \frac{\left(a+\frac{b}{x}\right.}{bc(2+n)\left(\right.}$$

Mathematica [A] time = 0.199432, size = 182, normalized size = 0.84

$$\frac{\left(a+\frac{b}{x}\right)^{n+1}\left(\frac{-a^2c^2(c+dx)-abcd(c(n+2)+d(2n+3)x)+b^2d^2(n+3)(c+d(n+2)x)}{bc^2(n+1)(c+dx)(bd-ac)} - \frac{bd^2(n+2)(bd(n+3)-3ac)\,{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c^2(n+1)(ac-bd)^2} - \frac{1}{x(c+dx)}\right)}{bc(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^n/(x^3*(c + d*x)^2),x]
```

```
[Out] ((a + b/x)^(1 + n)*(-(1/(x*(c + d*x))) + (-(a^2*c^2*(c + d*x)) + b^2*d^2*(3
 + n)*(c + d*(2 + n)*x) - a*b*c*d*(c*(2 + n) + d*(3 + 2*n)*x))/(b*c^2*(-(a*
c) + b*d)*(1 + n)*(c + d*x)) - (b*d^2*(2 + n)*(-3*a*c + b*d*(3 + n))*Hyperg
eometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c - b*d)])/(c^2*(a*c - b*d)^2
*(1 + n))))/(b*c*(2 + n))
```

Maple [F] time = 0.528, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\,(dx+c)^2}\left(a+\frac{b}{x}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^n/x^3/(d*x+c)^2,x)
```

```
[Out] int((a+b/x)^n/x^3/(d*x+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + \frac{b}{x}\right)^n}{(dx + c)^2 x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^n/((d*x + c)^2*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\frac{ax+b}{x}\right)^n}{d^2x^5 + 2\,cdx^4 + c^2x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(((a*x + b)/x)^n/(d^2*x^5 + 2*c*d*x^4 + c^2*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**n/x**3/(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{(dx+c)^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((a + b/x)^n/((d*x + c)^2*x^3), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
```

```

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
```

```
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
```

```
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
```

```
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
```

```
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
```

```
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
```

```
#               Albert Rich to use with Sagemath. This is used to
#               grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#               'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(
    flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```
def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
```

```
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
```

```
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)
```

```


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```